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Abstract

This paper shows that a firm prefers a process-based task assignment compared to a function based one if the tasks are from functional areas which are neither too complementary nor too substitutable. We consider several projects with contributions from several functional areas. The organization can be structured along processes like product lines (M-form) or along functional areas like marketing or production (U-form). The U-form enables cost savings due to specialization or scale economies. We show that the more effective incentives under the M-form might outweigh these savings if the functions are neither too complementary nor too substitutable.

Journal of Economic Literature Classification Numbers: D02, L23

Key words: Task Assignment, Organizational Form, Incomplete Contracts

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1 Introduction

This paper shows that a firm prefers a process-based task assignment compared to a function based one if the tasks are from functional areas which are neither too complementary nor too substitutable. We consider an organization or firm which undertakes several independent projects (e.g. distributes several products) each of which requires contributions from several functional areas (e.g. manufacturing and marketing). Under the unitary form (U-form), the firm is structured along functional areas so that there is, for example, a marketing department and a manufacturing department. Under the multi-level form (M-form), the firm has a process-based structure, it is organized along projects. For example, each product has its own department which is in charge of all tasks related to this product. While the U-form provides some cost savings due to specialization or economies of scale, the M-form enables more effective incentives in case of incomplete contracts. We show that this might outweigh the potential cost savings from the U-form, if the functions are neither too complementary nor too substitutable. Strong substitutability or complementarity of the functional areas makes it easier for the principal to implement her favorite effort levels. Incentives become less important and the M-form cannot be optimal in these cases.

We use a simple model of the assignment of tasks to analyze the optimal organizational form with the focus on the substitutability resp. complementarity of the functional areas. Our model considers a principal, e.g. a firm owner, who hires two agents for the implementation of two projects, e.g. the distribution of two different products. Each project consists of two tasks from different functional areas like manufacturing and marketing. The principal assigns the tasks to the agents. We assume that tasks cannot be split among several agents.[1] Under the U-form, each agent works on tasks from the same functional area, while under the M-form, each agent works on tasks from the same project.

The agents' effort is assumed to be non-contractible. Effort might be unobservable to a third party or unverifiable in court. The principal needs to provide incentives in order to induce the agents to spend effort. Moral hazard (hidden action problem) occurs. The agents are risk neutral and protected by limited liability.[2] Either the agent cannot conduct any payments

[1]Since Holmstrom and Milgrom (1991) as well as Besanko, Regibeau, and Rockett (2005) show that it is never optimal to split a task among several agents, our assumption seems justified.

[2]This moral hazard problem was introduced by Sappington (1983) for a single agent.

due to wealth constraints or ex post payments cannot be enforced so that the agent could break up the contract and walk away instead of paying. The principal faces a trade off between rent extraction and surplus maximization which occurs in our model under both organizational forms. The agents' effort choice is a non-cooperative game designed by the principal through the payment scheme. Winter (2004) shows that it might be necessary to discriminate between the agents if a unique Nash equilibrium in which all agents exert effort is desired. To avoid related issues, we assume that the principal can pick the equilibrium of her choice in case of mutliple equilibria. As a justification, think of the principal announcing the effort levels of her favorite equilibrium and the agents following her recommendation because they cannot gain from deviating unilaterally.

The output of the projects is assumed to be the only verifiable variable on which payments can condition. Under the U-form, each agent's payoff depends on both agent's actions, while under the M-form, it is independent of the other agent's actions.[3] Therefore, we have more effective incentives under the M-form. On the other hand, each agent is specialized in one functional area so that his effort costs are lower if working on a task from this area.[4] For example, the agents are a marketing director and a technical engineer specialized in manufacturing. A slightly different interpretation is to assume some economies of scale so that an agent working on two tasks form the same function has lower costs than an agent working on tasks from different functions. Under the U-form, the principal can gain from the resulting cost savings. The principal's trade off is to balance between cost savings under the U-form and more effective incentives under the M-form.

If there is a lot of complementarity between the tasks of a project, it is not expensive to provide incentives. Increasing incentives on one task results in increased incentives on the complementary task. The M-form's advantage of more effective incentives is less important and the U-form is optimal because of the cost savings. On the other hand, if there is a lot of substitutability, it is sufficient for the principal to induce high effort on one task per project. Incentives play a minor role and the U-form is optimal. Only in case of little substitutability and little complementarity, the M-form might be optimal.

[3] Corts (2006) applies the term individual accountability to the M-form and team accountability for the U-form.

[4] Note that the term specialization refers to the agents' effort cost functions in our model, while Holmstrom and Milgrom (1991) use it do describe that a task is not split among several agents.

Beginning with Chandler (1962), several papers deal with the potential advantages of the different organizational forms, especially the M- and U-form. Most of the given applications as in Besanko, Regibeau, and Rockett (2005), Corts (2006) and other papers describe a firm which is organized along process-based resp. functional lines, but there are other applications. For example, Maskin, Qian, and Xu (2000) apply the idea of M- and U-form to planned economies. While the Soviet Union had a centralized structure with ministries for the different industries which correspond to the functional areas, the Chinese economy is organized in decentralized regions corresponding to the projects. Furthermore, not only firms but any kind of organizaton or institution deals task assignment or composition of teams.

As Holmstrom and Tirole (1989) emphasize, the assignment of tasks is not necessarily the only difference between the two organizational forms. While the U-form is used to represent a centralized structure, the M-form describes a decentralized organization with several more or less independent units. To reflect this differences requires complex hierarchies as in, for example, Hart and Moore (2005). In Qian, Roland, and Xu (2006), the different levels of centralization imply differences between the two forms concerning the informational structure and the ability to coordinate actions. The top level manager has different roles under the different forms, which is also the case in Aghion and Tirole (1995). As Besanko, Regibeau, and Rockett (2005), we abstract from such organizational differences and restrict our attention to the assignment of tasks. We do not explicitly model (de)centralization, but use a simple one-layer hierarchy. Nevertheless, we continue to use the terms M- and U-form for process-based resp. functional organization structures. The cost savings under the U-form as well as the more effective incentives under the M-form might well be viewed as a result of different levels of centralization.

Different from our model, Corts (2006) or Besanko, Regibeau, and Rockett (2005) analyze M- and U-form with risk averse agents so that the principal faces a trade off between the allocation of risk and the provision of incentives as in the classical moral hazard model of Holmstrom (1979). Different form their approach, but similar to our limited liability assumption, Maskin, Qian, and Xu (2000) impose an upper bound on penalties when analyzing the organizational form.

It is common in the literature to assume that the U-form gives rise to economies of scale due to centralization, see for example Qian, Roland, and Xu (2006), or equivalently to assume that the M-form suffers from some diseconomies of span, as in Besanko, Regibeau, and Rockett (2005).

Under the M-form, a multi-task problem as analyzed in Holmstrom and Milgrom (1991) can occur. An agent who has to exert unobservable effort on several tasks allocates effort among the tasks in a way which optimizes the signal on which his wage is based. If this signal is only partially aligned with the organization's objective, this allocation is inefficient. In the model of Besanko, Regibeau, and Rockett (2005), the U-form can dominate the M-form because of what they call the incentive flexibility effect. Under the U-form, payment schemes are more flexible since the principal can influence the effort on every single task instead of whole projects only. This holds true for our model, but does not play a role for the optimality of either form since we do not have any kind of multi-task problem. Allowing for asymmetries or externalities could create such a multi-task problem and shift our results towards the U-form, as in Corts (2006). Differently from Besanko, Regibeau, and Rockett (2005), we have no correlation between the outputs, no functional asymmetry or cross-product externality which could favor either form.

Holmstrom and Tirole (1989) point out that a team problem as described in Alchian and Demsetz (1972) might occur under the U-form. Even if the project output reveals that there was shirking, the principal cannot detect who has shirked. Different from the M-form, free riding is possible under the U-form. As Holmstrom (1982) shows, the principal can solve the moral hazard in teams by breaking the budget balance condition. This enables a payment scheme which gives each agent a marginal reward equal to marginal costs for the efficient effort choice. In Besanko, Regibeau, and Rockett (2005), the principal can use such a payment scheme to overcome the team problem and extract the whole surplus through lump-sum payments of the agents. In our model, limited liability prevents the agents from such payments. Due to the additional possibility of free riding, moral hazard is more severe under the U-form than under the M-form so that the M-form provides more effective incentives.

