Diskussionsbeiträge des Fachbereichs Wirtschaftswissenschaft
der Freien Universität Berlin

Volkswirtschaftliche Reihe

2006/13

Monetary Policy Rules in a Two-Sector Small Open Economy

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ISBN 3-938369-38-8

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May, 2006

Abstract

The analysis of monetary policy rules has been confined to models not capable of examining situations where an economy is converging to a higher balanced growth path. For the small open economies having entered the European Union recently this is however a very relevant question. The main aim of their integration is convergence itself and most of the criteria they have to fulfil in order to become members of the euro zone are of monetary nature. It is thus of special interest for them whether and how the chosen strategy of monetary policy aiming at the fulfilment of the requirements they face affects the transition. In this paper a first attempt is made to compare monetary policy rules in a monetary model of small open economies, which builds essentially on the convergence literature. The results show that the economy behaves very differently in the transition under the different monetary policy rules examined.

*I wish to thank János Vincze, Lajos Zelkó, József Móczár, Dietmar Meyer, Ernő Zalai, Tamás Bánfi, Kornél Varsányi, Péter Szomorjai and Balázs Szentes for helpful comments and discussions. I received a lot of valuable comments on different conferences and my thesis draft defense, as well. In particular, I would like to mention the thorough contributions of my opponents, Zoltán Jakab, Péter Benczúr and Péter Pete. Special thanks go to Horst Tomann who was supervising my work for one year with much care and attention. For the same reason I am particularly indebted to Ákos Valentinyi whose constant help in the main phase of the research was invaluable. All the remaining errors are mine. Financial support of the German Academic Exchange Service (DAAD) and the Marie Curie Fellowship scheme of the European Commission (HPMT-CT-2001-00353) are gratefully acknowledged. The views expressed here are solely the responsibility of the author and should not be interpreted as reflecting the views of the European Community.

1 Introduction

The "eastern enlargement" of the European Union (EU) took place in May 2004. With this – in terms of population – biggest enlargement in the history of the EU ten countries became members. Most of the new members (Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia) are small open economies (Poland being the most notable exception) and except for Malta and Cyprus they are approaching the end of a postsocialist transition process from their centrally planned economies to market economies (the transition can be regarded as finished insofar as the fulfilment of the Copenhagen criteria are concerned, which was a condition of entering the EU). On accession the new members also became candidates for membership in the Economic and Monetary Union of Europe, since unlike some of the current members they do not have any opt-out clauses. The entry into the euro zone, however, requires the fulfilment of different eligibility criteria (the *convergence criteria of Maastricht*). Apart for the condition concerning the level of public debt and the budget deficit, these requirements are of monetary nature: the stability of the exchange rate against the euro, stability of the price level and a corresponding level for the long-run interest rates (which is connected to both) have to be satisfactorily achieved. Because of this monetary policy has a very important role to play in these countries. How should it be designed? What strategy would be preferable for these countries?

If we look at the current design of monetary policy in the ten countries, we find a very diverse picture. From giving up monetary autonomy completely and relying on the very strict fixed exchange rate system of a currency board (Estonia, Latvia among others) to a completely free float combined with inflation targeting (Poland, Czech Republic for example) different strategies are being followed.

Theory does not give too much guidance, either.[1] This may seem surprising at first sight, since there are many monetary general equilibrium models analysing the optimality and characteristics of monetary policy rules in several different set-ups, also for the case of a small open economy.[2] But most of these models abstract from production completely or include production using labour as the only input while physical capital does not explicitly ap-

[1] Though theoretical guidance would be especially important for these countries because for any econometric analysis the reliable and comparable time series are very short to get interpretable results, that is empirical studies cannot help much either.

[2] For closed economy see Galí (2002) or Woodford (2003); two-country models can be found in Obstfeld-Rogoff (1995 and 1998), a model for a small open economy is presented in Galí-Monacelli (2002).

pear. Moreover, they examine economies being close to their steady-states in low-inflation environments. None of these assumptions are a good approximation in the case of the new members (although low inflation has already been achieved in many of these countries). They are not supposed to be close to steady-state, but on their transition path towards it or in a catch-up process converging to a higher balanced growth path. This convergence is also one of the goals of their integration and is in my opinion a non-negligible characteristic of these economies.
Using models of convergence could be another approach. Here one can find growth models that build different frictions into an otherwise neoclassical framework in order to get a speed of convergence in the range of empirical estimates. (In open economies under neoclassical assumptions convergence would be immediate as it is optimal to jump to the balanced growth path making up for the missing resources by borrowing from abroad.) But these are usually non-monetary models.[3]
There are of course models including both capital and money explicitly, as well, but these are mainly dealing with closed economies and more importantly the basic model describing the real economy is not a convergence model, that is it is not capable of analysing the convergence behaviour of an open economy. The tools for empirically oriented modelling became primarily probably the dynamic stochastic general equilibrium approaches (DSGE-models) such as for example Smets–Wouters (2003) or Del Negro et. al. (2004) to mention only a few newer, well-known examples. These models are not aiming for modelling convergence either and for better empirical results they are assuming many different types of nominal rigidities. To my knowledge only Benczúr (2003) and Benczúr–Kónya (2004) and (2005)[4] use certain elements necessary for modelling convergence, but their goal differs from mine: instead of concentrating on the interactions of monetary policy and convergence and the role of monetary policy in it, they are examining the real effects of different nominal shocks.

The interaction between transition (real convergence) and monetary policy is not at all impossible. Already in 1979 Stanley Fischer showed in his article that the results of e.g. Sidrauski (1967) about the superneutrality of money hold true only in the steady-state. Superneutrality in the steady state also requires rather special assumptions, but even in his model where this holds, money affects transition: the speed of convergence is not independent of the growth rate of the nominal money supply. For the transition money is

[3] See for example Barro–Mankiw–Sala-i-Martin (1995), Chatterjee–Sakoulis–Turnovsky (2001), Lane (2001).

[4] The 2005 article is a later version of the 2004 paper.

superneutral only if the utility function is logarithmic, but for the constant elasticity of substitution form most commonly used in empirical applications it is not.

I believe that all three basic elements mentioned (*small open economies, convergence and monetary nature*) are essential for an analysis of optimal monetary policy in the case of these countries. So as a contribution to the existing body of literature this study tries to combine the three. In order to avoid making such a complex model very complicated and for having tractable results, I will try to keep the structure as simple as possible choosing the simplest setup and introducing money into it. I would like to note already here, that the model assumes *flexible prices*, so differs from the approach most widely used for empirical purposes. This was chosen in part for simplicity reasons, and partly because the introduction of sticky prices would require the assumption of monopolistic competition which is questionable in the case of small open economies where foreign competition is very fierce. A similar model is set up in Benczúr (2003) and Benczúr–Kónya (2005): the focus is precisely on small open economies, they use assumptions enabling the modelling of convergence, furthermore they choose the same way to introduce money into the model (they also apply the *money in the utility function approach*) and they do not have nominal rigidities either. Their goal is precisely to show that in their structure nominal shocks can have real effects even without nominal rigidities. Their model thus differs in its focus as written before and also the "convergence model" taken as basis here builds on a different assumption than the one used by them (here *constrained capital mobility* is supposed, while there investment incurs adjustment costs).

This is a theoretical paper, it does not include a detailed empirical application or analysis. Nevertheless, it is useful to link the questions examined by the model and the results to the actual economic relationships or so-called stylised facts. Being a complex phenomenon, convergence depends on numerous factors. The model presented here concentrates only on a single element of these: the potential role of monetary policy in bringing about convergence. Exactly because of this complex nature of the problem, it would be difficult to lend empirical footing to the conclusions of the model. It would however be undoubtedly interesting to examine whether these hold in reality or not. This could be the subject of another research programme, and is beyond the scope of the current paper. Therefore, I would like to confine to illustrating with some examples the empirical background of the questions analysed here.

The paper is structured as follows. In section 2, correspondingly I will try to give some illustrative examples of the analysed phenomena via a graphical comparison of macroeconomic time series as indicated. Section 3 then builds up the model, section 4 delivers the solutions of it and summarizes the conditions describing the competitive equilibrium. The dynamic analysis of the model's behaviour under different monetary policy rules follows in section 5, while section 6 concludes.

2 Empirical examples

Economic development is in general measured by the per capita level of the real gross domestic product (GDP). The relative position of a country can then be shown by relating this to a benchmark economy. Convergence or divergence will be accordingly expressed by the change of this ratio through time. Hence, to compare the development of certain economies, a comparable form of this measure is called for. The so-called Penn World Table (PWT) offers a useful database to this aim.[5] The PWT is built on an international comparison of prices. This is used to express the elements of the national accounts in purchasing power parity terms (that is, through a common system of prices, in a common "currency"), and thus enables a real quantity comparison in time and space, so between different countries too. It also provides information on relative prices within and between countries, demographic data and capital stock estimates. The database contains annual data for the period 1950-2000 (as a whole or in part) for 168 countries and 23 variables.

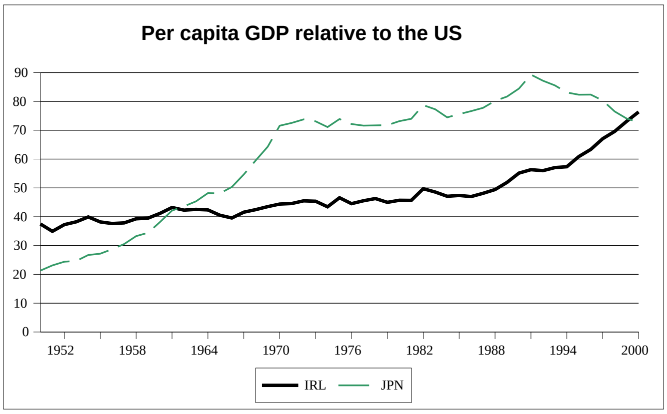
To illustrate the questions analysed here (in section 5) I chose Ireland and Japan, since for them data are available for the whole period[6], and they developed at a fast pace during these decades, making up for a large part of the distance in the per capita level of real GDP to the USA (which is typically taken as benchmark for such comparisons). The growth rate of the real GDP in Japan during the period was 4.81 percent on average, in Ireland 3.64 percent, while in the United States of America it only amounted to 2.27 percent. This means that these countries got closer to the economic performance of the USA in these fifty years, which is shown on figure 1.

