Heterogenous risk aversion, income inequality and tax policies in good times and bad times

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The role of risk aversion in the representative-agent model

- The representative-agent model
  - The degree of risk aversion affects the consumption growth

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\beta}$$

For instance, when $r>\rho$, the large degree of risk aversion reduces the rate of consumption growth

  - Instead, in the steady state the risk parameter does not affect the capital stock. $r^* = \rho$
- This is the case of aggregate capital stock. How about the individual capital stock?

The relationship between risk aversion parameter and individual capital

- Use the Japanese data collected by Osaka University
  - Total number of data is about 4000.
- The data of dependent variable:

  Stock: The present appraised value of all housing and properties which your entire household owns

  Flow: Salary or hourly wage for 2009, and the annual earned income before taxes with bonuses included (and also business income) for 2009.
- The data of independent variable:
  - Risk aversion parameter, time preference rate, age, school background, sex
  - I simultaneously measure the degrees of individual risk aversion as well as time preference in the switching analysis where I make use of the CRRA form or CARA form of utility function.

The questions which I use in the switching analysis

Let's assume there is **an instant lottery** with a 50% chance of winning 20,000 yen and a 50% chance of winning nothing. If the lottery ticket is sold as listed below, would you purchase a ticket? You may circle Option "A" to purchase the lottery ticket, or Option "B" not to purchase the lottery ticket. Please indicate which option you prefer for all 8 ticket prices.

	Which ONE do you prefer?	
Price of the ticket	buy the ticket	DO NOT buy the ticket
200	A	B
500	A	B
1000	A	B
2000	A	B
4000	A	B
7000	A	B
10000	A	B
15000	A	B

Let's assume you have a lottery ticket with a 50% chance of winning 20000 yen and a 50 % chance of winning nothing. Even if you win, **you can only receive the prize money one week from now**. You can either keep the lottery ticket yourself or you can sell it for cast immediately. If there is someone willing to buy this ticket from you right now for the prices listed below, would you sell the ticket or would you keep it knowing you have a chance to win the 20000 yen? You may circle Option "A" to sell the lottery ticket, or Option "B" not to sell the lottery ticket and keep it for yourself. Please indicate which option you prefer for all 8 ticket prices.

	Which ONE do you prefer?	
Price of the ticket	Sell the ticket	DO NOT sell the ticket
1000	A	B
2000	A	B
4000	A	B
7000	A	B
9000	A	B
10000	A	B
11000	A	B
15000	A	B

The relationship between risk aversion and assets

- As shown in an attached pdf, the degree of risk aversion strongly affects the individual assets regardless of the flow or the stock.
  - As larger the degree of risk aversion is, the smaller the level of individual asset is.
- From this result, I examine the following:

① Construct the dynamic model with n-persons whose risk parameters are different among agents.

② Examine the relationship between risk aversion and individual capital stock

③ Confirm the effects of tax policies on income distribution

④ Show the numerical examples concerning the Lorenz curve and the Gini coefficient, and furthermore show the impacts of tax policies on the welfare.

Set up 1

- Our economy is as follows:
  - There are n infinitely-lived agents where the felicity function and the initial holding of wealth are different among agents.
  - The population size in the whole economy is constant over time.
  - The economy is closed.
  - The commodity market is competitive.
- Taking account of the representative firm, we can show the rate of return to capital and wage as follows:

$$r = f'(K), \quad w = f(K) - Kf'(K) \qquad (1)$$

where K is the total capita-labor ratio.

- The full-employment condition is

$$K = \sum_{i=1}^{n} k_i \qquad (2)$$

where *ki* is the capital stock held by an agent *i*.

Set up 2

- The discounted sum of an agent's *i* utility over an infinite time horizen is

$$U_i = \int_0^\infty u^i(c_i)e^{-\rho t}dt$$

where c_i is the private consumption of an agent *i*.