Similarly, in Aghion and Tirole (1995), Corts (2006) and Besanko, Regibeau, and Rockett (2005) the incentives under the M-form are more effective than under the U-form. Maskin, Qian, and Xu (2000) consider a model in which the available information about the agents' performance is different for M- and U-form. The M-form enables more effective incentives if it provides more precise information. Together with the fact that it is quite easy to measure the impact of a certain product on the firm's profit but rather hard to measure the impact of a functional area, it seems reasonable that the

M-form enables more effective incentives than the U-form. Winter (2005) studies the impact of information about the other agents' efforts, especially due to collocation, and also finds that the M-form can provide more effective incentives. While Maskin, Qian, and Xu (2000) and Qian, Roland, and Xu (2006) assume that the organizational form changes the informational structure, we take the informational structure as given independent of the organizational form. To this respect, our model is similar to Besanko, Regibeau, and Rockett (2005).

Several trade offs between the two forms have been analyzed in the literature. In Corts (2006), the M-form provides better incentives, while the U-form helps to overcome the multi-task problem and improves the allocation of risk. Besanko, Regibeau, and Rockett (2005) analyze a similar trade off and show that asymmetries between functions and cross-product externalities might favor the U-form. In difference, our basic trade off is between cost savings due to scale economies or specialization under the U-form and more effective incentives under the M-form. This trade off is also used by, for example, Maskin, Qian, and Xu (2000). Similarly, Goldfain (2006) compares several organizational forms of a research project. Hiring a team of agents, which is comparable with our U-form, allows to gain from synergies but weakens the incentives in her model. Aghion and Tirole (1995) consider a similar trade off in a framework of information aquisition. Based on the difference between formal and real authority as introduced in Aghion and Tirole (1997) and its predecessors, the model of Aghion and Tirole (1995) shows that information acquisition increases the principal's overload under the U-form compared to the M-form. This is in line with the overload considerations of Williamson (1975). In Dessein, Garicano, and Gertner (2005), synergies are not due to the organizational form itself. The organizational form impacts the communication of private information, which is important for the possible implementation of synergies. Overload considerations as well as coordination problems do not play a role in our model. There is no private information or communication.

Most of the literature mentioned above simply assumes constant marginal returns to effort. Qian, Roland, and Xu (2006) assume complementary functions and substitutable projects, but follow a team-theoretic approach as discussed in Marschak and Radner (1972). That is, they abstract from any incentive problems and focus on coordination and communication. Winter (2005) considers incentives, but also assumes complementary functions and substitutable projects. In difference, our contribution is to explicitly model how effort spent on one functional area affects the marginal returns to effort of the other functional area and study the impact in the context of incentives.

The rest of the paper is structured as follows. Section 2 describes a multiple principal agent model which covers the different organizational forms. In section 3, the benchmark case of contractible effort is considered for exogenous as well as endogenous assignment of task. The case of non-contractible effort and exogenously given organizational form is analyzed in section 4 for the M-form and in section 5 for the U-form. In section 6, the assignment of tasks is endogenized in order to find the optimal organizational form. Section 7 concludes.

2 The Model

Consider a principal who undertakes two projects A and B. On each project, two tasks from different functional areas S and T have to be performed. This results in the four tasks AS, AT, BS and BT. We refer to AS and AT as the A-tasks, AS and BS as the S-tasks and so forth. For example, each project might represent a product while the tasks are production and marketing. If the principal distributes cars and chocolate, there are four tasks to be executed: production of cars, marketing of cars, marketing of chocolate and production of chocolate. The example is enhanced further below. For exogenous reasons, e.g. time constraints, the principal cannot work on the projects herself but hires two agents σ and τ. In principle, both agents are able to do each of the four tasks.

The timing is as follows: The principal offers a contract to the agents. The contract assigns the tasks to the agents and determines a payment scheme. If effort is contractible, it is determined in the contract. The agents accept if their participation constraints are fulfilled. In case of non-contractible effort, the agents choose their efforts simultaneously. Projects are undertaken, private costs occur and project outputs are realized. The payment scheme is executed. The details are given in the remaining section.

We assume that each task is assigned to exactly one agent. For each task he is assigned to, an agent chooses how much effort to spend on this task. To keep things simple, we assume a binary effort choice. The agent chooses between high effort e_h and low effort e_l with $e_h > e_l > 0$. Denote with $e_{AS}, e_{AT}, e_{BS}, e_{BT}$ the effort spent on the respective tasks and with e^{Si}, e^{Ti} the sum of effort agent i spends on the S-tasks resp. the T-tasks. If, for example, agent σ is assigned to the tasks AS and BS and exerts high effort on both tasks, we have $e^{S\sigma} = e_{AS} + e_{BS} = 2e_h$ and $e^{T\sigma} = 0$.

The agents incur private, unobservable effort costs which are assumed to be linear. The cost functions are

$$\begin{aligned} c_\sigma(e^{S\sigma}, e^{T\sigma}) &= \alpha e^{S\sigma} + \beta e^{T\sigma} \\ c_\tau(e^{S\tau}, e^{T\tau}) &= \beta e^{S\tau} + \alpha e^{T\tau} \quad . \end{aligned} \tag{1}$$

We assume each agent to be specialized in one function meaning that his effort costs on tasks with this function are lower. The parameters $0 < \alpha < \beta$ reflect the agents' specialization. For agent σ it is less costly to spend effort on the S-tasks than on the T-tasks, while for agent τ it is the other way around. The smaller α, the higher is the level of specialization. One possible interpretation is that the agent is more familiar with one of the functions and therefore needs less time to undertake the respective tasks. In this setting, we have to assume the principal to know who is specialized in which function if she wants to benefit from the specialization. But a similar idea is to assume some economies of scale so that an agent working on tasks from the same functional area has lower costs than an agent working on tasks from different functions. This does not require the principal to know the agents' abilities, but adds some notational complications with respect to the cost functions.[5] Up to some permutations, the results do not change.

Each project either succeeds or fails. If project A is successful, the principal receives an output $X > 0$. In case of failure, no output is generated. The success probability of project A is $\pi_A(e_{AS}, e_{AT})$ which depends on the effort spent on the A-tasks. It is

$$\begin{aligned} \pi_A(e_l, e_l) &= p_l \quad , \\ \pi_A(e_h, e_l) &= p_m \quad , \\ \pi_A(e_l, e_h) &= p_m \quad , \\ \pi_A(e_h, e_h) &= p_h \quad . \end{aligned} \tag{2}$$

Since the success probability is determined by the sum of effort spent on this project, the principal does not care about how a certain amount of effort might be allocated among the A-tasks. There is no multitask problem à la Holmstrom and Milgrom (1991). Due to the binary structure, the sum of effort is equivalent to the number of high effort levels. The succes probability

[5] If three tasks are assigned to one agent, the question arises which of the tasks are the low-cost tasks for this agent. No matter how this question is answered, such a solution turns out to be dominated with respect to the principal's payoff as well as the overall surplus.

does not depend on *who* spends effort on the project. The agents' specialization (resp. the scale economies) is reflected by the cost functions but does not influence the project output. Furthermore, the project is symmetric in functions. In case of one A-task undertaken with high effort and the other A-task undertaken with low effort, it makes no difference which of the two tasks is undertaken with high effort.

