The Design of Debt Clearing Markets: Clearinghouse Mechanisms in Pre-Industrial Europe

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Abstract

We examine the evolution of decentralized clearinghouse mechanisms that were in use throughout Europe from the 13th century to the 18th century; in particular, we explore the clearing of non- or limited-tradable debts like bills of exchange. We construct a theoretical model of these clearinghouse mechanisms and show that the specific decentralized multilateral clearing algorithms known as rescontre, skontrieren or virement des parties, used by merchants in this period, were efficient in specific historical contexts. Our analysis contributes to the understanding of both the emergence and evolution of these mechanisms during late medieval and early modern fairs, and their robustness during the 17th and 18th centuries.

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1 Introduction

Studying pre-industrial institutions is crucial in understanding the long run growth of Europe. During this era, the intensification of regional and long distance trade required the creation of institutional solutions to issues arising from more complex financial arrangements. Indeed, many of the institutions which form the foundation of the modern economy were first employed in more primitive forms during the late Middle Ages or the Early Modern eras: this is particularly true for institutions related to trade (Lopez 1976; Greif 2006).

An important step in the evolution of trade was the separation of quid and quo (Greif 2002, 2006b). That is, during this era, the payment for purchased goods no longer necessarily took place at the same time and location, but instead could take place at the next fair or market, possibly at a different location. Alongside this development appeared financial instruments like the cambium, letter obligatory, or the bill of exchange, which were promises of future payment at a specific time and place by the drawer or his representative or banker (Goldschmidt 1891; Schapps 1892; de Roover 1953). Access to such financial instruments was crucial to a merchant, since he could negotiate a time and place when and where he expected to be liquid again (e.g., after the successful trading of goods, or when he would cash in bills of exchange himself). Moreover, cashless payment was a good alternative to carrying heavy gold or silver coins along dangerous trading routes. In addition, it limited the dependency of merchants on the currency policies of local dukes. Finally, the use of cashless payment helped to overcome the money shortage of the Late Middle Ages caused by the scarcity of silver and gold bullion (Miskimin 1984; Spufford 1988).

The development of cashless trade made necessary the establishment of clearing institutions, through which liabilities could be settled for merchants and merchant bankers from all over Europe. However, any clearing institution faced a number of significant challenges. First, an agent may lack the liquid assets necessary for direct payments of debts, even though their net position is positive, a condition referred to as time mismatch. Second, there may be limited enforcement of contracts, where it may be difficult to compel payment of debts by
debtors. Indeed, these issues have been identified in the literature on contemporary payment mechanisms as key problems to solve (Kahn and Roberds 2009).

In the modern context, these problems are alleviated by the buying and selling of debt instruments. However, financial instruments, such as bills of exchange, were in general not tradable due to the limited enforceability of financial contracts across Europe (Usher 1943; Van der Wee 1963; North 1981; Munro 1994). Hence, settlement could not simply occur through the buying and selling of bills of exchange; instead, a specific institution designed to effectuate the efficient clearing of debt contracts in the presence of this constraint was necessary.

In fact, a self-enforcing clearing mechanism to offset non-tradable credits and debts at the end of a market period was developed during this era, known in the sources as rescontrire, rescontre, skontrieren, or vivre compte/des parties. During this procedure, each merchant revealed his debtors, and each creditor had to confirm his liabilities. The merchants then tried to offset credits with debts through a decentralized searching and matching procedure: in the first step, they cancelled reciprocal debts, in the second step, they used clearing cycles, and, in the third and final step, they used clearing chains. Once no more chains or cycles could be found, any liabilities still extant had to be paid in cash or new bills had to be drawn.

We show that this decentralized clearing procedure has several appealing properties. We first consider the case of balanced positions, where each agent’s liabilities are exactly offset by his assets. In this case, not only is the rescontre procedure sufficient to obtain an efficient outcome, but it is optimal for each agent to truthfully reveal his debts and credits.

For the more general state of affairs where a merchant may be a net creditor or a net debtor, the rescontre procedure is no longer necessarily efficient for general preferences. However, in the case where each agent is a either trustworthy debtor or not, the rescontre

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1 See Boerner and Ritschl (2009, 2011) for a discussion.
2 This could be achieved, for instance, by requiring each merchant to pay any outstanding debts at the end of the clearing process in cash.
procedure is efficient in reallocating debt. Finally, even when there are many types of agents, agents have incentives to reveal their positions truthfully.

These properties facilitated the spread of the rescontre procedure throughout Europe. At first, merchants and merchant bankers met at the end of a fair to settle bills of exchange and other liabilities. Later, permanent payment days were established and, during the 16th century, exclusive clearing fairs were initiated in Besancon, Piacenza, and Novi, where bills of exchange were settled from all over Europe (Endemann 1885; Neuhaus 1892; Ehrenberg 1896; Vigne 1903; Haristoy 1906; Prausnitz 1928; Carande 1943; Lapeyre 1955; Da Silva 1969; Gascon 1971; Boyer-Xambeu et al. 1986; Denzel 1994, 2009).

This clearing mechanism continued to develop at late medieval fairs and was perfected at the financial fairs of the late 16th and early 17th centuries, where it developed into a formal procedure embedded in a legal framework. It was the dominant debt-clearing mechanism in Europe’s most important trade centres: Germany, Italy, France, the Low Countries, and Spain. These markets and fairs not only built the central nodes of European finance and trade but also facilitated the acquisition of capital to finance trade with the New World (Ehrenberg 1896, Da Silva 1969, Boyer-Xambeu et al. 1986; van der Wee 1993, Pezzolo and Tattaro 2008, Marsilio 2008, Denzel 2009).

These clearing mechanisms waned in importance when the first exchange banks took over the clearing business and general endorsement of bills of exchange during the 17th century (Schneider 1989; Brübach 1994). Nevertheless, there is strong evidence that this decentralized procedure was still in use at many markets and fairs during the 17th and 18th centuries. In addition, traders who worked in the first stock exchange markets used similar mechanisms to offset payments for stocks or future contracts in cities like Amsterdam and London (Parker 1974; Gelderblom and Jonker 2005). Similarly, after the establishment of the London Clearing Club in 1775, banks began settling their liabilities in a similar way (Seyd 1872). Thus, this decentralized multilateral clearing mechanism was the backbone of financial clearing during more than 500 years of pre-industrial growth in Europe.
While scholars have long recognized that this clearing technology was used for cashless payment all over Europe in order to generate credit and solve problems of money shortage (Neuhaus 1892; Ehrenberg 1896; Boyer-Xambieu 1986; Brübach 1986; North 1996; Denzel 2005, 2009; Marsilio 2008), they have not analyzed the efficiency or strategic properties of the mechanism. Instead, legal and economic historians have focused on description, and have either assumed that mechanism was efficient due to its success, or argued that the mechanism was inefficient as the clearing was decentralized and seemed overly complex.

However, a wide literature on modern payment and clearing mechanisms does analyze the strategic behavior by agents induced by different mechanisms: see, e.g., Freixas and Parigi (1998), Furfine and Steam (1998), Kahn and Roberds (1998), and Angelini (1998). Moreover, the choice of clearing procedure can have significant effects on trade and welfare (Freeman 1996; Kahn and Roberds 2001). A central question in this literature is the use of gross versus net clearing procedures. In our context, the lack of a central institution, such as a public bank, with sufficient trust and enforcement power implies that gross clearing procedures were not feasible. This lack necessitated the use of a net clearing procedure which does not require a trusted central authority, e.g., the rescontre procedure. However, the use of a net clearing procedure implies that participants are at risk from the default of others. Thus, the allocation of liabilities for unpaid debts is a key question for any such clearing procedure.

In the rescontre procedure, the first step is to look for clearing cycles, where each agent holds debt from the next agent in the cycle. A cycle is cleared by canceling each of these debts, thus holding the net position of each agent constant. Hence, the question of how to allocate liabilities for unpaid debts does not arise when only using clearing cycles. We show that, in the case where positions were balanced, this mechanism was fully efficient. Furthermore, it was in the interest of every agent to participate. However, these results only hold in settings where agents positions were balanced.

If we assume that positions are unbalanced, then the use of chains is necessary in order
to facilitate clearing, as it was no longer necessarily the case that efficient clearing could be achieved solely through cycles. However, a natural question arises in such situations: If a creditor agreed to cancel a debt with his debtor in exchange for the debt of another, was his original debtor liable in the case that the new debtor was unable to pay? To facilitate clearing, it was essential that the answer to this question was negative, i.e., debt transfers were final. (This condition is referred to as *finality of payments* in the payments literature: see Kahn and Roberds (2007).) In such a setting, creditors then would be very careful about canceling such debts and taking on new debtors. Hence, it was vital that creditors were given the right to refuse chains, as otherwise they would have strong incentives not to participate in the clearing.

We show that, when an agent blocks a chain from clearing, the mechanism is no longer fully efficient in general. However, in the simple case where there are only two types of agents, i.e., where debtors are either “good” or “bad”, a mechanism which only clears chains that are approved by all the members of the chain is sufficient to obtain a Pareto efficient outcome. Moreover, even when there is a greater degree of heterogeneity among debtors, we show that simply clearing chains is enough to obtain a constrained form of efficiency. In this way, the rescontre procedure enables the allocation and clearing of debts in a heterogenous environment with limited enforcement options.

This paper contributes in several ways to the literature. Following a tradition established by Greif and others (Greif 1993, 2006; Greif et al. 1994), we provide a formal analysis of an important historical institution in order to understand long-run economic growth in Europe. In particular, we contribute to the understanding of the shaping of historical institutions through the lens of market design, and in so doing help to explain the design of pre-industrial financial institutions; these financial institutions were a key ingredient of European economic growth (de Roover 1953; van der Wee 1966; Munro 2002; Fratiani and Spinelli 2006). Second, we contribute to the payments literature by showing the existence of a simple clearing mechanism which allows for the efficient allocation of liabilities amongst
heterogenous agents, even in settings with limited enforcement power. Finally, it adds to the market design literature which analyzes allocation mechanisms when monetary transfers are not available (Abdulkadiroğlu and Sönmez 2002; Ehlers 2002; Roth et al. 2004, 2005; Roth 2007).

The remainder of the paper is structured as follows: Section 2 examines the history of the mechanism, from its early development in the late Middle Ages to its perfection in the 16th and 17th centuries. In addition, the role of exchange banks, endorsement of bills of exchange, the emergence of the mechanism on the first stock exchanges, and the use of the mechanism in the form of clearing clubs among banks are discussed. Section 3 studies the mechanism more formally. A theoretical game model is set up which analyzes the algorithm; the equilibrium outcomes are discussed. In Section 4 the theoretical predicted outcomes are discussed in their historical context. Both the successes and the vulnerabilities of the mechanism are elaborated in detail. Section 5 concludes. The proofs of the theorems in Section 3 may be found in the Appendix.

2 Historical Development

Section 2 provides a comprehensive historical analysis of the mechanism: Section 2.1 describes the early growth in its usage up to the 16th century, as well as the development of formal procedures and legislation during the 16th and early 17th centuries. Section 2.2 describes the rules and customary practices of the clearing procedure. Section 2.3 details the further development of the mechanism in its institutional and historical context. Finally, Section 2.4 provides an evaluation of the efficiency based on descriptive source analysis.

2.1 Early Development and the Formation of a Formal Procedure

Evidence of the use of debt clearing mechanisms such as rescontrire or rescontre procedures can be traced to the late Middle Ages. The earliest documents do not inform us about the
detailed structure or procedures of the clearing process, but rather just indicate its existence and probably its common use. The earliest sources are from Champagne fairs at the end of the 12th and beginning of the 13th century (Anschütz 1887, pp. 108f.; Haristoy 1906, pp. 284-88; Face 1957). The use of mutual settlement at the central fair locations of Europe can be again documented from the late 14th century and 15th century onwards: in Germany at Frankfurt, Brunswick, and later Leipzig (Orth 1765, p. 446, Wolf 1967, p. 205f., Bücher 1915, p. 212f. Brübacher 1994, p. 284, 307), in France at Lyon (Ehrenberg 1886, pp. 74-8; Vinge 1903, pp. 110ff.; Haristoy 1906, pp. 296-301; Bresard 1914, Gascon 1971, pp. 242-8; Boyer-Xambeu 1986, pp. 189-198), in the Netherlands at Antwerp and Bergen op Zoom (Ehrenberg 1896, pp. 10-13; van der Wee 1993, pp.151-158), and in Spain at the Castilian fairs of Medina del Campo (Rodriguez and Fernandez 1903, pp. 642f.; Carande 1943, pp. 195-234.; Martin 1965, pp. LXXXII-CIII; Vasquez de Prada 1991, pp. 117f). Early evidence from the 15th and 16th century for the use of the rescontre procedure can also be found in Mediterranean merchant cities with permanent wholesaler markets and finance locations such as Barcelona, Genoa, Naples, and Venice (Capmany 1792, p. 167; Lattes 1869, pp. 72; Ajello 1884, pp. 728ff.; Prausnitz 1928, p. 11).

The first regulations which describe the clearing procedure in detail are from 16th century Piacenza; these regulations provide further evidence of the common use of this clearing technology in many merchant cities and fair locations during the 15th and 16th centuries. These regulations, from 1597, were taken from the former clearing fairs of Besancon and are called the orders of Besancon. The clearing fairs at Piacenza were the first exclusive clearing fairs and were founded by the Genoese in 1537 to compete with and replace the clearing fairs of Lyon, which in turn go back to the second half of the 15th century. Because of this it has been argued that the clearing procedure described in the statutes of Besancon goes back to the customary practice applied by merchant bankers in Lyon. These orders from

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3This argument goes back to Endemann (1886, p. 174). Customary practices of the 16th century Lyon fair have been handed over by Fabiano (1556), Trenchant (1556), de Savonne (1567), and de Rubys (1604). Later authors includ Boyer (1627) and Clerac (1656). See Vigne (1903, pp. 118-128), Haristoy (1906, pp. 296-301), Bresard (1914), and Gascon (1971, pp. 242-8).
Besancon have been recorded by Raphael Turri in 1642, who describes the clearing process in detail. In addition, Peri (1638) and Scaccia (1645) also give descriptions of the clearing process (Neuhaus 1892; Endemann 1885 174ff.; Ehrenberg 1896, pp. 230-6; Prausnitz 1928; Da Silva 1969, Chapter 2).

These regulations can be seen as a blueprint of the procedures that can be found in many exchange laws during the 17th and 18th centuries in Europe. Similar procedures can be found in the fairs in Italy, such as those at Novi, Bozen, and Verona, which replaced Piacenza (Da Silva 1969; Denzel 2005, 2009). In France, Lyon remained the clearing centre for bills and other obligations, where the procedures were codified in detail in 1667 based on earlier attempts starting in 1642 (Haristoy 1906, pp. 303ff; Vigne 1903, pp. 118ff.). At the same time, Frankfurt, the most important fair location and finance centre in Germany, produced a more detailed exchange law, inspired by the codification in Lyon. Whereas Frankfurt already had a long tradition of using this clearing technique going back to the late 14th century, clearing regulations could only be found from 1578 and 1611, along with a more detailed version from 1666 (Brübach 1994, pp. 169-76). Other important clearing locations with regulations concerning the mechanism were Brunswick (1686, 1715) and Leipzig (1682, 1711).4 Further evidence is available from merchant cities such as Augsburg, Breslau, Nuremberg and others (Brübach 1994, p. 318; Neuhaus 1892, p. 11).5

2.2 Regulations and Customary Practices

Clearing took place about four times a year. This goes back to the tradition that fairs were held quarterly and, at the end of each fair, a financial clearing was organized.6 This pattern can be documented for Lyon, the Brabant fairs at Antwerp and Bergen, the Castilian fairs and all the Italian and most of the German fairs (Ehrenberg 1892, Da Silva 1969; van der

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4 Very short regulations for Leipzig exist for 1621 and 1676. See Hasse (1886, pp 277f.), Neuhaus (1892, p. 11), and Brübach (1994, pp. 310ff).

5 Beside the exchange laws themselves, contemporary writers such as Raumburger (1723) and Riccius (1783) provide more evidence on the use of the procedure in these places.

6 For the typical fair’s customs, see Verlinden (1971).
Wee 1963; Boyer-Xambeu et al. 1986). In cities in which fairs became permanent markets or financial clearing was separated from goods trading, the frequency was increased to monthly or even weekly meetings during the 17th and 18th centuries. This can be documented in the regulations of Frankfurt in 1776 and Leipzig in 1688, or in Augsburg where only a financial exchange was implemented (Neuhaus 1892, p. 11; Prausnitz 1928, p. 22; Brübach 1994, p. 175, 318). These clearing meetings lasted several days. In Besancon, for example, the meeting always took eight days, with a fixed ordering of events.

For most fairs, the regulations did not specify any restrictions on participation. The clearing was open to everybody; however, de Turri reports that in Besancon participants were merchant bankers and only they had the right to participate (Endemann 1889; p. 174, Prausnitz 1928, p. 17). Scaccia and Peri write that whoever wanted to participate had to pay a deposit (Endemann 1889, p. 174; Neuhaus 1892, p. 13; Ehrenberg 1896, p. 231). Sometimes merchants attending a fair were forced to take part in the clearing procedure. This was stipulated in the regulations of Bozen and Leipzig (Neuhaus 1892, p. 31, 39). These rules excluded cash payments or agreements about later payments ex ante by merchants who had not taken part in the mutual clearing procedure. Under the rules of Besancon, merchants who wanted to be paid in cash had to announce before the clearing process began (Ehrenberg 1892, p. 235; Prausnitz 1928; Neuhaus 1892). However de Turri, Scaccia, and Azorius, another writer from 1625, report that merchants had preferences for mutual clearing and were worried about receiving their payments in cash (Neuhaus 1892, p.19, 21; Endemann 1889, p. 179).

The clearing procedure worked as follows: merchant bankers met with their ledgers (scartefacci). In early Besancon there were at least 50-60 bankers; in Piacenza there were about 110 (Ehrenberg 1892, p. 229). Merchants who wanted to participate had to be present or have an official representative. This can be found in the statutes of Besancon and most

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7Permanent marketplaces document weekly meetings earlier on; for example, sources from Naples indicate weekly meetings as early as the 16th century. See Ajello (1884, pp. 728ff).
8See also the report by Raumburger (1723).
of the later regulations during the 17th and 18th centuries as well (Endemann 1885, p. 175; Brübach 1994, pp. 306f.; Neuhaus 1892, p. 30; Haristoy 1906, p. 304f). Once the set of participants had been determined, all creditors revealed their debtors. The debtors had to confirm or reject the debts. In the next step, each banker had to create a balance sheet with all the accepted claims, which had to be submitted in Besancon where officials would check the general balance. This practice cannot be documented for other later fairs. Only accepted bills were valid for the later mutual netting procedure. Rejected bills were discussed in front of the fair court and settled later; this regulation was also in use in all other fairs (Endemann 1885, p. 175ff; Prausnitz 1928, p. 17; Brübach 1994, p. 309). De Turri writes that not all bankers were honest: hence, some bankers were willing to go in front of the court in order to settle some bills later (Endemann 1885, p. 177).

In the next step the exchange rates for the bills were fixed in an official accounting currency. This was done by a group of bankers from different regions who had participated in the clearing process. At the Genoese fairs the bankers who set the exchange rates came only from different regions in Italy (Endemann 1885, p. 179f.; Ehrenberg 1896, p. 232), while in Lyon, they came from France, Germany and Italy (Gascon 1971, pp. 244f.). The regulations of Besancon state that exchange rates should be reasonable. However, based on these rates, other market prices were negotiated ex post (Ehrenberg 1896, p. 234f.). Having participants from different regions set exchange rates collectively was also typical for later regulations. In Frankfurt, where the fixing of the exchange rate can be documented from 1585 onwards, in 1625 legislation was passed after some bankers at previous fair meetings

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9It is not clear if this was all public or happened in a bilateral way: see Ehrenberg (1896, p. 234), Martens (1794, p. 16), Neuhaus (1892, p. 8), Prausnitz (1928, p. 17), Endemann (1886, p. 176), and Brübach (1994, p. 175).

10See the reports by Scaccia and de Turri (Endemann 1886, p. 176). The same practice was in use in Lyon earlier: see Gascon (1971, pp. 243f.). Later, the practice was used in the Frankfurt and Leipzig fairs: see Haristoy (1906, p. 304) and Brübach (1994, p. 310).

11This was reported by de Turri, Peri, and Scaccia. See Endemann (1886, p. 177) and Ehrenberg (1896, p. 234).

12The rejection of such bills can be documented for the fairs of Lyon from at least the second half of the 15th century. See Haristoy (1906 pp. 297f).

13De Rubys reports in 1604 that this procedure was already use at the Lyon fairs during the 16th century: see Vigne (1903, p. 121).
tried to manipulate exchange rates in their favor; it was stated that the exchange rates were to be decided by local exchange brokers and merchant bankers from different regions (Ehrenberg 1896, pp. 246f. Brübach 1994, p. 287, 311ff.).

In the final stage, merchants started the actual clearing process, sometimes with the help of brokers. This process was organized as a decentralized searching and matching procedure. The codified legal rules in various cities are rather cryptic in describing the process. However, contemporary witnesses paint a very lively picture of merchant bankers who were in search of finding clearing partners during the clearing time.

The decentralized clearing algorithm worked the following way. First, merchant bankers offset their positions in a reciprocal way. So if a merchant banker $i$ owes $j$ and $j$ at the same time owes $i$, the common amount could be directly cancelled. In the next step merchants tried to find clearing cycles and clearing chains. In a clearing cycle, for example, $i$ owes $j$, $j$ owes $k$, and $k$ owes $i$ a certain amount of money. The common amount then could also be cancelled. By contrast, in a clearing chain, where $i$ owes $j$ and $j$ owes $k$, only $j$ could clear his debts and the amount that could be cancelled out had to be transferred into a new debt relationship between $i$ and $k$. These clearing mechanisms could easily include seven or eight parties (Koenigk 1727, p. 66; Orth 1765, p. 481; E.B.A. 1773, p. 109f.). This clearing process was repeated again and again. For example, $k$ could use the transferred debt in the chain and bring it into another clearing chain or cycle. This sequential clearing can be documented in the ledger by Masse in 1611 (Brübach 1994, pp. 305f.). The clearing process lasted until the clearing time came to an end or no merchant could find another willing clearing partner.

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14 The help of brokers can be documented for the German fairs in Frankfurt, Brunswick and Leipzig. See Brübach (1994, p. 316f).
15 For example see the regulations of Frankfurt in 1611, Brunswick in 1686 and 1715, Leipzig in 1688, and Breslau in 1742; see Prausnitz (1928, p. 23) and Brübach (1994, p. 170).
16 Evidence can be found in de Turri and Scaccia for the earlier period. For the late 17th and 18th centuries, see Raumburger, Marberger, Riccius, and Zipffel (1701): see Endemann (1886, p. 178), Prausnitz (1928, pp. 17, 23), and Neuhaus (1892, p. 9).
17 Scaccia and de Turri report this for the Genoese fairs. See Neuhaus (1892, p. 8), Prausnitz (1928, p. 17), and Endemann (1882, p. 177).
18 For the Genoese fairs Peri gives an example. See Ehrenberg (1896, pp. 235f). For later fairs see, for example, descriptions by Koenigk and Riccius: see Prausnitz (1928 p. 23) and Neuhaus (1892, p. 35).
as every banker had reached his net position.

In the exchange regulations, differentiated rules arose about the constraints on participation in the clearing mechanism. In general, the rules gave the merchants the freedom to choose the clearing cycle or chains in which they wanted to participate. de Turri mentions this as an early practice. In addition, we can find a clause that everybody needs to agree to the specific clearing group in most of the exchange laws from the 17\textsuperscript{th} century.\textsuperscript{19} All these sources imply preferences of merchants over participation in different clearing cycles or chains. A writer of a merchant handbook in 1773 reports on the rationale behind this. The author writes “you need to be careful not replacing a good debtor with a bad debtor” (E.B.A. 1773, p. 112). Another regulation that indicates different preferences over the order of clearing procedures is legislation from Frankfurt in 1666 where bankers can first offset their liabilities before those of the customers that they represent (Brübach 1994, p. 176).

After the mutual netting was finished, merchants had to check if they still had outstanding credits or debts. All mutual settlements had to have been written in their scartefacci and having all participants do this made the clearing official.\textsuperscript{20} In general all transfers made by the mechanism were final. This meant if a debtor was transferred to a new creditor, the old creditor was no longer liable.\textsuperscript{21} Only in Besancon was it the customary practice that everyone within a clearing chain or cycle was collectively liable.\textsuperscript{22} Legislation ensured that the payment of debts through the clearing mechanism had the same legal status as paying cash.\textsuperscript{23}

\textsuperscript{19}See for example the exchange rules from Frankfurt in 1666, Leipziger exchange orders in 1688, Prussian exchange orders in 1684, the orders of Bozen in 1634, the orders of Brunswick in 1715, the orders of Augsburg in 1716, or the descriptions by Raumburger, Marberger, and Riccius: see Neuhaus (1892, p. 29) and Prausnitz (1928, p. 22).
\textsuperscript{20}de Turri mentions this regulation for Besancon. This regulation is mentioned for later fairs by Koenigk, Raumberger, and Marberger: see Prausnitz (1928, p. 17f). This regulation can be found in the statutes from Lyon in 1667, Leipzig in 1688 and 1715, Breslau, Augsburg, and Bozen: see Neuhaus (1892, p. 12, 31, 38), Prausnitz (1928, p. 17f., 23), and Brübach (1994, p. 314f).
\textsuperscript{21}For example see the clauses from Frankfurt in 1666, Leipzig in 1688, Breslau, Brunswick in 1686 and 1715, or reports by Riccius, Marberger, and Koenigk: see Prausnitz (1928, p. 24).
\textsuperscript{22}Contemporary witnesses Fabiani and Merenda inform us of this fact: see Ehrenberg (1896, pp. 235f.), Neuhaus (1892 p. 40), and Brübach (1994, p. 318).
\textsuperscript{23}For example see the exchange orders of Frankfurt in 1611 and 1666, Lyon 1667, Bozen 1634, Brunswick 1686, Leipzig 1688 or the contemporary witnesses Franck and Phoonsen (Neuhaus 1892, p. 40; Brübach 1994, p. 314f).
to the debts that were settled by the mechanism.\footnote{24} Factually, this meant that the payment could not be reversed. This way the mechanism was protected from an ex post unravelling of settled debts.

In the final step the remaining debts would be settled. Controversial bills which had been rejected by the debtors were judged by the local court (Ehrenberg 1896, p. 236). The payment of outstanding debts could be in cash or by drawing new bills with promises on future payments within a fixed period.\footnote{25} In Augsburg in 1778 it was forbidden to draw new bills on one’s own name; only cash was allowed (Neuhaus 1892, p. 32). Strict regulations were applied in case a debtor did not offset the outstanding debts by the end of the clearing days: Scaccia and de Turri wrote that participants who did not pay were excluded and declared bankrupt (Endemann 1886, p. 178; Neuhaus 1892, p. 9; Prausnitz 1928, p. 19).\footnote{26} Because of this, Scacchia concluded that merchant bankers with positive positions could easily find new debtors for their money (Endemann 1886, p. 175). By the end of the Besancon fair a general balance was produced by the officials to control the clearing. However such a general balance was not in use at other clearing locations (Endemann 1886 p. 177; Biener 1859 p. 50).

### 2.3 Further Development: Exchange Banks, Endorsement, Stock Exchange and Clearing Clubs

From the late Middle Ages there were attempts to establish public exchange banks, whose major task was to serve as clearing houses. This goes back to the first efforts in Venice during the 14\textsuperscript{th} and 15\textsuperscript{th} centuries, with the implementation in 1578 of the Rialto Bank and the even earlier establishment of exchange banks in several towns in Spain: the first

\footnote{1994, p. 171, 374). \footnote{24See the Besancon regulations, the Statuti di Genoa of 1589, Lyon in 1667, Leipzig in 1688, and the Frankfurt legislation of 1611 and 1666: see Neuhaus (1892 p. 8, 40) and Brübach (1994, p. 171, 309). \footnote{25See the statutes of Besancon and Frankfurt in 1666, and the contemporary witnesses de Turri, Raumburger and Franck (Endemann 1886, p. 178; Neuhaus 1892, p. 8, Biener 1859, p. 47; Prausnitz 1928, p. 18, 30; Brübach, p. 175f). \footnote{26This practice was also already known in Lyon: see Vigne (1904, p. 126) and Haristoy (1906, p. 300). Additionally, in the regulations from 1667 strict payment periods were fixed: see Haristoy (1906, pp. 305f).}
was the bank in Barcelona in 1401 (Usher 1943; Braudel 1966; Lane and Mueller 1985). Exclusive rights to execute the clearing process were given to most of these banks. However, from the fragmented data we possess we know that merchants were not interested in using these centralized institutions to balance their positions. For instance, after the founding of the bank in Venice, a regulation was passed in 1593 that forbade the use of the rescontre procedure: all bills had to be brought to the bank. This regulation was later rescinded and decentralized clearing was allowed again (Lattes 1869, pp. 170f.).

North of the Alps, efforts to install public clearing locations did not start until the beginning of the 17th century. The first bank was the Wisselbank of Amsterdam in 1609 (Van Dillen 1934; van der Wee 1977; de Vries and van der Woude 1997), which was a copy of the bank in Venice. More banks in the Netherlands were established during the 17th century, as well in Germany at Hamburg in 1619 and Nuremberg in 1621 (Sieveking 1934; Fuchs 1954). The success of these institutions was rather mixed. While the Amsterdam exchange bank was a major success as a multifunctional institution, its role as the exclusive clearing bank had to be legally enforced again and again. Even when the service was inexpensive or, until 1683, free, merchants were not willing to use the bank for clearing (van Dillen 1964, p.398; Sneller 1940, pp. 157-160). When the bank started its activities, private cashiers were forbidden to conduct the clearing business, but by 1621 the decision was reversed. In 1700, merchants were forced for amounts greater than 600 and later 300 guilders to use the bank (van Velden 1933). In Hamburg in 1693, they were forced to use the bank for amounts greater than 400 marks. Merchants were not allowed to use the direct clearing procedure (Prausnitz 1928, p. 26). In Nuremberg, bank orders from 1654 and 1721 inform us that merchants were not allowed to use the Skontration to settle their bills, but had to bring them to the exchange bank. In addition, the orders from 1654 indicate that merchants were only willing to hand in the bills which they could not easily clear or that were from unbalanced positions (Neuhaus 1992, p. 12; Prausnitz, p. 1928). However, decentralized clearing was again allowed later. In important clearing locations such as Frankfurt or Leipzig no clearing banks were introduced,
and merchant bankers relied on the decentralized practice in use. A similar story can be told for Lyon, France’s main clearing location, where several efforts to install a central bank before the 18th century failed (Velde 2009). One of the main reasons was a lack of trust in the activities of the bank. This can also be documented in the case of the banks of Nuremberg (Fuchs 1954, pp. 55-58).

Another important innovation which influenced the Skontration was the more frequent use of the endorsement of bills during the 16th and 17th centuries. Tradability of bills goes back to the late Middle Ages and can be documented for the 14th century (Munro 1994; Opitz 1967, van der Wee 1993). Thus it was quite natural for the decentralized clearing locations to be confronted with transferred bills.

The reaction to these bills was different from place to place. In Antwerp, with the growth of economic activities during the early 16th century, the transfer of financial instruments became standard and the court confirmed the use of transferable bills several times during the late 16th and early 17th centuries (van der Wee 1977, pp. 149-63). In Frankfurt, the outcome was the opposite (Schneider 1991, Brübach 1994, p. 171-4): the court declared, probably based on requests by fair bankers in 1620 and 1635, that the usage of endorsed bills was forbidden since it brought abuse, disorder, and confusion into the clearing mechanism. The judgment was anulled in 1666, after vehement protests by merchants who were accustomed to the endorsement technique from the financial centers of Europe like Lyon and Antwerp. However, regulations in Lyon from 1678 restricted the use of transfered bills from the Italian towns of Venice and Bozen (Haristoy 1906, pp. 309f.). This was probably due to the fact that these cities did not allow for endorsement (Schneider 1991, pp. 152f.). Hence, the town officials from Lyon feared these bills would create disorder in the clearing mechanism. A similar story of resistance to the endorsement technique, as documented in Frankfurt, can be reported from Italian fair locations. The well-established bankers at these Italian fairs strongly opposed the use of endorsed bills since it reduced the need for exchange fairs (Prausnitz 1928, p. 21; Neuhaus 1892 p. 11, Schneider 1991, pp. 150-3); the fact that endorsed
bills reduced the need for exchange fairs was noted by Raumburger in 1723 (p. 671).

Similar decentralized clearing procedures also appeared in the first stock exchange markets. In Amsterdam, traders of stocks and futures started to meet informally during the 1630s, probably once every month to offset their credits and debts in the form of stocks and futures (Geldbloom and Jonker 2005, pp. 204f.). The Amsterdam exchange, from probably the 1650s onwards, established fixed clearing days every fourth month to settle mutual obligations. Furthermore, during the 18th century speculators in English funds created their own clearing days, also four times a year (Wilson 1941, p. 83ff.). Finally, in London, regular clearing days were introduced, probably during the 1740s; however, traders in London met informally earlier in coffeehouses to offset their liabilities (Dickson 1967, p. 507ff.).

Similarly, national clearing clubs were introduced during the late 18th and 19th centuries between banking houses. It has been documented that some merchants attempted to establish clearing clubs at exchange fairs. In Frankfurt in 1642, 23 merchant bankers decided to meet regularly to clear their liabilities. Similar efforts can already be documented by Italian bankers, who started small clearing fairs during the 1570s, when the Spanish bankruptcy led to uncertainty among the merchant bankers (Denzel 1994, pp. 316f).

Such agreements can also be found by banking houses to clear liabilities in the late 18th century. In London in 1775, several private bankers agreed to build a clearing club, the London Clearinghouse, to settle their liabilities. Before this agreement, bankers had to present checks to each other and to settle individually. Anecdotal evidence suggests that cash messengers from different banks first met informally over breakfast and offset the liabilities so that they only had to cash in unbalanced positions later on.

The clearing procedure used by the banking houses was very similar to the procedure known from the clearing fairs. Each morning during the week, the representatives of the banks met and presented the financial obligations. During the day the representatives had time to verify the debts presented. In case the obligations were approved, the representatives offset their credits and debts. The remaining open positions had to be paid in cash (Seyd
1872, pp. 52ff.). The idea of such a clearing club was taken up in Dublin in 1845, in New York in 1853, Paris and Vienna in 1872, and Berlin in 1884 (Koch 1983).

2.4 Efficiency of the Mechanism: Evidence from Merchant Books and Contemporary Witnesses

Having outlined the historical evolution of these clearing mechanisms, it remains to be seen whether these mechanisms were efficient in the sense that a high clearing rate could be achieved without paying cash or writing new bills. The existence of the mechanism across Europe and the preference by merchants to use this mechanism instead of central public exchange banks indicates that this institutional solution must have worked relatively well. In addition, the large numbers of bills cleared this way gives further evidence (Ehrenberg 1896) that these clearing fairs were successful. Use of these decentralized clearing mechanisms was discontinued only due to economic crises, the economic or political abuse of the institution by manipulating exchange rates or delaying payment, or bankruptcy of major merchant bankers.27

Another way to evaluate the rescontre procedure is to analyze one of the few scartefacci which was handed down. The ledger of the 1632 autumn fair in Frankfurt of the merchant banker Johan Bodeck gives detailed evidence of the clearing process at work (Brübach 1994, pp. 331ff.). Bodeck was involved with 77 people during the clearing process, which included reciprocal clearing, cycles, and chains. The merchants and bankers involved were from all over northern Europe: Clearing partners came from Aachen, Amsterdam, Augsburg, Breslau, Dresden, Frankfurt, Hamburg, Leipzig, Strasbourg, Ulm and from other unknown locations. During the clearing, he transacted the total amount of 135,000 florins, but only 4,000 florins were paid in cash. The rest were cleared via the rescontre procedure. Thus, more than 97% could be cleared by mutual settlement. Taking this single booklet as a sample, the clearing

procedure was very successful.

We may also evaluate the decentralized clearing procedures by analyzing the reports of contemporary witnesses. While an early witness, the merchant Davanzati, in 1560 commented on Besancon with some ridicule (Ehrenberg 1896, p. 229), writers of the 17th and 18th centuries, such as Sotus, Molina, Azorius, Scaccia, and de Turri admired the clearing process. Scaccia and de Turri write that nothing was rarer than money at these clearing fairs, as merchant bankers only brought money for their expenses (Endemann 1886, pp. 178f.; Neuhaus 1892, p. 10; Haristoy 1906, p. 300). Only Koenigk, Raumburger, and Marberger (Prausnitz 1928, p. 25) mentioned that with the emergence of exchange banks, they were surprised that not more places had a centralized exchange bank since they were better than the decentralized clearing procedure. Based on these reports, the mechanism must have been rather successful.

However if the mechanism was so successful and led to a high clearing rate, why did some of the regulations force merchants to participate and not to pay cash in advance? Why did the statutes mention the freedom of merchants over the order and the clearing partnerships in which they could be involved? If the mechanism worked that well why did merchants have preferences about clearing different chains and cycles? Could the endorsement of bills of exchange interfere with the proper functioning of the mechanism? We now turn to a theoretical model of the mechanism to answer these questions.

3 Model

We consider a model with a finite set $I$ of agents. Each agent $i \in I$ is endowed with a vector of debts $(d^i_j)_{i,j \in I} \geq 0$ where $d^i_j$ represents the debt that $j$ owes $i$ (and where $d^i_i \equiv 0$). An

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28De Rubys and Clairac argues in a similar way for the Lyon fairs; see Haristoy (1906, p. 300) and Gascon (1971, p. 248).
allocation \( \mathbf{D} \equiv (d^i_j)_{i,j \in I} \) represents the debt structure for the economy. We will call

\[
\nu^i(\mathbf{D}) \equiv \sum_{j \in I} (d^i_j + d^j_i)
\]

the net position of trader \( i \). If the net position of trader \( i \) is 0, we will say that his position is balanced; an allocation \( \mathbf{D} \) is balanced if it balanced for each agent \( i \in I \). Two allocations \( \mathbf{D} \) and \( \tilde{\mathbf{D}} \) are net position equivalent if \( \nu^i(\mathbf{D}) = \nu^i(\tilde{\mathbf{D}}) \) for each \( i \in I \).

We will only consider mechanisms that leave unchanged every trader’s net position. Hence, each trader’s utility is defined only over the total number of outstanding commitments he has. We define the utility of trader \( i \) for an allocation \( \mathbf{D} \) as

\[
u^i(\mathbf{D}) \equiv -\sum_{j \in I} \alpha^i_j d^i_j
\]

where \( \alpha^i_j \) is the disutility for agent \( i \) of holding a debt from agent \( j \); we let \( \alpha^i_i \equiv 0 \). An allocation \( \mathbf{D} \) is Pareto efficient if there does not exist a net position equivalent allocation \( \mathbf{\tilde{D}} \) such that \( u^i(\mathbf{\tilde{D}}) \geq u^i(\mathbf{D}) \) for all \( i \in I \) and \( u^i(\mathbf{\tilde{D}}) > u^i(\mathbf{D}) \) for some \( i \in I \). The definition of efficiency here is standard, given the constraint that no agent’s net position changes: it simply requires that there is no other allocation which makes all agents weakly better off while making some agent strictly better off.

The disutility for agent \( i \) of being a creditor for agent \( j \) depends, in general, on both the identity of \( i \) and the identity of \( j \): some agent \( j \) may be a good credit risk and hence \( \alpha^i_j \) is low, while some other agent \( k \) may be a poor credit risk, and hence \( \alpha^i_k \) will be quite large. Alternatively, \( j \) may live in a town nearby to \( i \), so that \( i \) expects to see him again soon, while \( k \) may live far away, and so \( i \) will not have a chance to collect from \( k \) for a long time.

A clearing process \( \varphi \) is a game which, for any allocation \( \mathbf{D} \) and strategies by agents, transforms \( \mathbf{D} \) into a net-position equivalent allocation \( \varphi(\mathbf{D}) \). A clearing process is efficient

\[\footnote{If the trader is risk-neutral, the coefficient \( \alpha^i_j \) can be interpreted as the perceived risk by agent \( i \) that agent \( j \) will not pay him back if he is unable to settle this debt at this fair.}^29\]
if there exists a subgame perfect Nash equilibrium of the game induced by the clearing process $\varphi$ such that $\varphi(D)$ is efficient.

### 3.1 Balanced Positions

We first consider the case where the original allocation $D$ is balanced. Note that in this case, for any balanced allocation $D$, the only net-position equivalent Pareto optimal allocation is $\tilde{D} \equiv \{0\}_{i,j \in I}$. Hence, for the clearing process $\varphi$ to be Pareto efficient, it must be the case that $\varphi(D) = \{0\}_{i,j \in I}$ for some subgame perfect Nash equilibrium of $\varphi$.

We first consider the cycle removal clearing process, denoted $\varphi$, which consists of two stages:

**Stage 1:** Each agent $i \in I$ reports a set of debts $\tilde{d}_i$.

**Stage 2:** We proceed in a series of steps, indexed by $t$. The algorithm is initialized by letting $d_{ij}(0) = \tilde{d}_{ij}$ for each $i, j \in I$. At each step $t$, identify a cycle of agents $(i_1, i_2, ..., i_Z, i_{Z+1} = i_1)$ such that for each $z \in \{1, ..., Z\}$, $d_{iz_{z+1}}^{t+1} > 0$, where each agent agrees to clear the cycle. Once such a cycle has been identified, let

$$d_{iz_{z+1}}^{t+1}(t) = d_{iz_{z+1}}^{t+1}(t) - \min_{y \in \{1, ..., Z\}} d_{iy_{y+1}}^{t}(t - 1)$$

for all $z \in \{1, ..., Z\}$.

If no such cycle exists, the algorithm terminates; otherwise, proceed to step $t + 1$.

**Theorem 1.** The cycle removal clearing process $\varphi$ is efficient. In particular, for every allocation $D$, there exists a subgame perfect Nash equilibrium of the game induced by $\varphi$ where

1. Each agent truthfully reports his debts, i.e., $\tilde{d}_i = d_i$, and
2. Each agent agrees to every proposed cycle.
This theorem shows that there is a natural Nash equilibrium of the cycle removal clearing process such that debts will be efficiently cleared. Recall that, as discussed in the previous section, if debtors maintained that a creditor’s claim was invalid, that claim would only be adjudicated after the clearing process. Hence, it is important that the clearing process not give incentives to debtors to declare claims are invalid simply to manipulate the outcome of the clearing process, and Theorem 1 shows that this is indeed the case. Moreover, each agent will be happy to participate in the mechanism, as participating can only improve the outcome for the agent.\textsuperscript{31}

Furthermore, the algorithm should be very efficient, as once a cycle has been found, agents will, in equilibrium, agree to clear that cycle. While this seems intuitive, this result depends on the balancedness of the net-positions of all agents: since everyone’s position is balanced, by clearing cycles we eventually reach the unique efficient allocation \{0\}_{i,j \in I}. As we demonstrate in the next section, when positions are unbalanced, some agents may have strict preferences over which cycles clear.

The essence of the proof of Theorem 1 lies in the fact that there is a unique efficient allocation. First, it is straightforward that if agents report truthfully and approve every cycle, then the cycle removal clearing process yields the unique efficient allocation \{0\}_{i,j \in I}. This follows as if some agent \(i\) has a debt, then the agent \(j\) to whom \(i\) owes a debt owes a debt in turn to some other agent \(k\), as the position of \(j\) is balanced. Since all positions are balanced, this logic may be iterated to find an agent \(\ell\) to whom \(k\) owes a debt. Hence, since there are a finite number \(|I|\) of agents, at some point we must choose an agent that we have chosen before, and this forms a cycle, which is then removed. It then follows that each agent will wish to approve every cycle, as the result of the clearing process will be to leave that agent with no debtors. Finally, this implies that in the debt revelation stage each agent will wish to truthfully reveal his debts. As a corollary, this implies that each agent will wish

\textsuperscript{31}The Nash equilibrium identified in Theorem 1 is not unique; for instance, there exists a Nash equilibrium where every agent rejects every cycle. However, the equilibrium identified in Theorem 1 is likely to be focal, as it guarantees each agent his highest possible payoff.
to participate in the clearing process, since non-participation is equivalent to an agent not agreeing to participate in any cycle.

3.2 Unbalanced Positions

However, if agents do not have balanced positions (and are not required to pay remaining debts in cash), the nice properties of the cycle removal clearing process no longer hold. First, the cycle removal clearing process will not necessarily result in an efficient outcome, even if all agents report their debts truthfully and agree to every cycle proposed. Consider the example in Figure 1, where we use the convention that an arrow from $i$ to $j$ denotes that $d^j_i = 100$. Then the cycle removal clearing process may remove the cycle $(i, j, n, m, i)$, after which no other cycles can be removed; call this allocation $D'$. However, the cycle removal clearing process could also remove the cycle $(i, k, m, i)$ first, and then remove the only remaining cycle $(j, n, \ell, j)$; call this allocation $D''$. Hence, if $\alpha^j_i < \alpha^\ell_\ell$ and $\alpha^m_n < \alpha^m_\ell$, then every agent prefers $D''$ to $D'$: both $k$ and $\ell$ no longer have outstanding credits under the latter outcome, and both $j$ and $m$ have debtors they prefer.\footnote{Note that the fact that we wish to remove the smaller cycles first in this example does not generalize. In particular, note that if we replaced $k$ with a chain of several agents, it would be better to remove the larger cycle first.} Moreover, for other specifications of preferences, agents may block some cycles from clearing as they prefer a particular final allocation to another.

An alternative to using clearing cycles is using clearing chains. We define a \textit{chain} as a sequence of three agents $(i, j, k)$ such that $d^j_i > 0$ and $d^k_j > 0$. Given an allocation $D$ and...
a chain \((i, j, k)\), we say that we obtain the allocation \(f(D, (i, j, k))\) by clearing the chain 
\((i, j, k)\) from the allocation \(D\), where

\[
f_m^n(D, (i, j, k)) = \begin{cases} 
D_m^n - \min\{d_i^j, d_j^k\} & \text{if } n = j \text{ and } m = i \\
D_m^n - \min\{d_i^j, d_k^j\} & \text{if } n = k \text{ and } m = j \\
D_m^n + \min\{d_i^j, d_k^j\} & \text{if } n = k, m = i, \text{ and } i \neq k \\
D_m^n & \text{otherwise.}
\end{cases}
\]

Note that when a clearing a chain involves clearing mutual debts between two agents, the
two debts are simply cancelled.

As chains were a common clearing mechanism used in fairs where agents could have
unbalanced positions, a natural question is whether the use of chains is enough to obtain a
Pareto efficient allocation. In general, the answer is no. Consider Fig. 2, and suppose that
\(\alpha_k^j > \alpha_k^j\) and \(\alpha^\ell_i < \alpha^\ell_j\). Then to reach the Pareto efficient allocation requires a “swap” of
debts, so that \(k\) takes on the debt of \(j\) and \(\ell\) takes on the debt of \(i\).

However, historical evidence tells us that a creditor’s disutility from holding a debt was
primarily determined by the creditworthiness of the debtor, not idiosyncratic factors related
to both the debtor and creditor. Hence, it is reasonable to assume that \(\alpha_i^j \equiv \alpha_i^j\) for all
\(i, j \in I\). Note that, in our example above, this property did not hold: \(k\) preferred \(j\) as a
debtor while \(\ell\) preferred \(i\) as a debtor.

However, this same historical evidence warns us that agents will be unwilling to engage
in chain clearing if it results in the swapping of a “good debtor with a bad debtor”. Hence,
we shall say that a chain \((i, j, k)\) is \textit{locally inefficient} if \(\alpha^k_i < \alpha^k_j\). In other words, a chain \((i, j, k)\) is locally inefficient if transforming the debt \(i\) owes \(j\) and the debt \(j\) owes \(k\) into a debt that \(i\) owes \(k\) makes \(k\) better off. Hence, by clearing the chain \((i, j, k)\), we Pareto improve the allocation—note that \(i\) is indifferent over clearing \((i, j, k)\) and \(j\) is strictly better off as he no longer holds a debt from \(i\).

Clearing locally inefficient chains was a natural choice for a mechanism to clear debts, as finding such chains only requires “local” knowledge. In particular, if \((i, j, k)\) is a locally inefficient chain, then agent \(j\) is likely to be aware of its existence as \(j\) is aware of both his debtors and creditors. Furthermore, \(j\) has strong incentives to bring this chain to the attention of \(i\) and \(k\), as the clearing of this chain clearly benefits \(j\) since it reduces the debt of agent \(i\) to \(j\).

Unfortunately, it is not the case that clearing locally inefficient chains is sufficient to obtain a Pareto efficient allocation, as demonstrated in Fig. 3. Consider Fig. 3a, where \(\alpha_i = 5, \alpha_j = 3,\) and \(\alpha_k = \alpha_l = 2\). It is clear that there are no locally inefficient chains. However, the allocation depicted in Fig. 3a is not Pareto optimal, as the allocation depicted in Fig. 3b is a Pareto improvement. This example demonstrates that it may be possible to “trade” pairs of debts in such a way as to make both agents better off: in this example, \(l\) is better off as he prefers a 200 florin debt from \(j\) rather than 100 from \(i\) and 100 from \(k\), and \(k\) is better off as he now only has a single debt (from agent \(i\))—the other debt “traded” to \(k\) was a debt from \(k\), and hence was cancelled.

However, this counterexample relies on the existence of three types of debtors; if debtors are either strictly “good” or “bad”, then it is sufficient to clear locally inefficient chains. The following theorem shows that, when the disutility of a debt is determined solely by the identity of the debtor, it is sufficient to simply clear locally inefficient chains in order to obtain a Pareto efficient allocation.

\begin{theorem}
Suppose that \(\alpha_i^j \equiv \alpha_i \equiv \hat{\alpha} \) or \(\alpha_i^j \equiv \alpha_i \equiv \check{\alpha}\) for all \(i, j \in I\) such that \(i \neq j\). \hfill 33
\end{theorem}

\textsuperscript{33}Recall that \(\alpha^k_i \equiv 0\); this ensures that allocations where both \(i\) owes \(j\) and \(j\) owes \(i\) are locally inefficient.
for some $\hat{\alpha}$ and $\check{\alpha}$. Then the allocation $D$ is Pareto inefficient if and only if $D$ has a locally inefficient chain.

Theorem 2 states that for any Pareto inefficient allocation, there exists a chain which can be removed and, moreover, removing that chain results in a Pareto improvement for the agents involved in that chain. Furthermore, when the locally inefficient chain $(i,j,k)$ is cleared, the resulting allocation sets either $d^j_i = 0$ or $d^k_j = 0$. Hence, the number of non-zero elements of a Pareto inefficient allocation $D$ with the locally inefficient chain $(i,j,k)$ is strictly less than the number of non-zero elements of the allocation $f(D,(i,j,k))$. This observation, combined with Theorem 2, leads to the following corollary.

**Corollary 1.** Suppose that $\alpha^j_i \equiv \alpha_i \equiv \hat{\alpha}$ or $\alpha^j_i \equiv \alpha_i \equiv \check{\alpha}$ for all $i,j \in I$ such that $i \neq j$ for some $\hat{\alpha}$ and $\check{\alpha}$. From any allocation $D$, a Pareto efficient allocation $D'$ can be obtained from $D$ by clearing a finite number of locally inefficient chains.

Hence, we can expect a mechanism which only clears locally inefficient chains to eventually obtain a Pareto efficient allocation.

As an example, consider the debt structure $D$ as depicted in Fig. 1 and suppose that $\alpha_i = \alpha_j = \alpha_k = \hat{\alpha}$ and $\alpha_\ell = \alpha_m = \alpha_n = \check{\alpha}$ where $\hat{\alpha} > \check{\alpha}$, i.e., that $i$, $j$, and $k$ are low quality debtors and $\ell$, $m$, and $n$ are high quality debtors. In this case, the chain $(i,k,m)$ is locally inefficient, as $i$ no worse a debtor than $k$. The allocation $D(1) = f(D,(i,k,m))$ is a Pareto improvement, as both $k$ and $m$ are weakly better off, with $k$ being strictly
Figure 4: Clearing of locally inefficient chains. An arrow from $i$ to $j$ represents $d_{ij}^j = 100$.

better off: this new allocation is depicted in Fig. 4a. Now, since $i$ owes $m$ and $m$ owes $i$, a new locally inefficient chain $(i, m, i)$ exists: clearing this chain results in the allocation $D(2) = f(D(1), (i, m, i))$, depicted in Fig. 4b. Note that clearing these two chains was equivalent to clearing the $(i, k, m, i)$ cycle. The chain $(i, j, n)$ is also locally inefficient, and we may clear this chain to obtain the allocation $D(3) = f(D(2), (i, j, n))$, depicted in Fig. 4c. Finally, the chain $(n, \ell, j)$ is also locally inefficient, and by clearing this chain we obtain the allocation $D(4) = f(D(3), (n, \ell, j))$, depicted in Fig. 4d.

This final allocation $D(4)$ is Pareto efficient, as the chains $(i, n, j)$ and $(i, n, m)$ are not locally inefficient: both $j$ and $m$ prefer $n$ as a debtor to $i$. However, note that $D(4)$ is not the unique Pareto efficient allocation that is net position equivalent to $D$; for instance, the allocation $D'$ where the only non-zero entries are $d_{ij}^j = 100$ and $d_{mn}^n = 100$ is also net-position equivalent to $D$ and Pareto efficient. This non-uniqueness implies that there may be strategic considerations when deciding which chains to clear: agent $n$ would be better off if, in the third step of the clearing process, the locally inefficient chain $(\ell, j, n)$ was cleared instead of $(i, j, n)$; this would allow the chain $(\ell, n, \ell)$ to be cleared at a later step. In this case, the final allocation would be $D'$. However, Theorem 2 ensures that at each step, if we are not at a
Pareto efficient allocation, there will exist at least one locally inefficient chain; furthermore, some locally inefficient chain \((i, j, k)\) will clear, as \(i, j,\) and \(k\) will always be better off (in a myopic sense) by clearing that chain, and so will be strictly worse off if the clearing process ends. (Note that at each step of the clearing process, every agent is weakly better off; hence no agent will wish for the clearing process to end until at least every locally inefficient chain where he is a creditor is cleared.)

Furthermore, note that since it is only required to clear locally inefficient chains, it is not surprising that merchants were free to refuse to agree to the removal of a chain. So long as merchants agreed to clear some locally inefficient chain at any Pareto inefficient allocation, the allocation would evolve towards a Pareto efficient allocation. This policy of allowing refusal was also clearly beneficial to the fair itself: a merchant could attend the fair knowing that he would not be forced to take on any odious debts.

In particular, no merchant would wish to avoid the clearing process, since he could always retain his current set of debts and credits by simply refusing to clear chains. Furthermore, by only agreeing to clear locally inefficient chains, a merchant could guarantee that at every step of the process his position became (weakly) better. This also implies that merchants would be happy to provide accurate information to the mechanism. By doing so, a merchant could more easily identify locally inefficient chains that he was party to, and clearing such chains would improve the position of the merchant.

These results imply that simply removing locally inefficient chains is sufficient to obtain an efficacious reordering of debts so long as there is not too much dispersion in the quality of debtors. In particular, a Pareto efficient allocation can be achieved in the special case where there are only two debtor types.

More generally, there is a more restrictive notion of efficiency for which the chain removal mechanism is sufficient, even when debtors are of different types. We say that an allocation is constrained Pareto efficient if there does not exist a sequence of agents \(i_1, i_2, \ldots, i_Z, i_{Z+1} = j\) such that \(d_{i_{Z+1}}^{i_{Z+1}} > 0\) for all \(i = 1, \ldots, Z\) and \(\alpha_{i_1} \leq \alpha_{i_Z}\); in other words, an allocation is
constrained Pareto efficient if there does exist a sequence of debtors and a final creditor \( j \) such that the the first debtor is a better credit risk than the last debtor. For such an allocation, one could obtain a Pareto superior allocation by canceling the intervening debts and moving the debt of \( i_1 \) to \( j \). Constrained Pareto efficiency is a weaker notion of efficiency as it does not consider the possibility to "trade" sets of debts. Figure 3a is an example of a allocation that is not Pareto efficient but is constrained Pareto efficient.

It is straightforward to show that if an allocation is not constrained Pareto efficient, then that allocation includes a locally inefficient chain: that implies the following result.

**Theorem 3.** Suppose that \( \alpha_j^i \equiv \alpha_i \) for all \( i \in I \). Then, from any allocation \( D \), a constrained Pareto efficient allocation \( \tilde{D} \) can be obtained from \( D \) by clearing a finite number of locally inefficient chains.

4 Discussion of the Results

4.1 The Mechanism in Its Historical and Economic Context

We now consider these theoretical results in their economic and historical context. Theorems 1 and 2 predict efficient outcomes when agents are willing to clear cycles (when positions are balanced) and locally inefficient chains (when positions are unbalanced). Hence, we can rationalize the decentralized clearing procedures used, as their decentralized nature did not impede their efficiency. In order for the clearing process to be successful, it was sufficient that the merchants met at a specific time and worked through the various chains and cycles: no centralized authority was necessary. These meetings, along with the decentralized clearing procedures, are, as we have documented, the customary practice during the late Middle Ages; later, during the 16th and 17th century, these practices were codified into the exchange laws. Thus theory confirms the historically documented long run success of the rescontre procedure by contemporary witnesses.

However, the legal regulations also reflect the challenges towns faced over the centuries
related to the use of the clearing mechanism. For instance, one type of regulation deals with the separation of questionable bills of exchange. It is crucial that no questionable bills are transferred since merchants may then unknowingly take on disputed debts. Hence, it is quite natural that legislation governing debt clearing required that only acknowledged debts could be exchanged. The fear of using endorsed bills can also be understood by considering this factor: if the enforcement of these liabilities was not clear, merchants might be wary of clearing, and hence using endorsed bills might hinder the proper functioning of the mechanism, as judged by the Frankfurt court.

Another question faced by the designers of these clearing mechanisms was whether to require that merchants’ positions were balanced. When positions are balanced, our results show that a merchant should always willing to participate in clearing cycles. This logic helps us to understand the regulations on balancing positions at the Besancon and Lyon fairs, i.e., why these early fairs insisted on the payment of cash or the satisfactory drawing of new bills at the end of every fair. These regulations increased the willingness of the wealthy merchants to participate in the clearing mechanism, since this guaranteed that positions would effectively be balanced. An alternative regulation to achieve the same goal of balanced positions is to ask each fairgoer for an ex ante deposit; this method was eventually used in Besancon.

The requirement of balanced positions is also related to contemporary work on payment mechanisms: Studies of modern payment mechanisms have also identified that the enforcement of debt positions at the end of a dynamic process is critical to enable the granting of credit earlier and for more heterogeneous debtors (Freeman 1996; Kiyotaki and Moore 2000; Mills 2004). Furthermore, these studies have argued that balancedness in net clearing mechanisms reduces the incentive to default (Kahn et al. 2003).

However, when positions are unbalanced and enforcement policies are limited (as discussed above), the above logic no longer holds. If a merchant expects many unbalanced

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34 A merchant unable to comply with this regulation was declared bankrupt and excluded from any future clearing processes.
positions at the end, such as when many merchant bankers will not be able to balance their
debts, he may use only clearing cycles. By doing so, the merchant acquires no new debtors:
even a bankrupt clearing partner will not harm him, as the regulations clearly state that
merchants are not responsible for debts cleared at the fair. However, as we have demon-
strated, a merchant using only cycles might have preferences over the order of the clearing
process, as this can determine the outcome and his personal clearing rate. Moreover, when
positions are not balanced, cycles are not sufficient to obtain efficient outcomes, even in
relatively simple economies.

However, when a merchant uses chains in addition to cycles, he must be careful not
to pick a new partner who, as is written in the merchant booklet, turns out to be a bad
debtor. Consequently, we can rationalize the rules in the exchange regulations which give
each merchant the freedom to participate in any cycle or chain, and to choose in which
ordering he liked to clear. However, in such settings, we have shown that the efficiency
of the clearing process depends crucially on the degree of heterogeneity among debtors.
In this case, other historically observed regulations become relevant, as they reduce this
heterogeneity. For instance, one way to reduce heterogeneity is to create clearing clubs and
restrict them to members only; this way bad or uncertain debtors can be excluded. Another
option was to create sufficiently high deposits ex ante ensuring a minimal level of solvency
for merchants and hence reducing this heterogeneity.

Scholars who study modern payment mechanisms have also emphasized the role of re-
stricted access. In addition, they have stressed the role of alternative institutions, such as
banks; these alternative solutions can provide clearing services to less trustworthy customers
as they have lower monitoring costs (Kahn and Roberds 2002). Similar patterns can be
seen in the historical record of the period that we study. Merchant bankers offered bills
of exchange to a wide variety of local customers for whom the merchant could accurately
predict their creditworthiness. However, the creditworthiness of these customers was un-
known to merchants from other parts of Europe, and so these customers (and hence debts
involving them) could not be used in the clearing process. In comparison to their “local” customers, merchant bankers with factories all over Europe could be more easily identified as “good” types. For instance, a bill of exchange from the Fugger company was well-regarded throughout Europe (Kellenbenz 1990).

Thus, we might expect that a self-selection took place where a core of well-known merchant bankers met repeatedly at the same fair and could be generally identified as good types. In addition, some other agents participated who had only a limited reputation, and hence were not considered as creditworthy as the well-known merchants bankers. Nevertheless, these agents could, as outlined in the theory section, successfully clear a large portion of their debts. A core of returning merchant bankers can be documented at the Italian clearing fairs (Da Silva 1969). Such a process also took place in Antwerp, where a larger group of so-called cashiers performed the clearing procedure for a wide range of customers.35

We would also like to explain why, in some regulations, such as those of Bozen or Leipzig, merchants were forced to participate in the clearing procedure. One explanation is to interpret them as facilitating coordination: Merchants needed a focal time and location in order to coordinate for clearing purposes. Another possibility is that some participants would have preferred to clear in smaller groups (perhaps at a different time or place). However, this would have reduced the positive externalities for other potential participants, which was not in the interest of the fair organizers: the fair organizers preferred keeping as many debtor-creditor pairs as possible, as this attracted other merchants to the fair.

We have seen that the mechanism will, in many settings, produce an efficient clearing. However, there are a few instances in the historical record where the mechanism broke down, such as in the second half of the 16th century in Lyon. As demonstrated by Theorem 2, if there are both good and bad debtors, a simple increase in the number of bad debtors would not necessarily influence the clearing efficiency. However, an increase in the heterogeneity of debtors would reduce the clearing efficiency, possibly leaving participants with long, unre-

35This information is based on the unpublished dissertation manuscript of Daniel Velinov, who was so kind to gave us insights into the clearing markets of early 17th century Antwerp.
solved clearing chains (due to the hierarchy of preferences over debtors). While how much clearing efficiency will be reduced by such heterogeneity cannot be predicted ex ante since it depends on the complexity of the debt relationships, the example in Section 3 illustrates that such heterogeneity can significantly reduce clearing efficiency. Thus, a shock which creates greater heterogeneity among debtors will likely reduce the efficiency of the clearing mechanism, lowering the incentives for merchants to participate in the mechanism.

4.2 Alternative Institutions: Cash, Clearing Banks

The theoretically demonstrated efficiency and strategic properties provide an explanation for the empirical success of the rescontre procedure, and may also explain the limited success of alternative clearing forms. For instance, instead of a net clearing mechanism merchants could have used a gross clearing mechanism, such as simply paying cash. In this case, however, a merchant must carefully consider how much liquidity he needs to bring and how much cash he could expect from debtors paying back their debts. Bringing gold and silver coins to market was not only costly, but was also dangerous. In addition, in such a scenario we would also expect gridlock effects (Kahn et al. 2003) where merchants postpone their payments during the fair time until they received payments from their debtors to reduce the amount of liquidity they had to bring to the market. Thus, requiring cash payments at these fairs would have generated high liquidity costs.

A more sophisticated gross settlement mechanism would have been an exchange bank; such a bank could provide customers with sufficient liquidity and deal with the different types (and related risks) of creditors and debtors directly. An exchange bank can always achieve an efficient clearing because it works as an intermediary who takes in and hands out compensation directly. However, if merchants expected efficient or near-efficient clearing using the decentralized clearing procedure, then other factors were likely to determine whether exchange banks could replace the rescontre procedure. Three key issues are liquidity, confidentiality, and exchange rates, discussed below.
First, a public exchange bank must provide liquidity, which is costly, and accept transaction risk when providing bills of exchange. Liquidity could have come either directly from the bank itself or from holding deposits from merchants as customers. In the former case, the liquidity must be provided by the local government, and sufficient ex ante investment cannot be documented for early banks. In the latter case, the liquidity would be provided by the customers in form of deposits or paying back their debts from the bills directly to the bank. This would require customers to trust the public bank; however, foreign customers were unlikely to do so due to the limited enforcement mechanisms across state and city-state borders. By contrast, local merchants may have been willing to trust in local institutions. However, historical evidence does not reveal such behavior in general. There is evidence of early private banks in larger cities where locals were willing to deposit money, but many of these banks failed during the late medieval/early modern period (Usher 1943), exactly when clearing procedures became important. Furthermore, merchants were not willing to hand in their bills to the later public exchange banks, as discussed in Section 2.3. In addition, many exchange banks were chronically short of money (Fuchs 1954, pp. 55-8). Moreover, town officials, who governed these banks, were constantly passing new regulations which limited rights of their customers (Fuchs 1954, pp. 55-8). Hence, the unwillingness of merchants to deposit their bills of exchange into these banks comes as no surprise.\footnote{During the early 18\textsuperscript{th} century, Marberger (1717, pp. 98f.) comments on this problem (Schneider 1991, p. 142).}

Second, by using these new public institutions, merchants would have to reveal certain private business information to these banks, and in many cases the merchants probably did not wish to do so (Velde 2009). For example, in Nuremberg, merchants took to the bank bills which they could not clear easily; for these bills, they preferred to rely on the enforcement of the bank (Fuchs 1954, pp. 55-8). However, they handed in only these bills; decentralized clearing for the other bills was still preferred. Thus merchants preferred to use the decentralized clearing procedures and used the bank in selective (risky) cases only.

Third, the setting of exchange rates differed across the two mechanisms. At fairs,
change rates were negotiated by representatives of different merchant groups. By contrast, in dealing with exchange banks, merchants had to rely on the decision makers of the bank. Thus, based on this bargaining argument the participants of the fairs could expect better exchange rates at exchange fairs than at the public banks.

Thus, public exchange banks were unappealing to merchants for a number of reasons. For these reasons, much of the comparative analysis in the modern payments literature between gross and net clearing mechanisms is only of limited application: Exchange banks were unable to achieve the liquidity necessary for clearing, and, moreover, many of the benefits of the gross clearing procedures could not be realized because of the unwillingness of merchants to trust the bank due to issues of confidentiality and exchange rates.

We do not examine the later success of the exchange banks. The success of these new centralized institutions may have relied on effective legal enforcement of contracts and the ability of these banks to provide liquidity, neither of which is present during the majority of the historical period that we study. For further analysis on these topic see, for example, Quinn and Roberds (2006, 2007).

4.3 Impact on Trade

Trade has been identified as key for pre-industrial growth (Lopez 1976; Cipolla 1993; Epstein 2000; Greif 2006). Wholesale trade relied on the issuance of credit (de Roover 1953; Denzel 2008; Boerner and Ritschl 2009, 2011). However, the political fragmentation of Europe at that time meant that cross-border enforcement mechanisms were required in order to ensure the proper functioning of credit markets.

The willingness of merchants to engage in credit transactions relied on reputation or community responsibility. The latter institution allows for collective liability among community members, where all members of a city-state are collectively liable for the actions of its members. This allowed merchants to easily enter new markets, as other merchants could trust them due to this institution. Hence, the issuance of credit proliferated, and, moreover,
the vast majority of these credit transactions were executed as bills of exchange or similar financial instruments.

However, a general market for tradable bills was not yet established. Instead, the rescontre and other similar procedures developed. On the one hand, reputation-based business relationships and relationships backed up by community liability could be cleared with chains and cycles, while, on the other hand, credit relationships with merchants of unknown quality could also be incorporated into the clearing mechanism. Hence, different types of creditors could participate: Bills of exchange issued by good creditors were fully transferable during the procedure, while bills issued by others were only conditionally transferable or incorporated into clearing cycles.

Thus, the rescontre procedure is an institutional solution for resolving liabilities in an economic world in transition from a reputation-based system to a more anonymous trade-based system with limited centralized outside enforcement (Greif 2002, 2006). Not only was the procedure efficient for resolving debts, but its self-organizing nature required little in the way of a legal institutional framework. Thus, these clearing mechanisms made the bills more valuable for trade as they greatly reduced the liquidity and enforcement costs of using bills; moreover, these procedures imposed little additional costs on pre-industrial institutions.

5 Conclusion

This paper examines the evolution of a decentralized multilateral debt clearing algorithm for non- or limited-tradable debts during the late medieval and early modern period in Europe. We constructed a simple theoretical model of the mechanism and showed that the mechanism could lead to very high clearing rates without strong outside enforcement needs, helping us to understand why this mechanism was so commonly used throughout Europe for debt clearing.

We first examined the history of the mechanism. We analyzed the early emergence of this mechanism on Late Medieval markets and fairs and showed how the clearing algorithm
developed into a standardized procedure embedded in a legal framework during the early modern period. Moreover, we demonstrated the use of this clearing procedure throughout Europe. Finally, we identified the main elements of the procedure, based on legal sources and writings of contemporary witnesses.

We then constructed a model of the mechanism and identified the equilibrium outcomes and related strategic behavior of participants. We showed that the functioning of the mechanism depends on the balance of positions of merchants ex ante and the use of clearing cycles or clearing chains. If positions are balanced, then efficient clearing is always achievable with clearing cycles. If positions are unbalanced, it is necessary to use not only cycles but also clearing chains. Clearing a chain involves debt transfers which must be approved by all parties affected. We show that preferences over debtors influence the clearing behavior and type of efficiency which can be achieved. When there are only good and bad debtors, the clearing chains will lead to an efficient outcome in equilibrium. However, if more types are introduced, long chains may not be resolved due to the differences in debtor quality. Furthermore, whereas in the case of balanced positions the algorithm is deterministic, in case of unbalanced positions it is not. In particular, the clearing order may affect the distribution of debt transfers among merchants ex post.

Finally, we discussed these theoretical properties in their economic and historical context. We showed that the rescontre procedure was effective in the settings in which it was used; moreover, a number of regulations, such as requirements for participation, facilitated the proper functioning of the procedure. We also compared the rescontre procedure to alternative gross clearing options found in the historical literature, showing that gross clearing mechanisms, such as cash and clearing banks, could not be successfully implemented since the legal enforcement environment was too weak or costs were too high. Finally, we briefly discussed the benefits for trade of the rescontre procedure. In sum, this self-organizing procedure could achieve efficient clearing even in a world in a world with strong frictions and limited enforcement options.
These results also open up new questions for future research. The results here analyze the microeconomic properties of these clearing mechanisms. However, interesting questions related to trade and credit flows on a macroeconomic level could also be discussed. It is well known that all these clearing locations were linked and embedded in a common network: the evolution and robustness of this network should be further investigated.

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7 Appendix

7.1 Proof of Theorem 1

To prove the theorem, we proceed in two steps.

First, we show that if every agent reports truthfully, the mechanism will produce the outcome \( \tilde{D} = \{0\}_{i,j \in I} \) for any initial allocation \( D \). Suppose not. Then there exists at least one pair \( i_1, i_2 \) such that \( d_{i_1 i_2}^{i_2} > 0 \). But since the position of \( i_2 \) is balanced, there must exist an \( i_3 \) such that \( d_{i_2 i_3}^{i_3} > 0 \). In general, if \( d_{i_{k-1} i_k}^{i_k} > 0 \), there must exist an agent \( i_{k+1} \) such that \( d_{i_k i_{k+1}}^{i_{k+1}} > 0 \), as the position of \( i_k \) is balanced. But since there are only a finite number of agents, there must exist a \( \hat{k} \) such that \( \hat{k} = k \) for some \( k < \hat{k} \). But then a cycle exists, contradicting the assumption that all cycles have been removed.

Second, the result above implies that each agent obtains a payoff of 0 if every agent reports truthfully and agrees to every cycle. But 0 is the optimal payoff for each agent. Hence it is a best response for each agent to report truthfully and agree to every cycle if all other agents do so, and hence it a Nash equilibrium for each agent to report truthfully and agree to every cycle.

7.2 Proof of Theorem 2

If \( D \) has a locally inefficient chain \( (i,j,k) \), then \( f(D, (i,j,k)) \) (where \( f \) is as defined in the text) is a Pareto improvement, and so \( D \) is not Pareto efficient.

To prove the other direction, note the it is straightforward that any Pareto inefficient allocation contains a locally inefficient chain if \( |I| \leq 2 \). Suppose, as an inductive hypothesis,
that the result is true for all $I$ such that $|I| \leq N$ and consider a set $I$ such that $|I| = N + 1$. There are two cases:

**Case 1:** For all $i \in I$, $\alpha_i = \hat{\alpha}$. In this case, it immediately follows that any chain is locally inefficient.

Now suppose that $\mathbf{D}$ is Pareto inefficient. Then there must exist at least one agent $i$ who is both a creditor and a debtor, i.e., for whom there exist agents $h$ and $j$ such that $d_{h}^{i} > 0$ and $d_{j}^{i} > 0$. But then $(h, i, j)$ is a chain, and since all chains in this case are locally inefficient, we are done.

**Case 2:** For some $i \in I$, $\alpha_i = \check{\alpha}$. If no locally inefficient chain exists, then either $i$ is only a creditor or only a debtor, i.e., either $d_{j}^{i} = 0$ for all $j \in I$ or $d_{j}^{i} = 0$ for all $j \in I$; otherwise, there exists a locally inefficient chain $(h, i, j)$—note that $j$ must be weakly better off if such a chain is cleared, since if $i \in \check{I}$, we have that $\alpha_{h}^{j} = \alpha_{i} \geq \alpha_{h} = \alpha_{h}^{j}$.

Now suppose that $\mathbf{D}$ is Pareto inefficient. But if $i$ is only a creditor or only a debtor, then it must be the case that the allocation $\{d_{j}^{i}\}_{j,k \in I \setminus \{i\}}$ is Pareto inefficient as well. But then the allocation $\{d_{j}^{i}\}_{j,k \in I \setminus \{i\}}$ has a locally inefficient chain by the inductive hypothesis, and so we are done.

### 7.3 Proof of Theorem 3

Consider any allocation which is not constrained Pareto efficient. Then there exists a sequence of agents $i_1, \ldots, i_Z, j$ such that $d_{i_{z+1}}^{i_{z}} > 0$ and $\alpha_{i_{z}} \leq \alpha_{i_{z}}$. For $(i_{Z-1}, i_{Z}, j)$ to not be a locally inefficient chain, it must be the case that $\alpha_{i_{Z-1}} > \alpha_{i_{Z}}$. Similarly, it must be the case that $\alpha_{i_{Z-2}} > \alpha_{i_{Z-1}}$, $\alpha_{i_{Z-3}} > \alpha_{i_{Z-2}}, \ldots, \alpha_{i_{1}} > \alpha_{i_{2}}$. But then we have that $\alpha_{i_{1}} > \alpha_{i_{Z}}$ by transitivity; hence, no such sequence can exist if there does not exist a locally inefficient chain.

Hence, for any constrained Pareto inefficient allocation one can find a locally inefficient chain. Since clearing a locally inefficient chain always increase the number of zero entries in
the allocation matrix by at least one, clearing locally inefficient chains results in a Pareto inefficient allocation after a finite number of steps.