The figure also shows that while Ireland's convergence has been rather slow during most of the period and the spectular catch-up took place in the last decade, Japan converged fastly in the first two decades, its development

[5]Heston, Alan – Summers, Robert – Aten, Bettina: Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002 (accessible: http://pwt.econ.upenn.edu/php_site/pwt_index.php). About the methodology see Summers–Heston [1991], and for Version 6.1: http://pwt.econ.upenn.edu/Documentation/append61.pdf.

[6]Moreover, for both of them direct price comparison exercises have taken place, i.e. they were both benchmark countries in the PWT surveys (Japan in all of them, Ireland with the only exception of the first survey).

Figure 1: Per capita GDP in Ireland and Japan relative to that of the USA (in percent)



slowed down thereafter and the tendency even reversed in the last decade, when the growth rate of GDP in the USA exceeded that of Japan (see the table below showing average growth rates).

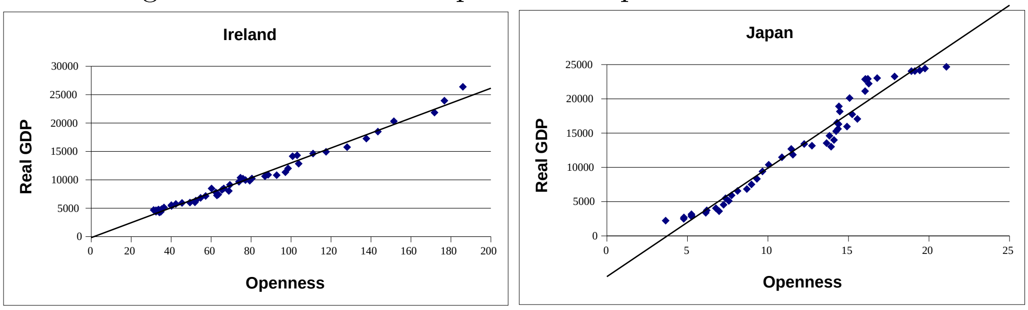
Decade	**GDP growth in Ireland**	**GDP growth in Japan**	**GDP growth in the US**
1950s	1.86%	7.13%	1.37%
1960s	3.46%	9.26%	2.87%
1970s	3.16%	3.08%	2.66%
1980s	3.51%	3.53%	2.15%
1990s	6.22%	1.05%	2.3%

Which factors examined in this paper can this convergence be related to? I would like to emphasize once more that I am not aiming for a thorough statistical analysis of the question, I just want to illustrate from the empirical side my theoretical results with the help of some examples. As we will see, the theoretical literature gives an important role for example to the openness of an economy when explaining convergence. Openness can support faster development through different channels. The data from the PWT seem to confirm this, the relationship is very close between the two factors, as shown also by the scatter diagram in figure 2.

The model shown here focuses on the relationship between convergence and monetary policy: it analyses, how monetary policy could further convergence. The results will differ under different monetary policy rules. Simplifying a bit, we could refer to two main conclusions.[7] According to one,

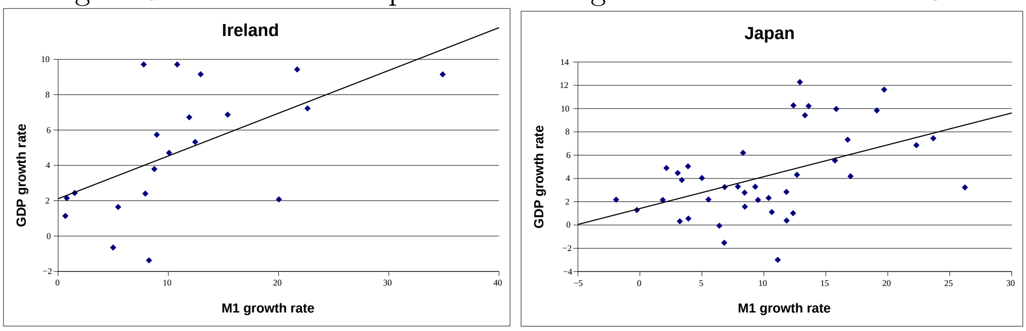
[7]See section 5 for the details or section 6 for a summary of the conclusions.

Figure 2: The relationship between openness and real GDP



(under money supply rule) the faster increase of the quantity of money leads to higher speed of convergence, supporting the catch-up process. The data seem to underpin this, both in the case of the more liquid component (M1), and of the broader monetary aggregate (M3). For the sake of conciseness, figure 3 presents the scatter diagrams only for M1.[8]

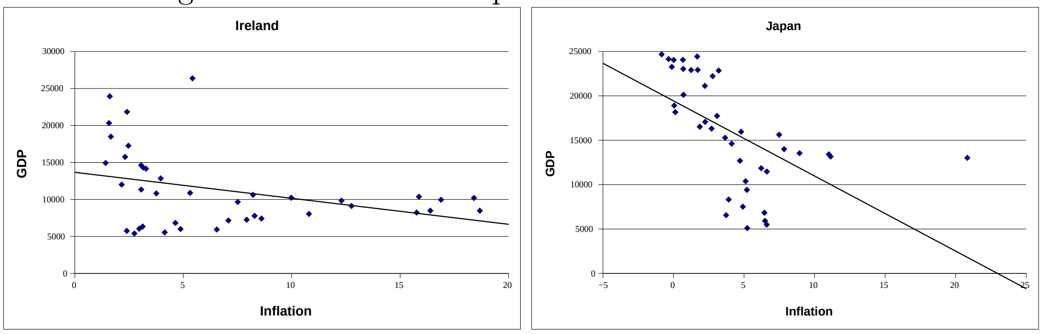
Figure 3: The relationship between the growth rates of M1 and GDP



The other (partly contradicting) conclusion refers to the negative relationship between inflation and convergence. Under the inflation targeting regime or a feedback rule for interest rates lower inflation can result in faster convergence. This means a similar, but negative relationship, which figure 4 presents. The figure shows inflation (calculated from the consumer price index data of the European Central Bank) and real GDP on a scatter diagram. The negative relationship between the variables is evident for both countries.

[8]The Penn World Table does not contain monetary data. The time series for the monetary aggregates are from the database of the Bank for International Settlements (BIS).

Figure 4: The relationship between inflation and GDP



The results of the model can thus also be motivated empirically. But providing them with a thorough empirical basis would be the subject of an independent econometric research. Therefore, in this section I was trying to simply illustrate them, and the figures presented should not be interpreted as providing statistical evidence for the results: they only serve these illustrative purposes.
In the next section I turn to the presentation of the model.

3 Economic Environment

My goal is to develop a model for analysing monetary policy in the case of converging small open economies. Within this model *convergence* means the process of reaching the steady state from the starting point: this is the path taking the economy from the state given by the initial conditions to the one characterised by a balanced growth path.[9] As argued in the introduction, I believe that for this we need a monetary model of a small open economy in transition. In this first and simplest attempt I chose the model setup used in Lane (2001), which is a modified version of the one by Barro–Mankiw–Sala-i-Martin (1995). This is the dynamic general equilibrium model of a

[9]Convergence in the model will thus correspond to the stable saddle path heading towards the steady-state. To examine this of course it needs to be uniquely determined, which holds in all the cases analysed here. More precisely convergence means approaching the steady-state on this path, the speed of moving towards the steady-state on this path is called the speed of convergence. Transition means moving on this path instead of being already in the steady-state, so I am not dealing with transition in the sense of moving between two saddle paths, so that the subsequent steady-states will also be different (transition could then bring the economy to a different path with a higher rate of balanced growth than is achievable through the path belonging to the given starting point).

small open economy in continuous time, where *capital mobility is constrained* (completely free in some areas, while completely prohibited in others), which enables a straightforward handling of the model structure, while slowing down convergence. (It can be seen from the article that calibrated versions of the model yield results for the convergence speed that are in line with empirical estimates under realistic parameter values.)
To "monetize" the model economy I chose the *money in the utility function approach* following an article of Stanley Fischer.[10]

The economy is populated with a large number of infinitely lived identical competitive households owning capital in the economy. With these resources they are operating the production technology themselves, so they are the producers in the economy, as well. Let us have a closer look at the productive activity conducted by the economic agents first.

3.1 Firms' Problem

We assume a large number of essentially identical households having access to the same production technology (this means that examining the problem of a representative producer is sufficient). They are selling their final product on a competitive market being small relative to it so having no market power. The production process consists of two phases. First intermediate products are being produced used later as inputs for producing the final products. We consider two main types of inputs: traded and nontraded ones (y_T and y_N in per capita terms). Both are produced using different physical capital goods (k_T and k_N, also in per capita form).[11] No labour input is employed in the production of the products. The production functions of the intermediate products have the following Cobb–Douglas form:

$$y_T = Ak_T^{\alpha} \quad y_N = Ak_N^{\alpha} \tag{1}$$

Here A is the exogenous productivity parameter of the economy and α is the capital share. Note, that we assumed α and A being the same in the production of the two inputs, which simplifies the analysis. By this assumption (the same capital intensity and productivity across sectors) the growth effects resulting from the variation in productivity between the traded and non-traded sectors are ruled out for now.

[10]Fischer (1979). This approach was introduced in Sidrauski (1967).

