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Platform Interconnection and Quality Incentives

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Abstract

We analyze competition between two platforms with positive network externalities. Platforms can choose to interconnect or alternatively, operate exclusively. We examine how this decision will affect pricing behaviour and incentives to invest in platform quality. We find that interconnection is a means to reduce externalities one side exerts on the other. It changes the mode of competition for subscribers and results in higher subscription prices. Further, even though interconnection allows for quality spillovers to the rival platform, it results in higher quality investment than the case of exclusive platforms. Coordination will facilitate collusion on the lowest quality levels possible if its provision is costly. For low quality costs it will lead to asymmetric networks. Therefore, interconnection without coordinated investment activities is welfare maximising.

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1 Introduction

In order to provide universal service, competing telephony suppliers, like traditional fixed line and mobile providers but also Voice-over-IP platforms, must be able to interconnect. Therefore, in Europe, the new regulatory framework of 2002 (Directive 2002/19/EC) encourages the use of standards to achieve interoperability.[1] This idea contradicts some network providers' claim of exclusive operation. Referring to Schumpeter, J.A. (1943) the providers argue that only the appropriability of innovation rents can stimulate investment incentives. Yet, also the opposite viewpoint based on Arrow, K. (1962) exists and finds competition necessary to spur innovation.

In this regard, European policy has - for the past few years - been concerned with the question of how to further investment incentives and the role competition plays. For the particular case of telecommunications and other network industries, the discussion ultimately draws attention to the question whether firms will consent to a common standard. This fundamentally affects the competitive environment since standardisation serves as a pre-requisite for interoperability and interconnection: It determines whether there will be intra- or inter-technology competition.

For this reason, we are interested in further examining the effect of firms' interoperability choices on competition and subsequent quality investments. Motivated by the particular example of telephony markets, we would like to explicitly consider two market characteristics: Network externalities and the two-sidedness of such markets. The latter characteristic describes the circumstance that markets can be regarded as platforms which serve as intermediaries between two distinct groups of users. It is then that there are not only intra-group but also inter-group externalities where one platform side affects the other. Referring to the particular example of international phone calls via Voice-over-IP, we find that this market structure can be applicable to telephony markets where a provider serves to issue calls from one particular group of users (e.g. telephone users of a certain country) to the other (e.g. users of a different country) and vice versa.[2]

In our model, we find that standardisation reduces price competition for subscribers and, therefore, leads to higher subscription prices. Higher quality incentives are aligned with higher prices as long as platforms do not coordinate their investments. Conversely, coordination bears the danger of collusion on minimum quality levels. Interestingly, this outcome depends on costs of quality provision: If quality provision is rather costly, collusive under-investment, indeed, can occur. Yet, for lower costs coordination results in the co-existence of highly differentiated platforms so that equilibrium market outcome is asymmetric. Concludingly, non-cooperative investments of interoperable platforms turns out to be welfare

[1] Common standards are published in the Directive 2002/19/EC on Access and Interconnection (related to network interconnection, this mainly refers to application interfaces and transmission protocols).

[2] It might also be a starting point to reflect interaction of national stock exchanges or other trading platforms.

maximising whereas coordinated investment activities take an ambiguous role: For low cost of quality provision, it produces the second-best outcome whereas it leads to a socially undesirable outcome for higher cost.

Our results rely on a multiplicative effect of quality in subscribers' utility function. Furthermore, we assume that inter-platform transactions, i.e. communication between users of competing platforms, can be characterized by the average transaction qualities of the two platforms involved. We believe that this might be adequate to represent the quality of Internet traffic in terms of speed, reliability etc., but it is an empirical question of how to best describe interacting qualities of two co-existent platforms. By our specification, consumers' willingness to pay is affected both by quality and the network size of one platform or, given a mutual agreement on interoperability, both platforms.

Formally, our basic model setup replicates competition between two existent platforms. These platforms can agree on a common standard.[3] If they do so, interconnection between platforms is possible. On the contrary, if they decide against a common standard they will operate independently and exclusively for their own customers. Platforms compete for subscribers via a fixed subscription fee corresponding to the prevalent offer of flatrates in reality. But before market competition takes place, platforms decide on the transaction quality they provide for their intermediation service.

This setup builds on seminal literature of two-sided markets by Rochet, J.-C. and Tirole, J. (2006) and, in particular, Armstrong, M. (2006) and Armstrong, M. and Wright, J. (2006). Their main interest lies in analysing platform pricing considering various market structures. In contrast to them, we restrict our view to duopolistic platform competition and the special case of subscription to one platform only. Further, our analysis is related to traditional literature on compatibility and standardization naming Farrell, J. and Saloner, G. (1985) and Katz, M.L. and Shapiro, C. (1985) as the most prominent examples. These works deal with the coordination problem of compatibility and investigate the social optimal degree of it. We, instead, abstract from the coordination problem of achieving a common standard and deal with the choice of full compatibility versus incompatibility only. Such restrained view serves our idea to explore the strategic effects of compatibility on competition. As we refer to telephone networks and discuss the implications of possible interconnection, we restrict attention to a two-way agreement on compatibility.[4] To incorporate quality aspects in our model, we introduce an additional investment stage before competition in the market takes place. Combined with the possibility of coordinating investments this builds on D'Aspremont, C. and Jaquemin, A. (1988). Fundmental to our equilibrium analysis is the concept of "fulfilled expectations" in terms of the expected network size. This concept has

[3] As our main aim is to examine strategic effects of a bilateral compatibility decision, we abstract from coordination problems and the idea of standards war.

[4] More precisely, standardisation or not.

been adopted by Katz, M.L. and Shapiro, C. (1985) and others before.[5]

In principal, also Schiff, A. (2003) and Baake, P. and Boom, A. (2001) contemplate the question of compatibility and competition. Schiff, A. (2003) even does so regarding two-sided markets. But he is concerned with finding an efficient market structure. Our approach instead, is closer to Baake, P. and Boom, A. (2001) who examine duopolistic competition referring to quality and compatibility as strategic choices. In terms of price competition, we come to the same conclusion as them and find that compatibility reduces price competition for subscribers. Yet, our outcomes differ with respect to quality incentives as we firstly, presume full instead of partial market coverage, and secondly, do not restrain our view on asymmetric outcomes by assumption. As a result of our model specification, our findings not only rely on the extent of network externalities, but, additionally, the cost of quality provision. According to Baake, P. and Boom, A. (2001), compatibility will always be induced by duopoly competition. We, instead, claim that it can occur in equilibrium only if competitors can cooperate in terms of investment. Then, standardisation and exclusivity are equivalent options for firms as soon as quality cost exceeds a certain threshold. Besides, we find that positive network externalities have a negative effect on quality incentives in case of exclusive platforms. With this, our findings vary from D'Aspremont, C. and Jaquemin, A.'s (1988): They claim that the lowest investment incentives occur for large spillovers. In our setup, spillovers - which occur for interconnected platforms - provoke the highest investment incentives. It is that they change the mode of platform competition and induce a quality race. In terms of welfare, we find that coordination is undesirable if the intention is to produce an optimal outcome. However, from the perspective of achieving second-best, coordinated investment activities can be socially desirable corresponding to the findings of D'Aspremont, C. and Jaquemin, A. (1988).

In the following, we explain our analysis in more detail and proceed as follows: Section 2 contains the basic setup, Section 3 looks at competition for subscribers in case of interconnected and non-interconnected platforms, Section 4 deals with the choice of quality investments, Section 5 looks at the compatibility decision, Section 6 discusses and Section 7 concludes.

2 The Model

We look at competition between two platforms $a \in \{A, B\}$ which serve as intermediaries between two different types of agents $i \in \{1, 2\}$. Agents obtain utility from possible transactions with (possibility of calling) opposite agents. Utilities are increasing in the expected number of agents they can reach on the other side. Also, utilities depend on the quality of transaction.

[5] We are fully aware of the fact that other expectations will lead to different outcomes.

