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Short communication

Comment on "The value of tax shields is NOT equal to the present value of tax shields"

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Abstract

Fernandez [2004; The value of tax shields is NOT equal to the present value of tax shields. *Journal of Financial Economics*, *73*, 145–165] claims to derive a formula for the valuation of debt tax shields for firms with cash flows that grow perpetually at a constant rate. We show that his formula is incorrect.

JEL classification: G31; G32; H43

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1. Introduction

In *The value of tax shields is NOT equal to the present value of tax shields*, Fernandez (2004) claims to derive the correct formula for the value of interest tax shields for a firm

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whose cash flows are fixed or growing perpetuities. Fernandez says that his result is valid under broad conditions, and his formula is in conflict with those contained in several previously published papers. If he were correct, Fernandez would indeed have a paper of great significance and practical importance in corporate finance. We show that Fernandez fails to correctly prove his result and that the earlier results have not been disproved.

Fernandez makes the valid point that tax shields may be valued as the difference between the discounted value of taxes paid by a levered firm and the value of taxes it would have paid as an unlevered firm. However, in his attempt to derive the new expression, he makes errors and unjustified assumptions.

2. The model and result

Before focusing on the key points, we note that Fernandez makes several unstated assumptions that are rather standard for the literature, but are important for deriving any results. These would include tax symmetry on gains and losses, zero personal taxes, equality of depreciation and investment, and the lack of non-debt tax shields. His paper also fails to describe the stochastic process followed by the cash flows of the firm, and neglects to even use expected value notation. Nor is it clear whether the debt is in the form of a perpetuity, or is of finite maturity but perpetually rolled-over. All of this makes it difficult at times to follow and assess his reasoning, but we will attempt to make the most charitable interpretation of his model.

The central claim in the paper is Eq. (28):

$$\mathrm{VTS} = \frac{DT\mathrm{Ku}}{(\mathrm{Ku} - g)}, \tag{28}$$

where here and below we employ the following elements of Fernandez's notation:

VTS, value of tax shields; D, value of debt; E, value of equity; ECF, expected cash flows to equity holders; T, corporate tax rate; Ku, expected return on unlevered equity; Ke, expected return on levered equity; Kd, expected return on debt; R_F, risk-free rate; g, expected growth rate of cash flows and debt level.

Eq. (28) is derived by subtracting Eq. (38) from Eq. (37):

$$[\mathrm{Ku} - \mathrm{Kd}(1 - T)] - \left(\frac{E}{D}\right)(\mathrm{Ke} - \mathrm{Ku}) = \left(\frac{\mathrm{VTS}}{D}\right)(\mathrm{Ku} - g) \tag{37}$$

$$[\mathrm{Ku} - \mathrm{Kd}(1 - T)] - \left(\frac{E}{D}\right)(\mathrm{Ke} - \mathrm{Ku}) = T\mathrm{Ku} \tag{38}$$

where Eq. (38) is obtained from Eq. (37) for the case $g = 0$, by substituting VTS = TD. The numbering of the equations is taken from the paper.

Beyond the general validity of Eq. (37), two claims are essential for deriving Eq. (28). First, it must be true that VTS = TD when growth equals zero. Second, it must be permissible to subtract an equation that applies to the 0 growth case from an equation that applies to the case with general growth rate g. We believe that neither of these results is established by Fernandez, and we address them in order.

The claim that VTS = *TD* for zero growth perpetuities dates to at least Modigliani and Miller (1963). The assumption generally made to establish this result is that the level of debt is fixed, rather than having an amount of debt that in expectation equals *D* for every period but is correlated with cash flows and equity values. Fernandez, on the other hand, rejects the argument that tax shield value is sensitive to debt strategy. He obtains *DT* as the difference in present value of taxes paid by the unlevered and levered firms. His misstep is in the following statement:[1] "The taxes paid by the levered company are proportional to ECF. Consequently, the taxes of the levered company have the same risk as the ECF and thus must be discounted at the rate Ke." The problem is, equity cash flow for a levered firm is not, in general, equal to taxable income. Any net issuance of debt increases ECF without increasing tax. This is true not only in the real world, but also in Fernandez's model. Then if the dollar value of debt outstanding is stochastic and positively correlated to levered equity value, taxes_L are of lower risk than ECF, and will have a different discount rate. In particular, the discount rate is lower in the usual case where $Ke > R_F$, owing, for example, to a positive CAPM beta.