[11]As this is a model in continuous time, all variables depend on time in a continuous way. I should actually write $y_T(t), y_N(t), k_T(t), k_N(t))$ where t is the index of time. To simplify the exposition I will use y_T, y_N, k_T, k_N. This also applies to the other variables used in the model later on: (M, p, m, X, c, d).

The final output is produced using the intermediate products as inputs in the following way:

$$y = y_N^{1-\theta} y_T^{\theta},$$

where θ can be interpreted as the degree of openness since it stands for the relative size of the traded goods' "sector". Substituting the production functions for the intermediate products (1), this becomes:

$$y = A k_N^{\alpha(1-\theta)} k_T^{\alpha\theta} \tag{2}$$

Output can be consumed or invested (equally used as consumption good or capital good).

Now we impose a borrowing constraint: the country can borrow at perfect capital markets but only limited amount not exceeding the value of capital in the traded sector because this is the only acceptable collateral[12]. If we denote the real per capita debt (borrowed funds) by d, we assume formally that $d \leq k_T$. Practically optimality will require then: $d = k_T$ to speed up growth and convergence as much as international capital markets make it possible. This also means that the net return on capital in the traded sector (its marginal product minus depreciation) has to equal the world real interest rate (in the sector open to international trade the rates of return at the domestic and international markets get equalized):

$$\alpha\theta A k_N^{\alpha(1-\theta)} k_T^{\alpha\theta-1} = \alpha\theta \frac{y}{k_T} = r^w + \delta \tag{3}$$

Here δ denotes the rate of depreciation and r^w is the world interest rate. Taking this condition and using the borrowing constraint we have:

$$k_T = d = \frac{\alpha\theta y}{r^w + \delta} \tag{4}$$

We can rewrite the production function accordingly:

$$y = B k_N^{\eta} \tag{5}$$

where $B = A^{\frac{1}{1-\alpha\theta}} \left(\frac{\alpha\theta}{r^w+\delta}\right)^{\frac{\alpha\theta}{1-\alpha\theta}}$ and $\eta = \frac{\alpha-\alpha\theta}{1-\alpha\theta}$, $0 < \eta < 1$. The assumption of constrained international capital mobility is useful as it slows down convergence in the case of an open economy and it makes the analysis simpler by reducing the number of variables in the model.

[12]Several reasons could be mentioned for this, for example it is easier to impose sanctions on this sector or this capital is useful also to foreign investors as opposed to the one in the nontraded sector. See Lane (2001) p.224. for more details.

3.2 The question of relative prices

In this setup there is actually no market where the capital goods would be rented: they are owned and used by the households. Similarly, there is no market where intermediate products are bought and sold, again, they are produced and used as inputs by the same households. But implicitly these markets and the prices that would prevail there are defined. It is possible to calculate shadow rental prices (implicitly we have already used the rental price of k_T for example) and also shadow relative prices of the three different products (y_T, y_N and y). This would give us an equivalent solution (the real rate of return on k_T will be the same) as I will show now.

If we look at the (implicit) problem of production separately, a representative productive unit (here the household) realizes its output as income and incurs the rental payments of the resources used in the production process as costs. There are actually three different production processes with the same objective of profit maximisation where all prices and quantities (r_N, r_T, p_N, p_T, k_N, k_T) must be nonnegative. The three different output goods define implicitly the price system in the economy: p_T will stand for the price of the tradable intermediate good, p_N for the price of the non-tradable one and p for the price of the final product. Here one price can be normalised to one as only the relative prices matter. The natural choice would be p_T, but in order to have the same structure where we have expressed everything in units of the single final good I will assume $p = 1$, so the price of the final product will be normalised.

In the "traded sector" the production problem is then the maximisation of the difference between income and expenses:

$$\max \; p_T A k_T^{\alpha} - r_T k_T$$

In the non-traded sector we have similarly:

$$\max \; p_N A k_N^{\alpha} - r_N k_N$$

The resulting first order conditions from the two problems are:

$$\alpha p_T A k_T^{\alpha-1} = \alpha p_T \frac{y_T}{k_T} \quad = \quad r_T \tag{6}$$

$$\alpha p_N A k_N^{\alpha-1} = \alpha p_N \frac{y_N}{k_N} \quad = \quad r_N \tag{7}$$

Let us now turn to the maximisation problem in the final goods' sector:

$$\max \; y_N^{1-\theta} y_T^{\theta} - p_N y_N - p_T y_T,$$

The resulting conditions here are of course that the prices of the inputs equal their marginal products:

$$\theta\left(\frac{y_T}{y_N}\right)^{\theta-1} = p_T \tag{8}$$

$$(1-\theta)\left(\frac{y_T}{y_N}\right)^{\theta} = p_N \tag{9}$$

Dividing (8) by (9) we have:

$$\frac{p_T}{p_N} = \frac{\theta}{1-\theta}\frac{y_N}{y_T}$$

We can express p_T from here while substituting (9) to get:

$$p_T = \frac{\theta}{1-\theta}\frac{y_N}{y_T}(1-\theta)\left(\frac{y_T}{y_N}\right)^{\theta} = \theta\left(\frac{y_T}{y_N}\right)^{\theta-1}$$

If we substitute this for p_T in (6) then we will finally have:

$$\alpha\theta\frac{y_T^{\theta}y_N^{1-\theta}}{k_T} = \alpha\theta\frac{y}{k_T} = r_T$$

This is the rate of return of capital in the traded sector which net of depreciation has to equal the world real interest rate in the model. As we can see, this really equals (3) from which (4) follows. Including the relative prices would lead to an equivalent formula for the real interest rate and thus would not change any results of the model.

3.3 Preferences – the problem of the households

We have already mentioned that there is a huge number of identical households with infinite life horizon in the economy. As they are the producers too, part of their income equals production (y). Note, that there is no labour input here, so profits may seem to be positive (there is a surplus over the value "paid" for using the inputs, see previous part), but actually this is the compensation of the household for its time spent by working (in self-employment). The households thus realise the value of the output produced as a whole. They also have the net stock of foreign debt (d in per capita terms), so funds borrowed from abroad is also a source of income ($\dot{d}$, a dot over a variable represents its time derivative, intuitively its change over time). Money is introduced through lump sum transfers of the government (x per

capita), also income for the households. The total income of the households is thus: $y + \dot{d} + x$.
This income can then be spent on consumption (c in per capita real terms), investment (I) and accumulating money (m denotes the real balances)[13], but the households also have to pay interest on the stock of debt with the interest rate being r^w. Thus the consumers' expenses are: $c + I + r^w d + \dot{m} + m\pi$. According to the law of motion for the capital stock it is increased by investment and reduced by depreciation, that is $\dot{k} = I - \delta k$, where δ is the rate of depreciation as before. Using this for substituting investment and taking everything in real terms (measured in units of the final good), the budget constraint of the representative household is the following:

$$y + \dot{d} + x = c + (\dot{k}_N + \dot{k}_T) + \delta(k_N + k_T) + r^w d + \dot{m} + m\pi$$

Here we supposed the same rates of depreciation for the two kinds of capital. Using (4) this becomes:

$$\dot{k}_N + \dot{m} = (1 - \alpha\theta)y + x - c - \delta k_N - m\pi$$

The household thus makes a decision about its consumption and saving first. Secondly it also makes a portfolio choice about what to use its savings for: accumulating capital stock or real money balances.

Let us now set up and then discuss the representative household's problem:

$$\max \int_0^\infty e^{-\rho t} \frac{(c^\beta m^\gamma)^{1-\sigma} - 1}{1-\sigma} dt \quad \text{where } \beta, \gamma, \sigma > 0,\ \beta + \gamma \leq 1 \tag{10}$$

$$\begin{array}{rl} \text{subject to} & \dot{k}_N + \dot{m} = (1 - \alpha\theta)Bk_N^\eta + x - c - \delta k_N - m\pi \\ & k(0) = k_N(0) + k_T(0) > 0 \\ & c, m, k_N, k_T, d \geq 0 \\ & r^w, P \geq 0 \text{ and } x \text{ given,} \\ \text{where} & k_N(0) + k_T(0) - d(0) < k_N^* \\ \text{and} & \lim_{t\to\infty} (k_N + k_T - d)e^{-r^w t} \geq 0 \text{ (no-Ponzi condition)} \end{array}$$

[13]Money is essentially a nominal category (usually denoted by M, understood here in per capita terms). The consumer can only make a decision about the stock of nominal money, but actually he is interested in the purchasing power of it. So real terms matter for the consumer's decision. Real balances are usually defined as $m = M/p$ where p is the price level, so $\dot{M}/p = \dot{m} + m\pi$ follows where $\pi = \dot{p}/p$ stands for the inflation rate. This is what appears in the real terms budget constraint. (Note, that the only essentially nominal categories are M and $X = xp$ in the budget constraint as we will see. X stands for the nominal per capita lump sum transfers of the government. The foreign debt, d is however understood in real terms.)

This is the well known dynamic optimisation problem in continuous time: the household aims at maximising its utility subject to certain constraints. The specification of the utility function has the constant elasticity of substitution (or constant relative risk aversion) form with the parameters σ, β and γ; ρ is the subjective discount rate.

With this specification utility is positive for any $c, m > 1$ and the marginal utility of both consumption and money is also positive with the above assumptions about the parameters (consuming one more unit or holding one more unit of real balances increases the utility of the agent). The concavity of the utility function in the two arguments is also guaranteed as $\beta(1-\sigma) < 1$ and $\gamma(1-\sigma) < 1$ that is satisfied. This means that the marginal utility is decreasing: the higher the level of consumption (or real balances) is, the less extra utility can be gained by consuming one more unit (or holding one more unit of real balances). The second order cross partial derivatives are positive (the derivative of the marginal utility of consumption with respect to the stock of real balances and the derivative of the marginal utility of money with respect to consumption, which are actually equal in this symmetrically structured model) when $\sigma < 1$ and negative if $\sigma > 1$. In the case of positive cross derivatives the marginal utility of consumption is an increasing function of the holdings of real balances ("complementarity") and vice versa. The opposite is true for negative cross partial derivatives ("substitutability").