- The flow budget constraint is:

$$\dot{k}_i = rk_i + w - c_i \quad (3)$$

- First order conditions are:

$$u_c^i(c_i) = q_i \text{ and } \frac{\dot{q}_i}{q_i} = \rho - r \quad (4)$$

- From these conditions we can show the following.

$$\frac{\dot{q}_1}{q_1} = \frac{\dot{q}_2}{q_2} = \ldots = \frac{\dot{q}_n}{q_n} \Rightarrow \frac{\dot{q}_1}{q_1} = \frac{\dot{q}_i}{q_i}, \qquad \frac{q_i}{q_1} = \frac{u_c^i(c_i)}{u_c^1(c_1)} = \Omega_i (= \text{constant}) \quad (5)$$

Set up 3

- Assuming that the utility function is CRRA type, the Euler equation is $$\frac{\dot{c}_i}{c_i} = \frac{r-\rho}{\beta_i} \qquad (6)$$
- From (1) and (3), capital accumulation equation is

$$\dot{k}_i = f(K) + (k_i - K)f'(K) - c_i \qquad (7)$$

Steady state 1

- The steady-state levels of aggregate capital is uniquely determined:

$$f'(K^*) = \rho \text{ and } K^* = \sum_{i=1}^{n} k_i^* \qquad (8)$$

- Summing up $\dot{k}_i = 0$ among all agents, the level of aggregate consumption in the steady state is

$$C^* = nf(K^*) + f'(K^*)K^*(1-n), \text{ and } C^* = \sum_{i=1}^{n} c_i^* \qquad (9)$$

- But, these conditions do not pin down the steady-state levels of individual capital stock because Ωi are undetermined.

$$\frac{u_c^i(c_i^*)}{u_c^1(c_1^*)} = \Omega_i (= \text{constant, and } \underline{\text{undetermined}}) \qquad (10)$$

Steady state 2

- Equation (10) says that the determination of the capital stock held by each individual in the long run needs to specify trajectory starting from a specific set of initial capital stocks.

- **<u>Lemma 1</u>**: Assuming that the economy converges to the specified steady-state equilibrium, we can show that

$$\Omega_i = \frac{u_{cc}^i(c_i^*)}{u_{cc}^1(c_1^*)} \frac{k_i^* - k_i(0)}{k_1^* - k_1(0)} \qquad (11)$$

- Proof: We make use of the linearization of the marginal utility around the steady state, which is substituted into equation (5). Then we obtain the equation (11).

Steady state 2

- Using (5) and Lemma 1, we can obtain:

$$\frac{\beta_1}{\beta_i} = \frac{k_i^* - k_i(0)}{k_1^* - k_1(0)} \frac{c_1^*}{c_i^*}, \qquad i = 2, 3, \ldots, n. \qquad (12)$$

- The level of individual consumption in the steady state is

$$c_i^* = f(K^*) + (k_i^* - K^*), \qquad i = 1, 2, \ldots, n \qquad (13)$$

- The 2n-equations in (8), (12) and (13) determine the capital stock and private consumption of n-persons in the steady state.

<u>Proposition 1:</u> The steady-state equilibrium is uniquely determined given the initial holdings of capital stock.

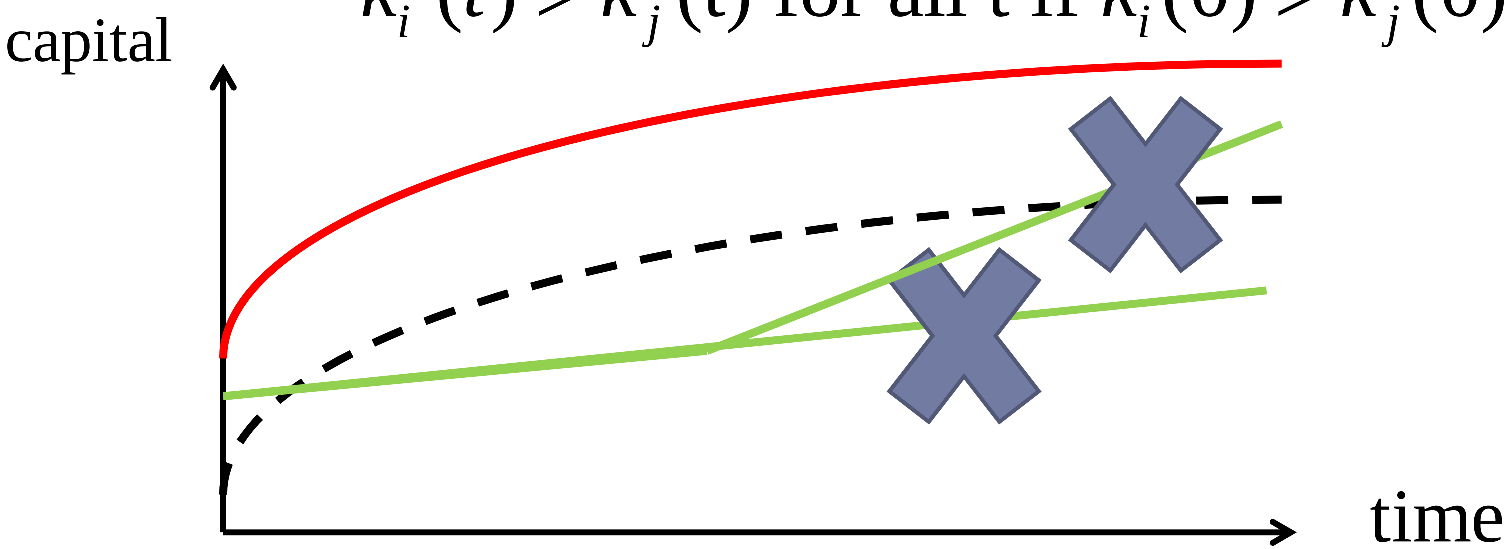
The impacts of capital stock at the initial period and risk aversion

- **Result 1.** Assuming that $\beta_i=\beta_j$ ($i , j = 1,2,\ldots,n$ and $i \neq j$), it holds that

$$k_i^* > k_j^* \text{ if } k_i(0) > k_j(0).$$

- **Result 2.** Assuming that $\beta_i=\beta_j$ ($i , j = 1,2,\ldots,n$ and $i \neq j$), it holds that

$$k_i(t) > k_j(\mathrm{t}) \text{ for all t if } k_i(0) > k_j(0).$$



- Now, assume that K*>K(0) is the good times and K*<K(0) is the bad times. That is, from Euler equation, we can say that

$$\text{Good times: } k_i^* > k_i(0), \quad \text{Bad times: } k_i^* < k_i(0) \text{ for all } i.$$

- **<u>Proposition 2.</u>** Assuming that ki(0)=kj(0) (i, j=1,2,…,n and i ≠ j), it holds that

$$\text{Good times: } k_i^* > k_j^* \text{ if } \beta_i < \beta_j$$

$$\text{Bad times: } k_i^* < k_j^* \text{ if } \beta_i < \beta_j$$

Intuition

- Good times
  - The consumption growth rates of all agents become positive for all time. Thus, the investment increases along time.

$$\frac{\dot{c}_t}{c_t} = \frac{r-\rho}{\beta_i}$$

  - The smaller the degree of risk aversion is, the greater the consumption growth rate is, which means the larger rate of investment. In this case, the steady-state level of capital stock is large.
- Bad times
  - The consumption growth rates are negative. Thus, the relationship is reversed.

The role of tax policies on inequality 1

- That is, we confirmed that individual capital stocks in the steady state are affected by the initial holdings of capital stock as well as risk aversion.
  - These effects are not observed in the representative-agent model.
- Hence, it would be useful to confirm the effects of tax policies such as consumption tax and labor income tax on individual capital stock.
  - Under the exogenous labor income, it has been well-known that the consumption tax rate and the labor income tax do not have any effects on the long-run level of capital stock in the representative agent model.

The role of tax policies on inequality 2

- Now, we modify the budget constraint as follows:

$$\dot{k} = (1-\tau_R) r k_i + (1-\tau_W) w - (1+\tau_C) c_i + \xi T_i,$$

τ_R, τ_W, τ_C are the rates taxes of return to capital, labor income and consumption.

- ξ is an indicator that represents whether the government conducts lump-sum transfer or not.
  - When ξ=1, the lump-sum transfer is conducted
  - When ξ=0, the lump-sum transfer is NOT conducted.
- The flow budget constraint is

$$\sum_{i=1}^{n} T_i = \tau_R r \sum_{i=1}^{n} k_i + \tau_W w n + \tau_C \sum_{i=1}^{n} c_i$$

- We assume that $T \equiv T_i = T_j \ (i, j = 1, 2, \ldots, n,$ and $i \neq j)$.

$$T = \frac{\tau_R r}{n} \sum_{i=1}^{n} k_i + \tau_W w + \frac{\tau_C}{n} \sum_{i=1}^{n} c_i \quad \Rightarrow T = \boxed{\frac{\tau_R r K}{n}} + \tau_W w + \boxed{\frac{\tau_C C}{n}}$$

- When n approaches to infinity, the effects of lump-sum transfer vanish.

Assumption

- Without the loss of generality, we assume the following.

<u>Assumption 1.</u> We assume that $\beta_i < \beta_{i+1}$ for all agents *i*.

- Good times: $k_i^* > k_{i+1}^*$
- Bad times: $k_i^* < k_{i+1}^*$

<u>Assumption 2.</u> The size of population is finite.

The effect of consumption tax 1

- Due to the limited time, we focus on the consumption tax because the intuition seems to be simple.
- Now, we assume that the tax rates of labor income and return to capital are zero. Thus, the budget constraint is:

$$c_i^* = \frac{1}{1+\tau_C}\left(\rho k_i^* + w^* + \frac{\zeta \tau_C C^*}{n}\right),$$

- The steady state level of aggregate consumption is uniquely determined by the aggregate capital stock.

$$C^* = \frac{\rho K^* + w^*}{1+\tau_C(1-\zeta)}$$

The effect of consumption tax 2

- Hence, the budget constraint can be rewritten as

$$\xi = 0 : c_i^* = \underbrace{\frac{1}{1+\tau_C}}_{\#1} (\rho k_i^* + w^*),$$

$$\xi = 1 : c_i^* = \underbrace{\frac{1}{1+\tau_C}}_{\#1} \left(\rho k_i^* + w^* + \frac{\tau_C(\rho K^* + w^*)}{n} \right),$$

- Taking account of the relative consumption, the effect given by #1 is cancelled out. For example, when ξ=0,

$$\frac{c_i^*}{c_1^*} = \frac{\rho k_i^* + w^*}{\rho k_1^* + w^*}$$

The effect of consumption tax 3

<u>Proposition 3.</u> Suppose that the government levies the consumption tax and does not conduct the lump-sum transfer. In this case, the consumption tax rate does not have any effects on individual capital stock.

Alternatively, suppose that the government conducts the lump-sum transfer. We can show the following.

$$\frac{\partial k_1^*}{\partial \tau_C} < 0, \frac{\partial k_n^*}{\partial \tau_C} > 0,$$

The level of capital stock held by an agent i (i=2,3,..,n-1) may increase or decrease.

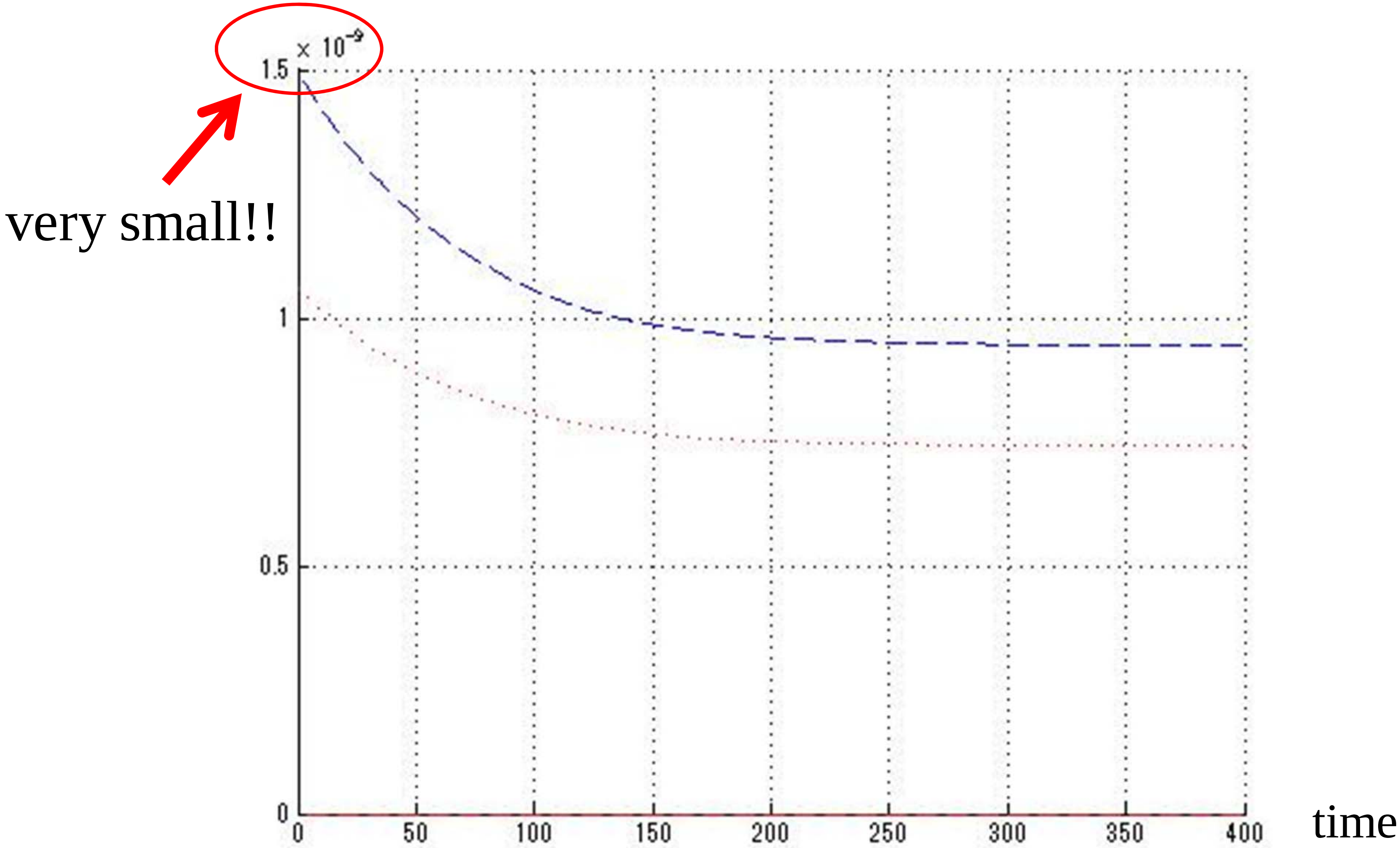
Intuition

- The level of capital stock held by the agent with lowest degree of risk aversion (i=1) increases, whereas that held by the agent with the highest degree of risk aversion (i=n) decreases.
- Good times:
  - The level of capital stock held by the richest (i=1) decreases, whereas that held by the poorest (i=n) increases.
  - It would modify the inequality by increasing the tax rate of consumption.
- Bad times:
  - The relationship of the above is reversed.
- not completed

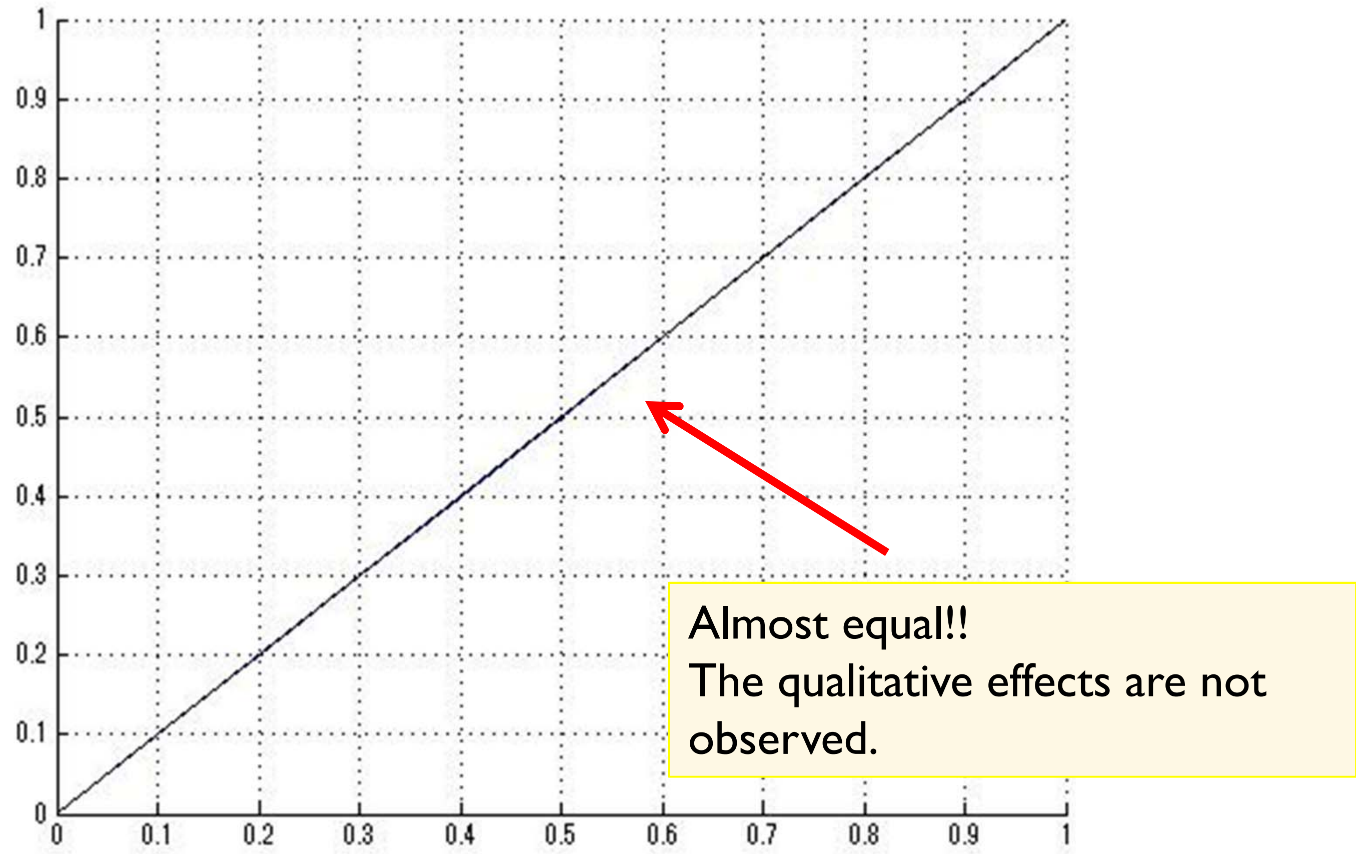
Numerical examples (not completed)

- Our findings are as follows:
  - The tax policies of consumption and labor income have the quantitative effects, but the qualitative effects are very limited.
  - Instead, the tax imposed in the return to capital greatly affects the income distribution. Furthermore, the inequality spreads along time regardless of whether the lump-sum transfer is conducted or not.
- I assume that the tax rates at the initial period of economy are all zero. And assume that the initial levels of capital stock held by individuals are the same where their risk aversion parameters are different. That is, the initial economy achieves the complete equal.
- We make use of the 10, 20, 30% of tax rates.