Therefore, we can at best hope that Eq. (28) for $g > 0$ is valid under a policy of deterministically growing debt level. In passing, we note that deterministically growing debt precludes the existence of fixed discount rates for the set (Ke, Ku, Kd),[2] but we won't dwell on that issue. Our hope in Eq. (28) instead is dashed by consideration of Fernandez's other crucial step. Combination of two equations representing entirely different worlds ($g = 0$ and $g > 0$) is unusual, and the stated justification is brief and rather cryptic: "If (*E*/*D*) is constant, the left-hand side of Eq. (36) does not depend on growth (*g*) because for any growth rate (*E*/*D*), Ku, Kd, and Ke are constant." Fernandez does not mean that (*E*/*D*) in fact is constant across growth rates; even his own examples fail to satisfy that condition. He means that *if* we, hypothetically, changed a firm's growth rate while also changing either *D* or FCF to maintain a constant (*E*/*D*) ratio, and preserved the returns to unlevered equity and debt across these two worlds, *then* the return to levered equity would be preserved as well. There is no further justification provided for this claim, which is far from being intuitive, much less obviously correct. The claim is equivalent to Fernandez's final result being true (if we accept Eqs. (37) and (38) for deterministic debt).

Rather than arguing the claim in the abstract, we mention an alternative approach that has been correctly proved. Sick (1990) used a certainty equivalent principle to value tax shields. Assuming fixed debt level (and the symmetric taxation of gains and losses we mentioned earlier), the certainty equivalent of each period's tax shield is $D_0 TR_F\ (1+g)^{t-1}$, yielding a discounted value $D_0 TR_F/(R_F - g)$. This result, of course, does not satisfy Fernandez's suggested property that appropriately controlled changes in *g* will leave Ke unchanged.[3,4] So, far from being a result that applies generally, there exists no set of assumptions under which Fernandez's formula is valid.

[1] Fernandez (2002, p. 4).

[2] Fixed discount rates require a process along the lines of a geometric random walk, and subtracting a fixed debt from a geometrically evolving firm value would result in a levered equity of non-constant risk.

[3] An exception is the trivial case where all required returns equal the risk-free rate.

[4] Another version of this comment, available from the authors, demonstrates that if a firm with deterministically growing risk-free debt were priced according to Fernandez's formula, arbitrage opportunities would be available using holdings in the levered and unlevered firm.

Fernandez attempts to provide an alternative justification for his main result by noting that in the case of an all debt firm, the following equation along with Kd = Ku generates Eq. (28):

$$D[\mathrm{Ku} - \mathrm{Kd}(1 - T)] - E(\mathrm{Ke} - \mathrm{Ku}) = \mathrm{VTS}(\mathrm{Ku} - g) \tag{36}$$

Aside from the fact that proving one's equation satisfies a very special case is not the same as proving it holds generally, the problem here is that a firm *cannot* be entirely debt financed in this sense. With debt growing at a fixed rate, the benefits from issuing future debt with its accompanying tax shields must reside in the value of equity, so that the special case is meaningless.

Finally, we would like to comment on the fact that existing methods sometimes generate VTS > *D*. Fernandez says this "hardly makes any economic sense." VTS is a present value of all future tax shields, including those based on debt that is yet to be issued. Outside the fixed *g* case, imagine a firm with no current debt but a policy that it will start issuing debt in two years. Obviously there is some value today associated with that future debt, so VTS > *D*. It is no great surprise that this relation can extend to the case of growth perpetuities.

3. Conclusion

While Pablo Fernandez makes some valid points in his provocative paper, his final result is incorrect. There is more work to be done in the area of tax shield valuation, but we need not throw away all the progress that has been made thus far.

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