The risk aversion parameter concerning the first argument is $1 - \beta + \beta\sigma$, while for the second one $1 - \gamma + \gamma\sigma$. The intertemporal elasticity of substitution for consumption and money is the inverse of the corresponding risk aversion parameter. In this deterministic model the risk aversion parameter does not have a great importance, but it is not true for the inverse of it. If the elasticity of substitution is greater than 1 (this is the case when $\sigma < 1$) then the agent is relatively willing to substitute between periods, so willing to let consumption (and here also the level of real balances) vary over time. In the other case of $\sigma > 1$ when the elasticity of substitution is less than 1, there is significant consumption (and real balance) smoothing.[14] I have not said much about the role of the other parameters β and γ so far: the higher the β/γ ratio is, the higher will be the marginal rate of substitution between consumption and real balances (the ratio of the marginal utility of consumption to the marginal utility of money) for given levels of c and m.

Let us turn to the constraints after having discussed the objective function. The first constraint is the budget constraint where we used (5). The second

[14] This also means that in the first case (low σ) the substitution effect will dominate if the interest rate rises (savings will increase), while in the second case (high σ) the income effect will dominate (savings will fall). For $\sigma = 1$ the two effects cancel out, savings will not change.

one is an initial condition: we have to start with a positive amount of capital, while the next two prescribe the nonnegativity of variables and prices taken as given by the consumers. The fifth condition is necessary for the borrowing constraint to be binding. To see this, imagine the opposite situation when $k_N(0)+k_T(0)-d(0) \geq k_N^*$ (asterisks denote steady-state values). Here the economy can jump instantaneously to the steady-state, since domestic resources suffice for financing k_N^* and foreign borrowing can be increased to finance k_T^*. With the constraint given above this is not possible and obviously this is the more interesting case to study in this context. The usual no-Ponzi condition (the last one) states that we cannot play Ponzi games, the present value of our net assets cannot be negative, so we have to be able to pay our debts back. Notice that this is actually not necessary here, as according to the borrowing constraint $d \leq k_T$, so the terms in brackets cannot be less than k_N, which is nonnegative.

3.4 Government

The government plays a very simple role in this model: it provides the economic agents with money. It has no other function, so I abstract from government purchases and bonds. This also means that its budget has to be balanced. It gives lump sum money transfers to the households, so its budget constraint with a balanced budget is $\dot{M} = X$ in nominal terms. Writing it in real terms we have:

$$\dot{m} + m\pi = x \tag{11}$$

4 Equilibrium

Let us now solve the household's problem. The corresponding Hamiltonian:

$$\mathscr{H}(c; k_N, m; \lambda) = e^{-\rho t}\frac{c^{\beta-\beta\sigma}m^{\gamma-\gamma\sigma}-1}{1-\sigma} + \lambda e^{-\rho t}[(1-\alpha\theta)Bk_N^{\eta} + x - c - \delta k_N - m\pi]$$

Here λ is the current value multiplier, this is the shadow price of an additional unit of income received in period t expressed in terms of utility in the current period (t). The following equations result from rearranging the first-order conditions:

$$\frac{\gamma c}{\beta m} = (1-\alpha\theta)\eta B k_N^{\eta-1} - \delta + \pi \tag{12}$$

$$(\beta - 1 - \beta\sigma)\frac{\dot{c}}{c} + (\gamma - \gamma\sigma)\frac{\dot{m}}{m} = -(1-\alpha\theta)\eta B k_N^{\eta-1} + \delta + \rho \tag{13}$$

To interpret equation (12), notice that the left-hand side of this equation is the ratio of the marginal utility of money to the marginal utility of consumption, while the right-hand side is actually the domestic nominal interest rate. The marginal rate of substitution between real balances and consumption therefore equals the relative price of these two. Equation (13) is a dynamic condition (the equivalent of an intertemporal Euler equation in the model). According to this the optimal growth path of consumption and real balances depends on the distance from the steady-state: on the right-hand side we have the difference of the real interest rate in the steady-state (ρ as we will see later) and the actual domestic real interest rate. If the difference in the returns is big, than the transition is faster, whereas close to the steady-state it is slower as the difference in the rates of returns is much smaller.
It is useful to take a look at the role of σ in this equation. If $\sigma < 1$ (the case of positive cross derivatives and little smoothing effect), then the signs of the coefficients of $\dot{c}/c$ and $\dot{m}/m$ differ, so these change in the same direction: if $\dot{m}/m$ is increasing (decreasing) then so is $\dot{c}/c$. On the other hand, if $\sigma > 1$ (negative cross derivatives, strong smoothing effect), the signs of the coefficients will be the same, and the quotients are going to change in the opposite direction: an increase (decrease) in the growth rate of real balances brings about a decrease (increase) in the growth rate of consumption. (These statements hold ceteris paribus as a decrease in the difference of the rates of return can offset this in both cases.) If $\sigma = 1$, then m (β) does not appear in this equation, the formula will simplify to $-\dot{c}/c = -(1-\alpha\theta)\eta B k_N^{\eta-1} + \rho + \delta$. Intuitively, in the first case consumption and real balances are "complementary" goods, so their rate of growth changes in the same direction in the optimum. In the second case they are "substitutes", so the two growth rates change in the opposite direction. In a resource constrained situation, consumption is limited as we can only consume output that is smaller due to the smaller amount of capital goods and because of the high rate of return on capital we are strongly motivated not to consume too much of it. Thus it is relatively easier to increase utility by increasing real money holdings, which can later be substituted for by consumption as it becomes relatively cheaper, so more available. In the last case this interaction is missing between holding money and the marginal utility of consumption (or vice versa), that's why the consumption path only depends on the difference between the steady-state and actual real interest rates, and the quantity of money will be determined separately (see equation (12) for this). Here consumption is clearly increasing by a decreasing rate during transition as the difference in returns is disappearing.
Of course the budget constraint has to be observed in the optimum, as well. To get an optimal solution we have a final necessary condition, the transver-

sality condition: $\lim_{t\to\infty} \lambda(k_T + k_N - d)e^{-\rho t} = 0$. This puts a restriction on the terminal value of the household's net capital stocks in terms of utility, because λ is the multiplier used in the Hamiltonian, which is equal to the marginal utility of consumption. According to this for an allocation to be optimal, either the present discounted value of the capital stock net of debts is zero, or if not, then the marginal utility of consumption is zero (the household could not increase its utility by selling its net assets and consuming more). Without this we clearly could not be in optimum. Because of the borrowing constraint it would actually suffice to write $\lim_{t\to\infty} \lambda k_N e^{-\rho t} = 0$ here, so the condition only refers to the stock of non-tradable capital in this model. In the equilibrium it is also true that all markets clear, that is

1. for the goods market: $\dot{k}_N = (1 - \alpha\theta)Bk_N^\eta - c - \delta k_N$;
2. for the money market: $\dot{m} + m\pi = x$.

The conditions describing the optimum of the consumer (12) and (13) actually give us the solution of the model (optimum conditions for consumers and producers combined). So the equilibrium in our case can be fully described by the transversality condition, the budget constraint of the government (this equals the clearing condition of the money market here), the market clearing condition for the goods market (these two assure the observance of the budget constraint of the representative consumer, as well) and equations (12) and (13). Let us repeatedly summarize these equations here.

$$\begin{aligned}
\lim_{t\to\infty} \lambda(k_T + k_N - d)e^{-\rho t} &= 0 \\
\dot{m} + m\pi &= x \\
(1 - \alpha\theta)Bk_N^\eta - c - \delta k_N &= \dot{k}_N \\
(1 - \alpha\theta)\eta Bk_N^{\eta-1} - \delta + \pi &= \frac{\gamma c}{\beta m} \\
(\beta - 1 - \beta\sigma)\frac{\dot{c}}{c} + (\gamma - \gamma\sigma)\frac{\dot{m}}{m} &= -(1 - \alpha\theta)\eta Bk_N^{\eta-1} + \delta + \rho
\end{aligned}$$

This system of equations is separable. For solving the system for the endogenous variables c, m, k_N, π we are going to use the last three equations. In the government's budget constraint x is then defined by m and π. (The role of the transversality condition is to rule out through a terminal restriction certain not optimal dynamic paths that would otherwise be possible.) This also means that we have three equations and four variables to define, so the system is underdetermined. We will need another equation, which will be a policy function describing economic (here monetary) policy: the monetary policy rule.

5 Dynamics of monetary policy rules

Let us now turn to the analysis of the transitional dynamics. The model described so far does not say anything about monetary policy, we have not included a policy function. This study however is concerned with comparing different monetary policy rules for the model economy, since I am interested in the question how the different strategies a national bank can follow affect the transition.
I will examine four different monetary policy strategies: money supply rule, nominal interest rate peg, inflation targeting (defined in a special way, as fixing the rate of inflation) and an interest rate feedback-rule.[15] The exact definitions of these will be given in the corresponding subsections below.