We assume

$$1 > p_h > p_m > p_l \geq 0 \tag{3}$$

so that π_A increases in the sum of effort spent on the A-tasks. Given AS is done with low effort, switching from low to high effort on AT increases π_A by $p_m - p_l$. Given AS is undertaken with high effort, a switch from low to high effort on AT increases the success probability by $p_h - p_m$. If

$$p_m > (p_h + p_l)/2 \quad , \tag{4}$$

we have $p_h - p_m < p_m - p_l$ and marginal returns to effort are decreasing. In this case, we define the two tasks to be *substitutes*. In case of

$$p_m < (p_h + p_l)/2 \quad , \tag{5}$$

marginal returns to effort are increasing and the two tasks are *complements*. To simplify calculations, we set

$$p_l = 0 \quad . \tag{6}$$

The two projects are completely identical so that project B's output is also $X > 0$ resp. zero and its success probability function $\pi_B(e_{BS}, e_{BT})$ is analog to π_A. This implies that the A-tasks are substitutes if and only if the B-tasks are substitutes and we can consider the functions S and T itself to be substitutes resp. complements. Note that the output of a project depends neither on the other project's output nor on the effort spent on the other project. This is reasonable in our example since the market success of chocolate is likely to be uncorrelated to the market success of cars. To avoid rather uninteresting corner solutions, we assume $p_h X > (\alpha + \beta)e_h$.

The project output is assumed to be verifiable so that payments can condition on it. Denote with v_j an unconditional transfer payment from the principal to agent j and with w_{ij} a payment from the principal to agent j paid if project i succeeds.[6] The whole payment scheme is given by the vector

[6] This payment scheme is equivalent to paying each agent for each project a wage which depends on the success or failure of this project. To verify this, rearrange the payoff functions. Furthermore, neither the principal's payoff nor the surplus can be increased by using a more advanced payment scheme.

$W = (v_\sigma, w_{A\sigma}, w_{B\sigma}, v_\tau, w_{A\tau}, w_{B\tau})$. We assume the agents to be of limited liability, maybe due to wealth constraints, so that all these payments have to be non-negative. We refer to v_j as agent j's basic wage and to w_{ij} as his success premium on project i. The principal and the agents are assumed to be risk neutral. Their payoff functions are composed of their expected benefits resp. payments and the private costs so that they are given by

$$U_\sigma = \pi_A(e_{AS}, e_{AT})w_{A\sigma} + \pi_B(e_{BS}, e_{BT})w_{B\sigma} + v_\sigma - \alpha e^{S\sigma} - \beta e^{T\sigma}, \quad (7)$$

$$U_\tau = \pi_A(e_{AS}, e_{AT})w_{A\tau} + \pi_B(e_{BS}, e_{BT})w_{B\tau} + v_\tau - \alpha e^{T\tau} - \beta e^{S\tau}, \quad (8)$$

and

$$\begin{aligned} U_P &= \pi_A(e_{AS}, e_{AT})(X - w_{A\sigma} - w_{A\tau}) \\ &+ \pi_B(e_{BS}, e_{BT})(X - w_{B\sigma} - w_{B\tau}) - v_\sigma - v_\tau \quad . \end{aligned} \quad (9)$$

The agents' outside options are assumed to be zero. The principal offers a contract to the agents who accept if and only if their participation constraints $U_\sigma, U_\tau \geq 0$ are fulfilled. A contract consists of an assignment of tasks and a payment scheme. If effort is contractible, it is determined in the contract as well. In case of non-contractible effort, the agents choose their efforts simultaneously. Given the assignment of tasks and the payment scheme, this is a non-cooperative game and we assume the agents to play a Nash equilibrium. Given an assignment of tasks, denote with e_i the vector of efforts agent i has to choose. For example, if agent σ is assigned to AS and AT, we have $e_\sigma = (e_{AS}, e_{AT})$. The equilibrium conditions are

$$\begin{aligned} e^*_\sigma &\in \operatorname*{argmax}_{e_\sigma} U_\sigma(e_\sigma, e^*_\tau) \\ e^*_\tau &\in \operatorname*{argmax}_{e_\tau} U_\tau(e^*_\sigma, e_\tau) \quad . \end{aligned} \quad (10)$$

For notational simplicity, we widely omit the asterisk. The equilibrium outcome is anticipated by the principal. When she designs the contract, in fact she designs the game by choosing an assignment of tasks and a payment scheme. In case of multiple equilibria, the principal determines which equilibrium is played. The principal offers a contract which maximizes her own payoff subject to the agents' participation constraints, the equilibrium conditions and the limited liability constraints. Such a contract is called optimal.

Overall expected surplus is

$$S = \pi_A(e_{AS}, e_{AT})X + \pi_B(e_{BS}, e_{BT})X - \alpha(e^{S\sigma} + e^{T\tau}) - \beta(e^{T\sigma} + e^{S\tau}) \quad . (11)$$

An assignment of tasks together with a combination of efforts which maximizes the surplus is called first best efficient.

The following sections analyze and compare different assignments of tasks, which describe different organizational structures of the principal's firm. We restrict our attention to situations in which each agent is in charge of two tasks.[7] If one agent receives both A-tasks while the other agent gets both B-tasks, this is called multi-level form or M-form. In our example, this form describes an organizational structure along product lines. One agent is completely responsible for cars, concerning marketing as well as production, while the other agent has to care about chocolate. Instead, if one agent receives both S-tasks while the other one gets both T-tasks, the structure is called unitary form or U-form. Each agent is in charge of a specific function, which we assume to be his low-cost function.[8] In our example, one agent is responsible for the production of both cars and chocolate while the other agent is concerned with the marketing of both products.

3 Contractible Effort

This section studies the benchmark cases of contractible effort. Lemma 1 and 2 take the organizational form as exogenously given, while Proposition 1 endogenizes the assignment of tasks.

Lemma 1 (M-Form) *Let the tasks be exogenously assigned according to the M-form. Performing all tasks with high effort creates the surplus*

$$S^{M4} := 2(p_h X - (\alpha + \beta)e_h) \quad , \tag{12}$$

while performing only the low-cost tasks with high effort results in the surplus

$$S^{M2} := 2(p_m X - \alpha e_h - \beta e_l) \quad . \tag{13}$$

Exerting high effort on all tasks maximizes the surplus if and only if

$$p_m \leq p_h - \frac{\beta(e_h - e_l)}{X} =: \bar{p}_m \tag{14}$$

while spending high effort only on each agent's low-cost task maximizes the surplus if and only if $p_m \geq \bar{p}_m$. The maximum surplus under the M-form is

$$S^M = \max\{S^{M2}, S^{M4}\} \geq 0 \quad . \tag{15}$$

Proof: see Appendix A.

[7] Any other organizational form within our framework can improve neither the principal's payoff nor the surplus.

[8] This is in line with the idea of scale economies determining the cost functions. In case of specialization, assigning each agent to his high-cost tasks could be viewed as a U-form as well and provides the analog results but with higher costs.

Due to the symmetry, the surplus-maximizing number of high effort levels per project is the same for both projects. If a project is executed with low effort on all tasks, it fails for sure since $p_l = 0$ so that a negative surplus is created. If p_m is small enough so that the functions are sufficiently complementary, it is surplus-maximizing to have all tasks done with high effort. Otherwise, it is better to have only one task per project done with high effort. Under the M-form, this should be the low-cost tasks in order to minimize the costs.

Lemma 2 (U-Form) *Let the tasks be exogenously assigned according to the U-form. Performing all tasks with high effort creates the surplus*

$$S^{U4} := 2(p_h X - 2\alpha e_h) \quad , \tag{16}$$

while performing only the low-cost tasks with high effort results in the surplus

$$S^{U2} := 2(p_m X - \alpha e_h - \alpha e_l) \quad . \tag{17}$$

Exerting high effort on all tasks maximizes the surplus if and only if

$$p_m \leq p_h - \frac{\alpha(e_h - e_l)}{X} =: \tilde{p}_m \tag{18}$$

while spending high effort only on one task per project maximizes the surplus if and only if $p_m \geq \tilde{p}_m$. The maximum surplus under the U-form is

$$S^U = \max\{S^{U2}, S^{U4}\} \geq 0 \quad . \tag{19}$$

Proof: see Appendix A.