The number of possible transactions is determined by the platforms' standardisation decision which will allow them to interconnect or not. If they agree on a common standard both intra- and inter-platform transactions can be realised. In other words, type 1 users of platform A can both connect to type 2 users of the same platform and the rival platform B and vice versa. The contrary holds if they decide against standardisation and therefore, operate independently. Then, only transactions between members of the same platform can take place. As a prerequisite for transactions agents need to subscribe to one of the platforms: For being part of platform $a \in \{A, B\}$ an agent of type $i \in \{1, 2\}$ has to pay the subscription fee p_i^a.

Platforms:
Within this framework, platforms compete for subscribers (in order to earn the subscription fee). They do so in a sequence of stages: As mentioned above, they first have to agree on a common standard or alternatively, decide to operate independently. Then, platforms simultaneously determine their qualities which we denote by q^A and respectively, q^B. These qualities affect the utility from transactions by dimensioning network externalities. We restrict our attention to qualities of limited positive value, i.e. $q^a \in [0, \overline{q}]$ with $\overline{q} < \frac{3}{4}t$. This way, we examine the effect of positive network externalities when there is duopolistic platform competition.[6] After qualities are determined, both platforms a simultaneously set the subscription fees p_i^a for users i.
It is assumed that a platform's cost arises due to provided transaction quality, but facilitating transactions between users does not incur additional cost. Throughout our analysis we use the example of $C(q^a) = \gamma\,(q^a)^2$. With this, each platform a's quality cost $C(q^a)$ is continuous, strictly increasing and convex in q^a, i.e. $C'(q^a) > 0$ and $C''(q^a) \geq 0$.
Furthermore, we assume that a platform's capacity is sufficient to host all agents. Then, a platform's profit is composed of the revenues from the number of agents 1 and 2 attracted and the cost to provide transaction quality. A platform a's profit function can be written as

$$\Pi^a = p_1^a n_1^a + p_2^a n_2^a - C(q^a) \tag{1}$$

with $a \in \{A, B\}$.

Agents:
There is a continuum of agents $i \in \{1, 2\}$ with a total mass of 1. Agents' subscription decisions are based on the payable fee, on individual tastes and, most importantly, the net benefits u_i^a they derive from joining a platform a. We model individual preferences for a platform with help of a Hotelling line where transportation cost t_i reflects the degree of

[6] In principal, negative externalities seem to be adequate to describe real-world phenomenon, too. For our particular example, negative effects could arise due to congestion or due to an increased probability of virus attacks etc.. Yet, we do not further investigate this phenomenon as tendencies of our results would not change. Reisinger, M. (2004) examines negative externalities from advertisements in the media market as another example.

product differentiation for the two existent platform markets.[7] We assume agents are uniformly distributed over the unit interval where platform A is located at 0 and B at 1. With this, total utility amounts to the net benefit of agent i belonging to platform a reduced by the subscription fee and the transport cost $t_i x_i$ where $x_i \in [0;1]$ represents the actual location of agent i, therefore,

$$U_i^a = u_i^a - p_i^a - t_i x_i \quad \text{for} \quad i \in \{1,2\}. \tag{2}$$

Net benefits are derived from possible transactions with the other type of agents. Therefore, they vary with the platforms' choice of using a common standard or not as much as with the quality which is provided for transactions. With a common standard, the possibility that platform a's type i-users interact with all type j-subscribers on the other side arises. If platforms decide to operate exclusively instead, only transactions between users of the same platform can take place.

We assume that transaction quality over all amounts to the average quality of the platforms involved. We believe this being appropriate to describe Internet transmission quality. Accordingly, we characterise inter-platform transactions by the sum of qualities q^A and q^B, and intra-platform transactions by twice quality q^A or q^B. Denoting the case of standardisation with the upper index S and of exclusivity with E, we specify net benefits of an agent i at platform a by

$$u_i^{a,S}\left(q^A, q^B, N_j^a\right) = \begin{cases} v_0 + 2q^a N_j^a + (q^A + q^B)(1 - N_j^a) & \text{if} \quad q^a \in \left[\underline{q}, \overline{q}\right] \\ 0 & \text{if} \quad q^a = 0 \end{cases} \tag{3}$$

in case of standardised platforms and by

$$u_i^{a,E}\left(q^a, N_j^a\right) = \begin{cases} v_0 + 2q^a N_j^a & \text{if} \quad q^a \in \left[\underline{q}, \overline{q}\right] \\ 0 & \text{if} \quad q^a = 0 \end{cases} \tag{4}$$

in case of platforms which operate exclusively, with $\underline{q} = 0 + \epsilon$, s.t. $\epsilon > 0$, $\lim \epsilon = 0$ and $i \neq j$, $a \neq b$.[8] With this, we specify the benefits from transactions presuming that consumers do not derive any benefit from mere subscription to a platform, without transactions. Note also that net benefits comprise baseline utility v_0 to ensure full participation of agents, i.e. $U_i^a \geq 0$.[9]

Besides, agents choose their preferred platform considering the expected market size on the opposite platforms' sides: Coherently, N_j^A represents the expected number of agents j on platform A while $N_j^B = 1 - N_j^A$ stands for the expected number of agents on platform B.

[7] Referring to network industries, one could also interpret them as switching cost, once subscription decisions have been taken.

[8] Hence, interconnection is modeled as a "quality spillover" to the other platform like in D'Aspremont, C. and Jaquemin, A. (1988).

[9] Therefore, v_0 is assumed to be constant and the same for all net utilities so as to not affect our results in the following.

We maintain the following additional assumptions throughout our analysis:

Assumption 1. $\{t_1, t_2\} < \frac{2}{3}v_0$

to make sure all agents subscribe to one platform in equilibrium. Further,

Assumption 2. $t_1 t_2 > (q^A + q^B)^2$

and

Assumption 3. $\{t_1, t_2\} \geq 1$.

With these, platforms' profits are always strictly concave in prices and thus, there exist unique equilibria in the market stage.[10] Additionally, we assume

Assumption 4. $0 < \gamma < \overline{\gamma} \equiv \frac{16}{9t_i}$ with $i = \{1, 2\}$.

Then quality provision is costly, but does not exceed a level at which firms can run profitably in equilibrium.

In sum, we consider the following sequence of decisions: In the first stage, platforms choose whether to conform to a common standard as a pre-requisite for interconnection or not. Then, they decide whether to cooperate in terms of quality investments and determine how much they invest in quality (assuming simultaneous moves). Finally, platforms determine subscription fees and agents choose which platform to subscribe to given their expectations about the number of subscribers on the opposite side will be fulfilled.[11]

We will determine Nash equilibria of the game by solving it backwards in the following.

3 Market shares and prices

Market shares are determined by identifying the marginal consumers x_i of the two market sides $i \in \{1, 2\}$, who are indifferent between joining network A or B, therefore, $U_i^A = U_i^B$. With this and $n_i = n_i^A + n_i^B = 1$, platform A's market share n_i^A of agents i amounts to $x_i = n_i^A$, while $n_i^B = 1 - n_i^A$. In general, the indifference condition yields

$$n_i^a = \frac{1}{2} + \frac{u_i^a - u_i^b + p_i^b - p_i^a}{2t_i}, \tag{5}$$

defining the relationship between net utilities, subscription prices and ex-ante differentiation leading to agents' subscription choices. As mentioned above, we assume fulfilled expectations in order to obtain an equilibrium outcome. This requires that users' expectations about the network size N_i^a will be fulfilled in equilibrium, thus, $N_i^a = n_i^a$ for all $i \in \{1, 2\}$ and

[10] This issue arises in case of no interconnection, see p. 8.

[11] Cf. Katz, M. and Shapiro, C. (1985).