5.1 Money supply rule

Money supply rule means nominal money supply targeting. We suppose that the national bank controls the nominal (per capita) money supply, so it can determine its growth rate. If we denote this growth rate by ω, then the growth rate of the real balances will be $\frac{\dot{m}}{m} = \omega - \pi$. This is the fourth equation in this first case, where the new variable ω is exogenous. Using this for substituting π in (12) and then using the result also in (13) we end up with the following system:

$$\begin{aligned} \frac{\dot{c}}{c} &= \frac{\gamma - \gamma\sigma}{\beta - 1 - \beta\sigma}\left[-(1-\alpha\theta)\eta B k_N^{\eta-1} + \delta - \omega + \frac{\gamma c}{\beta m}\right] + \\ &\quad + \frac{1}{\beta - 1 - \beta\sigma}\left[-(1-\alpha\theta)\eta B k_N^{\eta-1} + \delta + \rho\right] \qquad (14) \\ \frac{\dot{m}}{m} &= (1-\alpha\theta)\eta B k_N^{\eta-1} - \delta + \omega - \frac{\gamma c}{\beta m} \qquad (15) \\ \dot{k}_N &= (1-\alpha\theta) B k_N^{\eta} - \delta k_N - c \qquad (16) \end{aligned}$$

The steady-state is defined as the long-term equilibrium. Here the levels of the different variables reached are optimal, so no more change is required. This is now identical to the so-called balanced growth path, since here there is no growth in the long run (all factors affecting the long-run growth rate of the economy, e.g. rate of growth of the population, technological progress,

[15] The money supply rule is typically not used nowadays. I decided to include it in the analysis for two reasons. First, I wanted to get a more or less complete picture checking all the main rules. Secondly, its use has a long history and was analyzed in many papers, so this way the results can be compared. Especially, this is the rule examined in Fischer (1979) on whose work I was building when introducing money into the model.

have been assumed to be zero).[16] This means that in the steady state $\dot{m} = \dot{c} = \dot{k}_N = 0$. From the monetary policy rule we then have $\omega = \pi^*$, while $\dot{m}/m = \dot{c}/c = 0$ leads to $(1-\alpha\theta)\eta B k_N^{*\eta-1} = \delta + \rho$ according to the above equations. Hence, money is superneutral: changing the growth rate of the nominal money supply affects only inflation; for the real variables it does not matter, since they are uniquely determined by the parameters of the model (the equation given here defines k_N^*, then the resource constraint gives c^*). In this model the steady-state real interest rate is the discount rate ρ as shown by the above equation determining k_N^* (and as was referred to before, see section 4, p.16.). This means that the rate at which future utility is discounted equals the rate of return that can be generated by investing one more unit of capital, so there is no motivation to change the allocation (we are in the steady state). Throughout this section I will assume $\rho = r^w$, which means that the domestic economy is neither more nor less impatient than the world economy (since about the world economy we already assumed that it is in its steady state, so its real rate equals the subjective discount rate abroad). It can also be interpreted as having the same steady-states with access to international capital markets as when there is no international borrowing and lending.[17] Under this assumption we have the following steady-state values for our variables:

$$k_N^* = \left[\frac{A(1-\theta)\alpha}{r^w+\delta}\right]^{\frac{1}{1-\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha\theta}{1-\alpha}} \tag{17a}$$

$$k_T^* = \left[\frac{A\theta\alpha}{r^w+\delta}\right]^{\frac{1}{1-\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha\theta-\alpha}{1-\alpha}} \tag{17b}$$

$$\frac{k_T^*}{k_N^*} = \frac{\theta}{1-\theta} \tag{17c}$$

$$c^* = \left[\frac{A\alpha(1-\theta)}{r^w+\delta}\right]^{\frac{1}{1-\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha\theta}{1-\alpha}} \left\{(1-\alpha\theta)\left[\frac{\alpha(1-\theta)}{r^w+\delta}\right]^{-1} - \delta\right\} \tag{17d}$$

$$m^* = \frac{\gamma}{\beta} \cdot \frac{c^*}{r^w+\omega} \tag{17e}$$

To investigate on the stability of the steady-state and the transition path

[16]Otherwise the balanced growth path of the economy would be the steady state of an adequately transformed model. Actually the variables have to be divided by the long-run rate of growth and the equilibrium of the thus transformed system will be the steady state, when variables do not change any more. The original variables then increase by the rate of long-run growth, which can be the same for all variables, but also different, depending on the precise form of the model.

[17]Barro et. al. (1995), p.110.

we log-linearise the system around the steady state[18] resulting in:

$$\begin{pmatrix} \frac{dlog(c)}{dt} \\ \frac{dlog(m)}{dt} \\ \frac{dlog(k_N)}{dt} \end{pmatrix} = \begin{bmatrix} \frac{(\gamma-\gamma\sigma)(\omega+\rho)}{\beta-1-\beta\sigma} & -\frac{(\gamma-\gamma\sigma)(\omega+\rho)}{\beta-1-\beta\sigma} & -\frac{(\gamma-\gamma\sigma+1)(\eta-1)(\delta+\rho)}{\beta-1-\beta\sigma} \\ -(\omega+\rho) & \omega+\rho & (\eta-1)(\delta+\rho) \\ \delta-\frac{\delta+\rho}{\eta} & 0 & \rho \end{bmatrix} \begin{bmatrix} log(c/c^*) \\ log(m/m^*) \\ log(k_N/k_N^*) \end{bmatrix}$$

The trace of the transition matrix $trace = \rho + \frac{(\beta+\gamma)(1-\sigma)-1}{\beta-1-\beta\sigma}(\omega+\rho)$ is positive because of the assumptions of the model. Since the trace is the sum of the three eigenvalues its positivity excludes all roots being negative.
The determinant is: $det = \frac{(\varepsilon-1)(\delta+\rho)(\omega+\rho)}{\beta-1-\beta\sigma}\left(\delta - \frac{\delta+\rho}{\varepsilon}\right) < 0$. This is the product of the eigenvalues, which means that there is either one or three negative roots. But this last possibility has already been excluded, so there is only one negative (stable) root. The transition path leading to the steady state is thus uniquely determined.[19]
The characteristic equation (μ stands for the eigenvalue)[20]:

$$\begin{aligned} f(\mu,\omega) &= -\mu^3 + \mu^2\left[\rho + (\omega+\rho)\frac{(\beta+\gamma)(1-\sigma)-1}{\beta-1-\beta\sigma}\right] - \\ &- \mu\left[\rho(\omega+\rho)\frac{(\beta+\gamma)(1-\sigma)-1}{\beta-1-\beta\sigma} + \frac{(\gamma-\gamma\sigma+1)(\eta-1)(\delta+\rho)}{\beta-1-\beta\sigma}\left(\delta - \frac{\delta+\rho}{\eta}\right)\right] + \\ &+ \frac{(\eta-1)(\delta+\rho)(\omega+\rho)}{\beta-1-\beta\sigma}\left(\delta - \frac{\delta+\rho}{\eta}\right) = 0 \end{aligned}$$

Monetary policy is not superneutral along the transition path if $\frac{d\mu}{d\omega} \neq 0$ evaluated at the negative eigenvalue (the negative eigenvalue shows the speed of convergence). Using the implicit function theorem we know that

$$\frac{d\mu}{d\omega} = -\frac{\partial f/\partial\omega}{\partial f/\partial\mu}$$

[18] In the analysis I will follow the path laid out in Fischer (1979) where also this rule was analysed (but there the examination of local stability is done in a linearised version). Note that this way we approximate values close to the steady-state. Values far from it cannot be examined analytically, only numerically. This paper is confined to the analytical examination now, but later I would also like to analyse (a stochastic version of) the model by numerical methods.

[19] Negative roots are the stable ones representing convergence towards the steady-state, while positive roots show unstable, diverging paths. If there is only one negative root, this means that we can determine the stable transition path uniquely.

[20] The characteristic equation is actually a function of not only these two variables, but also η, which in turn depends on θ, the openness of the economy. But in order to keep the problem simpler and to get results for the effects of the monetary policy only, we regard η as constant or exogenous here.

Since $f(0,\omega) = det < 0$ and $f(\mu,\omega) = 0$ at the negative root, the function is decreasing there, so $\partial f/\partial\mu < 0$, which means that the sign of the derivative we are looking for is the same as the sign of $\partial f/\partial\omega$. We therefore have to differentiate $f(\mu,\omega)$ with respect to ω, which results in the following expression:

$$\frac{\partial f}{\partial \omega} = \left[\frac{(\beta+\gamma)(1-\sigma)-1}{\beta-1-\beta\sigma}\right](\mu^2-\mu\rho) + \frac{(\eta-1)(\delta+\rho)}{\beta-1-\beta\sigma}\left(\delta+\frac{\delta+\rho}{\eta}\right) \quad (18)$$

The characteristic equation can now be written:

$$\begin{aligned} f(\mu,\omega) \quad &= \quad -\mu(\mu^2-\mu\rho) - \mu\frac{(\gamma-\gamma\sigma+1)(\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)}{\beta-1-\beta\sigma} + \\ &+ \quad (\omega+\rho)\frac{\partial f}{\partial\omega} = 0 \end{aligned} \quad (19)$$

We can combine (18) and (19) to proceed to:

$$\begin{aligned} &\frac{\partial f}{\partial \omega}\left[\frac{\beta-1-\beta\sigma}{(\beta+\gamma)(1-\sigma)-1} - \frac{\omega+\rho}{\mu}\right] = \\ &= \quad (\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)\left[\frac{1}{(\beta+\gamma)(1-\sigma)-1} - \frac{\gamma-\gamma\sigma+1}{\beta-1-\beta\sigma}\right] \end{aligned} \quad (20)$$

When $\mu < 0$, the partial derivative is multiplied by a positive term on the left-hand side of (20), so its sign is that of the right-hand side, which can be shown to be negative when $\sigma \neq 1$. When $\sigma = 1$ (the case of a logarithmic utility function), then $\partial f/\partial\omega = 0$.
Hence, $d\mu/d\omega \leq 0$ evaluated at the unique negative root. For a logarithmic utility function, money is superneutral also along the transition path, but in any other cases it is not: the greater the growth rate of money, the greater in absolute value is the speed of convergence. Thus to speed up convergence we need monetary expansion, which will however also lead to higher steady-state inflation. This is a counterintuitive result at first sight, since there seems to be a consensus among economists that price stability is the best basis for economic growth, whereas in an inflationary environment the prospects for growth are fading. Here in this setup under money supply rule higher inflation seems to be the price for faster convergence.