Again, it is surplus-maximizing to perform all tasks with high effort if the tasks are sufficiently complementary. Otherwise, performing one task per project with high effort maximizes the surplus. Exerting low effort on all tasks creates again a negative surplus. Comparing the M- and U-form, the critical value of p_m is smaller in case of the M-form because the additional costs from having two instead of one high effort levels per project are higher under the M-form. Now endogenize the organizational form.

Proposition 1 (First Best) *Overall surplus is maximized if and only if the U-form is implemented together with the surplus-maximizing effort levels from Lemma 2. If effort is contractible, the principal implements a first best efficient solution.*

Proof: For any given effort combination, the expected output is independent of the organizational form, but effort costs are minimized if the U-form is chosen. Since efforts are contractible, there is no need to incentivize the agents. The principal can set the success premiums to zero and choose basic wages which make the agents' participation constraints binding. She always extracts the whole surplus so that it is her objective to maximize it. ■

If effort is contractible and the organizational form is endogenous, first best efficiency is always reached. The M-form is never efficient due to the higher effort costs. In difference to the U-form, the M-form does not allow to gain from the agents' specialization resp. the scale economies.

4 Multi-Divisional Organizational Form

From now on, we assume effort to be non-contractible. This section studies the impact of the multi-divisional organizational form or M-Form. Each agent is in charge of one of the projects. Without loss of generality, we assume that agent σ is assigned to the A-tasks and agent τ to the B-tasks. Throughout this section, we take this assignment as exogenously given.

The effort choice game of the agents is very simple under the M-form since there is no interaction between the two agents' decisions. We have $e_{Aj} = e^{j\sigma}$ and $e_{Bj} = e^{j\tau}$ for $j = S, T$. The equilibrium conditions boil down to the agents' incentive constraints

$$\begin{aligned} (e^{AS}, e^{AT}) &\in \operatorname{argmax} \pi_A(e^{AS}, e^{AT}) w_{A\sigma} - \alpha e^{AS} - \beta e^{AT} \\ (e^{BS}, e^{BT}) &\in \operatorname{argmax} \pi_B(e^{BS}, e^{BT}) w_{B\tau} - \beta e^{BS} - \alpha e^{BT} \quad . \end{aligned} \tag{20}$$

An effort combination fulfilling these conditions is also called incentive compatible. It can be implemented by the principal through the appropriate design of the contract, that is, the appropriate choice of the payment scheme.

Lemma 3 (Effort Decisions M-form) *Consider project A. Let v_σ be large enough to ensure agent σ's participation. If $(\alpha + \beta)p_m < \alpha p_h$, he always chooses the same effort level for both A-tasks. The principal can implement high effort on both tasks if and only if*

$$w_{A\sigma} \geq \frac{(\alpha + \beta)(e_h - e_l)}{p_h} \quad . \tag{21}$$

If $(\alpha + \beta)p_m \geq \alpha p_h$, high effort on both tasks can be implemented if and only if

$$w_{A\sigma} \geq \frac{\beta(e_h - e_l)}{p_h - p_m} \quad . \tag{22}$$

In case of

$$\frac{\alpha(e_h - e_l)}{p_m} \leq w_{A\sigma} \leq \frac{\beta(e_h - e_l)}{p_h - p_m} \quad , \tag{23}$$

the principal can implement high effort on the low-cost task AS and low effort on the high-cost task AT. Symmetric results hold for agent τ and project B.
Proof: see Appendix A.

Given the participation constraint is fulfilled, an agent's decision is determined by the success premium he receives for his project. If the agent exerts high effort on exactly one task, he does so on his low-cost task. If the tasks are highly complementary or the high-cost task is relatively cheap so that $(\alpha+\beta)p_m < \alpha p_h$, the agent never exerts different effort levels on his tasks.

Lemma 4 (Payoffs M-form) *Take the M-form as given. If the principal implements low effort on all tasks, she receives the whole surplus and her payoff is*

$$U_P^{M0} := -2(\alpha+\beta)e_l \quad < 0 \quad . \tag{24}$$

If $(\alpha+\beta)p_m \geq \alpha p_h$, she can implement high effort on the low cost tasks. She extracts the whole surplus and receives

$$U_P^{M2} \;\; := \;\; 2\left(p_m X - \alpha e_h - \beta e_l\right) \quad . \tag{25}$$

If she implements high effort on all tasks, she extracts the whole surplus if and only if

$$p_m \leq \hat{p}_m := \frac{p_h(\alpha e_h + \beta e_l)}{(\alpha+\beta)e_h} \quad . \tag{26}$$

Her payoff is

$$U_P^{M4} \;\; := \;\; 2\left(p_h X - \max\left\{\frac{p_h\beta(e_h-e_l)}{p_h-p_m}, (\alpha+\beta)e_h\right\}\right) \quad . \tag{27}$$

If the principal implements low effort on all tasks or high effort on the low cost tasks only, she receives the whole surplus. The principal covers the agents' costs in expectation, but does not need to provide further incentives. If the tasks are sufficiently complementary, this holds true also if the principal implements high effort on all tasks. Complementarity strongly incentivizes the agents and the principal can extract the whole surplus. But if there is little complementarity, she has to provide further incentives to implement high effort on all tasks. Due to limited liability, that means she has to offer the agents a positive share of the surplus.[9]

[9] Under unlimited liability, the principal could always combine an incentive compatible success premium with a basic wage which makes the participation constraint binding since she could, if necessary, choose a negative basic wage.

Proposition 2 (Optimal Contracts M-form) *Under the M-form, there is a unique critical value p_m^* so that the following holds:*

(i) If $p_m \leq p_m^*$, it is optimal for the principal to implement high effort on all tasks.

(ii) If $p_m \geq p_m^*$, it is optimal for the principal to implement high effort on the low-cost tasks only.

(iii) It is

$$0 = p_l \leq \hat{p}_m \leq p_m^* \leq \bar{p}_m \leq p_h \tag{28}$$

where $\hat{p}_m$ is the kink of U_P^{M4}, that is, it is the largest p_m for which the principal can implement hight effort on all tasks whithout offering the agents a positive rent share. $\bar{p}_m$ as defined in Lemma 1 is the maximum p_m for which high effort on all tasks is surplus-maximizing.

(iv) If the principal implements an optimal effort combination, her payoff $U_P^M := \max\{U_P^{M2}, U_P^{M4}\}$ is positive.

Proof: see Appendix A.

If the tasks are highly complementary (so that $p_m \leq \hat{p}_m$), the principal can maximize the surplus (given the M-form) and extract it completely. Due to the complementarity, it is surplus-maximizing to implement high effort on all tasks. If there is less complementarity (so that $p_m \geq \hat{p}_m$), it is still surplus-maximizing to implement high effort on all tasks. But the principal can no longer extract the whole surplus. Alternatively, the principal can implement high effort only on the low-cost tasks which creates a smaller surplus but enables her to extract it completely. The principal faces a trade off between surplus maximization and rent extraction due to limited liability. As long as there is enough complementarity (so that $p_m \leq p_m^*$), this trade off is solved in favor of surplus maximization. High effort on all jobs is implemented. The principal receives a smaller share but of a larger surplus. But the less complementarity (resp. the more substitutability) is present, the harder it is to incentivize the agents and the smaller is the principal's rent share if she implements high effort on all tasks. At the same time, the surplus created if she implements high effort only on the low cost task differs less from the surplus created if all tasks are performed with high effort. If there is enough substitutability (so that $p_m \geq p_m^*$), the trade off is solved in favor of rent extraction. The principal implements high effort only on the low-cost tasks and extracts the whole surplus. If there is even higher substitutability (so that $p_m \geq \bar{p}_m$), it is surplus-maximizing to have high effort on the low-cost tasks only and the trade off vanishes. The following section shows similar results for the U-form.

5 Unitary Organizational Form

This section studies the impact of the unitary organizational form or U-form in case of non-contractible effort. Each agent is in charge of the tasks from the same function, which we assume to be his low-cost task. Agent σ is assigned to the S-tasks and agent τ to the T-tasks. Throughout this section, we take this assignment as exogenously given.

Again, there is no interaction between the projects. An agent's effort choice on one project is independent of his choice on the other project. Different from the M-form, there is some interaction between the agents. If the functions are sufficiently complementary, an agent who knew the other agent's effort choice on a project would prefer the same effort level on this project. In equilibrium, the agents choose the same effort levels on a project. In case of sufficiently substitutable functions, each agent prefers an effort level different from the other agent's choice and in equilibrium, we have different effort levels on a projects' tasks.

Lemma 5 (Effort Decisions U-form) *Let v_σ, v_τ be large enough to ensure the agents' participation. The principal can implement high effort on both A-tasks if and only if*

$$w_{A\sigma}, w_{A\tau} \geq \frac{\alpha(e_h - e_l)}{p_h - p_m} \quad . \tag{29}$$

She can implement high effort on AS and low effort on AT if and only if

$$\begin{aligned} w_{A\tau} &\leq \frac{\alpha(e_h - e_l)}{p_h - p_m} \\ w_{A\sigma} &\geq \frac{\alpha(e_h - e_l)}{p_m} \quad . \end{aligned} \tag{30}$$

The principal can implement high effort on AT and low effort on AS if and only if (30) holds with $w_{A\sigma}$ and $w_{A\tau}$ interchanged. Symmetric results hold true for project B.
Proof: see Appendix A.

Under the U-form, the principal disposes of more effective instruments compared to the M-form. She can target single tasks instead of whole projects only. Therefore, she can always implement any effort combination. But since we do not have a multi-task problem, this turns out not to be a comparative advantage of the U-form.

Lemma 6 (Payoffs U-form) *Take the U-form as given. If the principal implements low effort on all tasks, she receives the whole surplus and her payoff is*

$$U_P^{U0} := -4\alpha e_l \quad < 0 \quad . \tag{31}$$

If the principal implements high effort on exactly one task per project, her payoff is

$$U_P^{U2} \;=\; 2\left(p_m X - \alpha(e_h + e_l)\right) \tag{32}$$

and she extracts the whole surplus. If she implements high effort on all tasks, she extracts the whole surplus if and only if

$$p_m \leq \check{p}_m := \frac{p_h e_l}{e_h} \quad . \tag{33}$$

Her payoff is

$$U_P^{U4} \;=\; 2\left(p_h X - \max\left\{\frac{2p_h\alpha(e_h - e_l)}{p_h - p_m}, 2\alpha e_h\right\}\right) \quad . \tag{34}$$

Proposition 3 (Optimal Contracts U-form) *Under the U-form, there is a unique critical value p_m^{**} so that the following holds:*

(i) It is

$$U_P^{U2} \geq U_P^{U4} \iff p_m \geq p_m^{**} \tag{35}$$

with equality if and only if $p_m = p_m^{**}$.

(ii) It is

$$0 = p_l \leq \check{p}_m \leq p_m^{**} \leq \tilde{p}_m \leq p_h \tag{36}$$

where $\check{p}_m$ is the kink of U_P^{U4}, that is, it is the largest p_m for which the principal can implement hight effort on all tasks whithout offering the agents a positive rent share. $\tilde{p}_m$ as defined in Lemma 2 is the maximum p_m for which high effort on all tasks is surplus-maximizing.

Proof: see Appendix A.

There are parameter constellations for which $U_P^{U2}, U_P^{U4} < 0$ so that the principal prefers to cancel the projects. If a cancellation is impossible, she might prefer to implement low effort on all tasks even though this generates a negative surplus. We skipped these details and restricted the proposition to the results we need later on for a comparison of M- and U-form. Besides, the results are quite similar to those of the M-form with $\check{p}$ instead of $\hat{p}$ and $\tilde{p}$ instead of $\bar{p}$. For $p_m^{**} < p_m < \tilde{p}_m$, limited liability creates some distortion. To implement high effort on all tasks would create a larger surplus, but the principal had to share it with the agents. If $p_m \notin (p_m^{**}, \tilde{p}_m)$, first best efficiency is reached.

6 Optimal Organizational Form

In this section, we endogenize the organizational form. When the principal designs the contract, she chooses the assignment of tasks which is in fact the organizational form. We restrict our analysis to a principal who chooses between M-form and U-form since any other organizational form can increase neither the surplus nor the principal's payoff. The U-form allows the principal to gain from the agents' specialization resp. the economies of scale, while the M-form provides more effective incentives. To see the latter, suppose for the moment $\alpha = \beta$ which eliminates the effects of the specialization. To implement high effort on one task per project, it is sufficient to cover the agents' costs without providing further incentives and both forms result in the same payoffs. But under the M-form, the principal needs smaller expected wage payments to incentivize the agents to exert high effort on all tasks. Incentives are more effective under the M-form so that this form is optimal. But if there is some specialization (resp. scale economies) so that $\beta > \alpha$, the U-form provides some cost savings. A trade off occurs between effort costs (which favor the U-form) and incentives (which favor the M-form). The following Lemma compares the organizational forms for given effort combinations.

Lemma 7 (Given Effort Combination) *If the principal implements high effort on exactly one task per project, her payoff under the U-form is always larger than under the M-form, that is $U_P^{U2} > U_P^{M2}$. If the principal implements high effort on all tasks so that she receives a payoff U_P^{M4} under the M-form and U_P^{U4} under the U-form, there is a unique critical value p_m^I so that the following holds:*

(i) If $\beta > 2\alpha$, it is $U_P^{U4} > U_P^{M4}$.

(ii) If $\beta = 2\alpha$, it is $U_P^{U4} > U_P^{M4}$ if $p_m < p_m^I$ and $U_P^{U4} = U_P^{M4}$ if $p_m \geq p_m^I$.

(iii) *If* $\beta < 2\alpha$, *it is* $U_P^{U4} \geq U_P^{M4} \iff p_m \leq p_m^I$ *with equality if and only if* $p_m = p_m^I$.

(iv) *If* $\beta \leq 2\alpha$, *it is* $\check{p}_m \leq p_m^I \leq \hat{p}_m$.

Proof: see Appendix A.

To implement high effort on one task per project, the principal can simply cover the agents' costs and does not need to provide further incentives. Since effort costs are smaller under the U-form, this form is preferred. Given that high effort on all tasks is implemented, the cost savings under the U-form might be outweighed by the more effective incentives under the M-form if there is not too much specialization or complementarity. To endogenize the effort combination, the following lemma is helpful.

Lemma 8 (Optimal Form) *Remember that the principal's payoff is* U_P^{M4} *if she implements high effort on all tasks under the* M*-form and* U_P^{U2} *if she implements high effort on one task per project under the* U*-form. There is a unique critical value* p_m^{II} *so that* $U_P^{M4} \geq U_P^{U2}$ *if and only if* $p_m \geq p_m^{II}$ *with equality if and only if* $p_m = p_m^{II}$. *It is*

$$p_m^I \leq p_m^{II} \iff p_m' := \frac{2\alpha(e_h - e_l)p_h}{(\alpha+\beta)e_h} - \frac{\beta e_h - \alpha e_l}{X} \geq 0 \quad . \tag{37}$$

Proposition 4 (Optimal Form) *The* U*-form is optimal if and only if*

$$2\alpha \leq \beta \quad or \quad p_m \leq p_m^I \quad or \quad p_m \geq p_m^{II} \quad . \tag{38}$$

The M*-form is optimal if and only if*

$$2\alpha \geq \beta \quad and \quad p_m^I \leq p_m \leq p_m^{II} \quad . \tag{39}$$

If there is a lot of specialization resp. economies of scale (so that $\beta > 2\alpha$), the U-form is always optimal independent of the complementarity of the tasks. This is different in case of a more moderate level of specialization. First, consider the case $p_m^I \leq p_m^{II}$. If the tasks are highly complementary (so that $p_m < p_m^I$), the principal optimally chooses the U-form and implements high effort on all tasks. The complementarity strongly incentivizes the agents. As long as $p_m \leq \check{p}_m$, she extracts the whole surplus, while in case of $\check{p}_m < p_m < p_m^I$ she has to offer the agents a positive share. As soon as $p_m > p_m^I$, she still implements high effort on all tasks, but prefers to use the M-form. The generated surplus is smaller than under the U-form, but due to the more effective incentives she can extract a larger share. But if there is too much substitutability (so that $p_m > p_m^{II}$), it is no longer profitable to implement high effort on all tasks. The principal implements high effort on one task per project. The comparative advantage of the M-form is lost and the U-form is optimal. In case of $p_m^{II} < p_m^I$, the M-form is never optimal. If there is so little complementarity that a principal who implements high effort on all tasks prefers the M-form, this is already enough substitutability to make it profitable to implement high effort on one task per project and again the U-form is optimal.

We have shown that a necessary condition (besides $\beta \leq 2\alpha$) for the M-form to be optimal is $p_m^I \leq p_m^{II}$, which is equivalent to $p_m' \geq 0$ according to Lemma 7. To interpret this condition, note that p_m' is increasing in p_h and X, decreasing in β and e_l and decreasing if e_h, e_l are increased keeping $e_h - e_l$ constant. If p_h and X are small, it is not beneficial to implement high effort on all tasks so that there is no need for incentives and the M-form is not used. To increase e_l, e_h keeping $e_h - e_l$ constant does not change the equilibrium conditions for implementing high effort on all tasks, but increases the effort costs. Potential cost savings become more important, which favors the U-form. If e_l is increased while keeping e_h constant, the effort costs for implementing high effort on all tasks remain the same while $e_h - e_l$ decreases. The equilibrium conditions for implementing high effort on all tasks change, it becomes less expensive to incentivize the agents. The potential advantage of the M-form withers.

7 Conclusion

This paper provides a simple model of the assignment of tasks, which compares different organizational forms. While the M-form is a process-based organizational form, the U-form is a function-based one. The U-form allows for some cost savings from the agents' specialization or economies of scale, but on the other hand provides less effective incentives compared to the M-form. The M-form can be optimal if and only if the functional areas are neither too complementary nor too substitutable.

To summarize, the principal favors the M-form if the following is fulfilled: First, possible cost savings from specialization resp. scale economies have to be small, due to little specialization and generally small effort cost. Second, it has to be attractive to implement high effort on all tasks since marginal returns to effort $(p_h - p_m)X$ are large, implying that there is not too much substitutability. Third, it has to be expensive to incentivize the agents to spend high effort on all tasks, due to large marginal effort costs (driven by

$e_h - e_l$) and little complementarity. Under these conditions, the incentive effects of the M-form outweigh the cost savings of the U-form.

In our model, we restrict the differences between the two organizational forms to the assignment of tasks. But the economies of scale we assume for the U-form might well be a result of a more centralized structure which on the other hand enables some free riding. The M-form provides more effective incentives since any kind of team problem is absent, which fits into the interpretation of a more decentralized structure. Even though we do not model (de)centralization explicitly, we are in line with the idea of the organizational form representing different levels of centralization.

Our main results still hold true if we allow for $p_l \geq 0$. If $p_l > 0$, the principal needs to provide incentives not only to implement high effort on all tasks but also to implement high effort on one task per project. In case of extreme complementarity or substitutability, the U-form remains optimal since it is relatively easy for the principal to implement the desired effort combinations. In case of less extreme complementarity resp. substitutability, incentives are harder to provide so that it might be optimal to use the M-form. Furthermore, there might be a set of intermediate p_m for which the principal chooses not to provide any incentives and implement low effort on all tasks so that the U-form is clearly optimal. This set of p_m is empty if p_l is very small or even zero.

In order to obtain clearcut results about the substitutability resp. compementarity of the functional areas, we have restricted the model to a simple one-layer hierarchy. A model with a more complex hierarchical structure which allows to model the amount of (de)centralization explicitly could provide further insights to the differences between M- and U-form and combine our results with, for example, the overload considerations of Aghion and Tirole (1995) or Dessein, Garicano, and Gertner (2005). Other possible extensions include sequential effort choices or collusion of the agents. These are left for future research.

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Appendix

A Proofs

Proof of Lemma 1:
It is straightforward to calculate S^{M2} and S^{M4}. Due to the symmetry, the surplus-maximizing effort combination has the same number of high effort levels for both projects. If a project's tasks are undertaken with different effort levels, effort costs are minimized if the agent in charge of this project works with high effort on his low-cost task. To work with low effort on all tasks creates a negative surplus. The only remaining candidates for a maximum of the surplus are S^{M2} and S^{M4}. $\bar{p}$ is the intersection of S^{M2} and S^{M4} with $S^{M2} \leq S^{M4} \iff p_m \leq \bar{p}$ with equality if and only if $p_m = \bar{p}$. It is $S^M = \max\{S^{M2}, S^{M4}\} \geq 0$ since $S^{M4} \geq 0$ by assumption. ■

Proof of Lemma 2:
The proof is analog to the the proof of Lemma 1. It is straightforward to calculate S^{U2} and S^{U4}. To work with low effort on all tasks creates a negative surplus. $\tilde{p}$ is the intersection of S^{U2} and S^{U4} with $S^{U2} \leq S^{U4} \iff p_m \leq \tilde{p}$ with equality if and only if $p_m = \tilde{p}_m$. It is $S^U = \max\{S^{U2}, S^{U4}\} \geq 0$ since $S^{U4} \geq 0$ by assumption. ■

Proof of Lemma 3:
Due to the M-form, we have $e_{AS} = e^{S\sigma}$ and $e_{AT} = e^{T\sigma}$. The equilibrium conditions of the effort choice game are simply the agents' incentive constraints. For agent σ, this is

$$(e^{AS}, e^{AT}) \in \operatorname{argmax} \pi_A(e^{AS}, e^{AT})w_{A\sigma} - \alpha e^{AS} - \beta e^{AT} \tag{40}$$

since all other terms of his payoff function are independent of his choice. The principal can implement any effort combination which fulfills (40). For the agent, it is alway strictly dominated to choose $e^{AS} = e_l, e^{AT} = e_h$ since the agent can reach the same success probability less costly with $e^{AS} = e_h, e^{AT} = e_l$. Straight forward calculations show that the choice $e^{AS} = e_h, e^{AT} = e_l$ is incentive compatible (that is, fulfilling (40)) if and only if

$$\frac{\alpha(e_h - e_l)}{p_m} \leq w_{A\sigma} \leq \frac{\beta(e_h - e_l)}{p_h - p_m} \quad . \tag{41}$$

Such a $w_{A\sigma}$ exists if and only if $(\alpha + \beta)p_m \geq \alpha p_h$. If this is the case, $e^{AS} = e^{AT} = e_h$ is incentive compatible if and only if

$$w_{A\sigma} \geq \frac{\beta(e_h - e_l)}{p_h - p_m} \quad . \tag{42}$$

If $(\alpha + \beta)p_m < \alpha p_h$, agent σ chooses between $e^{AS} = e^{AT} = e_h$ and $e^{AS} = e^{AT} = e_l$. The former is incentive compatible if and only if

$$w_{A\sigma} \geq \frac{(\alpha + \beta)(e_h - e_l)}{p_h} \quad . \tag{43}$$

Due to the symmetry, everything is analog for agent τ and project B. ■

Proof of Lemma 4:
Given an effort combination $e_{AS}, e_{AT}, e_{BS}, e_{BT}$ to be implemented, the principal faces the maximization problem

$$\max_{W} U_P \tag{44}$$

subject to the agents' incentive constraints

$$\begin{aligned} (e^{AS}, e^{AT}) &\in \operatorname{argmax} \pi_A(e^{AS}, e^{AT})w_{A\sigma} - \alpha e^{AS} - \beta e^{AT} \\ (e^{BS}, e^{BT}) &\in \operatorname{argmax} \pi_B(e^{BS}, e^{BT})w_{B\tau} - \beta e^{BS} - \alpha e^{BT} \quad , \end{aligned} \tag{45}$$

the participation constraints $U_\sigma, U_\tau \geq 0$ and the limited liability constraints $w_{ij}, v_j \geq 0$.

The principal cannot gain from choosing $w_{\sigma B} > 0$ or $w_{\tau A} > 0$. These payments do not incentivize the agents and the participation constraints can be ensured via v_σ, v_τ as well. We set $w_{A\tau} = w_{B\sigma} = 0$. Due to the symmetry, the principal can choose $v_\sigma = v_\tau := v$ and $w_{A\sigma} = w_{B\tau} := w$. Given an effort combination to be implemented, it is optimal for the principal to choose the smallest $w \geq 0$ which ensures incentive compatibility and the smallest $v \geq 0$ which ensures the agents' participation. To find w, see Lemma 3. We plug this w into $U_\sigma = 0$ and solve for v. If the result is non-negative, it is the optimal basic wage and the agents' participation constraints are binding. If the result is negative, we set $v := 0$ and the participation constraints are not binding.[10]

To implement low effort on all tasks, it is optimal to choose $w = 0$ and $v = (\alpha + \beta)e_l$, which results in a payoff

$$U_P^{M0} := -2(\alpha + \beta)e_l \quad < 0 \tag{46}$$

which equals the generated surplus. Let $p_m < \alpha p_h/(\alpha + \beta)$. According to Lemma 3, it is impossible to implement different effort levels for an agent's

[10]Under unlimited liability, the principal could also choose a negative v and therefore ensure that the participation constraints are binding.

tasks. To implement high effort on all tasks, it is optimal to choose $w = (\alpha + \beta)(e_h - e_l)/p_h$ and $v = (\alpha + \beta)e_l$. The resulting payoff is

$$U_P^{M4c} = 2\,(p_h X - (\alpha + \beta)e_h) \geq 0 \quad . \tag{47}$$

Let $p_m \geq \alpha p_h/(\alpha+\beta)$. To implement high effort on all tasks, it is optimal to choose $w = \beta(e_h - e_l)/(p_h - p_m)$ and $v = \max\{0, (\alpha + \beta)e_h - p_h w\}$. The resulting payoff is

$$U_P^{M4} = 2\left(p_h X - \max\{\frac{p_h \beta(e_h - e_l)}{p_h - p_m}, (\alpha + \beta)e_h)\}\right) \quad , \tag{48}$$

which has a kink in

$$\hat{p}_m := \frac{p_h(\alpha e_h + \beta e_l)}{(\alpha + \beta)e_h} \tag{49}$$

and equals the generated surplus if and only if $p_m \leq \hat{p}_m$. Since $\alpha p_h/(\alpha+\beta) < \hat{p}_m$, we can combine U_P^{M4c} and U_P^{M4}. A principal who implements high effort on all tasks receives a payoff U_P^{M4}.

But if $p_m \geq \alpha p_h/(\alpha + \beta)$, the principal can implement high effort on the low-cost tasks only. To implement this, it is optimal to choose $w = \alpha(e_h - e_l)/p_m$ and $v = (\alpha + \beta)e_l$. The payoff is

$$U_P^{M2} = 2\,(p_m X - \alpha e_h - \beta e_l) \tag{50}$$

which equals the generated surplus. ■

Proof of Proposition 2:
Due to the symmetry, it is optimal for the principal to implement the same effort combination on both projects. There are three candidates for an optimum: low effort on all tasks, high effort on the low cost tasks, high effort on all tasks. To find the optimum, compare the payoffs from Lemma 4.

Consider the payoffs as functions of p_m. For the moment, make the ad hoc assumption that it is never optimal to implement low effort on all tasks and compare the two remaining candidates. It is optimal to implement high effort only on the low-cost tasks if $U_P^{M2} \geq U_P^{M4}$ and $p_m \geq \alpha p_h/(\alpha + \beta)$. The function U_P^{M4} is continuous and has a kink in $\hat{p}_m$. U_P^{M4} is a positive constant for $p_m \leq \hat{p}_m$. It is monotone decreasing in p_m for $p_m \geq \hat{p}_m$ and approaches $-\infty$ for $p_m \to p_h$. The function U_P^{M2} is continuous and monotone increasing in p_m. It is negative for $p_m = 0$ and positive for $p_m = p_h$. Therefore, we have a unique intersection p^*. We have $U_P^{M2} \geq U_P^{M4}$ if and only if $p_m \geq p^*$ with equality if and only if $p_m = p^*$. For $p_m = \alpha p_h/(\alpha+\beta)$, we have $U_P^{M4} \geq U_P^{M2}$ so that $p^* > \alpha p_h/(\alpha+\beta)$ and U_P^{M2} can be implemented whenever $U_P^{M2} \geq U_P^{M4}$. In summary, high effort on the low cost tasks is optimal if and only if $p_m \geq p_m^*$ and we have shown (ii).

Remember that $\bar{p}_m$ is the intersection of U_P^{M2} and $2(p_h X - (\alpha+\beta)e_h)$. Since $\bar{p}_m \geq \hat{p}_m$, we have $\hat{p}_m \leq p_m^* \leq \bar{p}_m$, which is (iii). At the intersection p_m^*, U_P^{M4} is decreasing.

If $p_m < \alpha p_h/(\alpha+\beta)$, high effort on the low cost tasks only cannot be implemented. If $\alpha p_h/(\alpha+\beta) \leq p_m$, we have $U_P^{M4} \geq U_P^{M2} \iff p_m \leq p_m^*$. Together, it is optimal to implement high effort on all tasks if and only if $p_m \leq p_m^*$, and we have shown (i).

The principals payoff is $U_P^M := \max\{U_P^{M2}, U_P^{M4}\}$. U_P^M has its minimum in p_m^* and $U_P^{M2}(p_m^*) \geq U_P^{M2}(\hat{p}_m) \geq 0$ implies $U_P^M \geq 0$. Our ad hoc assumption is justified and we have also shown (iv). ■

Proof of Lemma 5:
Due to the U-form, we have $e_\sigma = (e_{AS}, e_{BS})$, $e_\tau = (e_{AT}, e_{BT})$, $e_{AS}+e_{BS} = e^{S\sigma}$ and $e_{AT} + e_{BT} = e^{T\tau}$. The equilibrium conditions can be rewritten as

$$e_{AS}^* \in \operatorname*{argmax}_{e_{AS}} \pi_A(e_{AS}, e_{AT}^*) w_{A\sigma} - \alpha e_{AS} \tag{51}$$
$$e_{AT}^* \in \operatorname*{argmax}_{e_{AT}} \pi_A(e_{AS}^*, e_{AT}) w_{A\tau} - \alpha e_{AT} \tag{52}$$
$$e_{BS}^* \in \operatorname*{argmax}_{e_{BS}} \pi_B(e_{BS}, e_{BT}^*) w_{B\sigma} - \alpha e_{BS} \tag{53}$$
$$e_{BT}^* \in \operatorname*{argmax}_{e_{BT}} \pi_B(e_{BS}^*, e_{BT}) w_{B\tau} - \alpha e_{BT} \quad . \tag{54}$$

For notational simplicity, we suppress the asterisk. Consider project A. Given $e_{AS} = e_h$, agent τ's best response (which maximizes his payoff because it fulfills (52)) is $e_{AT} = e_h$ if and only if

$$p_h w_{A\tau} - \alpha e_h \geq p_m w_{A\tau} - \alpha e_l \tag{55}$$

which is equivalent to

$$w_{A\tau} \geq \frac{\alpha(e_h - e_l)}{p_h - p_m} \quad . \tag{56}$$

If $w_{A\tau} \leq \alpha(e_h - e_l)/(p_h - p_m)$, his best response is $e_{AT} = e_l$. Given $e_{AS} = e_l$, agent τ's best response is $e_{AT} = e_h$ if and only if

$$p_m w_{A\tau} - \alpha e_h \geq -\alpha e_l \tag{57}$$

which is

$$w_{A\tau} \geq \frac{\alpha(e_h - e_l)}{p_m} \quad . \tag{58}$$

If $w_{A\tau} \leq \alpha(e_h - e_l)/p_m$, his best response is $e_{AS} = e_l$. Agent σ's best responses are constructed analog. Combining the agents' best responses shows that there is a Nash equilibrium with $e_{A\sigma} = e_{A\tau} = e_h$ if and only if

$$w_{A\tau}, w_{A\sigma} \geq \frac{\alpha(e_h - e_l)}{p_h - p_m} \quad . \tag{59}$$

An equilibrium with $e_{A\sigma} = e_h, e_{A\tau} = e_l$ exists if and only if

$$\begin{aligned} w_{A\sigma} &\geq \frac{\alpha(e_h - e_l)}{p_h - p_m} \\ w_{A\tau} &\leq \alpha(e_h - e_l)/p_m \quad . \end{aligned} \tag{60}$$

For an equilibrium with $e_{A\sigma} = e_l, e_{A\tau} = e_h$, replace $w_{A\sigma} \leftrightarrow w_{A\tau}$ so that the agents change their roles. Due to the symmetry, everything is completely analog for project B.

