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WACC and a Generalized Tax Code

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ABSTRACT *Valuation of firms is generally based on the WACC approach which typically neglects personal income taxes. This paper extends this approach to incorporate personal income taxes and develop a generalized valuation formula which can be used for any taxation system. The approach is illustrated for four different taxation systems highlighting the importance of considering personal taxes.*

KEY WORDS: WACC, tax shield, imputation system

1. Introduction

Any method of evaluation of a firm needs to take taxes into account. If, in particular, interest payments are tax-deductible, the value of an unlevered firm will differ by the so-called tax shield from the value of a levered firm. One of the most frequently used methods of valuation is the weighted average cost of capital (WACC) approach. In this approach, the tax advantages from debt are taken into account by using as discount rates the WACC instead of the unlevered cost of capital.

The first WACC formula was presented by Modigliani and Miller (1963). Their idea was based on a constant expectation of future cash flows as well as 'constant interest bill'. In this case the leverage ratio is a random variable. Miles and Ezzell (1980) pointed out that with constant leverage ratios this approach will not yield the correct value of the firm. For the case of a perpetual rent they provided a different WACC formula.

Although the WACC approach can be found in almost every paper on valuation less is known how the approach has to be adjusted in order to account for tax systems which differ from the one assumed by Modigliani and Miller (1963). Both authors focused on the corporate income tax of the US tax system but neglected personal income taxes. Paying attention to personal income taxes with an allowance for corporate income taxes (imputation system) can change the level of the tax shield. Therefore, valuing a firm without the personal income tax may give wrong results.

Miller (1977) investigated a two-period model with personal and corporate income taxes but focussed on equilibrium considerations. His two-period model uses the so-called classical system where the corporate tax is not deductable in the personal tax calculation. DeAngelo and Masulis (1980) extended Miller's model to include such non-debt corporate tax shields as depreciation deductions and investment credits.

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In a more recent paper Cooper and Nyborg (1999) analyze a similar generalization of the WACC approach with a tax system which does not distinguish between EBIT and cash flows. Thus the tax base of the corporate income tax is not appropriate. Furthermore, the authors consider a only one-period model.

Taking the Miles–Ezzel paper as a starting point the objective of our paper is to present a multi-period WACC model that not only will consider personal income taxes but will also generalize the tax system such that characteristics of most existing national systems can be reflected. Thus, our model is formulated to consider any tax system with the tax system of the US, Japan or most European countries being special cases. The paper is organized as follows: we develop for different tax systems the appropriate expressions for the taxes of the unlevered firm and some combined tax rates (Section 2). These expressions are used in the general valuation formula and it is proven that correct valuation of the levered firm will be obtained in the difference tax systems (Section 3). Finally, our approach is illustrated by valuing a hypothetical firm using the different tax systems highlighting substantial differences in valuation results.

2. The Model

We consider a firm existing over T time periods with $t = 0, 1, \ldots, T$. The future is uncertain. Let $\mathrm{E}[\tilde{X}|\mathcal{F}_t]$ denote the conditional expectation of a random variable $\tilde{X}$ given the information at time t. Although this notation conceals a rather difficult mathematical concept (for details see Duffie (1988, p. 130)) an explanation will not be given since only standard calculation rules for conditional expectations will be used.

The unlevered firm generates cash flows before taxes $\widetilde{CF}_t$ over its lifetime. Earnings before interest and taxes are denoted by $\widetilde{EBIT}_t$. $\tilde{E}_t^U$ and $\tilde{E}_t$ describe the face value of equity ('paid in capital') of the unlevered and levered firm respectively.

The levered firm raises risk-free debt $\tilde{D}_t$ (bonds) at time t. In the case of a finite lifetime the firm agrees to retire the debt completely by the end of the planning period, thus $\tilde{D}_T = 0$. Although the creditor faces no default risk, there exists no fixed schedule for the retirements of debt. Interest payments $\tilde{Z}_t$ are determined from the debt $\tilde{D}_{t-1}$.