It is not easy to give an intuitive explanation for this result. First of all, it is not obvious whether the rise in ω will induce a more than proportional increase in inflation or a less than proportional one. Fischer starts from noting that the necessarily increasing inflation devalues the money stock at disposal, so its real value will decrease causing a lower value of wealth. This will result in a lower rate of consumption, enabling a faster convergence. Fischer however mentions that this logic is not really right as this would imply the result also for the logarithmic utility function where the effect is not present. He suspects the second order cross derivatives to be the channel of transmission, but there is no simple relationship as the sign of this cross derivative depends on the parameter σ and can be different. (Fischer (1979), pp.1438-1439.)

An increase (decrease) in the growth rate of the money supply will most likely increase (decrease) $\dot{m}/m$, except for the case when the increase (decrease) of inflation exceeds the change of ω. In empirical works consumption smoothing is a usual result, that's why $\sigma > 1$ is often assumed. In this case the growth rate of consumption changes differently relative to that of the real balances ceteris paribus, as seen in section 4 (p.16.), that is $\dot{c}/c$ decreases (increases), which enables a faster (slower) convergence. On the other hand as known from the literature (see e.g. Walsh (2003), p.55.), the equilibrium in a money in the utility function model is unique with $u_{cm} > 0$, which however holds for $\sigma < 1$ here. Then the two growth rates change in the same direction, that is in this case probably the increase of ω causes a more than proportional increase in inflation (for example because inflation is typically higher in this case), that is $\dot{m}/m$ will decrease in the end, and so will $\dot{c}/c$ allowing for faster convergence. In the case of a logarithmic utility function, that is $\sigma = 1$ there is no relationship between the growth rates of consumption and real balances. We have seen that the path of consumption is determined solely by the difference in the rates of return, so it cannot be influenced by changing ω (which means that convergence cannot be affected either).

5.2 Fixing the nominal interest rate

In this case the monetary authority controls the nominal interest rate instead of deciding on the growth rate of the money supply. We know that the domestic nominal interest rate is $(1-\alpha\theta)\eta B k_N^{\eta-1} - \delta + \pi \left(= \frac{\gamma c}{\beta m}\right)$, let us denote it by i. In the case of an interest rate rule an adequate level of i is chosen and then kept constant. Now this will be our policy function representing a fourth relationship between the variables.
Differentiating (12) with respect to time and using the constancy of i we have:

$$\frac{\dot{c}}{c} = \frac{\dot{m}}{m}$$

Using this in (13) results in the following two-dimensional system:

$$\frac{\dot{c}}{c} = \frac{1}{(\beta+\gamma)(1-\sigma)-1}\left[-(1-\alpha\theta)\eta B k_N^{\eta-1} + \delta + \rho\right] \tag{21}$$

$$\dot{k}_N = (1-\alpha\theta)Bk_N^{\eta} - \delta k_N - c \tag{22}$$

The steady-state is the same as in the previous case (19.o.), where π^* depends on the chosen nominal interest rate ($\pi^* = i - (1-\alpha\theta)\eta B k_N^{*\eta-1} + \delta$). In the steady-state money is superneutral, the real variables are determined independently of monetary policy. The chosen nominal interest rate uniquely determines the steady-state inflation rate. The steady-state real balances (m^*) will then be determined according to (17e) with π^* standing instead of ω now:

$$m^* = \frac{\gamma}{\beta} \cdot \frac{c^*}{r^w + \pi^*} \tag{23}$$

The log-linearised system:

$$\begin{pmatrix} \frac{dlog(c)}{dt} \\ \frac{dlog(k_N)}{dt} \end{pmatrix} = \begin{bmatrix} 0 & \frac{-(\eta-1)(\delta+\rho)}{(\beta+\gamma)(1-\sigma)-1} \\ \delta - \frac{\delta+\rho}{\eta} & \rho \end{bmatrix} \begin{bmatrix} log(c/c^*) \\ log(k_N/k_N^*) \end{bmatrix}$$

The trace of the transition matrix is simply $trace = \rho > 0$ and the determinant is $det = \frac{(\varepsilon-1)(\delta+\rho)}{(\beta+\gamma)(1-\sigma)-1}(\delta - \frac{\delta+\rho}{\varepsilon}) < 0$. This means that we have a positive and a negative eigenvalue, so an unstable and one stable saddle-path. The negative eigenvalue (μ) gives the speed of convergence along the stable transition path:

$$2\mu = \rho - \left\{\rho^2 - 4\frac{(\eta-1)(\delta+\rho)}{(\beta+\gamma)(1-\sigma)-1}\left(\delta - \frac{\delta+\rho}{\eta}\right)\right\}^{\frac{1}{2}}$$

As we can see, in this case monetary policy is superneutral also along the transition path (the convergence speed does not depend on the variable of monetary policy, the nominal interest rate). However, the speed of convergence depends on the degree of openness of the economy. Since $\frac{\partial\eta}{\partial\theta} < 0$ and $\frac{\partial\mu}{\partial\eta} > 0$, the more open the economy is (the higher is θ), the higher in absolute value is the speed of convergence (the higher is $|\mu|$).

To sum up, under interest rate rule money is completely neutral, there is nothing the monetary authority can do to influence convergence. What it can and should use its policy for is influencing the nominal variables and especially the inflation rate. As we saw, nominal variables depend solely on monetary policy. It means that many steady-state equilibria can exist with the same values for the real variables and differing nominal ones. In a model with a more detailed specification here the not traditional channel of the transmission mechanism takes on special relevance: expectations about future inflation are crucial in determining which of the possible equilibrium outputs will be realised (but for simplicity the model used here abstracts from an explicit inclusion of expectations). So by setting the nominal interest rate this should be taken into account. What matters for the convergence is the degree of openness, a more open economy have better chances to catch up more quickly than the more closed ones.

Here the intuition for the neutrality result is simple: through the fixing of the nominal interest rate the marginal rate of substitution between money and consumption (thus the ratio of holdings of real balances and consumption, as well) will also be fixed. Because of this, real balances and consumption grow at the same rate and there is no way to induce any change in this relationship. The system will contain only real variables (c and k_N) as m can be expressed by using c in a very simple way.
The result concerning the effect of openness on convergence can also be easily explained: the more open the economy is (the higher is the share of the traded goods' "sector" in output, k_T), the higher is the amount of funds borrowed from abroad ($d = k_T$). This implies a higher rate of investment into capital used in the traded goods' production, so the accumulation of this type of capital (and thus also convergence) can be faster.

5.3 Inflation targeting

In the case of inflation targeting (in its simple form assumed here) the monetary authority controls inflation directly. Again, it decides on a level regarded as adequate and then keeps inflation constant at that level.[21] We again start with differentiating (12) with respect to time, which using the constancy of π now gives the following result:

$$\frac{\dot{c}}{c} - \frac{\dot{m}}{m} = \frac{(\eta-1)(1-\alpha\theta)\eta B k_N^{\eta-1}\left[(1-\alpha\theta)Bk_N^{\eta-1} - \delta - \frac{c}{k_N}\right]}{(1-\alpha\theta)\eta B k_N^{\eta-1} - \delta + \pi}$$

[21]The definition used here is a very simple, special definition of inflation targeting. We could not include forms taking expectations into account, however, as the model is deterministic.

We can again use this in (13) to get a two-dimensional system:

$$\begin{aligned}
\frac{\dot{c}}{c} &= \frac{(\gamma-\gamma\sigma)(\eta-1)(1-\alpha\theta)\eta B k_N^{\eta-1}\left[(1-\alpha\theta)Bk_N^{\eta-1}-\delta-\frac{c}{k_N}\right]}{[(\beta+\gamma)(1-\sigma)-1][(1-\alpha\theta)\eta B k_N^{\eta-1}-\delta+\pi]}+ \\
&+ \frac{1}{(\beta+\gamma)(1-\sigma)-1}\left[-(1-\alpha\theta)\eta B k_N^{\eta-1}+\delta+\rho\right] \qquad (24) \\
\dot{k}_N &= (1-\alpha\theta)Bk_N^{\eta}-\delta k_N - c \qquad (25)
\end{aligned}$$

The steady-state is again the same (19.o.) with π^* chosen by the monetary authority and m^* determined by (23) as in the previous section, while the log-linear approximation has now the form:

$$\begin{pmatrix} \frac{dlog(c)}{dt} \\ \frac{dlog(k_N)}{dt} \end{pmatrix} = \begin{bmatrix} \frac{(\gamma-\gamma\sigma)(\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)}{[(\beta+\gamma)(1-\sigma)-1](\rho+\pi)} & \frac{[\rho(\gamma-\gamma\sigma-1)-\pi](\eta-1)(\delta+\rho)}{[(\beta+\gamma)(1-\sigma)-1](\rho+\pi)} \\ \delta-\frac{\delta+\rho}{\eta} & \rho \end{bmatrix} \begin{bmatrix} log\left(\frac{c}{c^*}\right) \\ log\left(\frac{k_N}{k_N^*}\right) \end{bmatrix}$$

In this case $trace = \rho + \frac{(\gamma-\gamma\sigma)(\eta-1)(\delta+\rho)(\delta-\frac{\delta+\rho}{\eta})}{[(\beta+\gamma)(1-\sigma)-1](\rho+\pi)}$. If $\sigma > 1$ (the case of negative second order cross partial derivatives), then this is surely positive, otherwise we cannot be sure about the sign in general. The determinant $det = \frac{(\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)}{(\beta+\gamma)(1-\sigma)-1} < 0$, which means that the steady-state is saddle-point stable. The negative eigenvalue gives us the speed of convergence:

$$2\mu = trace(\eta,\pi) - \sqrt{trace(\eta,\pi)^2 - 4det(\eta)}$$

As we can see, in this case monetary policy is not superneutral along the transition path, the chosen inflation rate influences the speed of convergence. Because of the uncertainty about the sign of the trace and the term $(\gamma-\gamma\sigma)$, however, it is difficult to have a clear-cut result about this relationship if the trace is positive. If $\sigma < 1$ *and* the trace is negative (which is not necessarily true even with this assumption), then surely $\frac{\partial\mu}{\partial\pi} > 0$. In this case the higher the inflation rate is, the smaller in absolute value is the convergence speed. Although for $\sigma > 1$ not, but for $\sigma = 1$ we also have a unique result in line with the preceding: here money is superneutral in transition, the choice of inflation rate does not affect the speed of convergence (the case of a logarithmic utility function).