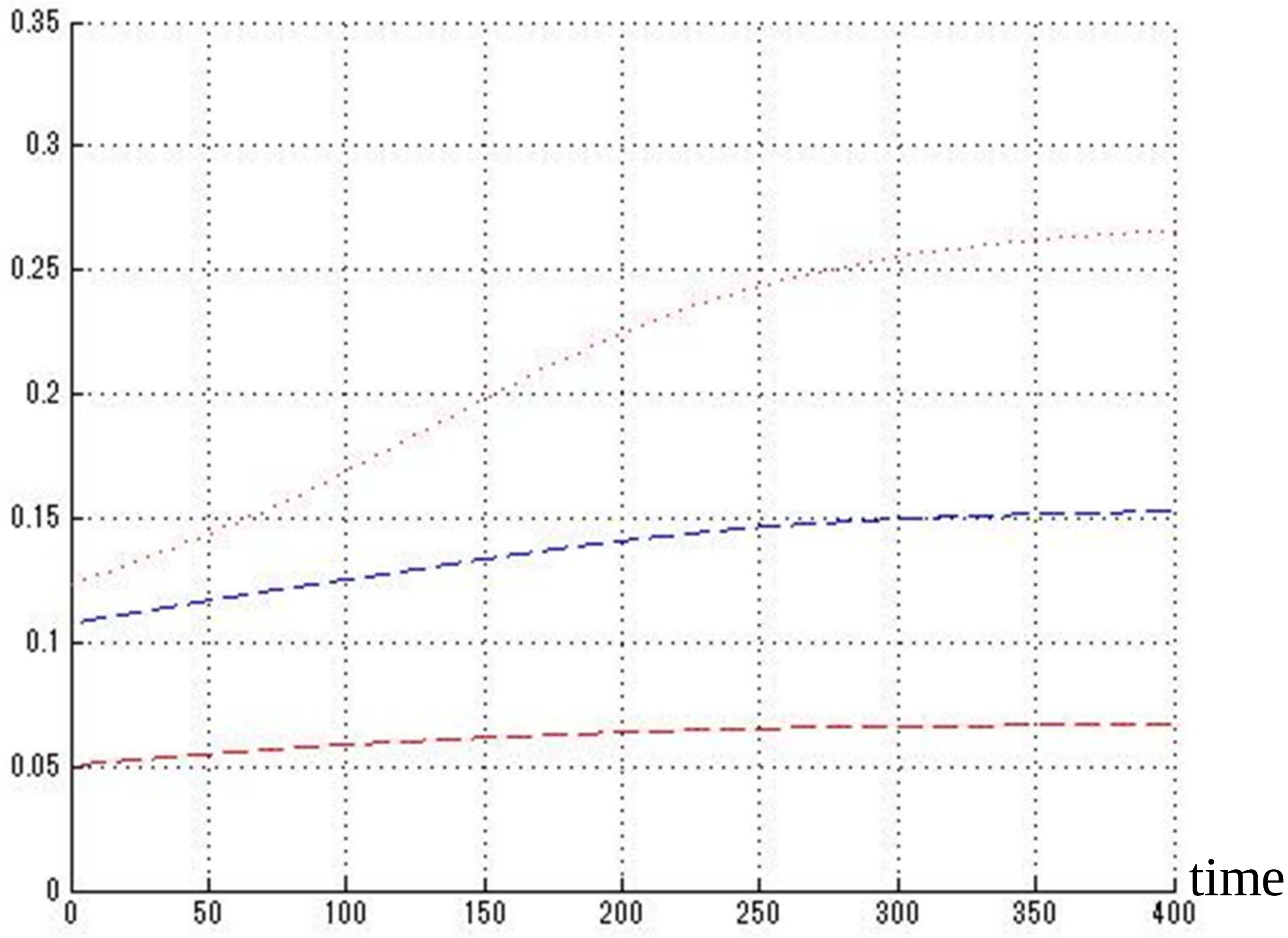
Gini coefficient along time in the case of consumption tax



Lorenz curve in the case of consumption tax



Gini coefficient along time in the case of the tax on return to capital



Lorenz curve in the case of the tax on return to capital