■

Proof of Lemma 6:
The proof is quite analog to the proof of Lemma 4. Given an effort combination $e_{AS}, e_{AT}, e_{BS}, e_{BT}$ to be implemented, the principal faces the maximization problem

$$\max_W U_P \tag{61}$$

subject to the equilibrium conditions (51)-(54), the participation constraints $U_\sigma, U_\tau \geq 0$ and the limited liability constraints $w_{ij}, v_j \geq 0$.

Due to the symmetry, the principal can choose $v_\sigma = v_\tau := v$ and $w_{A\sigma} = w_{B\tau} := w_1$ and $w_{B\sigma} = w_{A\tau} := w_2$. Given an effort combination to be implemented, it is optimal for the principal to choose the smallest $w_1, w_2 \geq 0$ which fulfill the equilibrium conditions and the smallest $v \geq 0$ which ensures the agents' participation. To find w_1, w_2, see Lemma 5. We plug these w_1, w_2 into $U_\sigma = 0$ and solve for v. If the result is non-negative, it is the optimal basic wage and the agents' participation constraints are binding. If the result is negative, we set $v := 0$ and the participation constraints are not binding.

To implement low effort on all tasks, it is optimal to set $w_1 = w_2 = 0$ and $v = 2\alpha e_l$. The resulting payoff is

$$U_P^{M0} := -4\alpha e_l \quad < 0 \tag{62}$$

which equals the generated surplus. To implement high effort on exactly one task per project, it is optimal to choose $w_1 = \alpha(e_h - e_l)/p_m, w_2 = 0$ and $v = 2\alpha e_l$. The payoff is

$$U_P^{U2} = 2\left(p_m X - \alpha(e_h + e_l)\right) \quad . \tag{63}$$

which equals the generated surplus. To implement high effort on all tasks, it is optimal to choose $w_1 = w_2 = \alpha(e_h - e_l)/(p_h - p_m)$ and $v = \max\{0, 2\alpha e_h - p_h w_1\}$. The resulting payoff is

$$U_P^{U4} = 2\left(p_h X - \max\{\frac{2p_h\alpha(e_h - e_l)}{p_h - p_m}, 2\alpha e_h)\}\right) \tag{64}$$

which has a kink in

$$\check{p}_m := \frac{p_h e_l}{e_h} \tag{65}$$

and equals the generated surplus if and only if $p_m \leq \check{p}_m$. ■

Proof of Proposition 3:
The proof is quite analog to the proof of Proposition 2. U_P^{U4} is a positive constant for $p_m \leq \check{p}_m$. It is monotone decreasing in p_m for $p_m \geq \check{p}_m$ and approaches $-\infty$ for $p_m \to p_h$. The function U_P^{U2} is continuous and monotone increasing in p_m. It is negative for $p_m = 0$ and positive for $p_m = p_h$. Therefore, we have a unique intersection p^{**} with $U_P^{U2} \leq U_P^{U4}$ if and only if $p_m \leq p^{**}$ with equality if and only if $p_m = p^{**}$, and we have shown (i).
Remember that $\tilde{p}_m$ is the intersection of U_P^{U2} and $2(p_h X - 2\alpha e_h)$. Since $\tilde{p}_m \geq \check{p}_m$, we have $\check{p}_m \leq p_m^{**} \leq \tilde{p}_m$, which is (ii). At the intersection p_m^{**}, U_P^{U4} is decreasing. ■

Proof of Lemma 7:
Since $U_P^{U2} \geq U_P^{M2}$, a principal who implements high effort on one task per project always prefers the U-form. Now consider a principal who implements high effort on all tasks. In case of $\beta > 2\alpha$, we have $U_P^{U4} > U_P^{M4}$, which is (i). Now let $\beta \leq 2\alpha$. It is straightforward that $\check{p}_m \leq \hat{p}_m$. For $p_m \leq \check{p}_m$, it is $U_P^{U4} > U_P^{M4}$ and both functions are constant. For $p_m \geq \hat{p}_m$, it is $U_P^{U4} \leq U_P^{M4}$ with equality if and only if $\beta = 2\alpha$. Both functions are decreasing. For $\check{p}_m < p_m < \hat{p}_m$, U_P^{M4} is constant while U_P^{U4} is decreasing. It is implied that for $\beta < 2\alpha$, there is a unique intersection p_m^I with $U_P^{M4} \geq U_P^{U4} \iff p_m \geq p_m^I$ with equality if and only if $p_m = p_m^I$ and we have shown (iii). For $\beta = 2\alpha$, we have $U_P^{U4} > U_P^{M4}$ if $p_m < \hat{p}_m$ and $U_P^{U4} = U_P^{M4}$ if $p_m \geq \hat{p}_m$. This is (ii) with $\hat{p}_m = p_m^I$. Combining the properties of p_m^I for $\beta < 2\alpha$ and $\beta = 2\alpha$, we get (iv). ■

Proof of Lemma 8:
Since U_P^{U2} is strictly increasing, negative in $p_m = 0$ and positive in $p_m = p_h$ while U_P^{M4} is decreasing, positive in $p_m = 0$ and approaches $-\infty$ for $p_m \to p_h$ we have a unique intersection of U_P^{M4} and U_P^{U2} which we denote with p_m^{II}. It is straightforward to calculate

$$p_m^I \quad = \quad p_h \frac{(\beta - \alpha)e_h + 2\alpha e_l}{(\alpha + \beta)e_h} \quad . \tag{66}$$

Furthermore, there is a unique intersection of U_P^{U2} and $2(p_h X - (\alpha + \beta)e_h)$, which is

$$p_m^\dagger := p_h - \frac{\beta e_h - \alpha e_l}{X} \tag{67}$$

so that

$$p_m^\dagger - p_m^I = p_m' = \frac{2\alpha(e_h - e_l)p_h}{(\alpha + \beta)e_h} - \frac{\beta e_h - \alpha e_l}{X} \quad . \tag{68}$$

There are two cases to analyze. If U_P^{M4} is strictly decreasing in p_m^{II}, we have

$$p_m^I \leq \hat{p}_m < p_m^{II} < p_m^\dagger \tag{69}$$

and $p_m' > 0$. If U_P^{M4} is constant in p_m^{II}, we have

$$p_m^{II} = p_m^\dagger \leq \hat{p}_m \tag{70}$$

so that

$$p_m^{II} - p_m^I = p_m' \quad . \tag{71}$$

In summary, we have

$$p_m^{II} \geq p_m^I \iff p_m' \geq 0 \quad . \tag{72}$$

■

Proof of Proposition 4:
The M-form guarantees a payoff $U_P^M \geq 0$ so that it is never optimal to implement low effort on all tasks, which results in a negative payoff. It is $U_P^{M2} < U_P^{U2}$. The U-form is optimal if and only if $\max\{U_P^{U4}, U_P^{U2}\} \geq U_P^{M4}$. The M-form is optimal if and only if $\max\{U_P^{U4}, U_P^{U2}\} \leq U_P^{M4}$. The results follow from Lemma 7 and Lemma 8. ■

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