The speed of convergence depends also on the degree of openness of the economy. We know that $\frac{\partial\eta}{\partial\theta} < 0$, but with determining the sign of $\frac{\partial\mu}{\partial\eta}$ we have similar problems as in the previous case. Again, if $\sigma < 1$ *and* the trace is negative then we know for sure that $\frac{\partial\mu}{\partial\eta} > 0$, so the more open the economy is (the higher is θ), the higher in absolute value is the speed of convergence (the higher is $|\mu|$). Otherwise we cannot be sure about the relationship.

In the case of the inflation targeting regime we thus have results being consistent with intuition: lower inflation can help convergence under certain conditions. This then contradicts the results of the section analysing the money supply rule. That strategy resulted in a trade-off between convergence speed and inflation, here this is not present, we have the opposite case. But without knowing more about our parameter values we cannot be sure about these relationships now. As far as openness is concerned, we have the usual result here too, but again only under the same conditions and not as a general case.

The intuition is similar to what we had before: the increase of inflation reduces the real value of our money holdings, which for the case of $\sigma < 1$ (when we can be sure about the effect) because of "complementarity" implies the fall of c. The saving rate will thus increase, convergence will be faster. In the case of a logarithmic utility function the path of consumption is determined independently of money as seen before, so then we cannot see any relationships between the speed of convergence and the variable of monetary policy (the inflation rate in this case) either.

5.4 Interest rate feedback-rule

In this case the monetary authority controls the nominal interest rate again, but changing it in line with inflation and not keeping it constant. It means that the central bank has a reaction function in inflation: $i = g(\pi)$ and $g'(\pi) > 0$, so when inflation is higher, the nominal interest rate is raised and vice versa. This is going to be the fourth equation in this case analysed as last. When $g'(\pi) > 1$ we call the monetary policy active (interest rate is raised more than inflation increases), when $g'(\pi) < 1$ then it is passive (the change of the interest rate is less than that of inflation).[22]
This means using equation (12) that $\frac{\gamma c}{\beta m} = g(\pi)$ and differentiating this with respect to time we have the following equation:

$$\frac{\dot{c}}{c} - \frac{\dot{m}}{m} = \frac{g'(\pi)\dot{\pi}}{g(\pi)}$$

It is also true using (12) that $(1-\alpha\theta)\eta B k_N^{\eta-1} - \delta + \pi = g(\pi)$. Differentiating this with respect to time while expressing k_N as a function of π and using

[22]The differentiation between active and passive economic policy was first used by Leeper (1991) in his paper analysing the interactions of monetary and fiscal policy. Their formal definitions in the form used here, as well were given by Benhabib, Schmitt-Grohé and Uribe who examined their characteristics concerning the uniqueness and stability of equilibrium in more articles, see e.g. Benhabib et. al. (2001a), (2001b) and (2002).

the resource constraint we get:

$$
\begin{aligned}
\dot{\pi} &= \frac{\eta-1}{g'(\pi)-1}(1-\alpha\theta)\eta B k_N^{\eta-1}\frac{\dot{k}_N}{k_N} = \\
&= \frac{(\eta-1)[g(\pi)+\delta-\pi]}{g'(\pi)-1}\left\{\frac{g(\pi)+\delta-\pi}{\eta} - c\left[\frac{g(\pi)+\delta-\pi}{(1-\alpha\theta)\eta B}\right]^{\frac{1}{1-\eta}} - \delta\right\}
\end{aligned}
$$

Using these two equations in (13) we end up again with a two-dimensional system:

$$
\begin{aligned}
\frac{\dot{c}}{c} &= \frac{(\gamma-\gamma\sigma)g'(\pi)(\eta-1)[g(\pi)+\delta-\pi]\left\{\frac{g(\pi)+\delta-\pi}{\eta} - c\left[\frac{g(\pi)+\delta-\pi}{(1-\alpha\theta)\eta B}\right]^{\frac{1}{1-\eta}} - \delta\right\}}{[(\beta+\gamma)(1-\sigma)-1]g(\pi)[g'(\pi)-1]} + \\
&+ \frac{\rho+\pi-g(\pi)}{[(\beta+\gamma)(1-\sigma)-1]} \qquad (26) \\
\frac{\dot{\pi}}{\pi} &= \frac{(\eta-1)\left[\frac{g(\pi)+\delta}{\pi}-1\right]}{g'(\pi)-1}\left\{\frac{g(\pi)+\delta-\pi}{\eta} - c\left[\frac{g(\pi)+\delta-\pi}{(1-\alpha\theta)\eta B}\right]^{\frac{1}{1-\eta}} - \delta\right\} \qquad (27)
\end{aligned}
$$

The steady-state is again the same (19.o.), but here the monetary variables remain undetermined if only the reaction function g is specified. For any inflation rate $m^* = \frac{\gamma c^*}{\beta g(\pi^*)}$, where $g(\pi^*) = \rho + \pi^*$. The log-linear approximation has now the form:

$$
\begin{pmatrix} \frac{dlog(c)}{dt} \\ \frac{dlog(\pi)}{dt} \end{pmatrix} = \begin{bmatrix} \frac{(\gamma-\gamma\sigma)g'(\pi)(\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)}{[(\beta+\gamma)(1-\sigma)-1]g(\pi)[g'(\pi)-1]} & \frac{[g(\pi)-\rho]\left\{\frac{(\gamma-\gamma\sigma)g'(\pi)\rho}{g(\pi)}-[g'(\pi)-1]\right\}}{(\beta+\gamma)(1-\sigma)-1} \\ \frac{(\eta-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\eta}\right)}{[g'(\pi)-1][g(\pi-\rho]} & \rho \end{bmatrix} \begin{bmatrix} log\left(\frac{c}{c^*}\right) \\ log\left(\frac{\pi}{\pi^*}\right) \end{bmatrix}
$$

In this case $trace = \rho + \frac{(\gamma-\gamma\sigma)g'(\pi^*)(\varepsilon-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\varepsilon}\right)}{[(\beta+\gamma)(1-\sigma)-1]g(\pi^*)[g'(\pi^*)-1]}$. If $\sigma > 1$ and the monetary policy is active *or* if $\sigma < 1$ and the monetary policy is passive, then this is surely positive, otherwise in general we cannot determine the sign. The determinant is $det = \frac{(\varepsilon-1)(\delta+\rho)\left(\delta-\frac{\delta+\rho}{\varepsilon}\right)}{(\beta+\gamma)(1-\sigma)-1} < 0$, we have a saddle-point stable steady-state again. The speed of convergence is now:

$$
2\mu = trace(g(\pi),\eta) - \sqrt{trace(g(\pi),\eta)^2 - 4det(\eta)}
$$

As we can see, in this case monetary policy is not superneutral along the transition path, the chosen reaction function (and so the inflation rate, as well) influences the speed of convergence. Because of the uncertainty about the sign of the trace and the term $(\gamma - \gamma\sigma)$, however, we do not have a general result about this relationship. If $\sigma < 1$ *and* the monetary policy

is active *and* the trace is negative (which is not necessarily true even with these assumptions) *or* if $\sigma > 1$ and the monetary policy is passive *and* the trace is negative (again, this is not necessarily true), then surely $\frac{\partial \mu}{\partial g(\pi)} > 0$. In this case the higher the inflation rate is, the smaller in absolute value is the convergence speed.
The degree of openness of the economy also affects the speed of convergence. We know that $\frac{\partial \eta}{\partial \theta} < 0$, but with determining the sign of $\frac{\partial \mu}{\partial \eta}$ we have similar problems here. Again, only in the same two cases as for the influence of monetary policy can we say something about this relationship: then $\frac{\partial \mu}{\partial \eta} > 0$, so the more open the economy is (the higher is θ), the higher in absolute value is the speed of convergence (the higher is $|\mu|$).

This rule though looking quite similar to the interest rate targeting produces very different results: money is not superneutral in the transition. Furthermore, here the monetary variables and the inflation rate are undetermined highlighting the possibility of multiple equilibria and the importance of expectations. Apart for this we find stronger similarities with the results under inflation targeting: lower inflation speeds up convergence, but only under certain parameter values. In general we cannot be sure about the effects of monetary policy. The same is true for the effects of openness, but under some specific circumstances we have the usual result that more open economies are likely to experience faster convergence.
We have another general result in line with the previous cases for a logarithmic utility function: if $\sigma = 1$ then money is superneutral also in the transition, the monetary authority cannot influence the speed of convergence with means at its disposal.

Except for this last case, it is not obvious to explain this quite complicated result. Let us first have a look at the second case, where $\sigma < 1$, so we have positive cross derivatives. With active monetary policy the nominal interest rate is increased more than the change in inflation, so real returns rise. Here the substitution effect dominates, so the agents are likely to save more and consume less. If the inflation rate is higher, the real value of money holdings will fall and this would cause the consumption also to fall because its marginal utility decreases ("complementarity"). Moreover, the ratio c/m has to increase, because the nominal interest rate is higher. This means that consumption falls less relative to the decrease in the real balances.
In the other case when $\sigma > 1$ we have a negative relationship between money holdings in real terms and consumption ("substitutability"). If monetary policy is passive, the rise in the nominal interest rate will fall short of the change in the inflation rate, so real returns fall. Here the income effect dominates, so this will again induce increased saving and decreased consumption.