Since we want to relate the book value of the levered firm to the book value of the unlevered firm we need to assume that investments (and therefore cash flows), EBITs, and retained earnings are the same for both firms. This implies

$$\tilde{E}_t^U = \tilde{E}_t + \tilde{D}_t \tag{1}$$

Notice that the above equation only says that the book values of the levered and the unlevered firm are identical. If the investors retire the debt at time t the face value of the levered firm's equity will change. Nothing, however, has been said about the market value of both firms and in particular about their difference, i.e. the tax shield.

For the purpose of this paper we consider an arbitrary tax system which will include both corporate and personal income taxes without emphasizing the fact that differences may exist in tax bases and accounting principles. We will consider both the classical tax system, a tax system where the corporate tax is partially or fully deductible from the personal tax base (imputation system) as well as a mixed system as it currently exists in Germany.

Referring to the papers of Miles and Ezzell (1985) and Löffler (2001) we define $\tilde{T}_t$ as the sum of corporate and personal taxes related to the firm's cash flows which is given in

$$\tilde{T}_t = \tilde{T}_t^U(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U, \widetilde{EBIT}_t) - \tau\tilde{Z}_t, \quad t \geq 1 \tag{2}$$

In this equation the tax payments of the unlevered firm $\tilde{T}_t^U(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U, \widetilde{EBIT}_t)$ is a linear function of free cash flows (minus changes in paid in capital) as well as corporate and personal tax rates and their interaction. τ represents the net tax saving per currency unit of interest paid and could also be zero as well as negative. Expression (2) will serve as a basis to cope with the different tax systems as found in Europe, New Zealand and the USA.

2.1 The Classical System

We start with the classical system that can be found among others in the USA, Denmark, the Netherlands, and in Switzerland. Typically, the earnings of the firm are taxed both at the firm's and the shareholders' level. With τ_F being the corporate tax rate corporate taxes amount to

$$\tilde{T}_{F,t} = \tau_F(\widetilde{EBIT}_t - \tilde{Z}_t) \tag{3}$$

The payment to the shareholders are $\widetilde{CF}_t - (\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t) - \tilde{T}_{F,t}$. The creditors receive $\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t$. Furthermore, the shareholders receive a tax relief if equity ('paid in capital') is lowered by $\tilde{E}_{t-1} - \tilde{E}_t$. We assume that dividends are taxed at the personal income tax rate τ_S while interest is taxed at τ_B. If we bear in mind that both shareholders and creditors receive income in form of dividends and interest payments, their personal taxes will be

$$\tilde{T}_{P,t} = \tau_S(\widetilde{CF}_t - (\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t) - (\tilde{E}_{t-1} - \tilde{E}_t) - \tilde{T}_{F,t}) + \tau_B\tilde{Z}_t$$

Notice, that personal income taxes depend on the difference $\tilde{E}_t - \tilde{E}_{t-1} - (\tilde{D}_{t-1} - \tilde{D}_t)$ which in turn is according to (1) the same as $\tilde{E}_t^U - \tilde{E}_{t-1}^U$ for the unlevered firm. Adding corporate and personal taxes and rearranging terms yields (2) with the specifications for

$$\tilde{T}_t^U = \tau_S(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U) + \tau_F(1-\tau_S)\widetilde{EBIT}_t \quad \text{and} \quad \tau = -\tau_B + \tau_S + \tau_F(1-\tau_S) \tag{4}$$

Equation (4) underlines the significant impact of personal taxes: without taxing dividends the combined tax rate τ would be reduced by $\tau_S(1-\tau_F)$. This point is of particular relevance to the discussion of taxation of dividends currently going in the US. On the other hand, if in a classical system dividends and interest payments are not taxed, (4) reduces to $\tau = \tau_F$ which represents the model of Miles and Ezzell (1980).

2.2 The Imputation System

If the corporate taxes are added to the personal tax base and are then completely deductible from the personal income taxes to be paid, we speak of an imputation system. Thus corporate income taxes are nothing else than prepayments on personal income taxes. Such imputation systems are currently used in New Zealand and Belgium. The sum of personal taxes amounts then to

$$\tilde{T}_{P,t} = \tau_S(\widetilde{CF}_t - (\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t) - (\tilde{E}_{t-1} - \tilde{E}_t) - \tilde{T}_{F,t} + \tilde{T}_{F,t}) + \tau_B\tilde{Z}_t - \tilde{T}_{F,t}$$

Again, personal income taxes depend on the difference $\tilde{E}_t - \tilde{E}_{t-1} - (\tilde{D}_{t-1} - \tilde{D}_t)$ which in turn is according to (1) the same as $\tilde{E}_t^U - \tilde{E}_{t-1}^U$ for the unlevered firm. By adding the firm tax as given in (3) we again obtain (2) with the specifications for

$$\tilde{T}_t^U = \tau_S(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U) \quad \text{and} \quad \tau = -\tau_B + \tau_S \tag{5}$$

In a partial imputation system some portion ($\gamma_1 > 0$) of the dividends received will be added to the tax base and another portion ($\gamma_2 > 0$) of the dividends is granted as tax relief from the personal income taxes. This type of tax regime can be found in Canada and the UK. For this special case the sum of the personal taxes amounts to

$$\tilde{T}_{P,t} = (\tau_S \gamma_1 - \gamma_2)(\widetilde{CF}_t - (\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t) - (\tilde{E}_{t-1} - \tilde{E}_t) - \tilde{T}_{F,t}) + \tau_B \tilde{Z}_t$$

As above personal income taxes depend on the difference $\tilde{E}_t - \tilde{E}_{t-1} - (\tilde{D}_{t-1} - \tilde{D}_t)$ which in turn is according to (1) the same as $\tilde{E}_t^U - \tilde{E}_{t-1}^U$ for the unlevered firm. Adding (3) and some rearrangement yields (2) with specifications for

$$\begin{aligned} \tilde{T}_t^U &= (\tau_S \gamma_1 - \gamma_2)(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U) + (1 + \gamma_2 - \gamma_1 \tau_S)\tau_F \widetilde{EBIT}_t \quad \text{and} \\ \tau &= 1 - \tau_B - (1 - \tau_F)(1 + \gamma_2 - \gamma_1 \tau_S). \end{aligned} \tag{6}$$

2.3 A Mixed System (The German System)

A more complicated tax regime is the current German tax system. This system has two kinds of corporate income taxes: a trade tax ('Gewerbesteuer') and a corporate tax ('Körperschaftsteuer'). The trade tax is similar to the corporate income tax, but only half of the interest payments is deductible. Let τ_g be the tax rate the trade taxes can be expressed as

$$\tilde{T}_{g,t} = \tau_g \left(\widetilde{EBIT}_t - \frac{1}{2} \tilde{Z}_t \right).$$

A particular feature of the German tax system is that trade taxes are fully deductible from the corporate tax base which yields

$$\tilde{T}_{k,t} = \tau_k (\widetilde{EBIT}_t - \tilde{Z}_t - \tilde{T}_{g,t})$$

with τ_k being the rate of the corporate tax.

Adding both taxes produces the following expression for the firm's combined income taxes

$$\tilde{T}_{F,t} = (\tau_g + \tau_k(1 - \tau_g))\, \widetilde{EBIT}_t - \left(\frac{\tau_g}{2} + \tau_k \left(1 - \frac{\tau_g}{2} \right) \right) \tilde{Z}_t. \tag{7}$$

Turning to the personal income tax it must be noted that creditors and shareholders and treated differently: while interest payments are fully taxed, dividends are taxed only at 50% of the dividend payments actually received.[1] With τ_e representing the personal income tax rate (for both creditors and shareholders) the equation for personal income taxes can be expressed as

$$\tilde{T}_{P,t} = \frac{\tau_e}{2}(\widetilde{CF}_t - (\tilde{D}_{t-1} + \tilde{Z}_t - \tilde{D}_t) - (\tilde{E}_{t-1} - \tilde{E}_t) - \tilde{T}_{F,t}) + \tau_e \tilde{Z}_t.$$

Once again personal income taxes depend on the difference $\tilde{E}_t - \tilde{E}_{t-1} - (\tilde{D}_{t-1} - \tilde{D}_t)$ which in turn is according to (1) the same as $\tilde{E}_t^U - \tilde{E}_{t-1}^U$ for the unlevered firm. Adding (7) to the last equation and rearranging terms leads to (2) with specifications for

$$\begin{aligned} \tilde{T}_t^U &= \frac{\tau_e}{2}(\widetilde{CF}_t - \tilde{E}_{t-1}^U + \tilde{E}_t^U) + \left(1 - \frac{\tau_e}{2}\right)(\tau_g + \tau_k(1 - \tau_g))\, \widetilde{EBIT}_t \quad \text{and} \\ \tau &= -\frac{\tau_e}{2} + \left(1 - \frac{\tau_e}{2}\right)\left(\frac{\tau_g}{2} + \tau_k\left(1 - \frac{\tau_g}{2}\right)\right). \end{aligned} \tag{8}$$

3. The WACC Formula

We now turn to the market values of the levered and the unlevered firm. Denote by V_0^L the value of the levered firm and by V_0^U the value of the unlevered firm. Investment in stocks in the capital market will be taxed at τ_S. As in Miles and Ezzell (1980) we assume that γ^U is not only the expected return of the company but the discount rate of any single cash flow. The assumption that γ^U is constant is made for reasons of simplicity but can be dropped without changing results substantially. We furthermore assume that the riskless discount rate after personal income tax is r_f.

Let l_t be the (deterministic) leverage ratio of the levered firm at time t, i.e. the quotient of the market value of debt and the market value of the levered firm. This quote does not need to be constant. Now the following holds:

PROPOSITION 1 (WACC formula) *If the cost of capital of the unlevered firm is constant the value of the levered company is given by*

$$V_0^L = \sum_{t=1}^{T} \frac{E[\widetilde{CF}_t - \tilde{T}_t^U]}{(1 + WACC_0)(1 + WACC_1) \cdots (1 + WACC_{t-1})} \tag{9}$$

where

$$1 + WACC_s = (1 + r^U)\left(1 - \frac{\tau r_f}{1 + r_f} l_s\right), \quad s = 0, 1, \ldots \tag{10}$$

Proof Our model coincides with the one of Löffler (2001) if the firm income tax is τ, the discount rate of the cash flows of the unlevered firm is r^U, the riskless interest rate is r_f and the cash flows after tax of the unlevered firm are $\widetilde{CF}_t - \tilde{T}_t^U$. For a proof of the WACC formula see Löffler (2001). □

The more interesting application is a formula for a firm having an infinite lifetime.

PROPOSITION 2 (infinite lifetime) *If it is further assumed that $g_t = g$ and leverage ratios as well as cost of capital are constant, the value of the levered firm is given by*

$$V_0^L = \frac{V_0^U}{1 - \dfrac{r_f}{r^U - g} \dfrac{1 + r^U}{1 + r_f} \tau l}. \tag{11}$$

Proof Applying the formula for a geometric sum and using the expressions for V_0^U gives the desired result. □

4. An Illustration

In order to stress that our approach is of significance for practical purposes we will calculate the weighted average cost of capital for some selected countries. In particular we will look at the USA which has the classical system. While a full imputation system is found in New Zealand, Canada has a partial imputation system. Finally we will focus on Germany which has a tax regime that combines characteristics of both the classical and the imputation system. At last we compare our results with those derived by the traditional Miles–Ezzell formula. We also maintain a website (www.wacc.info) where WACC formulas for several other national tax systems can be found.[2]

In order to control for non-tax variables the after tax riskfree rate is set at $r_f = 4\%$, the after tax capital cost of the unlevered firm at $r^U = 10\%$ and the leverage ratio at $l = 75\%$.

United States of America The USA with its classical system has a federal income tax rate of up to 35%. State and local income tax rates range from less than 1% to 12%. Since state and local taxes are deductible when computing federal taxable income, effective corporate income tax rates range from 35% to 43%. The average rate is approximately 40%. While personal income tax rates run up to 35%, dividends and capital gains are taxed at a flat rate of 15%. Local taxes may also apply and can amount up to 12%. For the purpose of our illustration we use as an average effective personal income tax rate on dividends 20% and 40% on interest income. Hence, according to (4) we obtain a value of

$$\tau^{USA} = 12.00\%$$

New Zealand Firms in New Zealand are currently taxed at a rate of 33%. Personal income tax rates on dividends and interest can be as high as 40%. Due to (5) of the imputation system we get

$$\tau^{NZ} = 0\%$$

regardless of the actual personal income tax rate.

Canada A partial imputation system exists in Canada that currently has a federal tax rate of 22%, including a surtax.

Depending on the province, the effective corporate tax rate generally ranges from 31% to 39%.[3] The average effective corporate tax rate is therefore approximately 35%. The personal federal tax rate can be as high as 29%. Additional provincial or territorial tax rates apply and range from about 10% in Alberta to 18% in Newfoundland and Labrador. Hence, the average personal tax rate equals approximately 43%. Interest income is fully taxed. However, calculating income tax on dividends is more complicated. In general, a person receiving dividends from a Canadian corporation must first determine the taxable amount of dividends by multiplying the dividends actually received by 125%. This taxable amount is then subject to personal income taxation, i.e. $\gamma_1 = 1.25$ in (6). The person is then allowed to deduct a federal dividend tax credit equal to 13.33% of the taxable amount of dividends. This leads to about 17% of dividends actually received, i.e. $\gamma_2 = 0.17$ in (6). The tax credit reduces both federal and provincial or territorial taxes. Using these tax rates we finally arrive at

$$\tau^{CA} = 15.89\%$$

Germany The actual trade tax rate varies from municipality to municipality and ranges from 9% to 25%. As an average we use 17%. The corporate tax rate (including a solidarity surcharge) is 26.4%. The personal income tax rate (again including the solidarity surcharge) peaks at 47.5%. From all this we get

$$\tau^{D} = 1.15\%$$

Using the $\tau^{(\cdot)}$ derived for the different countries and inserting them into (10) produces values for WACC as summarized in Table 1. These results indicate variations which may lead to substantially different valuations. Furthermore, Table 1 includes the results given by Miles–Ezzell which lead to a substantially lower WACC flowing from the fact that personal income taxes are not taken into account in their approach.

Table 2 contains the corresponding results for the infinitely living firm with values of the levered and unlevered firm being used instead of $\tau^{(\cdot)}$. As above, significant differences result from not taking personal income taxes into account.

Table 1. WACC for different countries

Country	Tax system	τ	WACC
US	Classical	12.00%	9.62%
NZ	Full imputation	0.00%	10.00%
CA	Partial imputation	15.89%	9.50%
D	Mixed	1.15%	9.96%
	Miles–Ezzell	40.00%	8.73%

Table 2. Values of the infinitely living firm ($g = 3\%$) for different countries

Country	Tax system	$V_0^L / V_0^U - 1$
US	Classical	5.75%
NZ	Full imputation	0.00%
CA	Partial imputation	7.76%
D	Mixed	0.52%
	Miles–Ezzell	22.15%

5. Conclusions

Taking the Miles–Ezzell model as a point of departure we have extended their approach by not only including personal income taxes but also developing a generalized valuation formula which can accommodate different taxation systems. The results obtained in the numerical illustration for different countries underline the relevance of our approach.

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Notes

[1] This particular feature of the German tax system is also used in Austria and France. In Italy, a 60% exemption will be available for dividends paid.

[2] The following description of the national tax codes where taken from *Internal Revenue Service* (2004), *Inland Revenue* (2004), *Canada Revenue Agency* (2004), *KPMG* (2004a), *KPMG* (2004b) and Canefield (1999).

[3] There is a reduced tax rate for manufacturing firms ranging from 27% to 38%.

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