$a \in \{A, B\}$. These subscription choices are considered by the platforms when determining profit maximising price levels, i.e. equilibrium prices. Explicitly, each platform a's profit maximisation concern is described as

$$\max_{p_1^a, p_2^a} \pi^a = p_1^a n_1^a + p_2^a n_2^a - C(q^a).$$

Standardisation: Given that platforms agreed to use a common standard, users' subscription choices are described by (3) combined with (5) under fulfilled expectations. This yields the number of a platform's subscriptions with respect to those of its competitor. Simultaneously considering these conditions for platforms A and B returns market shares

$$n_i^{A,S}\left(t_i, q^A, q^B, p_i^{A,S}, p_i^{B,S}\right) = \frac{1}{2} + \frac{q^A - q^B + p_i^{B,S} - p_i^{A,S}}{2t_i} \quad \text{and} \tag{6}$$

$$n_i^{B,S}\left(t_i, q^A, q^B, p_i^{A,S}, p_i^{B,S}\right) = \frac{1}{2} + \frac{q^B - q^A + p_i^{A,S} - p_i^{B,S}}{2t_i} \quad \text{for} \quad i \in \{1, 2\}. \tag{7}$$

Inspecting (6) and (7) the following is immediate:

Lemma 1. *Given a common standard, each platform's market shares are independent of the opposite side's prices. I.e. competition for i-type users takes place within one market side and depends on prices p_i^A and p_i^B only.*

Hence, network externalities and competition persist within one particular market side i. This also becomes obvious when looking at the first-order conditions to obtain profit maximising platform prices. These write

$$\frac{\partial \pi^{A,S}}{\partial p_i^{A,S}} = n_i^{A,S} + p_i^{A,S} \frac{\partial n_i^{A,S}}{\partial p_i^{A,S}} = 0, \tag{8}$$

$$\frac{\partial \pi^{B,S}}{\partial p_i^{B,S}} = n_i^{B,S} + p_i^{B,S} \frac{\partial n_i^{B,S}}{\partial p_i^{B,S}} = 0 \tag{9}$$

for $i \in \{1, 2\}$ describing two systems of two simultaneous conditions. Solving these systems for equilibrium prices and re-inserting these values into (6) and (7) yields equilibrium market shares. Expressing quality differences by $\Delta_q = q^A - q^B$ we can, therefore, state:

Proposition 1. *Given interconnection there exists a unique equilibrium in the market stage where prices are given by*

$$p_i^{A,S} = t_i + \frac{1}{3}\Delta_q \qquad \text{and} \qquad p_i^{B,S} = t_i - \frac{1}{3}\Delta_q \tag{10}$$

and market shares by

$$n_i^{A,S} = \frac{1}{2} + \frac{1}{6t_i}\Delta_q \qquad \text{and} \qquad n_i^{B,S} = \frac{1}{2} - \frac{1}{6t_i}\Delta_q. \tag{11}$$

Proof: See Appendix.

Hence, it is solely a comparative advantage in qualities which feeds back into higher market shares and prices, also in line with Lemma 1.[12]

Exclusivity: Similar to the previous case, condition (5) determines users' subscription choices with net benfits described by (4) when platforms act exclusively. Solving the conditions simultaneously under fulfilled expectations allows to determine platform A's market shares $n_i^{A,E}\left(q^A, q^B, p_1^{A,E}, p_1^{B,E}, p_2^{A,E}, p_2^{B,E}\right)$ and $n_i^{B,E}\left(q^A, q^B, p_1^{A,E}, p_1^{B,E}, p_2^{A,E}, p_2^{B,E}\right)$ for platform B, more precisely,

$$n_i^{A,E} = \frac{t_1 t_2}{T}\left(\frac{1}{2} + \frac{q^A - q^B + p_i^{B,E} - p_i^{A,E}}{2t_i}\right) + \frac{1}{2T}\left(q^A + q^B\right)\left(-2q^B + p_j^{B,E} - p_j^{A,E}\right),$$

$$n_i^{B,E} = \frac{t_1 t_2}{T}\left(\frac{1}{2} + \frac{q^B - q^A + p_i^{A,E} - p_i^{B,E}}{2t_i}\right) + \frac{1}{2T}\left(q^A + q^B\right)\left(-2q^A + p_j^{A,E} - p_j^{B,E}\right)$$

with $T = t_1 t_2 - (q^A + q^B)^2$, $i = \{1, 2\}$ and $i \neq j$.[13] Examining these, we conclude:

Lemma 2. *Given exclusivity, each platform's market shares are determined by subscription prices of both sides. I.e. a platform's market share of i-type users is determined by competition within and between market sides and depends on all prices p_i^A and p_i^B for $i = \{1, 2\}$.*

Here, in contrast to the case of platforms with a common standard, competition and externalities affect the equilibrium outcome of both platform sides instead of one particular market side i. For platforms' reaction functions concerning prices $p_i^{A,E}$ and $p_i^{B,E}$, it immediately follows that

$$n_i^{A,E} + p_i^{A,E}\frac{\partial n_i^{A,E}}{\partial p_i^{A,E}} + p_j^{A,E}\frac{\partial n_j^{A,E}}{\partial p_i^{A,E}} = 0 \quad \text{and} \tag{12}$$

$$n_j^{A,E} + p_j^{A,E}\frac{\partial n_j^{A,E}}{\partial p_j^{A,E}} + p_i^{A,E}\frac{\partial n_i^{A,E}}{\partial p_j^{A,E}} = 0 \tag{13}$$

with $i, j \in \{1, 2\}$ and $i \neq j$ yielding a system of four simultaneous conditions. Solving these we find:

Proposition 2. *Given exclusivity, there exists a unique equilibrium where prices are given by*

$$p_i^{A,E} = t_i - \frac{2}{3}(q^A + 2q^B) \quad \text{and} \qquad p_i^{B,E} = t_i - \frac{2}{3}(2q^A + q^B) \tag{14}$$

[12] The equilibrium outcome reflects price competition á la Bertrand as found in a one-sided market

[13] Taking the second derivatives gives rise to Assumption 2: More precisely, we check the strict concavity of profit functions with respect to the two prices the platform charges itself, Assumption 2, i.e. $T > 0$, ensures negative definiteness of the corresponding Hessian.

and market shares by

$$n_i^{A,E} = \frac{t_1 t_2}{T}\left(\frac{1}{2} + \frac{\Delta_q}{6t_i}\right) - \frac{1}{3T}\left(q^A + q^B\right)\left(q^A + 2q^B\right), \tag{15}$$

$$n_i^{B,E} = \frac{t_1 t_2}{T}\left(\frac{1}{2} - \frac{\Delta_q}{6t}\right) - \frac{1}{3T}\left(q^A + q^B\right)\left(2q^A + q^B\right) \tag{16}$$

with $T = t_1 t_2 - (q^A + q^B)^2$.

Proof: See Appendix.

Hence, prices for exclusive platforms are negatively affected by higher quality levels.[14] Yet, regarding market shares, quality effects are not as clear. Indeed, market shares as given in (15) and (16) consist of two terms: The first captures standard competitive effects within a Hotelling framework. As a result, comparative quality advantages Δ_q, or, respectively, $-\Delta_q$ will positively affect a platform's market shares. In contrast to that, the total level of platforms' qualities have a negative impact on market shares in the second term. This results from mutual effects between platform markets: The size of user group i affects user j's subscription decision and vice versa. This leads to stronger competition for subscribers the higher network externalities given that higher qualities increase network externalities.

Comparing, in general, the equilibrium outcomes of platform competition with and without the possibility of interconnection, we can summarise in terms of prices and the direct effect of qualities:

Corollary 2. *Price competition in case of exclusivity is stronger than in case of standardisation, i.e.* $p_i^{a,E} < p_i^{a,S}$ *for* $i \in \{1, 2\}$*. Moreover, a platform's own quality* q^a *affects prices negatively in case of exclusivity, but positively in case of standardisation, i.e.* $\frac{\partial p_i^{a,E}}{\partial q^a} < 0$ *and* $\frac{\partial p_i^{a,S}}{\partial q^a} > 0$.

In other words, price competition is stronger and leads to lower prices in case of exclusive than in case of standardised platforms. As stated before, this result is driven by the way network externalities influence the outcome. Also referring to Lemma 1 and Lemma 2 we found that in case of standardised platforms there are only network effects within a group whereas there are network effects within a user group i and between user groups of opposite platform sides in case of exclusive platforms. We can, therefore, conclude that

[14] Further rearranging first-order conditions additionally shows:

Corollary 1. *The impact of ex-ante differentiation* t_i *and qualities on prices can be described as follows:*

$$\frac{p_1^A + p_1^B}{p_2^A + p_2^B} = \frac{t_1 - (q^A + q^B)}{t_2 - (q^A + q^B)}.$$

This indicates that market price levels will be higher the less platforms compete (the higher t_i). Further, it reveals that higher qualities will always lead to a lower price structure on both sides of the platforms. Note also, that there is room for subsidising one side of the platform while extracting profits from the other side also iff $t_i < q^A + q^B < t_j$.

standardisation eliminates feedback effects.[15] Even though a common standard eliminates feedback effects and correspondingly, for exclusive platforms, there is a negative impact of total qualities on platforms' market shares, we find:

Lemma 3. *A quality advantage affects market shares positively and more significantly under exclusivity. Thus,* $\frac{\partial n_i^{a,E}}{\partial q^a} > \frac{\partial n_i^{a,S}}{\partial q^a} > 0$ *for* $q^a > q^b$.

I.e. that for the equilibrium outcome, in principal, total qualities have a positive effect on market share. Considering that only in case of exclusivity, total qualities exert a negative impact on market shares it is surprising that market shares as a whole, here, are raised with higher qualities to a further extent than standardised platforms. We conclude that direct network externalities within a platform side are relatively stronger for exclusive than for standardised platforms due to full appropriation of these.[16]

4 Quality Investments

Based on the outcome of the precedingly solved market stage we examine incentives for quality investments. We consider both non-cooperative and cooperative behaviour in this framework following D'Aspremont, C. and Jaquemin, A. (1988). Note that for the remaining part of this analysis we simplify our setup by assuming $t_1 = t_2 = t$ in order to ease computation and readability of results.[17]

4.1 Non-cooperative quality investments

To determine platforms' quality choices we look at each platform a's reduced profit function $\pi^a\left(t, \gamma, q^A, q^B\right)$. The profit-maximising quality is determined by

$$q^{a^*} = arg\max_{q^a} \pi^a(t, \gamma, q^a, q^{b*}) \quad s.t. \quad q^a \in [0, \overline{q}] \, .$$

Standardisation: With equilibrium market shares and prices from (10) and (11) a platform a's profit amounts to

$$\pi^{a,S} = \frac{1}{9t}\left(3t + \Delta_q\right)^2 - \gamma\left(q^a\right)^2 \tag{17}$$

with $a, b \in \{A, B\}$, $a \neq b$ and $\Delta_q = q^a - q^b$. Further examining a platform a's reaction to changes in its own quality shows that its profits are not necessarily concave in its quality q^a. Instead of looking at first-order conditions, we, therefore, define Nash equilibria in terms of

[15] This turns the platform market into a regular one-sided market.

[16] Further comparative statics analysis in t_i does not yield surprising results. For either standardisation choice a higher degree of ex-ante differentiation t_i will lead to higher prices and lower elasticity of market shares.

[17] We claim that tendencies of results should not alter with this simplification as there are no real opposing effects of switching cost t. Moreover, as we refer to telephone users in different countries, we claim that mostly, individual tastes do not vary that much from country to country.

mutual best-responses. As cost and, correspondingly, the functional form of $\pi^{a,S}$ depends on γ, equilibrium outcomes vary with the cost level γ and the degree of horizontal differentiation t:

Lemma 4. *The following types of equilibria emerge when platforms agree on a common standard, but do not coordinate quality provision:*

i. If $\gamma \leq \frac{4}{9t}$, equilibrium qualities are given by

$$q^{A,S^*} = q^{B,S^*} = \overline{q}.$$

ii. If $\gamma > \frac{4}{9t}$, the equilibrium qualities in pure strategies are given by

$$q^{a,S^*} = q^{b,S^*} = \frac{1}{3\gamma}$$

for $a, b \in \{A, B\}$ and $a \neq b$.

Proof: See Appendix.

Looking at Looking at (10) and (11) gives the intuition for this result: Obviously, platforms gain only if they outperform their rival. Therefore, they enter a quality race when competing with each other. As long as quality cost are relatively moderate such quality competition induces both platforms to provide the largest quality level allowed. If quality provision is more costly, investment occurs until marginal revenues equate marginal cost. Yet, considering that extra gains occur for comparative advantages only, these outcomes do not constitute profit-maximising equilibria. Interestingly, outcomes represent a prisoners' dilemma situation as a consequence of market stage competition.[18]

Exclusivity: Analogue to the case of standardisation, we use equilibrium market shares and prices given in (14) to (16) to state platform a's profit function as

$$\pi^{a,E} = \frac{2}{T}\left[t - \frac{2}{3}(q^a + 2q^b)\right] \cdot \left[\frac{t^2}{2} + \frac{t\left(q^a - q^b\right)}{6} - \frac{1}{3} \cdot \left(q^a + q^b\right)\left(2q^b + q^a\right)\right] - \gamma(q^a)^2 \quad (18)$$

with $a, b \in A, B$ and $a \neq b$.[19] We derive the profit function with respect to $q^{a,E}$ to obtain a platform a's reaction function. Using $q^A = q^B = q$, we conclude:

Lemma 5. *Suppose firms operate exclusively and determine qualities non-cooperatively,*

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[18] For a symmetric outcome, platforms do not gain from providing higher qualities since only differences will matter. Given symmetry, profit maximisation becomes a question of cost minimisation leading to minimum quality levels.

[19] The corresponding second-order conditions are $t_1 + t_2 < 18t_1t_2$ which always holds for all $t \geq 1$, see Assumption 3.

then there exists a unique equilibrium of symmetric quallity provision with

$$q^{A,E^*} = q^{B,E^*} = \underline{q}.$$

Proof: See Appendix.

Essentially, this occurs because higher qualities raise network effects: Even though stronger direct network effects make each platform more valuable for users it leads to stronger price competition with a substantial decrease in prices due to feedback effects from one platform side to the other. This decrease in subscription prices due to a higher quality always dominates the gain in market share. Therefore, raising its quality will not be profitable for a platform at any cost level. It will withdraw from investment activity as much as possible in order to reduce network effects and soften competition.[20]

With this, we finally, can compare equilibrium quality levels given platforms' decision whether to conform to a common standard:

Proposition 3. *Equilibrium quality levels are higher for standardised than for exclusive platforms, i.e.* $q^{a,S^*} > q^{a,E^*}$.

Proof: See Appendix.

Reasons for this outcome have already been given above. When platforms have a common standard, they will attempt to gain a comparative advantage so that quality provision is above the minimum level. In contrast to that, when platforms operate exclusively, they aim to reduce network effects between user groups so as to alleviate fiercer price competition in case of incompatibility. For that reason they keep qualities at a minimum.[21] (This outcome is somewhat surprising as a common standard allows for quality spillovers: Results contradict D'Aspremont, C. and Jaquemin, A. (1988) and the frequently used claim to further investment incentives by allowing for the appropriation of investments.)

4.2 Cooperative quality investments

Still following D'Aspremont, C. and Jaquemin, A. (1988), we look at quality outcomes when platforms coordinate themselves during the investment stage while competition for subscribers remains non-cooperative. Then, quality choices depend on joint profits $\pi^{AB} = \pi^A + \pi^B$ given platforms' decision about a common standard.

Standardisation: We obtain joint profits $\pi^{AB,S}$ in case platforms conform to a common standard by using (17). Looking at first and second derivatives shows that joint profits are only concave in qualities if $\gamma \geq \overline{\gamma}_{AB} \equiv \frac{2}{9t}$. If this condition holds, profit maximising

[20] In fact, platforms would aim to reduce dominant indirect network externalities to a certain extent, so that - if allowed - the would provide for negative qualities.

[21] In fact, if we permitted negative qualities such as conscious delay or interruption of transmission, the equilibrium qualities would amount to $q^{A,E} = q^{B,E} = -\frac{1}{6\gamma}$.

quality levels q^A and q^B, as usual, are derived by help of first-order conditions. Otherwise, profits are convex. Then, we conclude on equilibrium quality levels considering functional properties of $\pi^{AB,S}$, in particular, $\frac{\partial \pi^{AB,S}}{\partial (q^a)^2} > 0$ and $\frac{\partial \pi^{AB,S}}{\partial q^A q^B} < 0$. Denoting the provided level of qualities in case of coordinated investment with q^a_{AB} and q^b_{AB} for $\{a,b\} \in \{A,B\}$ and $a \neq b$, we find:

Lemma 6. *The following types of equilibria emerge when platforms agree on a common standard and coordinate their quality provision:*
1. If $\gamma < \overline{\gamma}_{AB}$, coordinated investment will lead to the highest quality differentiation feasible with qualities

$$q^a = \underline{q} \quad \text{and} \quad q^b = \overline{q}$$

.

2. If $\gamma \geq \overline{\gamma}_{AB}$ there is collusion on minimal quality levels s.t.

$$q^{A^*}_{AB} = q^{B^*}_{AB} = \underline{q}$$

.

Proof: See Appendix.

It shows that coordination can resolve the prisoner's dilemma situation of non-cooperative quality investments leading to collusion on cost minimising quality levels if joint profits are concave. If, however, joint profits are convex in a quality q^a, maintaining networks with maximal vertical differentiation proves to be the profit maximising strategy. It is that with $\gamma < \overline{\gamma}_{AB}$ additional cost of raising quality q^a is lower than the marginal return. Since $\frac{\partial \pi}{\partial q^a \partial q^b} < 0$ it is that a higher quality q^a raises joint profits the most, the lower the other platform's quality q^b with $\{a,b\} \in \{A,B\}$ and $a \neq b$.

Exclusivity: Joint profits $\pi^{AB,E}$ are determined considering (18). With the negative effect of a higher quality on a platform's profits, quality incentives are the same for individual and joint profit maximisation, therefore:

Lemma 7. *In order to maximise joint profits, exclusive platforms refrain from quality investment as much as possible, s.t.*

$$q^{A,E^*}_{AB} = q^{B,E^*}_{AB} = \underline{q}.$$

Proof: See Appendix.

Such investment behaviour is driven by the unambiguous and dominant negative price effect for higher qualities due to externalities opposite user groups exert on each other.
Comparing the outcomes of cooperative and non-cooperative investment behaviour, we find:

Proposition 4. *Let $\gamma < \overline{\gamma}_{AB}$. Then, the joint amount of platforms' investments is higher when platforms standardise, i.e. $q^{A,S^*}_{AB} + q^{B,S^*}_{AB} > q^{A,E^*}_{AB} + q^{B,E^*}_{AB}$. For $\gamma \geq \overline{\gamma}_{AB}$, neither* *standardised nor exclusive platforms have an incentive to invest above the minimum level, i.e.* $q_{AB}^{a,S^*} = q_{AB}^{a,E^*} = \underline{q}$ *with* $a \in \{A, B\}$.

Proof: See Appendix.

This result is rather unsurprising: In case platforms operate exclusively higher qualities are unambiguously associated with decreasing profits. Therefore, only minimum qualities will be provided. In case platforms operate on a common standard, the outcome depends on the cost of quality provision: If quality provision is costly, only minimum qualities in order to minimise cost will be provided. But for lower cost, there is the possibility of higher joint profits by exploiting network effects: Then, maintaining large asymmetries proves itself profit maximising and aggregate qualities exceed the provided quality level of exclusive platforms.

5 Standardisation or Exclusivity

In our basic setup platforms' choice whether to use a common standard or not is the initial stage. For expositional purposes we additionally look at the possibility of standardisation after qualities are chosen. With this, we can compare the outcome in terms of standardisation from a long-term view (our original setup) and from a short-term view. To find out whether platforms favour standardisation or not we compare platforms' profits in light of this decision. If equilibrium profits of standardised platforms are higher than the ones of exclusive platforms, they will agree to a common standard. Note that we abstract from side-payments or other details concerning the design of contracts between platforms when examining coordinated investment activities. Comparing total industry profits already indicates whether agreeing to a common standard will cause larger gains for both platforms.

5.1 Short-Term Decision

Comparing profits after competition in the market stage took place as stated in (17) and (18), we find that

$$\pi^{a,S} > \pi^{a,E}$$

for all $a \in \{A, B\}$. This holds no matter whether the same or differing qualities are provided. The reason is that the higher a platform's quality the larger is its competitivity towards its rival. In case of standardised platforms this affects both the platform's market shares and its prices positively. These positive effects due to higher competitivity are, in aggregate, offset by stronger competition due to a higher quality level in case of exclusivity: The negative impact of higher qualities on prices outweighs a slightly stronger increase in a platform's market share compared to the case of a common standard:

Lemma 8. *Platforms are mutually interested in standardisation and they would always agree on a common standard in the market stage.*

With a common standard both platforms acquire higher profits as prices for some given qualities are always higher compared to exclusivity. Even though quality investments of exclusive platforms solely benefit the investing platform, there are fewer gains from higher qualities. This is due to positive network externalities between user groups on opposite sides and inter-platform competition.

5.2 Long-Term Decision

Platforms' quality choices and, accordingly, its profits vary in the scale of quality cost measured by γ. By comparing profits after qualities were determined we conclude:

Proposition 5. *Choosing a common standard and collusion in qualities is always an option for platforms. For cost with $\gamma \geq \overline{\gamma}_{AB}$ this option is equivalent to operating exclusively.*

Proof: See Appendix.

Hence, if $\gamma \geq \overline{\gamma}_{AB}$, platforms merely aim to maintain platform usage but do not have any incentives for further quality improvements. This can be realised by cooperating in terms of qualities when platforms agreed on a common standard, and likewise by operating exclusively. Below $\overline{\gamma}_{AB}$, it is also collusion of standardised platforms which will unambiguously return the highest level of total industry profits. But then, increasing returns to scale are exploited by feigning monopolistic tendencies as far as quality provision is concerned. This leads to more than minimal quality provision.

6 Welfare

We compare the social efficiency of the various outcomes we analysed in the previous section. Welfare is, as usual, defined as the sum of agents' surpluses, thus

$$W = \pi^A + \pi^B + n_1^A U_1^A + n_2^A U_2^A + n_1^B U_1^B + n_2^B U_2^B.$$

With this, welfare implications are driven by agents' net utilities reduced by their transportation cost t and the cost of quality provision. [22] The above stated welfare function reflects higher social efficiency due to improved consumer welfare if we consider results of the quality stage.[23] Let W^S and W^E denote social welfare in case of non-cooperative quality decisions with and without a common standard. We compare this outcome to $W^{S,AB}$ and

[22] Note that due to the assumptions made there is no additional insight by analysing short-term outcomes, i.e. equilibrium outcomes after market stage, but before quality competition. Here, with symmetric outcomes, social surplus is always higher in case of standardisation due to additional network benefits in subscribers' utilities.

[23] As we restricted our attention to symmetric outcomes, there is the suspicion that within a Hotelling model of differentiation total surplus remains the same in all cases but prices determine the distribution of surplus. Yet, this reasoning does not apply for our setup as agents' utility raises with higher total quality, thus, welfare comparisons might point to a socially desirable outcome in terms of investment.

$W^{E,AB}$ denoting coordinated quality investments in view of standardisation or exclusivity. We find:

Proposition 6. *A common standard is socially most desirable as long as platforms do not coordinate their quality provision. For low cost of quality provision with* $\gamma < \overline{\gamma}_{AB}$ *a common standard with coordinated quality provision proves itself more efficient than exclusive arrangements. If quality provision is rather costly with* $\gamma \geq \overline{\gamma}_{AB}$ *both coordination in qualities and exclusivity are socially undesirable.*

Proof: See Appendix.

In other words, the social efficiency of a common standard and cooperative quality provision depends on the cost level: For rather low cost, standardisation always proves itself more beneficial to welfare than exclusivity due to investment incentives. Indeed, quality competition of standardised platforms yields the highest quality provision and correspondingly, the highest consumer surplus. It is, therefore, welfare maximising. Coordinated quality provision proves itself the second-best outcome, as it results in high quality differentiation where one platform makes greatest efforts to provide high transaction quality. Higher social welfare is achieved compared to the case of platform exclusivity: The ensuing market structure actually produces incentives to disinvest in qualities so that exclusivity becomes socially least desirable in terms of qualities and overall welfare. If quality provision is rather costly, coordination of quality investment bears the collusive danger of maintaining lowest quality levels.

7 Discussion

The results we obtained apply for our specific setup with duopolistic platform competition and full participation. In this section, we discuss these assumption pointing to further aspects which seem interesting to analyse.

Local and long-distance competition

Considering that we relate our specific setup to competition of VoIP-providers in the long-distance market, the objection that such providers compete both in long-distance and local markets at the same time is immediate. Our approach simply looked at platform interaction from the perspective of one particular market (the market for long-distance calls). If we extended our framework to consider local competition, it should be most likely that direct network externalities within one market side would gain importance. Provided that these tendencies are not overwhelming, still results would be driven by the interplay and existence of network effects within and between markets driven by the decision to enable interconnection or not. Hence, qualitative results should not be affected.

Asymmetric competition, sequential quality choices and partial participation

Our results build on symmetric starting points for platform competitors. We motivated this situation by referring to the situation of new telecommunication platforms like VoIP providers. Given prospects of converging telecommunications, i.e. growing competition between differentiated platforms, there are various possibilities of extending our discussion. E.g. when switching cost play a substantial role, sequential decisions seem more adequate. Given sequential quality choices and alleviating restrictions on quality investments, entry deterrence becomes an issue. We find that, in this context, an exclusively operating platform requires lower quality effort to deter its rival's entry than a standardised platform. This result is not surprising since interoperability allows for a certain degree of "free-riding" on the other platform's quality. With this, more effort is required to induce market exit of the competitor.

One could, further, imagine market expansion with the arrival of new telecommunication services. From a consumption perspective, one could, in this regard, discuss partial market coverage. We suppose that such a situation could make collusion on symmetric quality levels more difficult because it would reinforce competition for the 'users at the margin'. We would expect further asymmetries induced by a change in quality incentives. Yet, the question whether assuming full or partial market coverage is more appropriate is an empirical question and crucially depends on our market definition and the country we look at. Theoretically, it remains an open question whether such a situation could resolve collusion on low qualities.

Entry and coalitions

In light of the discussion of market dynamics, entry of additional platforms is an aspect to look into. Here, it might be interesting to raise the question in what way coalitions will arise to lower entry barriers.

Details of standardisation agreements

So far, we have looked at platforms' agreement on a common standard or not. Considering the possibility of asymmetric outcomes and coordination, commitment and the design of contracts will play an essential role to realise the depicted outcomes. Here, bargaining, outside options and other contractual aspects are an area of further research.

8 Conclusion

In this paper, we examined how common standards as a prerequisite for interconnection will affect platform competition. We compared pricing behaviour of platforms operating under a common standard and platforms operating exclusively. We looked at incentives for higher network quality as a result of market stage competition and the possibility of coordination. Restricting our view to market stage competition, we found that platforms would agree on a common standard in order to mitigate platform competition: This would lead to softer

competition for subscribers and higher prices compared to the case when platforms operate exclusively. Such phenomenon occurs as in case of exclusivity externalities between platform sides exert a downward pressure on prices. When enabling interconnection, this effect is eliminated. Nevertheless, choosing interconnection is not necessarily platforms' equilibrium choice if the outcome of quality investments are considered. Then, only in case of coordinated quality provision, platforms find a common standard profitable.
This outcome contrasts the political target of achieving higher quality provision corresponding to higher social welfare in our setup. Here, the highest quality provision is guaranteed if platforms agree on interconnection, but do not coordinate their investment activities. It is then that competing platforms enter a quality race leading to high qualities of both platforms. Most surprisingly, exclusivity of platforms cannot spur quality incentives even though it enables full appropriability of quality investments. The reason is that higher qualities lead to reinforced competition for subscribers and lower subscription prices. Facing this situation, platforms would prefer to disinvest in order to soften competition. From a welfare perspective, quality coordination, particularly, as standardised platforms are concerned, might not lead to the socially most desirable outcome. Yet, its social desirability varies with the actual level of quality cost: If rather low, coordination leads to the second-best outcome where platforms provide different quality levels above the minimum quality. But if quality provision is rather costly, coordination bears the danger of collusive least-quality provision.

These results suggest that there is not necessarily a need to reinforce interoperability between competing networks. Yet, one should be aware of possible collusion on low qualities as a result of platforms' long-term decisions. Hence, our results are, in principal, in line with present European policy which does not explicitly interfere in terms of interoperability, but still recommends it. Yet, the interdependency with collusion in investment and, in particular, the role of quality cost, do not seem to be explicitly considered. Our findings imply that cooperation in terms of quality investments cannot achieve the most efficient outcome and even could lead to collusive underinvestment if quality provision is rather costly. Yet, considering the possibility of quality advances at low cost, allowing for coordination could ensure the second-best outcome with respect to welfare and consumer surplus. It suggests that prohibiting cartels is an appropriate measure from the perspective of welfare maximisation if we assume high investment cost. Yet, it raises the question whether this attitude should be reconsidered, explicitly dealing with cost measures and the underlying market structure, aiming to achieve the second best outcome.
With this, our findings leave the design of interconnection agreements to balance the aims of lower prices, higher qualities and lower cost to a further discussion. Considering the uprise of numerous co-existent networks, nowadays, the emergence and effect of coalitions also could be discussed. As far as pricing is concerned, we imagine discriminatory prices, but also transfer payments between platforms and agents aspects relating our analysis further to real-world situations.

9 Appendix

Proof of Proposition 1:

Utilities of subscribers in case of interconnected platforms can be described as

$$U_i^{A,S} = v_0 + 2q^A n_j^{A,S} + (q^A + q^B)(1 - n_j^{A,S}) - p_i^{A,S} - t_i x_i, \tag{19}$$

$$U_i^{B,S} = v_0 + 2q^B n_j^{B,S} + (q^A + q^B)(1 - n_j^{B,S}) - p_i^{B,S} - t_i(1 - x_i) \tag{20}$$

with $n_i^{B,S} = 1 - n_i^{A,S}$. Market shares are determined by identifying the marginal consumer i with $i \in \{1, 2\}$ who is indifferent between network A and B, i.e. $U_i^{A,S} = U_i^{B,S}$. This yields conditions as described in (5). Solving these conditions simultaneously leads to

$$n_i^{A,S}\left(q^A, q^B, p_i^{A,S}, p_i^{B,S}\right) = \frac{1}{2} + \frac{q^A - q^B + p_i^{B,S} - p_i^{A,S}}{2t_i},$$
$$n_i^{B,S}\left(q^A, q^B, p_i^{A,S}, p_i^{B,S}\right) = \frac{1}{2} + \frac{q^B - q^A + p_i^{A,S} - p_i^{B,S}}{2t_i}$$

also given in (6) and (7). These results have to be taken into account when platforms set prices. The platforms' profit considerations can be written as

$$\max_{p_1^{a,S}, p_2^{a,S}} \pi^{a,S} = p_1^{a,S} n_1^{a,S} + p_2^{a,S} n_2^{a,S} - C(q^a)$$

for $a \in \{A, B\}$. Then, the first-order conditions with respect to prices $p_i^{A,S}$ and $p_i^{B,S}$ can be stated as

$$\frac{1}{2} + \frac{q^A - q^B + p_i^{B,S} - 2p_i^{A,S}}{2t_i} = 0 \quad \text{and} \tag{21}$$

$$\frac{1}{2} + \frac{q^B - q^A + p_i^{A,S} - 2p_i^{B,S}}{2t_i} = 0. \tag{22}$$

Solving simultaneously two systems of two first-order-conditions results in equilibrium

$$p_i^{A,S} = t_i + \frac{1}{3}\left(q^A - q^B\right),$$
$$p_i^{B,S} = t_i + \frac{1}{3}\left(q^B - q^A\right).$$

as given in (10). Inserting these values into (6) and (7) returns market shares as given in (11), i.e.

$$n_i^{A,S} = \frac{1}{2} + \frac{1}{6t_i}\left(q^A - q^B\right),$$
$$n_i^{B,S} = \frac{1}{2} + \frac{1}{6t_i}\left(q^B - q^A\right).$$

q.e.d

Proof of Proposition 2:

If platforms operate exclusively, agents' utilities are given by (2) and (4). Market shares are determined by the indifference condition $U_i^{A,E} = U_i^{B,E}$ and amount to

$$n_i^{A,E} = \frac{t_1 t_2}{T}\left[\frac{1}{2} + \frac{q^A - q^B + p_i^{B,E} - p_i^{A,E}}{2t_i}\right] + \frac{1}{2T}\left(q^A + q^B\right)\left(-2q^B + p_j^{B,E} - p_j^{A,E}\right),$$
$$n_i^{B,E} = \frac{t_1 t_2}{T}\left[\frac{1}{2} + \frac{q^B - q^A + p_i^{A,E} - p_i^{B,E}}{2t_i}\right] + \frac{1}{2T}\left(q^A + q^B\right)\left(-2q^A + p_j^{A,E} - p_j^{B,E}\right)$$

with $T = t_1 t_2 - (q^A + q^B)^2$. With profits

$$\pi^{A,E} = p_1^{A,E} n_1^{A,E} + p_2^{A,E} n_2^{A,E} - (q^A)^2 \quad \text{and} \tag{23}$$
$$\pi^{B,E} = p_1^{B,E} n_1^{B,E} + p_2^{B,E} n_2^{B,E} - (q^B)^2 \tag{24}$$

the respective FOCs for profit maximisation with respect to prices $p_1^{A,E}$, $p_1^{B,E}$, $p_2^{A,E}$ and $p_2^{B,E}$ according to (12) to (13) can be explicitly stated as

$$\begin{aligned}
&\tfrac{t_1 t_2}{T} n_1^{A,S} + \tfrac{1}{2T}\left(q^A + q^B\right)\left(-2q^B + p_2^{B,E} - p_2^{A,E}\right) - \tfrac{t_2}{2T} p_1^{A,E} - \tfrac{1}{2T}(q^A + q^B) p_2^{A,E} = 0,\\
&\tfrac{t_1 t_2}{T} n_1^{B,S} + \tfrac{1}{2T}\left(q^A + q^B\right)\left(-2q^A + p_2^{A,E} - p_2^{B,E}\right) - \tfrac{t_2}{2T} p_1^{B,E} - \tfrac{1}{2T}(q^A + q^B) p_2^{B,E} = 0,\\
&\tfrac{t_1 t_2}{T} n_2^{A,S} + \tfrac{1}{2T}\left(q^A + q^B\right)\left(-2q^B + p_1^{B,E} - p_1^{A,E}\right) - \tfrac{t_1}{2T} p_2^{A,E} - \tfrac{1}{2T}(q^A + q^B) p_1^{A,E} = 0,\\
&\tfrac{t_1 t_2}{T} n_2^{B,S} + \tfrac{1}{2T}\left(q^A + q^B\right)\left(-2q^A + p_1^{A,E} - p_1^{B,E}\right) - \tfrac{t_1}{2T} p_2^{B,E} - \tfrac{1}{2T}(q^A + q^B) p_1^{A,E} = 0.
\end{aligned}$$

Rearranging in terms of prices yields the following system of simultaneous equations to solve:

$$\frac{1}{T}\begin{bmatrix} 2t_2 & -t_2 & 2(q^A + q^B) & -(q^A + q^B) \\ t_2 & -2t_2 & (q^A + q^B) & -2(q^A + q^B) \\ 2(q^A + q^B) & -(q^A + q^B) & 2t_1 & -t_1 \\ (q^A + q^B) & -2(q^A + q^B) & t_1 & -2t_1 \end{bmatrix}\begin{bmatrix} p_1^{A,E} \\ p_1^{B,E} \\ p_2^{A,E} \\ p_2^{B,E} \end{bmatrix}$$
$$= \begin{bmatrix} t_1 t_2 + t_2(q^A - q^B) - 2q^B(q^A + q^B) \\ -t_1 t_2 + t_2(q^A - q^B) + 2q^A(q^A + q^B) \\ t_1 t_2 + t_1(q^A - q^B) - 2q^B(q^A + q^B) \\ -t_1 t_2 + t_1(q^A - q^B) + 2q^A(q^A + q^B) \end{bmatrix}$$

Solving for individual prices and rearranging yields

$$p_i^{A,E} = t_i - \frac{2}{3}(q^A + 2q^B)$$
$$p_i^{B,E} = t_i - \frac{2}{3}(2q^A + q^B)$$

as given in (14). From there, calculating price differences is straightforward with

$$p_i^{B,E} - p_i^{A,E} = \frac{2}{3}(q^B - q^A) \quad \text{and} \quad p_i^{A,E} - p_i^{B,E} = \frac{2}{3}(q^A - q^B).$$

Considering this, we insert the equilibrium prices into market shares described by the indifference conditions at the beginning of the proof and obtain

$$\begin{aligned} n_i^{A,E} &= \frac{t_1 t_2}{T}\left[\frac{1}{2} + \frac{q^A - q^B}{6t_i}\right] - \frac{1}{3T}\left(q^A + q^B\right)\left(2q^B + q^A\right), \\ n_i^{B,E} &= \frac{t_1 t_2}{T}\left[\frac{1}{2} + \frac{q^B - q^A}{6t_i}\right] - \frac{1}{3T}\left(q^A + q^B\right)\left(2q^A + q^B\right) \end{aligned}$$

as in (15) and (16). To ensure the existence equilibria, concavity of profits in prices is a sufficient condition, therefore, we assume $t_1 t_2 > (q^B + q^A)^2$ so that $T > 0$.

q.e.d

Proof of Lemma 3:
In order to show $\frac{\partial n_i^{a,E}}{\partial q^a} > \frac{\partial n_i^{a,S}}{\partial q^a} > 0$ for $q^a > q^b$ we compare equilibrium market shares for symmetric qualities $q^a = q^b$ with the outcome for $q^a > q^b$:
It is obvious from (11), (15) and (16) that for $q^a = q^b$ equilibrium market shares amount to $n_i^a = 1/2$. In case of $q^a > q^b$ we claim that

$$n_i^{a,E} > n_i^{a,S}.$$

Given the equilibrium outcomes (11), (15) and (16), we can restate this claim as

$$\Leftrightarrow \frac{t_1 t_2}{T} n_i^{a,S} - \frac{1}{3T}\left(q^a + q^b\right)\left(q^a + 2q^b\right) > n_i^{a,S}$$

Rearranging and inserting (11) yields

$$t_i > -\left(q^a + q^b\right)$$

which holds by assumption.

q.e.d

Proof of Lemma 4:
If $\gamma \leq 4/9$ marginal revenues always exceed marginal cost, i.e. $\partial R^{a,S}/\partial q^a \geq \partial C^{a,S}/\partial q^a \geq 0$. With that, best response of a platform a to its rival's quality q^b can be described as $q^{a^*}\left(q^b\right) = \overline{q}$ for $a \in \{A, B\}$. There is no incentive to deviate as $\pi^{a,S}\left(\overline{q}, q^b\right)|_{q^{b^*}(q^a)} \geq \pi^{a,S}\left(q^a, q^b\right)|_{q^{b^*}(q^a)}$ for all $q^a \in [0, \overline{q}]$. Therefore, $q^{a,S^*} = q^{b,S^*} = \overline{q}$ constitutes a Nash equilibrium. Analogously to the above described case, mutual best responses of a platform

a in case of $\gamma > 4/9$ can described as

$$q^{a^*} = q^{b^*} = \frac{1}{3\gamma}$$

where platforms do not have a unilateral incentive to deviate.

q.e.d

Proof of Lemma 5:

To derive results for exclusively operating platforms, let us look at the effect of a higher quality q^a on a platform a's profit as given in (18):

$$\begin{aligned}\frac{\partial \pi^a}{\partial q^a} =& \frac{-4(q^A+q^B)}{T^2}\left[t - \frac{2}{3}(q^a+2q^b)\right]\left[\frac{t^2}{2} + \frac{t\left(q^a-q^b\right)}{6} - \frac{1}{3}\left(q^a+q^b\right)\left(2q^b+q^a\right)\right] \\ &+ \frac{2}{T}\left[-\frac{2}{3}\left(\frac{t^2}{2} + \frac{t\left(q^a-q^b\right)}{6} - \frac{1}{3}\left(q^a+q^b\right)\left(2q^b+q^a\right)\right)\right] \\ &+ \frac{2}{T}\left[t - \frac{2}{3}(q^a+2q^b)\right]\left[\frac{t}{6} - \frac{2}{3}q^a - q^b\right] - 2\gamma q^a < 0\end{aligned}$$

assuming a symmetric equilibrium, i.e. $q^a = q^b = q$, and given the restricted quality range of $q^a \in [0, \overline{q}]$ and differentiability within $(\underline{q}, \overline{q}\,]$. With $\pi^a(0,\cdot) < \pi^a(q,\cdot)$, corner solution $q^{A,E^*} = q^{B,E^*} = \underline{q}$ results in equilibrium.

q.e.d

Proof of Proposition 3:

The result directly follows from Lemma 4 and 5. Comparing equilibrium quality levels q^{a,S^*} with q^{a,E^*} with $a \in \{A, B\}$ immediately shows $q^{a,S^*} > q^{a,E^*}$.

q.e.d

Proof of Lemma 6:

For standardised platforms, joint profits amount to

$$\begin{aligned}\pi^{AB,S} &= \frac{1}{9t}\left(3t + q^A - q^B\right)^2 + \frac{1}{9t}\left(3t + q^B - q^A\right)^2 - \gamma\left((q^A)^2 + (q^B)^2\right) \\ &= 2t + \frac{2}{9t}\left(q^A - q^B\right)^2 - \gamma\left((q^A)^2 + (q^B)^2\right).\end{aligned}$$

Let us first check on the concavity of the profit function in its qualities q^A and q^B by considering its Hessian:

$$H(q^A, q^B) = \begin{bmatrix} \frac{4}{9t} - 2\gamma & -\frac{4}{9t} \\ -\frac{4}{9t} & \frac{4}{9t} - 2\gamma \end{bmatrix}$$

The profit function is strictly concave if $H(q^A, q^B)$ is negative definite, i.e. it requires

$$\left(\frac{4}{9t} - 2\gamma\right)^2 - \left(\frac{4}{9t}\right)^2 > 0.$$

This condition holds for $\gamma > \frac{2}{9t}$. Therefore, the profit function is concave for $\gamma \geq \frac{2}{9t}$ for which we can derive profit maximising choices by regular first-order conditions:

$$\frac{\partial \pi^{AB,S}}{\partial q^A} = \frac{4}{9t}\left(q^A - q^B\right)^2 - 2\gamma^A = 0 \quad \text{and} \tag{25}$$

$$\frac{\partial \pi^{AB,S}}{\partial q^B} = \frac{4}{9t}\left(q^B - q^A\right)^2 - 2\gamma^A = 0 \quad . \tag{26}$$

Solving these, we obtain

$$q_{AB}^{A^*} = q_{AB}^{B^*} = \underline{q}.$$

In case of $\gamma < \frac{2}{9t}$, we have a closer look at the the second-order conditions in terms of platform qualities. It is that

$$\frac{\partial^2 \pi^{AB,S}}{\left(\partial q^a\right)^2} > 0 \quad \text{if} \quad \gamma < \frac{2}{9t}$$

but

$$\frac{\partial^2 \pi^{AB,S}}{\partial q^a \partial q^b} < 0 \quad \text{if} \quad \gamma < \frac{2}{9t}$$

revealing convexity in one platform's quality q^a, but strategic substitutability between the two qualities q^A and q^B. With that, maximal quality differentiation is profit maximising provided that quality provision is coordinated. Therefore,

$$q_{AB}^{a,S^*} = \underline{q} \quad \text{and} \quad q_{AB}^{b,S^*} = \overline{q}$$

with $\{a, b\} \in \{A, B\}$ and $a \neq b$.

q.e.d

Proof of Lemma 7:
See Lemma 5.

Proof of Proposition 4:
Results follow immediately from Lemma 6 and Lemma 7.

q.e.d

Proof of Proposition 5:
It is that if $\gamma < \overline{\gamma}_{AB}$ the following order of total industry profits holds:

$$\pi^{AB,S} > \pi^{A,E} + \pi^{B,E} = \pi^{AB,E} > \pi^{A,S} + \pi^{B,S}.$$

If $\gamma \geq \overline{\gamma}_{AB}$ the ranking of profits changes to

$$\pi^{AB,S} = \pi^{A,E} + \pi^{B,E} = \pi^{AB,E} > \pi^{A,S} + \pi^{B,S}.$$

q.e.d

Proof of Proposition 6:
The welfare function as stated in section 6 can be simplifed to

$$W = n_1^A u_1^A + n_2^A u_2^A + n_1^B u_1^B + n_2^B u_2^B - 2t - \gamma\left((q^A)^2 + (q^B)^2\right).$$

We use this formulae to calculate welfare using the equilibrium qualities calculated in Section 4. Then, if $\gamma < \overline{\gamma}_{AB}$ we obtain

$$W^S > W^{S,AB} > W^{E,AB} = W^E.$$

For higher cost levels characterised by $\gamma \geq \overline{\gamma}_{AB}$ this ranking changes to

$$W^S > W^{S,AB} = W^{E,AB} = W^E.$$

q.e.d

References

[1] Armstrong, M., 2006. Competition in Two-sided Markets. RAND Journal of Economics 37, 668-691.

[2] Armstrong, M. and Wright, J., 2007. Two-sided Markets, Competitive Bottlenecks and Exclusive Contracts. Economic Theory 32(2), 353-380.

[3] Arrow, K., 1962. Economic welfare and the allocation of ressources for invention. In: Nelson, R. (Eds.), The Rate and Direction of Inventive Activity, New Jersey, 609-625.

[4] Baake, P. Boom, A., 2001. Vertical product differentiation, network externalities, and compatibility decisions. International Journal of Industrial Organization 19(1-2), pp. 267-284.

[5] Crémer,J.; Rey, P. and Tirole, J., 2000. Connectivity in the commercial Internet. The Journal of Industrial Economics 48, 433-472.

[6] Distaso, W.; Lupi, P. and Manenti, F., 2006. Platform competition and broadband uptake: Theory and empirical evidence from the European union. Information Economics and Policy 18(1), 87-106.

[7] D'Aspremont, C. and Jaquemin, A., 1988. Cooperative and Noncooperative R&D in Duopoly with Spillovers. American Economic Review 78 (5), 1133-1137.

[8] Farrell, J. and Saloner,G., 1985. Standardization, Compatibility, and Innovation. RAND Journal of Economics 16, 70-83.

[9] Foros, O. and Hansen, J., 2001. Competition and compatibility among Internet Service Providers. Information Economics and Policy 13, 411-425.

[10] Katz, M.L. and Shapiro, C., 1985. Network Externalities, Competition, Compatibility. American Economic Review 75 (3), 424-440.

[11] Rochet, J.-C. and Tirole, J. 2003. Platform Competition in two-sided markets. Journal of the European Economic Association 1, 990–1092.

[12] Schiff, A., 2003. Open and closed systems of two-sided networks. Information Economics and Policy 15(4), 425-442.

[13] Schumpeter, J.A. 1943. Capitalism, socialism and democracy. London.

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