As real balances decrease with inflation, this causes the marginal utility of consumption and thus consumption to increase. It is not obvious which effect dominates, but the c/m ratio has to increase again, so it is more likely that consumption will increase.
With logarithmic utility function monetary policy has surely no impact on the speed of convergence, since the path of consumption is determined by the difference in returns only.

6 Conclusion

The goal of the model presented here was the examination of the question relevant for the new member states of the European Union with the help of mixing the convergence literature and literature of monetary policy, whether it matters for convergence what monetary policy rule is followed. On the basis of the analysis in this paper the answer is definitely yes: the model economy behaves very differently in transition under the different monetary policy rules assumed, although the equilibrium values of the real variables are the same in each case.

I shortly summarize (first in two tables) the main results.

	Dimension	**Steady state**
Money supply rule	3: c, k_N, m	money is superneutral
Interest rate targeting	2: c, k_N	money is superneutral
Inflation targeting	2: c, k_N	money is superneutral
Feedback-rule	2: c, π	money is superneutral, nominal variables not uniquely defined!

	Transition: for faster convergence ...
Money supply rule	faster monetary expansion
Interest rate targeting	money is superneutral here too!
Inflation targeting	lower inflation rate (not a general result!)
Feedback-rule	lower inflation (not a general result!)

As can be seen also from the tables, in the case of a *money supply rule* the system describing the behaviour of the model economy during transition is three-dimensional with the variables c, k_n, m, in all the other cases we have a two-dimensional system. The steady-state inflation rate is determined through the chosen value for the growth rate of the nominal money supply (ω) and through this all monetary variables, while the real ones are determined independently of money by the parameters of the model (so as in all of the cases, money does not matter in the steady-state). The model is dynamically stable, there is a unique stable transition path. Money is not superneutral during transition: after controlling for openness (considering it as constant), the speed of convergence is higher, the higher is the growth rate of the money supply. This calls for increasing the nominal money supply, we should not however forget that this results in a higher steady-state inflation rate, as well.

The *interest rate rule* stands out leading to very different results from the others. We have a two-dimensional system here in c, k_N, which is saddle-point stable (there is again one stable transition path), but in this case money is superneutral also in transition. What matters is the degree of openness: the greater it is, the faster convergence will be (this is the standard result we get in the examined cases).

Under *inflation targeting* the system is again two-dimensional in the same variables (c, k_N). Here the monetary authority controls inflation directly and through the choice of its steady-state level (π^*) the monetary variables are also determined in the steady-state. The system is saddle-point stable. We do not have clear-cut results here however. Money matters for the transition, but it is not obvious how, under certain conditions a higher inflation rate will reduce the speed of convergence after controlling for openness.

For the *feedback-rule* we have very similar results: a two-dimensional system but now in c, π, one stable saddle-path and uncertainty about the effects of monetary policy in transition (that are however present). Again under certain conditions a higher inflation rate results in slower convergence with openness kept constant. Both cases are however contradicting the results under the money supply rule, where higher money growth rate (and so higher inflation) can lead to faster convergence. The only difference from the inflation targeting apart for the different system variables is that monetary variables remain undetermined in the steady-state in this case.

In all three cases where monetary policy affects convergence, for a special case of the utility function ($\sigma = 1$, logarithmic form) the superneutrality of money holds also in the transition. Also, in the three cases examined (fixing the nominal interest rate, inflation targeting and the interest rate feedback-rule) openness always furthered, speeded up convergence, but under the last two rules this relationship was not generally true (it held only for certain parameter values). The result is consistent with the view of economists: a higher degree of openness enables a faster adjustment. The mechanism behind the result in the model is also clear: a more important role of k_T (which shows a greater openness here) enables a greater reliance on foreign sources, thus increasing the available resources and leading to faster convergence.

The analysis of this simple model illustrates that the dynamic behaviour of the economy can differ to a great extent under different monetary policy rules already in a very simple setup. The model presented combined the elements regarded as inevitable for examining the question (small open economy, possibility of analysing convergence, monetary policy), but many other, also important elements have still been missing. The most relevant among them for a small open economy is probably the exchange rate, since in these countries the exchange rate channel is often considered to be a more important channel of monetary transmission than the interest rate channel. An obvious extension is thus the analysis of similar problems in models where the exchange rate can also explicitly be taken into account. The model shown here is however not eligible for including the exchange rate, as resulting from the assumption used to model convergence (constrained capital mobility) the variables that could potentially be affected by the exchange rate fall out of the model. Hence, to explicitly introduce the exchange rate a different setup capable for analysing convergence has to be used. It would be possible for example to use a convergence model, where investment incurs adjustment costs (Benczúr (2003), Benczúr–Kónya (2004), these are monetary models), or the credit supply curve is increasing (with higher indebtedness the interest rate on the new debts is higher), such as Chatterjee et. al. (2001) (where both assumptions are used, but in the framework of a real model).

There are other obvious extensions. It can be worth modifying the model by assuming different productivity parameters for the production technology of the sectors (different As) and examine whether the Balassa–Samuelson effect is valid in this two-sector model. An important extension would be the numerical examination of the model to see the complete transition path. But this is more interesting in a stochastic model, so first it could be justifiable to set up and solve a version of the model explicitly allowing for uncertainty.

The model shown here however already in its present form highlights the differences in the behaviour of the model economy under applying different monetary strategies and due to the simple structure a local analysis is also analytically possible. Since the exchange rate is not (and as I said before, could not be) explicitly included, it can be used to characterise such a situation in the first place, where changes in the exchange rate are not essential for the economic decisions – such as the situation after the introduction of the euro, where the currency used by the main trading partners is not different from the one used in the respective country. The analysis of the model is therefore despite all the listed shortcomings in any case theoretically interesting and justifiable.

References

[1] Barro, Robert J. – Mankiw, Gregory N. – Sala-i-Martin, Xavier (1995): *Capital Mobility in Neoclassical Models of Growth*, American Economic Review, Vol. 85, 1, pp.103-115., March

[2] Benczúr, Péter (2003): *Nominális sokkok átmeneti reálhatása egy két-szektoros növekedési modellben*, MNB Füzetek, 2003/9, November

[3] Benczúr, Péter – Kónya, István (2004): *Nominal growth in a small open economy*, manuscript, for the October 2004 conference titled Economic Growth and Development by the Institute of Economics of the MTA (September)

[4] Benczúr, Péter – Kónya, István [2005]: *A flexible price theory of equilibrium real exchange rates and output*, manuscript (September), for the September 2005 4th Macroeconomic Policy Research Workshop on Nominal Exchange Rates and the Real Economy of the National Bank of Hungary, downloaded from: http://english.mnb.hu/Engine.aspx?page=mnben_konf_fomenu&ContentID=6894

[5] Benhabib, Jess – Schmitt-Grohé, Stephanie – Uribe, Martin (2001a): *Monetary Policy and Multiple Equilibria*, American Economic Review, Vol. 91, March, pp.167-186.

[6] Benhabib, Jess – Schmitt-Grohé, Stephanie – Uribe, Martin (2001b): *The Perils of Taylor Rules*, Journal of Economic Theory, Vol. 96, pp.40-69.

[7] Benhabib, Jess – Schmitt-Grohé, Stephanie – Uribe, Martin (2002): *Avoiding Liquidity Traps*, Journal of Political Economy, Vol. 110, No. 3, pp.535-563.

[8] Chatterjee, Santanu – Sakoulis, Georgios – Turnovsky, Stephen (2001): *Unilateral Capital Transfers, Public Investment, and Economic Growth*, European Economic Review, Vol. 47, No. 6, pp.1077-1103.

[9] Del Negro, Marco – Schorfheide, Frank – Smets, Frank – Wouters, Raf (2004): *On the Fit and Forecasting Performance of New Keynesian Models*, Federal Reserve Bank of Atlanta, Working Paper, 2004-37 (December)

[10] Fischer, Stanley (1979): *Capital Accumulation on the Transition Path in a Monetary Optimizing Model*, Econometrica, Vol. 47, 6 (November)

[11] Galí, Jordi (2002): *New Perspectives on Monetary Policy, Inflation, and the Business Cycle*, NBER, Working Paper, 8767

[12] Galí, Jordi - Monacelli, Tommaso (2001): *Monetary Policy and Exchange Rate Volatility in a Small Open Economy*, NBER, Working Paper, 8905

[13] Lane, Philip R. (2001): *International trade and economic convergence: the credit channel*, Oxford Economic Papers 53, pp.221-240.

[14] Leeper, Eric M. (1991): *Equilibria under 'active' and 'passive' monetary and fiscal policies*, Journal of Monetary Economics, Vol. 27, pp.129-147.

[15] Obstfeld, Maurice - Rogoff, Kenneth (1995): *Exchange Rate Dynamics Redux*, Journal of Political Economy, Vol. 103, 3, pp.624-660.

[16] Obstfeld, Maurice - Rogoff, Kenneth (1998): *Risk and Exchange Rates*, NBER, Working Paper, 6694

[17] Sidrauski, Miguel (1967): *Rational Choice and Patterns of Growth in a Monetary Economy*, American Economic Review, Papers and Proceedings, 57, pp.534-544.

[18] Smets, Frank – Wouters, Raf (2003): *An estimated stochastic dynamic general equilibrium model of the euro area*, Journal of Economic Association, Vol. 1, No. 5 (September), pp.1123-1175.

[19] Summers, Robert – Heston, Alan [1991]: *The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988*, Quarterly Journal of Economics, May, pp.327–368.

[20] Walsh, Carl E. (2003): *Monetary Theory and Policy*, second edition, MIT Press, Cambridge, MA

[21] Woodford, Michael (2003): *Interest and Prices, Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton