Analyzing Patterns of Economic Growth: A Production Frontier Approach

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Abstract

Economic growth is best understood as a combination of high and low growth regimes. This paper deals with the sources of growth around growth regimes change. To that end the derivation of structural breaks in growth rates series is combined with nonparametric growth accounting, which allows the decomposition of productivity changes into technological progress and efficiency changes. Even medium-run growth rate changes are mainly the result of productivity changes. Growth spurts due to technological progress happen only in developed countries. Growth spurts in developing countries are catch-up growth episodes based on efficiency improvements. Factor accumulation is of minor importance.

Keywords: Growth, Structural Breaks, Data Envelopment Analysis JEL Classification: O11, O47

1 Introduction

Growth rates in virtually all countries are highly unstable over time (Easterly et al., 1993; Pritchett, 2000). Acknowledging this important fact, a new empirical literature on economic growth is emerging that emphasizes the existence of and the reasons for major *turning points* in growth rates series instead of restricting the analysis to differences in long-run average growth rates. The present paper contributes to this literature: it identifies statistically significant shifts in the average growth rates of income per capita for a large number of countries and explores the relative importance of factor accumulation, efficiency changes and technological changes as proximate causes for the observed transitions.

The motivation for this paper is a contribution by Jones and Olken (2005), who investigate the proximate causes for significant transitions between high growth and low growth episodes via growth accounting. They find that growth accelerations and decelerations differ with regard to the relative importance of changes in factor accumulation, which is significantly more important for decelerations than for accelerations. However, factor accumulation plays a surprisingly small role for both types of growth transitions: it explains less than ten percent of the differences in growth rates in the event of an acceleration and about thirty percent in the event of a deceleration. Rather, both types of growth transitions coincide with major shifts in total factor productivity. While the importance of total factor productivity changes for long run growth is by now widely accepted (Caselli and Wilson, 2004; Easterly and Levine, 2001; Hall and Jones, 1999; Prescott, 1998) and consistent with the neoclassical growth models (Barro and Sala-i Martin, 2004; Solow, 1956), the dominant role of productivity changes in the short run is surprising. Transitional dynamics in the neoclassical growth models are driven by changes in the capital stock. Poverty trap models often focus on a nonconvexity in factor accumulation to explain why some nations fail to escape poverty (Acemoglu and Zilibotti, 1997; Murphy et al., 1989). Finally, there is agreement that industrialization in the initial phase is about capital accumulation (Galor and Moav (2004), Porter (1990, chap. 10)). Therefore, one would expect to see an important role for capital accumulation in initializing episodes of fast economic growth in particular in low income countries.

This paper reassesses the findings by Jones and Olken (2005). It applies nonparametric instead of traditional parametric growth accounting and thus renders unnecessary implicit assumptions such as a Cobb-Douglas production technology or fully competitive markets. Only mild assumptions like free disposal or constant returns to scale are needed. As a further advantage nonparametric growth accounting makes allowance for inefficiencies in production, thereby enabling the further decomposition of changes in total factor productivity into changes in the efficiency of production and technological changes. This paper adds to the existing literature on nonparametric growth accounting by reporting confidence intervals for changes in efficiency, technology and factor accumulation and by explicitly incorporating the assumption of no technological regress in the bootstrap procedure. With regard to Jones and Olken's (2005) original contribution four further refinements are made: First, a more powerful variant of the Bai-Perron procedure is used to derive the structural breaks (Bai and Perron, 2006). Second, each growth episode is required to last for at least eight years to ensure that growth transitions are not confounded with business cycles. Such a confusion may occur in Jones and Olken (2005) because growth spells are allowed to be as short as two years. Third, production is specified in terms of capital per worker instead of capital per inhabitant and, fourth, the data coverage is increased by using the Penn World Tables version 6.2. A final contribution of this paper is the implementation of the Bai-Perron procedure as a new Stata command.

Despite the differences in methodology, this paper confirms the weak role of capital accumulation in growth transitions. The average growth acceleration results from efficiency improvements with little effects of technological change and capital accumulation. However, the average masks important differences according to the state of development. While the conclusion holds for low income countries, growth accelerations in middle and high income countries are not only explained by efficiency changes, but also by technological improvements and capital accumulation. Both factors become relatively more important the higher the state of development. As in Jones and Olken (2005) growth decelerations are different from accelerations in that they are more strongly affected by the formation of capital. Yet, deteriorations in the efficiency of production remain the key cause for the breakdown of growth. Unlike in the case of accelerations, the proximate causes of growth decelerations do not hinge on the level of development of the respective countries. These results survive a number of robustness tests.

The remainder of the paper is organized as follows. The related literature is surveyed in Section 2. The methodology is discussed in Section 3. Results and robustness tests are presented in sections 4 and 5. Section 6 concludes.

2 Related Literature

The research program for analyzing growth transitions is linked to Pritchett (2000), who argues that traditional growth regressions in the style of Kormendi and Meguire (1985), Barro (1991), Mankiw et al. (1992), or Islam (1995) are largely uninformative because highly unstable growth rates are regressed on highly persistent explanatory variables. As a consequence, the results are not robust to slight alterations of the estimation framework and only limited policy conclusions can be drawn.¹ According to Pritchett (2000), a more promising way to uncover determinants of growth is to shift the focus to episodes with similar characteristics and ask, for example, what happens before growth accelerates or decelerates, what happens to growth if major policy reforms are undertaken or what distinguishes the reaction of a successful country from that of a less successful one in the presence of similar shocks. The resulting literature on growth transitions has so far quite strictly adhered to this program.

When analyzing growth transitions, a definition of growth spells, i. e. periods during which the growth rate remains reasonably stable, is required. Three different approaches have been suggested in the literature: the threshold, the episodic, and the statistical approach.² In the threshold approach, successive years are classified as high or low growth spells if the average growth rate during these years exceeds or falls below a previously defined magnitude. Usually, the average refers to periods of four to eight years (Aizenman and Spiegel, 2010; Arbache and Page, 2010; Hausmann et al., 2005; Imam and Salinas, 2008; Jong-A-Pin and De Haan, 2011). The episodic approach is similar to the threshold approach, but focuses on longer periods, e. g. 10 to 15 years. Moreover, the episode selection is not necessarily based on calculations, but may simply rely on common knowledge such as dividing time series into the period before and after 1975 to capture the growth slowdown in the 1970s (Rodrik, 1999; Sahay and Goyal, 2006). In the statistical approach growth episodes are derived using well defined statistical testing procedures that allow for one (Ben-David and Papell, 1998) or several structural breaks (Jones and Olken, 2005). Combinations in particular of the threshold and statistical approach have been applied, too (Berg et al., 2008).³

Given the growth spells, some authors have applied regressions akin to crosscountry growth regressions to uncover the reasons for different resilience to shocks

¹Similar criticism has been raised by Levine and Renelt (1992), and Easterly et al. (1993), but the research program is attributable to Pritchett (2000).

²Sahay and Goyal (2006) use a similar classification but assign the existing literature somewhat differently to the categories.

³? and ? take a different approach and interpret the observed instability of growth rates within a Markov switching model of growth.

(Rodrik, 1999), while others have employed correlation analysis to single out factors that are different across good and bad growth spells (Sahay and Goyal, 2006). The most common approach, however, is to use discrete choice models in an attempt to find events after which a growth transition is likely. While there is evidence that terms of trade shocks, economic reforms, financial liberalization and policy changes play some role, the ultimate reasons for growth transitions remain largely a mystery. There are numerous contributions to this literature, among others Aizenman and Spiegel (2010), Arbache and Page (2010), Becker and Mauro (2006), Dovern and Nunnenkamp (2007), Hausmann et al. (2005), Hausmann et al. (2006), and Jong-A-Pin and De Haan (2011). Berg et al. (2008) extend this literature and look directly at the duration of growth spells employing duration analysis.

Jones and Olken (2005) contribute to the preceding literature in that they apply the statistical approach to detect growth episodes. After the identification of growth spells, however, they use growth accounting to explore the contribution of factor accumulation versus total factor productivity to growth transitions. In order to gain more insight into total factor productivity changes, a further decomposition into technological and efficiency changes is desirable. An analytical tool to determine the relative importance of the two components is data envelopment analysis (DEA), which dates back to Farrell (1957) and which has been introduced into macroeconomic productivity analysis by Färe et al. (1994). Kumar and Russel (2002) show that income changes can be decomposed into changes in efficiency, technology and factor accumulation if one is willing to assume constant returns to scale. They use this nonparametric growth accounting to analyze the contribution of each factor to the emerging bimodal distribution of labor productivity across countries. DEA in macroeconomics has subsequently been extended into two directions: First, the Kumar-Russel type of analysis has been applied to extended time periods or has taken into account an increasing number of production factor (Henderson and Russell, 2005; Salinas-Jimenez et al., 2006). Second, the statistical properties of the DEA estimators⁴ have been taken into consideration, albeit this development has largely been restricted to studies focusing on the decomposition of productivity only (Enflo and Hjertstrand, 2009; Henderson and Zelenyuk, 2007).

In terms of the reviewed literature this paper can be integrated as follows: The statistical approach is used to determine episodes of high and low growth. After that, nonparametric growth accounting, including the derivation of confidence intervals, is

 $^{{}^{4}}$ These have been developed in particular in a series of papers by Simar and Wilson. Cf. section 3.2.1.

applied to derive the proximate causes of growth transitions.

3 Methodology

Identification of Structural Breaks 3.1

Consider the following model for the growth rate of GDP per capita:

$$g_t = \beta_i + u_t, \ t = T_{i-1} + 1, \dots, T_i.$$
(1)

Within the growth regime labeled i the annual growth rate g_t equals the regime-specific mean growth rate β_i plus a stationary error term u_t , which may have a different distribution across regimes. Suppose it is known that the growth rate series contains m structural breaks points denoted by $(T_1, ..., T_m)$ and that each of the m+1 growth regimes is required to last for at least h > 1 periods. In the Bai-Perron (BP) procedure (Bai and Perron, 1998, 2003a,b, 2006) the coefficients $\hat{\beta} = (\hat{\beta}_1, ..., \hat{\beta}_{m+1})$ are estimated by minimizing the total sum of squared residuals S_T for the *m*-partition $(T_1, ..., T_m)$, which is given by

$$S_T = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [g_t - \beta_i]^2.$$
(2)

The break points $(\hat{T}_1, ..., \hat{T}_m)$ are estimated such that S_T with the associated leastsquares estimate $\hat{\beta}$ is minimized over all conceivable *m*-partitions while taking account of the minimum duration requirement h for each regime.^{5,6}

In order to derive the required number of breaks, Bai and Perron (1998) suggest a sequential testing procedure based on the $\sup \mathbf{F}_T$ test statistic. Intuitively, the $\sup F_T(\ell|\ell+1)$ testing procedure tests the null hypothesis of $m=\ell$ breaks against the alternative hypothesis of $m = \ell + 1$ breaks and rejects the null if the additional break point reduces the total sum of squared residuals by a sufficiently large amount. Starting with the null hypothesis of m = 0 breaks, the number of breakpoints is increased one by one until the sup $F_T(\ell+1|\ell)$ test fails to reject the null hypothesis of ℓ breaks. The critical values are simulated and depend on ℓ and a so-called trimming

⁵Appendix A reviews the details the empirical implementation of the BP procedure in more

detail. ⁶This paper follows Jones and Olken (2005) and assumes that the log of GDP per capita is integrated of order one. Since the question of deterministic versus stochastic trend in log GDP series is not yet settled, it might be worthwhile to apply the Kejriwal and Perron (2010) testing procedure in future work, because it allows the investigation of structural breaks both in the presence of I(0) and I(1) errors.

parameter ε equalling h/T. One drawback of this testing procedure is the frequently low power of the test zero against one break point if more than one break is present. Since the power of the double maximum test, which tests the null hypothesis of m = 0breaks against the alternative hypothesis of an unknown number of breaks up to M(i.e. $1 \le m \le M$) is almost as high as if the null is tested against the true number of breaks, Bai and Perron (2006) suggest to adapt the first step of sequential testing procedure. Instead of testing zero against one break point, the double maximum test should be applied in the first step to test the null of m = 0 against the alternative of $1 \le m \le M$. If the null hypothesis is rejected, testing should be continued using the $\sup F_T(\ell | \ell + 1)$ testing procedure. This alternative is referred to as the udmaxL testing procedure in the following.

3.2 Nonparametric Growth Accounting

A nonparametric approach to growth accounting is used in this paper to evaluate the relative contributions of changes in factor accumulation, efficiency of production and technological progress to the observed growth rate changes in growth transitions. Unlike standard growth accounting (Solow, 1957), this approach does not need an assumption about the form of the production function (except for the returns to scale) and the form of technological progress, nor does it require the assumption of perfect competition and constant factor shares.⁷ Since it explicitly allows for the possibility of non-efficient production, it can distinguish between catch-up growth due to efficiency improvements and growth due to real innovations. Nonparametric growth accounting is based on data envelopment analysis and Malmquist productivity indices. The following exposition draws on Färe et al. (1994), Kumar and Russel (2002) and

⁷In parametric growth accounting usually a Cobb-Douglas production function and constant factor shares are assumed. Often the capital share is set to 1/3 for all countries. For a critique of the Cobb-Douglas assumption confer for instance Duffy and Papageorgiou (2000). The appropriate way of measuring factor shares and whether the assumption of constant factor shares is warranted is discussed among others in Krueger (1999), Bentolila and Saint-Paul (2003), Crafts (2003) or Gollin (2002). The effect of imposing a constant elasticity of substitution between capital and labor equal to one via the assumed Cobb-Douglas production function is reviewed in Nelson (1973) and Rodrik (1997). If growth accounting were to be based on an alternative production function than Cobb-Douglas, the question of factor-augmenting technological progress would have to be addressed (Acemoglu and Autor, 2010). Both types of growth accounting rely on aggregate production functions, a concept subject to considerable debate (Felipe and Fisher, 2003).

Ray (2004, chap. 2) unless otherwise noted.⁸

3.2.1 Data Envelopment Analysis

Each country j (j = 1, ..., J) in period t produces the single output aggregate GDP (Y_t^j) using aggregate capital (K_t^j) and aggregate labor (L_t^j) as inputs. Assuming a convex technology with constant returns to scale, free disposability of inputs and outputs, and ruling out technological regress,⁹ the production possibility set of the world in period t (\mathcal{T}_t) encompasses all convex combinations of ever observed input-output bundles until period t. Formally:

$$\mathcal{T}_{t} = \{ (Y, K, L) \in \mathbb{R}^{3} : K \geq \sum_{\tau \leq t} \sum_{j} \mu_{\tau}^{j} K_{\tau}^{j} \wedge L \geq \sum_{\tau \leq t} \sum_{j} \mu_{\tau}^{j} L_{\tau}^{j}$$

$$\wedge Y \leq \sum_{\tau \leq t} \sum_{j} \mu_{\tau}^{j} Y_{\tau}^{j}, \mu_{\tau}^{j} \geq 0 \ \forall j, \tau \}.$$

$$(3)$$

The upper boundary of this production possibility set represents the world technology frontier. Each country's actual output is related to the world technology frontier by means of the distance function, which is defined as follows:

$$D_t^j(K_t^j, L_t^j; Y_t^j) = \inf\left\{\phi_t^j : \left(K_t^j, L_t^j; \frac{Y_t^j}{\phi_t^j}\right) \in \mathcal{T}_t\right\}.$$
(4)

The inverse of the distance function indicates by how much output could be increased with the chosen input mix and still remain technologically feasible. In this sense, it indicates the efficiency of production.¹⁰ Obviously, feasible production can only have $D_t(\bullet) \leq 1$, with $D_t(\bullet) = 1$ meaning that production takes place on the world technology frontier and is thus fully efficient.

The world technology frontier is not directly observable, but has to be estimated from the observed input-output combinations. DEA analysis, which essentially wraps the data in the "tightest fitting convex cone" (Kumar and Russel, 2002) and constructs the best-practice frontier as the boundary of this set, is one popular technique to do so.

⁸Growth accounting based on stochastic frontier analysis was considered as an alternative. Like the DEA approach, it allows the decomposition of productivity into efficiency and technology. Unlike DEA, it allows for a stochastic error term. However, in a long panel like in this article technological change and time-varying efficiency levels have to be allowed for. In the context of stochastic frontiers, this is only possible by severely restricting the evolution of the efficiency term such that the time path is either equal across countries or smooth over time (Kumbhakar and Lovell (2000)). Neither assumption is suited for an analysis that focuses on the behavior of growth components in the presence of structural breaks.

⁹In order to rule out technological regress, the formulation suggested by Henderson and Russell (2005) is used.

¹⁰Efficiency of production refers to *proportional* changes of inputs and outputs. Hence, efficient production in DEA does not necessarily mean Pareto-efficiency.

Formally, for each country the distance functions are estimated by solving the linear programming problem in (5). The estimated distance functions uniquely determine the estimated world technology frontier.

$$D_t^j(K_t^j, L_t^j; Y_t^j) = \min \ \phi_t^j \text{ subject to } \frac{Y_t^j}{\phi_t^j} \le \sum_{\tau \le t} \sum_j \mu_\tau^j Y_\tau^j,$$

$$K_t^j \ge \sum_{\tau \le t} \sum_j \mu_\tau^j K_\tau^j, L_t^j \ge \sum_{\tau \le t} \sum_j \mu_j^\tau L_\tau^j,$$

$$\mu_\tau^j \ge 0 \ \forall j, \tau.$$
(5)

3.2.2 Tripartite Decomposition

In order to derive the decomposition of income changes between two periods into changes attributable to efficiency change, technological change and capital accumulation, two features of the DEA framework are exploited. First, each country's production in period t is expressed as the distance function times the world technology frontier, and second, aggregate inputs and output are converted into input and output per worker using the constant returns to scale assumption. Hence, GDP per worker \tilde{y}_t is produced using capital per worker \tilde{k}_t . Given the distance functions from above, dropping country superscripts and using $D_t = \phi_t$, output per worker at capital intensity \tilde{k}_t is related to the world technology frontier of period t ($\tilde{\mathbf{y}}^t(\tilde{k}_t)$) via $\tilde{y}_t(\tilde{k}_t) = \phi_t \tilde{\mathbf{y}}^t(\tilde{k}_t)$.

With some rearranging the growth factor of output per worker from t to t + 1 can be expressed as

$$\frac{\tilde{y}_{t+1}(\tilde{k}_{t+1})}{\tilde{y}_t(\tilde{k}_t)} = \frac{\phi_{t+1}}{\phi_t} \frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^t(\tilde{k}_{t+1})} \frac{\tilde{\mathbf{y}}^t(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^t(\tilde{k}_t)}.$$
(6)

According to (6) changes in output per worker as measured by the growth factor are the result of changes in efficiency (first term), changes in technology measured at the second-period capital intensity (second term) and changes in the capital intensity measured in relation to the first-period world technology frontier (third term). Of course, changes in technology could be measured at the first-period capital intensity and changes in capital intensity could be measured using the second-period world technology frontier. There is no good reason for either alternative, but the decompositions yield different results unless technical progress happens to be Hicks-neutral. This ambiguity is usually solved by employing the Fisher ideal decomposition, i. e. by taking the geometric average of the two measures. Since the structural breaks are derived in terms of GDP per capita, the proposed decomposition is extended to incorporate changes in the labor force participation rate (lfp). The final decomposition becomes

$$\frac{y_{t+1}}{y_t} = \frac{\phi_{t+1}}{\phi_t} \left(\frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^t(\tilde{k}_{t+1})} \frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_t)}{\tilde{\mathbf{y}}^t(\tilde{k}_t)} \right)^{1/2} \left(\frac{\tilde{\mathbf{y}}^t(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^t(\tilde{k}_t)} \frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_t)} \right)^{1/2} \frac{lfp_{t+1}}{lfp_t}$$

All components of (6) are Malmquist indices and can be expressed solely in terms of distance functions and observed output.¹¹ However, it is necessary to know the efficiency of production in period t relative to the world technology frontier of period t + 1 and vice versa. These counterfactual distance functions are obtained by appropriately adjusting the reference technology in the linear program (5) and solving the resulting two problems in addition to the standard linear programs for each country and period.¹²

3.2.3 Inference

DEA is a very flexible tool to derive production frontiers and by now many statistical properties are well established. These properties are derived under the assumption that all observed input-output combinations are technically attainable, so that no allowance for measurement errors is made. However, under this assumption the estimated efficiency scores are consistent and their rate of convergence in the two-inputsone-output case is comparable to that of parametric estimates (Simar and Wilson, 2000). Unfortunately, the estimated efficiency scores are upward biased because they are derived in relation to the best-practice, i. e. observed, world technology frontier, which due to the finiteness of the sample may miss some more efficient input-output combinations shaping the true world technology frontier. Furthermore, as DEA makes no allowance for measurement errors and as the world technology frontier is effectively determined by the small subset of efficient observations, the method is sensitive to outliers. Therefore, results should always be checked for robustness (Simar and Wilson, 2008).

Asymptotic sampling distributions for the DEA estimator are difficult to derive analytically, so that statistical inference relies on bootstrap methods. In the present framework naive bootstrapping does not consistently mimic the data generating process due to the bounded nature of the efficiency estimate. Therefore, this paper uses the smoothed bootstrap introduced by Simar and Wilson (1998), which bootstraps on the radial inefficiencies ϕ and assumes that these are homogeneously distributed

¹¹Malmquist indices are ratios of distance functions, which are representations of technologies based on input and output data only. Cf. Caves et al. (1982) and Färe et al. (1994).

¹²Cf. Appendix D for the counterfactual linear programs and the tripartite decomposition based on distance functions.

over the input-output space.¹³ The problems related to the bounded nature of the efficiency scores are overcome by drawing the bootstrap efficiency scores from a smoothed distribution of efficiency scores instead of the empirical one. The smoothed distribution is based on a kernel density estimation and the Silverman reflection method. In order to bootstrap the Malmquist indices inherent in the tripartite decomposition, the procedure needs to be adapted to account for the possibility of temporal correlation between efficiency scores (Simar and Wilson, 1999). One further adjustment is introduced in this paper: in order to mimic the assumption of no technological regress in the bootstrap, it is modified in such a way that each bootstrap for period t+1 always includes all bootstrap observations drawn for period t in addition to the newly generated bootstrap observations for t + 1.¹⁴ Given the bootstrapped distance functions, the parameters of interest like the bias of the estimate, the bias-corrected estimate or the boundaries of confidence intervals, can be derived for each bootstrap replication and for the bootstrap as a whole. Let $\hat{\xi}$ denote the estimated quantity of interest, $\hat{\xi}_{h}^{*}$ the bootstrap estimate of the quantity for replication $b, \hat{\xi}$ the bias-corrected quantity and B the number of bootstrap replications. Then the bias and bias-corrected values are estimated as

$$\widehat{bias}_B[\hat{\xi}] = \frac{1}{B} \sum_{b=1}^B \hat{\xi}_b^* - \hat{\xi} \text{ and}$$
(7)

$$\hat{\hat{\xi}} = 2\hat{\xi} - \frac{1}{B}\sum_{b=1}^{B}\hat{\xi}_{b}^{*}.$$
(8)

Bias-correction should only be applied if the bias-corrected estimator has a smaller mean-square error (s^2) than the original one. Therefore, bias-correction is only applied if the ratio

$$r = \frac{1/3 * (\widehat{bias}_B[\hat{\xi}])^2}{s^2}$$
(9)

exceeds unity. Finally, confidence intervals are constructed based on the sorted differences $(\hat{\xi}^* - \hat{\xi})$ so that they account both for the statistical variation and the inherent bias of the estimates.

¹³Whether the homogeneity assumption that is standard in this literature is a valid one, is an open question. It implies that there are no systematic differences in the efficiency levels of rich and poor countries. The only study addressing this question is, to the knowledge of the author, the one by Henderson and Zelenyuk (2007). This study finds that efficiency in developing countries is systematically lower than in developed countries. Therefore, future research and refinements of the bootstrap procedure in the country context are certainly required.

¹⁴The calculations are carried out in R using the FEAR package 1.15 distributed by Wilson (2008). The adjustment refers to the subcommand malmquist.components. It is not clear from the contributions whether other studies such as Henderson and Russell (2005) or Jerzmanowski (2007) used such an adjustment, too.

4 Data and Results

4.1 Data

The data is taken from the Penn World Tables (PWT) version 6.2 (Heston et al., 2006b). Income per capita is expressed in international prices of the year 2000 and is based on the Laspeyeres deflator (RGDPL in PWT notation).¹⁵ Aggregate output is obtained by multiplying income per capita with the population size (POP), aggregate investment by multiplying the investment share (ki) with aggregate output. Aggregate labor is approximated by the number of workers in each country and the labor force participation rate is taken to be the ratio of workers to total population.¹⁶ Human capital is not taken into account because data is only available after 1960 and only for a subset of countries.¹⁷ Following Jones and Olken (2005) the aggregate capital stock is derived using the perpetual inventory method (Nehru and Dhareshwar, 1993) with an assumed depreciation rate of seven percent.¹⁸

With regard to the sample the following choices are made: for each country a minimum of 30 observations is required in order to ensure a sufficient number of data points for the calculations. Moreover, only countries with at least 20 observations in PWT version 6.1 are used, because many of the additional countries introduced in version 6.2 suffer from implausibly high historical levels of income (Heston et al., 2006a). Only countries with a population exceeding one million in the final year of available data are included to avoid biased DEA estimates due to the prevalence of an "atypical" production structure. Since there are not enough data points for united Germany, data for the former West Germany between 1950 and 1989 is included. Gabon is excluded as it is an obvious outlier.¹⁹ These rules leave 105 countries for the analysis.

¹⁵In order to mitigate the substitution bias inherent in long time series that are deflated by a Laspeyeres index, it would be preferable to use RGDPCH, which is a chain index number for income per capita.(Summers and Heston, 1991; Schreyer, 2004). Unfortunately, the investment share needed for the derivation of the capital stock is only available in terms of the Laspeyres index.

index. 16 The number of workers equals RGDPCH * POP/RGDPWOK in PWT notation. For Taiwan, the number of workers is extrapolated from 1999 onwards based on the assumption that the labor force participation rate remains unchanged.

¹⁷Since the human capital stock evolves very slowly, human capital is unlikely to have large impacts for short-term growth events. Therefore, losing a large number of observations is too high a price to pay for its introduction. Cf. Jones and Olken (2005) who also find negligible effects of human capital.

¹⁸The initial capital stock is calculated using the geometric mean of the investment rate in the first ten years of the data series to approximate the growth rate before the initial observation. In case of a negative investment rate, a rate of zero is assumed.

¹⁹See Appendix C.

4.2 Structural Breaks

The structural breaks are derived using the udmaxL testing procedure. The minimum duration of a growth regime is set to 8 years in order to strike a balance between too long a duration requirement that would make it likely to miss breaks and too short a duration requirement that would reduce the power of the testing procedure too much (Berg et al., 2008).²⁰ Moreover, a maximum of three breaks is allowed.²¹ Separate covariance matrices are calculated for each growth regime to control for potential heteroscedasticity. Breusch-Godfrey tests indicate that autocorrelation is of minor importance (Greene, 2003, chap. 12).²² The calculations are carried out in Stata using a newly written command.²³

Table 1 summarizes the results. In total, 97 breaks are detected. A break is called an upbreak or a growth acceleration if the average growth rate after the break exceeds the one before the break. Otherwise, the break is classified as a downbreak or growth deceleration. The upper part of Table 1 indicates that downbreaks are more common than upbreaks (60 % versus 40 % of all cases). Numerous structural breaks are observed in all regions of the world. However, whereas in Asia and in Oceania upbreaks and downbreaks are equally common, Europe, North and South America, and Africa experience more decelerations than accelerations. According to the middle part of Table 1, most structural breaks happened in the 1970s and 1980s. It should be noted, however, that the number of structural breaks in the 1950s and 1990s is low by construction due to the minimum duration requirement. Even for a time series starting in 1951 and ending in 2004 the earliest admissible break point is 1958 and the latest is 1996.²⁴ Since 36 time series start only in 1960 or later, the admissible break points in the 1960s are also seriously restricted. Despite these reservations regarding the relative importance of structural breaks in different decades, the large number of downbreaks recorded in the 1970s is in accordance with the occurrence of a major productivity slowdown in industrialized countries during that era. Moreover, 23 of the 29 recorded downbreaks during that era occurred in Europe and North America,

²⁰The power of the testing procedure varies across countries because the minimum duration requirement in combination with time series of different lengths implies varying trimming parameters. However, if trimming parameters are kept fixed, the minimum duration of growth regimes varies as in Jones and Olken (2005). In their setting the minimum duration of a growth regime may be as short as two years.

²¹This restriction is without limited consequences because the udmaxL procedure would never opt for more than three breaks even if more breaks were allowed.

 ²²See Section 5 for a robustness test using the heteroscedasticity and autocorrelation consistent variance estimator.
 ²³The ado-file is available upon request together with an introductory note. The procedure has

²³The ado-file is available upon request together with an introductory note. The procedure has been implemented following existing implementations in RATS and GAUSS.

²⁴Recall that these are the last observations belonging to the former regime.

	Structural Breaks by Region								
	Total	Africa	Asia	Europe	North America	South America	Oceania		
Total number of breaks	97	22	20	25	17	11	2		
Upbreaks	39	9	10	8	7	4	1		
Downbreaks	58	13	10	17	10	7	1		
		G .							
		Str	uctural Br	eaks by D	ecade				
	Total	1950s	1960s	1970s	1980s	1990s			
Total number of breaks	97	5	15	32	29	16			
Upbreaks	39	5	10	3	9	12			
Downbreaks	58		5	29	20	4			
	Structural Breaks by Initial Income								
	1	Fotal	High Ir	ncome	Middle Incom	ne Low	/ Income		
Total number of breaks	of	97	28	3	24		45		
Upbreaks		39	7		7		25		
Downbreaks		58	21		17		20		
The structural h	reaks are d	derived usin	o the udma	axL testing	z procedure usi	ing a minin	um duratic		

 Table 1: Summary Statistics for Structural Breaks

The structural breaks are derived using the udmaxL testing procedure using a minimum duration requirement of 8 periods. The level of significance is 0.1. The recorded break years are the final years of the previous growth regimes.

i. e. in the regions where most industrialized countries are found. The lower part of Table 1 classifies the structural breaks according to the stage of development of the respective countries in the year preceding the break. In order to account for economic progress in the period under consideration, a dynamic definition of the state of development similar to that suggested by Becker and Mauro (2006) is used. All countries with at least half of the US per capita income are considered high income countries. The middle income countries comprise all countries with an income per capita that is at least as high as half of the leader middle income country. All other countries are classified as low income countries. Growth accelerations are mainly a feature of low and middle income countries, more than 80 % of the detected upbreaks occur here. Growth decelerations are more evenly distributed across income categories. Overall, the results support the common finding in the related literature that, generally, countries are not locked in growth traps, but that they fail to sustain a growth acceleration once it occurs (Berg et al., 2008; Hausmann et al., 2005; Jerzmanowski, 2006; Rodrik et al., 2004).

Figure 1 illustrates that the calculated break points are usually related to major events. For instance, in China the drift of the time series increases after 1977, which coincides with Deng Xiaoping's ascension and the start of economic reforms such as



Figure 1: Examples of Structural Breaks

the liberalization of agriculture and the opening of the economy. In Mexico, the deceleration of growth after 1981 can be linked to the severe currency crisis starting in that year whereas the deceleration after 1973 in Portugal heralds the turbulent time after a bloodless military coup. In Poland, the first turning point 1978 coincides with the beginnings of the Solidarnosc Movement and severe price increases, whereas the upbreak after 1991 can be related to the economic and political reforms after the fall of communism. Poland also illustrates the trade-off introduced by imposing a minimum duration requirement for each regime: the method identifies well defined break points, but misses short-lived events that are very close to each other. In the case of Poland, the growth acceleration between 1982 and 1988 is not picked up.

4.3 Proximate Causes of Growth Transitions

In order to account for the sources of growth transitions, nonparametric growth accounting is carried out for the five year period before and for the five year period after each break.²⁵ The reported yearly contributions of capital accumulation, efficiency change and technological change to yearly economic growth is the geometric average

²⁵If the break year is, say, 1960, the regime before the break comprises $g_{56}, ..., g_{60}$ and the regime after the break comprises $g_{61}, ..., g_{65}$. g_{56} denotes the growth rate from 1955 to 1956.

of the corresponding numbers for the five year period.²⁶ For ease of comparison with traditional growth accounting studies, the results are presented as growth rates, so that slight rounding errors owing to the conversion from growth factors to rates may occur. Across countries arithmetic means means are reported.²⁷ Before proceeding with the analysis, it is worthwhile to have a look at the estimated world technology frontiers. Figure 2 plots these in the (\tilde{k}, \tilde{y}) -space for the years 1950, 1975 and 2004. The notion of Hicks-neutral technological progress, one of the assumptions of standard growth accounting, is not supported, as can be seen by the much more pronounced outward shift of the production frontier at high levels of capitalization.

In Table 2 the available information for the proximate causes of growth in the five years preceding an upbreak is reported. In the first column the average yearly per-capita growth rate is decomposed into contributions made by efficiency, technology, capital deepening and labor force participation according to equation 7, whereby each estimate denotes the percentage points of growth that is generated by the respective component. Before an upbreak yearly income per capita declines by 0.546percentage points on average. The negative growth rate is the result of deteriorating efficiency: had it no been for changes in efficiency, the annual growth rate would have been 1.079% owing to capital accumulation, 0.043% owing to increased labor force participation and an additional 0.671% owing to technological progress. However, less efficient production decreases the growth rate by 2.277%. Based on 1000 bootstrap replications, columns two and three report the estimated bias and standard error of the bootstrap estimates. Since the bias greatly exceeds the standard error both for efficiency and technology, it makes sense to use the bias-corrected estimates instead of the original one. These are reported in column five. According to the bias corrected estimates technological change contributes only 0.322% to growth and thus roughly half of the uncorrected value. The contribution of capital accumulation remains essentially unchanged whereas efficiency is estimated to decrease growth only by -1.944%. Finally, columns six and seven report the bootstrapped 90% confidence intervals for efficiency and technological change, and capital deepening. The contribution to growth is significantly different from zero for each of the components. In the following only the bias-corrected values and confidence intervals are reported. The re-

²⁶Since the Fisher type indices do not satisfy the circularity test, the results depend on the order of calculation (Coelli, Rao, and Battese, Coelli et al., chap. 4.5).

²⁷Using arithmetic means implies that the relative weights of the economies are ignored, which is particularly problematic when the focus is on standardized magnitudes. If groups of countries are compared at a specific point in time, it therefore makes sense to use weighted averages (Simar et al., 2007; Henderson and Zelenyuk, 2007). However, since in this paper the behavior in different years is summarized, there is no obvious weight that should be used.



Figure 2: World Technology Frontiers and the Type of Technological Progress

lation between the estimated bias and standard error continues to favor this approach.

Table 3 summarizes the decomposition results for growth around upbreaks. The first column in the upper panel of the table refers to the proximate causes of growth preceding an upbreak and corresponds to table 2. The second column shows that after an upbreak the average annual growth of per capita income jumps to almost 5%. While the contribution of all components increases, it is the altering role of efficiency changes that stands out: improvements in efficiency alone generate more than two percentage points of the observed growth rate. Technological change occurs faster after an upbreak than before, but the confidence intervals indicate that this change is not statistically significant. The contribution of capital accumulation to growth increases after an upbreak from 1% to 1.7% and is significantly higher than before. The third column reports the changes before and after the break. Growth per capita increases by 5.5%. 4.2% and thus three quarters of this increases can be traced back to improved efficiency of production. Capital accumulation accounts for only 0.6% or less than 12 percent of the increase and the contributions of labor force participation and faster technological progress are even more limited. The confidence intervals indicate that the difference in the contributions of efficiency and capital accumulation between the two periods is significant, whereas that of technological change is not. Changes in

Upbreaks	Estimate	Bias	Sigma	Bias Corrected	Lower bound	Upper Bound
Income per Capita	-0.546					
Efficiency Change	-2.277	-0.333	0.005	-1.944	-2.207	-1.678
Technological Change	0.671	0.349	0.005	0.322	0.048	0.597
Capital Deepening	1.079	-0.002	0.001	1.081	1.027	1.140
Labor Force Participation	0.043					
Observations	39					
The estimates are the es	timated aver	age annu	al growth r	ates for the res	pective qua	antities.

Table 2: Proximate Causes of Growth Prior to an Upbreak

labor force participation are not estimated and therefore do not come with confidence intervals.

Is the relative contribution to growth around upbreaks sensitive to the state of development of the economies? The lower part of Table 3, which reports the differences in per-capita growth and the respective contributions for high, middle and low income countries, reveals that the sources of growth differ between groups of countries. First, the less developed a country, the larger the increase in the growth rate tends to be. Second, efficiency improvements contribute to the accelerated growth rate in all countries, but more so the less developed the economy. Around 88% of the difference in growth is due to efficiency improvements in low income countries compared to roughly 55% in middle income and 25% in high income countries. The importance of technology is the reverse: technological change explains almost 36% of the increased growth rate in high income countries and around 25% in middle income countries, whereas it makes no significant contribution to the growth difference in low income countries. Capital deepening is a significant source of an accelerating growth rate in all types of countries. However, its relative contribution is the highest in high income countries (22%) and lowest in low income countries (9%).

Table 4 replicates Table 2 for downbreaks. In the average downbreak the growth rate falls from 4.8% to -0.87%. Three quarters of this fall are explained by deteriorations in efficiency, roughly one quarter is explained by slower capital accumulation. Technological progress makes no significant contribution to growth rates changes whereas the contribution of labor force participation increases. The increase in labor force participation can be traced back to middle and low income countries and possibly indicates that households have to make up income losses at the individual level by higher participation in the labor market at the household level. The magnitude of the decline in per capita growth rates is almost comparable to the magnitude of the increases in upbreaks in high and low income countries, but it is noticeably larger

	Before		After		Difference	e	
Income per Capita	-0.546		4.964		5.510		
Efficiency Change	-1.944		2.228		4.172		
	[-2.207]	-1.678]	[2.008]	2.461]	[3.823]	4.511]	
Technological Change	0.322		0.657		0.335		
	[0.048]	0.597]	[0.417]	0.878]	[-0.010]	0.699]	
Capital Deepening	1.081		1.717		0.636		
	[1.027]	1.140]	[1.652]	1.782]	[0.547]	0.724]	
Labor Force Participation	0.043		0.334		0.290		
Observations	39		39		39		
Differences	High Income		Middle Ir	Middle Income		Low Income	
Income per Capita	3.389		5.237		6.180		
Efficiency Change	0.838		2.929		5.454		
Efficiency Change	0.838 [0.071]	1.603]	2.929 [2.464]	3.369	5.454 [5.018]	5.907]	
Efficiency Change Technological Change	0.838 [0.071 1.229	1.603]	2.929 [2.464 1.328	3.369]	5.454 [5.018 -0.193	5.907]	
Efficiency Change Technological Change	0.838 [0.071 1.229 [0.468]	1.603] 2.011]	2.929 [2.464 1.328 [0.896	3.369] 1.754]	5.454 [5.018 -0.193 [-0.636	5.907] 0.247	
Efficiency Change Technological Change Capital Deepening	0.838 [0.071 1.229 [0.468 0.750	1.603] 2.011]	$2.929 \\ [2.464 \\ 1.328 \\ [0.896 \\ 0.725]$	3.369] 1.754]	5.454 [5.018 -0.193 [-0.636 0.579	5.907] 0.247	
Efficiency Change Technological Change Capital Deepening	$\begin{array}{c} 0.838 \\ [0.071 \\ 1.229 \\ [0.468 \\ 0.750 \\ [0.584 \end{array}$	1.603] 2.011] 0.912]	2.929 [2.464 1.328 [0.896 0.725 [0.589	3.369] 1.754] 0.874]	5.454 [5.018 -0.193 [-0.636 0.579 [0.451	5.907] 0.247 0.711]	
Efficiency Change Technological Change Capital Deepening Labor Force Participation	$\begin{array}{c} 0.838 \\ [0.071 \\ 1.229 \\ [0.468 \\ 0.750 \\ [0.584 \\ 0.512 \end{array}$	1.603] 2.011] 0.912]	$\begin{array}{c} 2.929 \\ [2.464 \\ 1.328 \\ [0.896 \\ 0.725 \\ [0.589 \\ 0.190 \end{array}$	3.369] 1.754] 0.874]	5.454 [5.018 -0.193 [-0.636 0.579 [0.451 0.257	5.907] 0.247 0.711]	

Table 3: Bias Corrected Estimates and Confidence Intervals: Upbreaks

This table reports the average annual growth rates for the respective quantities. For efficiency changes, technology changes and capital deepening the lower and upper confidence intervals based on 1000 repetitions are reported in brackets.

in middle income countries. Slower capital accumulation plays a relatively more important role for downbreaks than for upbreaks, a feature that is particularly evident for low and middle income countries. The relative importance of capital deepening is 23% and 20% around downbreaks compared to 9% and 13% compared to upbreaks.²⁸ However, similar to upbreaks, the main change occurs in the efficiency of production, which plummets in middle and low, and falls in high income countries and which explains more than 80% and 63% of the difference in growth rates, respectively. Technological changes explain less of the growth rate after the break, but the decline in explanatory power is not significant.

²⁸To the extent that downbreaks are associated with civil unrest or wars, the numbers understate the contribution of capital accumulation since destruction of the capital stock exceeding that of normal depreciation rates is not considered.

	Before		After		Difference	э
Average Annual Growth of Income per Capita	4.797		-0.867		-5.663	
Efficiency Change	2.220		-2.119		-4.339	
	[2.076]	2.376]	[-2.222]	-2.004]	[-4.524]	-4.156]
Technological Change	0.255		0.185		-0.070	
	[0.119]	0.380]	[0.068]	0.291]	[-0.232]	0.105]
Capital Deepening	2.106		0.781		-1.326	
	[2.046]	2.165]	[0.742]	0.818]	[-1.394]	-1.254]
Change in Labor Force Participation	0.188		0.319		0.130	
Observations	58		58		58	
	High Income		Middle Income		Low Income	
Differences	High Inco	ome	Middle Ir	ncome	Low Inco	me
Differences Average Annual Growth of Income per Capita	High Inco -3.898	ome	Middle Ir -7.051	ncome	Low Inco -6.372	me
Differences Average Annual Growth of Income per Capita Efficiency Change	High Inco -3.898 -2.490	ome	Middle Ir -7.051 -5.761	ncome	Low Inco -6.372 -5.106	me
Differences Average Annual Growth of Income per Capita Efficiency Change	High Inco -3.898 -2.490 [-2.826	-2.149]	Middle Ir -7.051 -5.761 [-6.014	-5.520]	-6.372 -5.106 [-5.410	me -4.838]
Differences Average Annual Growth of Income per Capita Efficiency Change Technological Change	High Inco -3.898 -2.490 [-2.826 -0.196	-2.149]	Middle Ir -7.051 -5.761 [-6.014 -0.086	-5.520]	-6.372 -5.106 [-5.410 0.068	-4.838]
Differences Average Annual Growth of Income per Capita Efficiency Change Technological Change	High Inco -3.898 -2.490 [-2.826 -0.196 [-0.549	-2.149] 0.146]	Middle Ir -7.051 -5.761 [-6.014 -0.086 [-0.290	-5.520] 0.118]	-6.372 -5.106 [-5.410 0.068 [-0.181	-4.838] 0.335]
Differences Average Annual Growth of Income per Capita Efficiency Change Technological Change Capital Deepening	High Inco -3.898 -2.490 [-2.826 -0.196 [-0.549 -1.076	-2.149] 0.146]	Middle Ir -7.051 -5.761 [-6.014 -0.086 [-0.290 -1.415	-5.520] 0.118]	-6.372 -5.106 [-5.410 0.068 [-0.181 -1.507	-4.838] 0.335]
Differences Average Annual Growth of Income per Capita Efficiency Change Technological Change Capital Deepening	High Inco -3.898 -2.490 [-2.826 -0.196 [-0.549 -1.076 [-1.210	-2.149] 0.146] -0.944]	Middle Ir -7.051 -5.761 [-6.014 -0.086 [-0.290 -1.415 [-1.519	-5.520] 0.118] -1.301]	-5.106 [-5.410 0.068 [-0.181 -1.507 [-1.623	-4.838] 0.335] -1.394]
Differences Average Annual Growth of Income per Capita Efficiency Change Technological Change Capital Deepening Change in Labor Force Participation	High Inco -3.898 -2.490 [-2.826 -0.196 [-0.549 -1.076 [-1.210 -0.050	-2.149] 0.146] -0.944]	Middle Ir -7.051 -5.761 [-6.014 -0.086 [-0.290 -1.415 [-1.519 0.289	-5.520] 0.118] -1.301]	-6.372 -5.106 [-5.410 0.068 [-0.181 -1.507 [-1.623 0.190	-4.838] 0.335] -1.394]
DifferencesAverage Annual Growth of Income per CapitaEfficiency ChangeTechnological ChangeCapital DeepeningChange in Labor Force Participation Observations	High Inco -3.898 -2.490 [-2.826 -0.196 [-0.549 -1.076 [-1.210 -0.050 21	-2.149] 0.146] -0.944]	Middle Ir -7.051 -5.761 [-6.014 -0.086 [-0.290 -1.415 [-1.519 0.289 16	-5.520] 0.118] -1.301]	Low Inco -6.372 -5.106 [-5.410 0.068 [-0.181 -1.507 [-1.623 0.190 21	-4.838] 0.335] -1.394]

 Table 4: Bias Corrected Estimates and Confidence Intervals: Downbreaks

This table reports the average annual growth rates for the respective quantities. For efficiency changes, technology changes and capital deepening the lower and upper confidence intervals based on 1000 repetitions are reported in brackets.

4.4 Discussion

The previous section confirms Jones and Olken's (2005) surprising result that capital accumulation is not driving medium-term growth rate changes. Despite using a different testing procedure for structural breaks, longer time-series and the less restrictive nonparametric growth accounting framework, this paper, too, finds that capital accumulation explains only 12% of the growth rate changes around upbreaks and 23% around downbreaks. It also confirms that upbreaks and downbreaks are asymmetric events in the sense that capital accumulation explains significantly more of the growth rate change around downbreaks than around upbreaks. Therefore, the basic conclusions are similar to those by Jones and Olken (2005). The number of growth accelerations in low income countries suggests that countries do not remain in poverty traps. The focus on capital accumulation to explain short-term growth, which is present in many models such as those on industrialization, poverty traps but also the neoclassical growth model, does not quite get to the heart of explaining growth transitions. The asymmetry of upbreaks and downbreaks casts doubt on attempts to explain growth accelerations and decelerations within the same modeling framework.

The decomposition of productivity changes into efficiency changes and changes in technology and the distinction of countries according to their state of development offers further insights. Based on the numbers for total averages only, technological improvements seem unimportant in the context of medium-term growth rate changes. Changes in total factor productivity appear to reflect almost entirely changes in the efficiency of production. However, a different assessment follows if growth accelerations are analyzed according to the state of development of the respective countries at the timing of the breaks. Whereas technological improvements are indeed irrelevant for low income countries, this does not hold for middle and high income countries. These countries benefit from technological progress around upbreaks and it is quite possible that the enhanced production possibilities gained by technological improvements are the ultimate reason behind the accelerated growth rate. The fact that the contribution of capital accumulation to accelerations of the growth rate increases with technological improvements may further indicate that technological progress is of the embodied type. Hence, the endogenous growth framework with embodied technological progress may be a promising modeling framework for these countries and may be very informative at suggesting appropriate policies to achieve growth accelerations.

Clearly, the driving forces of growth accelerations are very different for the less developed countries. Growth accelerations in this country group are essentially improvements in the efficiency of production with not much else going on. Hence, the reallocation of resources appears to be a central element of what is happening. How this reallocation is achieved is an open question and should be the focus of future studies. Jones and Olken (2005) suggest that openness and the composition of manufacturing are essential. Yet, these kind of changes are not sufficient to predict growth accelerations (Hausmann et al., 2005), so that more encompassing explanations are required. The literature also acknowledges that initiating growth accelerations is different from sustaining them (Rodrik, 2005). The present framework points at one possible reason. If low income countries initially grow on the intensive margin by improving the efficiency of production, at some point these benefits will be reaped and the countries will have to switch to the extensive margin and either accumulate capital or innovate. It is conceivable that the inability of many poor countries to sustain growth accelerations is a consequence of the countries' failure to undergo this change.

Downbreaks differ from upbreaks in two respects: first, less rapid capital accumulation is a non-negligible part of the explanation and second, massive falls of productive efficiency occur across all country groups. However, the decrease of efficiency might be overstated. The calculations are based on per worker values, which themselves are constructed using labor force participation rates that do not account for unemployment or hours worked. Therefore, if unemployment during downbreaks increases or if hours worked fall, output per worker and hence efficiency is underestimated.²⁹ Still, the direction of the potential error is not unequivocal because the same argument implies that capital per worker is understated or lies idle, which leads to overestimated efficiency scores. Obviously, a better way to account for capacity utilization is desirable, but it is quite likely that efficiency changes will continue to play a major role.³⁰ The reasons for the observed decline in efficiency are of major interest and should be the focus of additional research. Based on the existing literature, conceivable explanations include civil conflict, bad macroeconomic management (e.g. hyper inflation), adverse terms of trade shocks coupled with an inflexible production structure, price shocks, inflexibility due to vested interest group or demography to name just a few (Hausmann et al., 2006; Feyrer, 2009; Funke et al., 2008; Becker and Mauro, 2006). Since many of these aspects are difficult to measure, the most rewarding way forward appears to be a series of case studies to find out the common factors present in all countries.

5 Robustness Checks

Due to the well known sensitivity of DEA analysis to atypical observations, the previous results have to be checked for robustness. This section analyzes the consequences of altering the assumptions used in the BP procedure and the derivation of the capital stock, of extending the accounting period around growth transitions and of eliminating frontier-defining countries from the sample. Table 5 shows how the contributions of efficiency, technology and capital deepening change across growth transitions in high, middle and low income countries. The bias-correction of the estimates is based on 200 bootstrap replications.

Consider the robustness with regard to the BP assumptions first. The structural breaks are derived using heteroscedasticity and autocorrelation consistent standard errors, a minimum duration requirement of five years or a constant trimming parameter of 0.1, respectively. HAC standard errors imply additional breaks, but do not significantly influence the conclusions otherwise. The number of break points is also increased if the minimum duration of growth spells is reduced or if a constant trimming

²⁹However, it might be argued that unemployment should be considered in the efficiency on an economy-wide level.

³⁰Jones and Olken (2005) employ electricity consumption to assess capacity utilization more directly. Their results are not sensitive to this change.

Table	5:	Robustness	Checks
20010	. .	100000000000000	01100110

			Upbreaks		D	ownbreal	κs
		High	Middle	Low	High	Middle	Low
Base	Income per Capita Efficiency Change Technological Change Capital Deepening Labor Force Part. Observations	$3.39 \\ 0.84^* \\ 1.23^* \\ 0.75^* \\ 0.22 \\ 7$	5.24 2.93^{*} 1.33^{*} 0.73^{*} 0.14 7	6.18 5.45^* -0.19 0.58^* 0.09 25	-3.90 -2.49^{*} -0.20 -1.08^{*} 0.28 21	-6.87 -5.70^{*} 0.03 -1.42^{*} 0.21 17	-6.49 -5.13* -0.02 -1.51* 0.23 20
HAC	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	$3.39 \\ 0.86^* \\ 1.21^* \\ 0.74^* \\ 7$	5.44 3.38^{*} 0.94^{*} 0.77^{*} 8	5.96 5.16^{*} -0.18 0.66^{*} 27	-3.90 -2.50* -0.18 -1.08* 21	-6.75 -5.70^{*} 0.13 -1.38^{*} 18	-6.54 -4.98* -0.01 -1.76* 23
h = 5	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	2.82 1.07^{*} 0.84^{*} 0.59^{*} 7	$6.23 \\ 4.95^* \\ 0.70^* \\ 0.33^* \\ 11$	$7.00 \\ 6.02^* \\ 0.67^* \\ 0.05 \\ 30$	-4.02 -2.35* -0.54* -1.02* 23	-7.07 -6.41^{*} 0.20 -1.01^{*} 17	-7.72 -6.53^{*} 0.32^{*} -1.61^{*} 26
$\varepsilon = 0.1$	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	2.82 1.09^{*} 0.83^{*} 0.58^{*} 7	6.42 4.92^{*} 0.95^{*} 0.33^{*} 10	$7.92 \\ 7.14^* \\ 0.60^* \\ 0.06 \\ 33$	-4.16 -2.44* -0.57* -1.02* 22	-7.24 -6.01* 0.09 -1.47* 16	-7.60 -6.07* 0.24 -1.85* 24
acc = 8	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	$2.81 \\ 0.28 \\ 1.21^* \\ 0.66^* \\ 7$	$\begin{array}{c} 4.45\\ 2.70^{*}\\ 0.76^{*}\\ 0.73^{*}\\ 7\end{array}$	$5.44 \\ 4.35^{*} \\ 0.22 \\ 0.45^{*} \\ 25$	-3.62 -1.92* -0.45* -1.19* 21	-5.62 -4.51^{*} 0.00 -1.43^{*} 17	-5.46 -4.32* 0.13 -1.48* 20
acc = regime	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	$1.74 \\ 0.01 \\ 0.75^* \\ 0.17^* \\ 7$	5.82 3.55^{*} 0.61^{*} 1.20^{*} 7	5.07 4.79^{*} 0.26^{*} -0.72^{*} 25	-3.71 -2.51* -0.01 -1.25* 21	-4.94 -4.04* 0.18* -1.99* 17	-4.49 -2.09* 0.09 -2.97* 20
$\delta = 0.1$	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	$3.39 \\ 0.09 \\ 1.93^* \\ 0.80^* \\ 7$	5.24 2.93^{*} 1.03^{*} 1.02^{*} 7	$\begin{array}{c} 6.18 \\ 5.45^* \\ -0.39 \\ 0.77^* \\ 25 \end{array}$	-3.90 -2.20* -0.32 -1.26* 21	-6.87 -5.24* -0.07 -1.79* 17	-6.49 -4.81* 0.00 -1.86* 20
$\delta = 0.05$	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	3.39 2.73* -0.33 0.39* 7	5.24 3.23^{*} 1.28^{*} 0.47^{*} 7	$6.18 \\ 5.26^* \\ 0.13 \\ 0.45^* \\ 25$	-3.90 -2.64* -0.23 -0.88* 21	-6.87 -5.90* -0.04 -1.15* 17	-6.49 -5.40* -0.01 -1.24* 20
Mild Sample	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	3.29 1.59^{*} 0.70^{*} 0.48^{*} 6	5.24 2.99^{*} 1.21^{*} 0.79^{*} 7	6.18 5.38^* -0.06 0.52^* 25	-3.85 -2.59* -0.13 -0.98* 20	-6.76 -6.00^{*} 0.35^{*} -1.35^{*} 16	-6.49 -5.41* 0.24 -1.48* 20
Strict Sample	Income per Capita Efficiency Change Technological Change Capital Deepening Observations	3.29 1.96^{*} 0.14 0.65^{*} 6	5.24 4.35^{*} -0.07 0.69^{*} 7	$\begin{array}{c} 6.18 \\ 5.28^* \\ -0.15 \\ 0.71^* \\ 25 \end{array}$	-3.85 -2.18* -0.41* -1.10* 20	-6.74 -5.66^{*} 0.06 -1.44^{*} 14	-6.32 -4.55^{*} 0.10 -2.06^{*} 18

parameter is imposed.³¹ While the conclusions remain broadly similar, the importance of capital accumulation for middle and low income countries during upbreaks is reduced even further, possibly indicating that unsustained (i. e. shorter) growth accelerations rely to a greater extent on short-run efficiency improvements. Regarding downbreaks, technological change in the case of h = 5 contributes positively and significantly to the growth rate deceleration in low income countries.³² This finding is highly implausible and closer inspection reveals that it is due to a recorded downbreak for Gambia in 1966. This downbreak, however, is a highly questionable event because before the growth deceleration the growth rate jumps erratically between -10% and 27%, so that it is difficult to see growth in that period as moving around a well defined average.

By focusing on the five year period before and after a break only, it is conceivable that the importance of capital deepening is missed due to lagged effects of investment (Jones and Olken, 2005). Therefore, the accounting period around growth transition is extended to eight years or the whole duration of the spell, respectively. In both cases, efficiency changes become insignificant for high income countries, corroborating the notion that once efficiency reserves have been used up, long-run growth requires technological progress and cannot be achieved by accumulating capital indefinitely. Somewhat unexpectedly, capital accumulation contributes negatively to growth accelerations in low income countries when long-run averages are considered. This implies that in absolute numbers more of the growth rate is explained by capital deepening before an upbreak than afterwards.³³ One explanation might be that many low income countries start out with extremely low capital stocks when the whole regime prior to an upbreak is considered and thus find themselves in the region where the world technology frontier is the steepest. As a consequence even modest capital stock extensions have larger growth implications than anywhere else. This feature is what should be expected if capital has diminishing marginal returns. In any case, the limited relevance of capital accumulation and the importance of technology for growth at the extensive margin are confirmed.

A key difficulty in cross-country growth accounting is the need to impute the

³¹Notice that the latter effectively reduces the minimum duration of growth spells even further to 3.4 or five years depending on the number of available observations. 3 breaks had to be discarded in the growth accounting because the next break happened before the end of the accounting period of 5 years.

³²Although the contribution of technological progress to growth transitions often varies in significance, it seldom is significant with a deviating sign compared to the baseline case. Only these cases are commented on. ³³Capital accumulation contributes positively to growth both before and after the break, though.

capital stock of countries. Calculations based on the perpetual inventory method are particularly sensitive to the assumed depreciation rate so that the effects of alternative depreciation rates equalling 5% and 10% are evaluated as robustness checks. Compared to the baseline case the implied differences are vast: the initial capital stock for the United States ranges from approximately 70% to 130% of the base estimate. Since the imputed capital stock enter directly into the world technology frontier, it is clear that its shape will be altered: higher (lower) depreciation rates imply lower (higher) capital stocks so that the technology frontier becomes steeper (flatter). Notwithstanding, the results for growth decelerations remain stable as do the results for growth accelerations in middle and low income countries. Unfortunately, the implications for growth accelerations in high income countries are less robust. It appears that a lower depreciation rate increases the contribution of efficiency at the expense of the contribution of technology and capital accumulation to the extent that technological progress is no longer a source of the observed accelerations. The sensitivity of the results in high income countries can be traced back to first, most accelerations happening at the beginning of the sample period and thus at a time where the huge differences in the implied initial capital stocks still have strong implications, and second, to countries being relatively close to and thus very dependent on the exact shape of the technology frontier. While, the sensitivity of the results is disconcerting, there is evidence that depreciation rates in rich economies are higher than 5 % (Hulten and Wykoff, 1981; Timmer et al., 2007) so that the basic conclusions warrant some confidence.

Finally, to rule out that the technology frontier is an artefact of some atypical observations, the robustness of the results is checked by eliminating frontier-defining countries from the sample. The observations to drop are selected in two different ways. In a mild version, the technology frontier is calculated separately for each year and the countries that span the frontier in that particular year are dropped for that particular year. A more demanding definition eliminates frontier-defining countries forever.³⁴ All results pass the mild robustness test. In the strict version, technological change loses its significance around upbreaks for both high income and middle income countries, which follows from the United States of America being eliminated from the sample. Given that the United States is among those countries that offer the highest quality of data and that is the least likely to be an outlier, the dropping rules in the strict version appear to be overly hard.

³⁴This amounts to estimating the frontier for t and dropping the fully efficient countries to obtain the reduced sample S_t . In t+1, the frontier is estimated using S_t and the observations from t+1. Observations from t+1 that lie on the frontier are dropped to generate S_{t+1} . By this definition, past technological advances are "forgotten" if they are not replicated by other countries.

Summing up, most results are robust to a variety of specification changes. In particular, the limited importance of capital accumulation and the dominating role of efficiency improvement for growth accelerations in middle and low income countries is highly robust. The results for high income countries are somewhat more sensitive, but there are good reasons to trust the findings that these countries benefit from technological improvements and capital accumulation in growth accelerations. The results for downbreaks are even more robust. Growth decelerations are explained by lower contributions of capital accumulation and by huge efficiency deteriorations.

6 Conclusion

In this paper the proximate causes of significant growth rate changes within countries have been analyzed with a special focus on the relative importance of factor accumulation versus productivity changes in order to test the robustness of a recent finding by Jones and Olken (2005): namely that total factor productivity improvements not only drive long-term growth, but also short-term growth events. Methodologically, nonparametric growth accounting has been applied because this helps to avoid a number of assumptions implicit in parametric growth accounting. Moreover, productivity changes can be attributed to changes in efficiency and changes in technology.

Despite the tendency of nonparametric growth accounting to find an increased role for factor accumulation compared to traditional growth accounting (Henderson and Russell, 2005; Jerzmanowski, 2007) and despite the finding that the Hicks-neutral technological progress assumption of parametric growth accounting is not justified, the present study confirms that even short-run growth transitions are mainly productivity events. Depending on the level of development at least three quarters of growth accelerations and decelerations are explained by efficiency and technological changes. In contrast to predictions by neoclassical growth models, capital accumulation contributes the most to growth rate increases in high income countries and the least to those in low income countries. Growth accelerations in low income countries are mainly due to improvements in the efficiency of production. Technological progress benefits middle and high income, but not low income countries. Growth decelerations are the result of slower capital accumulation and deteriorating efficiency levels. Unlike before, the level of development has only minor impacts on the relative contributions of the different drivers of growth. Finally, like Jones and Olken (2005) the present study corroborates the view that growth accelerations are different from deceleration in that the importance of capital accumulation is significantly higher in the latter. These results are robust to a number of specification changes.

The most lasting impression of the accounting exercise is the dominance of efficiency changes to explain growth transitions. Therefore, the next logical step is to search for the sources of efficiency changes. The literature has identified a multitude of factors that may influence the efficiency of production at the economy-wide level. Among these are the sectoral composition of production, the skill composition in the economy, the prevailing regulations and laws, the organization of vested interests, the integration into the world economy and thereby the ability to benefit from spillovers or scale economies, the prevalence of violent conflicts and rent-seeking, the availability of a well-functioning financial system or a reasonable level of trust between market participants to name just a few (Acemoglu and Zilibotti, 2001; Edwards, 1993; Frankel and Romer, 1999; ?; ?; ?; Murphy et al., 1989; Prescott, 1998). It is an open question whether there are typical patterns which countries experiencing significant and lasting efficiency changes have in common thus lending themselves to become blueprints of reforms or whether changes are country-specific and not easily transferable (Rodrik, 2005; Williamson, 1990). Most likely, a fruitful analysis will require more detailed data on economic reforms and institutional changes than are currently available. It might also be beneficial to focus directly on breaks in the efficiency scores rather than choosing the indirect way using growth transitions. In any case, a more thorough understanding of the mechanisms that determine the efficiency of production is needed to understand the forces holding back countries from prosperity.

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A Multiple Structural Breaks Estimation in Stata Using the Bai and Perron Methodology

Abstract: One important topic in the context of macroeconomic time series is the possible prevalence of one or multiple structural breaks. Bai and Perron (1998, 2003a, 2003b, 2006) have developed a methodology for finding multiple structural breaks and testing their significance in a linear regression model. This paper shortly reviews their methodology and introduces a Stata command that implements it.

A.1 Introduction

One important topic in the economics and statistics literature concerns structural change. A typical analysis looks at macroeconomic time series and asks whether structural changes have occurred at exogenously determined break dates or whether a single change has happened at an unknown break date. In these cases, the Chow test and the Andrews-Ploberger test apply, respectively (Greene, 2003, chap. 7). For a long time little has been known about an appropriate way to handle multiple structural breaks with unknown break points. However, in a series of influential papers, Bai and Perron (1998, 2003a, 2003b, 2006) have developed a methodology that allows consistent estimation of break dates in the presence of multiple unknown structural breaks along with testing procedures and algorithms to select the appropriate number of breaks. This paper reviews their methodology with a special emphasis on practical implementation issues and implements a stata command that allows estimation and sequential testing of multiple breaks in a pure linear structural change model.

A.2 Estimation and Testing

A.2.1 Estimating break points

In this note, the following linear regression model with m breaks and m + 1 regimes is considered:

$$y_t = \delta_j + u_t, \quad t = T_{j-1} + 1, \dots, T_j$$
 (S1)

for j = 1, ..., m + 1. y_t is the observed stationary dependent variable of a time series at time t that is regressed on a regime-specific constant δ_j (j = 1, ..., m + 1) yielding a model with regime-specific means in each resulting data segment. The disturbance term u_t has an expected value of zero, but may exhibit different variances across segments. Autocorrelation in the residuals is allowed. The total number of available observations is T. The purpose is to estimate the unknown break points $T_1, ..., T_m$ together with the unknown regression coefficients $\delta_1, ..., \delta_{m+1}$. $T_0 = 0$ and $T_{m+1} = T$ is assumed. The convention throughout this note is that T_j denotes the last observation belonging to regime j.

The method of estimation is based on the least-squares principle. For each *m*-partition $(T_1, ..., T_m)$ the coefficients δ_j (j = 1, ..., m+1) minimize the sum of squared residuals. Formally,

$$S_T(T_1, ..., T_m) = \operatorname*{argmin}_{\delta_1, ..., \delta_{m+1}} \sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (y_t - \delta_j)^2.$$
(S2)

The estimated break points minimize the sum of squared residuals over all conceivable *m*-partitions subject to the constraint that a minimum length of h > 1 between breaks is respected. Hence,

$$(\hat{T}_1, ..., \hat{T}_m) = \underset{T_1, ..., T_m}{\operatorname{argmin}} S_T(T_1, ..., T_m)$$

s. t. $T_j - T_{j-1} \ge h$ for $j = 1, ..., m + 1$. (S3)

Thus, the final solution globally minimizes the sum of squared residuals both with respect to the break dates and with respect to the regression coefficients.

In practice, the global minimizers of the objective function are derived by summarizing the sum of squared residuals in a suitable way and by applying a dynamic programming algorithm afterwards. Both steps serve to avoid a curse of dimensionality problem. First, the upper-triangular $(T \times T)$ matrix M is defined. The entry $M[t_1, t_2]$ stores the sum of squared residuals (SSR) that result if y_t is regressed on a constant using observations $t_1, ..., t_2$. The SSR for every conceivable m-partition $S_T(T_1, ..., T_m)$ can be derived by summing up the SSR for each associated segment so that the essence of equation (S2) is implemented. In order to avoid too many matrix inversions, the SSR are obtained using the updating formula for recursive residuals suggested by Brown et al. (1975).³⁵

The optimal m-partition is found by solving the following recursive problem:

$$SSR(T_{m,T}) = \min_{mh \le t \le T-h} [SSR(T_{m-1,t}) + SSR(t+1,T)].$$
 (S4)

 $SSR(T_{r,n})$ denotes the SSR associated with the *optimal* partition of the time series

 $^{^{35}}$ It is not even necessary to calculate all entries of M because certain entries are not permissible due to the minimal length requirement for each regime. However, this refinement is not implemented.

containing r breaks and using the first n observations, SSR(t+1,T) denotes the SSR for the data segment starting in (t+1) and lasting until T. It is easiest to understand the logic of the procedure by following its empirical implementation. Two further $(m+1) \times T$ matrices L and B are defined. Matrix L records the minimal estimated SSR for a partition running from period 1 to the column number for a given number of breaks, which equals the row number minus one. Matrix B stores the associated break dates following the same conventions. It follows that the first line of matrix L contains the estimated SSR for a sample running from period 1 to T, 1 to (T-1) etc. with no break and is therefore equal to the first line of matrix M. The first line in matrix B is empty because no breaks are involved.

The second line of matrix L contains the minimal estimated SSR for a sample running from 1 to T with one structural break, the minimal estimated SSR for a sample running from 1 to (T-1) with one structural break and so on. The structural break is chosen such that the estimated SSR is minimized and that the minimum duration requirement h for each regime is respected. Hence, for the entry L[2,T] the resulting SSR is compared for all conceivable break dates ranging from h to (T-h) and the break date leading to the smallest SSR is selected. This break date \hat{T}_1 is recorded in B[2,T] while the associated SSR is recorded in L[2,T]. The other entries are derived accordingly. For instance, for L[2, (T-1)] and B[2, (T-1)] the resulting SSR is evaluated for possible break dates ranging from h to (T-h-1).

The derivation of the third lines in L and B illustrates the working of the recursive procedure. Suppose the aim is to derive L[3,T] and B[3,T]. Since $\hat{T}_1 < \hat{T}_2$ and since the minimum duration requirement for each regime has to be respected, the second break can happen between period 2h and (T - h). The resulting SSR for each conceivable second break date T_2 in the sample running from 1 to T is given by $L[2,T_2] + M[T_2 + 1,T]$. It automatically incorporates the optimal one-break partition for the sample spanning the observations from 1 to T_2 . After the SSR has been derived for every conceivable second break date \hat{T}_2 and the SSR are recorded in B[3,T]and L[3,T], respectively. All other entries in the second line are derived accordingly. The same routine is repeated until m breakpoints are imposed upon the time series. Once matrices L and B are derived, it is easy to read off the optimal break points. If m break points are estimated, \hat{T}_m is recorded in B[m + 1, T]. The next break point \hat{T}_{m-1} is found in $B[m, \hat{T}_m]$, \hat{T}_{m-2} is available in $B[m - 1, \hat{T}_{m-1}]$. Going back step by step the first break is obtained from $B[2, \hat{T}_2]$.

A.2.2 Test Statistics

The test statistics in the presence of multiple structural are derived under the shrinking shift asymptotic framework so that the consistency results do not directly refer to the break dates, but to the break fractions $\lambda_j = (T_j/T)$ for j = 1, ..., m.³⁶ Therefore, the test statistics are not expressed in terms of the *m*-partition $(T_1, ..., T_m)$, but in terms of the *m*-partition $(\lambda_1, ..., \lambda_m)$. Furthermore, the asymptotic distributions depend on the trimming parameter $\varepsilon = h/T$, which is the asymptotic equivalent of the minimum duration requirement and is necessary for the break fractions to be asymptotically distinct and bounded from the boundaries of the sample.

The first test statistic is called the $\sup F_T$ test statistic and forms the basis for all following tests. The $\sup F$ test tests the null hypothesis of no structural break (m = 0) against the alternative hypothesis that m = k structural breaks are present. This test is particularly useful if one has a fairly good idea a priori as to how many breaks to expect. The $\sup F_T(k)$ test statistic is the supremum of all the standard Fstatistics testing the equality of means across regimes over all admissible k-partitions. Asymptotically, the $\sup F_T(k)$ test statistic is equivalent to the standard F-statistic that results if the consistently estimated break fractions $(\hat{\lambda}_1, ..., \hat{\lambda}_k)$ are used for its construction. Hence, only the following formula needs to be evaluated in practice:

$$\sup F_T(k) = F_T(\hat{\lambda}_1, ..., \hat{\lambda}_k) = \frac{1}{T} \left(\frac{T - (k+1)}{k} \right) \hat{\delta}' R' (R\hat{V}(\hat{\delta})R')^{-1} R\hat{\delta}.$$
 (S4)

Thereby R is the conventional restrictions matrix such that $(R\delta)' = (\delta_1 - \delta_2, ..., \delta_k - \delta_{k+1})$ and such that the estimated break fractions are respected. $\hat{V}(\hat{\delta})$ is the covariance matrix of $\hat{\delta}$ that - if desired - can be made robust to serial correlation and heteroscedasticity. $\hat{V}(\hat{\delta})$ is estimated by the standard OLS covariance matrix using all observations if the errors are assumed to be identically distributed across segments. It is estimated as the standard OLS covariance matrix using the data for each segment separately if the errors are assumed to have different variances across regimes but are serially uncorrelated. If both serial correlation and different variances across regimes prevail, the covariance matrix is estimated using the quadratic spectral kernel based method introduced by Andrews (1991). In this case prewhitening as in Andrews and Monahan (1992) is recommended. The value of the sup $F_T(k)$ test statistics is compared to simulated critical values, which depend both on k and ε . A large test statistic indicates that the break points significantly improve the fit of the model. Hence, in

 $^{^{36}}$ However, Bai and Perron (1998) show that with high probability the deviation between the estimated and true break dates is bounded by some constant. Therefore, in empirical applications the estimated break dates may be used with some confidence.

these cases the null hypothesis tends to be rejected.

In many interesting applications the number of breaks is not known beforehand. In this case the double maximum tests allows to test the null hypothesis of no break (m = 0) against the alternative of an unknown number of breaks up to K. The test-statistic is defined as largest sup $F_T(k)$ statistics for k = 1, ..., K or formally as

$$UDmax F_T(K) = \max_{1 \le k \le K} F_T(\hat{\lambda}_1, ..., \hat{\lambda}_k).$$
(S5)

The critical values depend on K and ε . As before, a large test statistic indicates that the null hypothesis should be rejected. The power of the double maximum test exceeds that of the supF test if k used in the latter does not correspond with the true number of breaks.

Ultimately, if the number of breaks is not known beforehand, the aim is to find the appropriate number of breaks by testing. One easy way to derive the appropriate number of breaks is to calculate the Bayesian information criterion (BIC) and choose the number of break points which minimize the associated BIC. Formally, for a series with k break points the BIC is defined as

$$BIC(k) = \ln\left(\frac{\hat{u}'\hat{u}}{T}\right) + \frac{(2k+1)\ln(T)}{T},$$
(S6)

with \hat{u} denoting the estimated residuals accounting for the k breaks. It should be noted that this version of the *BIC* penalizes each estimated coefficient and each estimated break point, hence the factor (2k + 1) in the second term. The *BIC* performs reasonably well in the absence of serial correlation, but tends to opt for too many breaks in the presence of it.

A more refined method for determining the appropriate number of structural breaks is the supF_T($\ell + 1|\ell$) sequential testing procedure, which is called supFL in the following. Here the null hypothesis of $k = \ell$ breaks is tested against the alternative that $k = \ell + 1$ breaks are present. Starting with $\ell = 0$ and increasing the number of break points one by one until the null is accepted, the number of required breaks is derived systematically. The test is implemented as follows. If the number of breaks under the null equals ℓ , an additional break point is introduced into each of the $\ell + 1$ data segments and the corresponding supF_T(1) test statistics are derived.³⁷ The

 $^{^{37}}$ If the minimum duration requirement precludes the introduction of an additional break point, supF_T(1) equals zero by definition.

largest supF_T(1) test statistic across all segments is selected and compared against the critical values that depend on k and ε .³⁸ The null is rejected in favor of a model with $(\ell + 1)$ breaks if the overall minimal value of the SSR is sufficiently smaller the the SSR from the model with ℓ breaks.

In some instances it may be difficult to reject the null of zero against one break, but easy to reject the null of zero against a higher number of breaks. In these cases the supFL testing procedure breaks down. Since the power of the double maximum test is almost as high as the power of a test of no breaks against the alternative specifying the true number of breaks, Bai and Perron (2006) recommend to adjust the supFL procedure and use the double maximum test in the first step when the null hypothesis of m = 0 breaks is tested. After this altered first step the test proceeds exactly like the supFL test. This test is called the udmaxL test. All tests presented in this section are implemented in the stata command *sbbpm*. In order to achieve as much power as possible, the covariance matrix should be corrected for heteroscedasticity and serial correlation whenever necessary.

A.3 Stata Implementation

A.3.1 Syntax

sbbpm depvar timevar, [minspan(#) maxbreaks(#) alpha(#) trimming(#) het(string)
prewhit(#) method(string)]

A.3.2 Description

The stata command sbbpm fits a pure multiple structural change model for means using the methods suggested by Bai and Perron. As a minimum it requires the name of the dependent and the time variable. The dependent variable must not contain any missing values. The data may be tsset beforehand.

A.3.3 Options

minspan(#) specifies the minimal length h that needs to be respected between breaks. The default value is 5.

maxbreaks(#) specifies the maximum number of breaks M that the time series may contain. The default value is 5.

³⁸Notice that this is equivalent to calculating the sup $F_T(\ell+1)$ test statistic using the whole time series.

alpha(#) specifies the significance level for the tests. The values 0.1, 0.05, 0.025 and 0.01 are allowed. The default value is 0.1.

trimming(#) specifies the trimming parameter ε that is needed for the critical values. The values 0.05, 0.10, 0.15, 0.20 and 0.25 are allowed. The default value is 0.1. However, it is strongly recommended to adjust this default value to match $\varepsilon = h/T$ because otherwise wrong critical values are applied in the tests.

het(string) specifies the assumptions made with respect to the distribution of the data and the errors across segments. If there is no serial correlation, different distributions for the data, but identical distributions for the errors across segments, *iid* should be selected. With no serial correlation in the errors, but different data and error distributions across segments, the heteroscedasticity consistent covariance matrix *hc* should be selected. If in addition autocorrelation is assumed, the relevant option is the heteroscedasticity and autocorrelation consistent matrix *hac*. The default value is *iid*.

prewhit(#) specifies whether the heteroscedasticity and autocorrelation consistent matrix should be derived using prewhitening. If prewhitening should be used, the value 1 has to be entered. 0 denotes no prewhitening. The default value is 1.

method(string) specifies the methods to be applied. Possible entries are: bonly, sup, udmax, bic, supseq, udseq. In this order they indicate to calculate the breaks only, to apply the supF_L or the UDmaxF_T test, to report the BIC, to apply the original sequential supF_T($\ell + 1|\ell$) testing procedure or the udmaxL sequential testing procedure. The default value is bonly.

A.3.4 Saved Results

sbbpm saves in e(). The following is a complete list of saved results. The returned results vary depending on the chosen method.

Scalars

e(baserss)	SSR for model with no breaks
e(bicbreak)	Final number of breaks chosen according to BIC
e(ellfinal)	Final number of breaks chosen with sequential methods

Matrices

e(delta) Reports the regime-specific means for all breaks up to maxbreak breaks

e(var)	Reports the regime-specific variance for all breaks up to $maxbreak$ breaks
e(intervalrss)	Matrix M
e(bestrss)	Matrix L
e(lastbreak)	Matrix B
e(breaks)	Reports the break periods counted from 1 onwards for up to $maxbreak$ breaks
	according to the number of the observations
e(dates)	Reports the break dates for up to maxbreak breaks
	according to the time variable
$e(\texttt{supF_res})$	Reports the $\sup F_L$ test statistics
e(udmax)	Reports the largest supF-statistic for each number of
	breaks up to maxbreak breaks
e(udres)	Reports the results of the double maximum test
e(bic)	Reports the BIC for all breaks up to maxbreak breaks
e(supFL)	Reports the results of the sequential \sup_{L} test

B Structural Breaks

Country	Break 1	Break 2	Break 3	Regime 1	$\frac{\text{Regime}}{2}$	Regime 3	Regime 4
Australia	1961			1.39	2.28		
Austria	1973			4.83	2.13		
Belgium	1959	1974		2.24	4.54	1.87	
Burkina Faso	1996			0.24	3.50		
Bolivia	1958			-2.08	0.58		
Brazil	1980			4.49	0.40		
Botswana	1989			8.30	3.85		
Canada	1961			1.34	2.34		
Switzerland	1973			3.32	0.80		
Chile	1971	1985		2.29	-0.70	4.24	
China	1977			2.63	8.36		
Cote d'Ivoire	1989			2.56	-1.66		
Cameroon	1986	1994		2.44	-5.87	2.63	
Congo, Republic of	1984			4.92	-2.79		
Colombia	1967	1979		1.45	3.47	1.08	
Costa Rica	1978			3.01	0.89		
Denmark	1973			3.20	1.51		
Dominican Republic	1991			2.37	4.29		
Algeria	1987			2.11	0.10		
Ecuador	1981			3.35	-0.41		
Spain	1974	1984		5.94	0.61	2.70	
Finland	1974			4.35	1.93		
France	1973			4.07	1.80		
United Kingdom	1982			2.01	2.52		
West Germany	1960			6.70	2.58		
Guinea	1994			-0.55	3.28		
Greece	1962	1973	1996	4.35	7.65	0.79	3.65
Guatemala	1980	1988		1.98	-2.04	0.67	
Hong Kong	1988			6.93	2.32		
Honduras	1982			1.15	-0.21		
Haiti	1980			4.08	-1.01		
Hungary	1988	1996		3.37	-0.82	4.50	
Indonesia	1968			0.36	3.73		
India	1994			2.29	4.73		
Ireland	1993			2.76	6.67		
Iran	1976	1989		5.73	-5.19	3.51	
Israel	1973			5.09	1.40		
Italy	1974			4.87	1.87		

Table 6: Structural Breaks in Growth

Table 6 continued

Country	Break 1	Break 2	Break 3	Regime 1	Regime 2	Regime 3	Regime 4
Jamaica	1972	1980		4.28	-3.29	0.95	
Jordan	1965			5.20	-0.65		
Japan	1958	1970	1991	6.14	9.44	3.30	0.79
Korea, Republic of	1962			0.91	6.11		
Lesotho	1978			4.37	2.31		
Morocco	1958			-0.28	2.74		
Madagascar	1971			0.95	-1.79		
Mexico	1981			3.44	0.36		
Mozambique	1995			0.38	6.39		
Mauritius	1960			-4.29	3.75		
Malawi	1995			1.81	-0.26		
Malaysia	1970			2.77	4.88		
Nigeria	1960			4.56	0.46		
Nicaragua	1976			2.74	-2.38		
Netherlands	1974			3.23	1.57		
Norway	1986			3.28	2.29		
New Zealand	1966			2.48	1.21		
Pakistan	1962	1988		-0.13	3.79	1.42	
Panama	1981			3.61	1.44		
Peru	1975			3.21	-0.46		
Philippines	1977			3.16	0.94		
Poland	1978	1991		7.16	-1.05	4.06	
Portugal	1973			5.90	2.25		
Paraguay	1973	1981		1.06	6.12	-0.33	
Romania	1979			8.09	0.95		
Rwanda	1994			-1.34	11.89		
Singapore	1996			5.34	1.37		
Sierra Leone	1987			0.25	-3.81		
El Salvador	1978	1989		1.99	-1.84	1.90	
Sweden	1970			3.15	1.64		
Togo	1969			5.54	-1.44		
Thailand	1958			-2.36	4.75		
Trinidad &Tobago	1981	1989		4.72	-5.27	5.87	
Taiwan	1962	1996		4.36	7.00	3.42	
Uganda	1988			-0.56	3.79		
United States	1961			1.58	2.41		
Venezuela	1977			2.82	-0.96		
South Africa	1983	1995		1.99	-0.49	2.48	
Zambia	1976			3.54	-1.33		

C Gabon



Figure 3: Observations and Frontier-Defining Countries in 1976

D Details of Nonparametric Growth Accounting

The two counterfactual distance functions that need to be solved for nonparametric growth accounting are

$$D_{t}^{j}(K_{t+1}^{j}, L_{t+1}^{j}; Y_{t+1}^{j}) = \min \ \phi^{j} \text{ subject to } \frac{Y_{t+1}^{j}}{\phi^{j}} \leq \sum_{\tau \leq t} \sum_{j} \mu_{\tau}^{j} Y_{\tau}^{j},$$
$$K_{t+1}^{j} \geq \sum_{\tau \leq t} \sum_{j} \mu_{\tau}^{j} K_{\tau}^{j}, L_{t+1}^{j} \geq \sum_{\tau \leq t} \sum_{j} \mu_{j}^{\tau} L_{\tau}^{j}, \qquad (10)$$
$$\mu_{\tau}^{j} \geq 0 \ \forall j, \tau,$$

 $\quad \text{and} \quad$

$$D_{t+1}^{j}(K_{t}^{j}, L_{t}^{j}; Y_{t}^{j}) = \min \ \phi^{j} \text{ subject to } \frac{Y_{t}^{j}}{\phi^{j}} \leq \sum_{\tau \leq t+1} \sum_{j} \mu_{\tau}^{j} Y_{\tau}^{j},$$
$$K_{t}^{j} \geq \sum_{\tau \leq t+1} \sum_{j} \mu_{\tau}^{j} K_{\tau}^{j}, L_{t}^{j} \geq \sum_{\tau \leq t+1} \sum_{j} \mu_{j}^{\tau} L_{\tau}^{j}, \qquad (11)$$
$$\mu_{\tau}^{j} \geq 0 \ \forall j, \tau.$$

The individual elements of the tripartite decomposition in equation (6) are calculated using the following formulas, which are based on the actual and the counterfactual distance functions:

$$\frac{\phi_{t+1}}{\phi_t} = \frac{D^{t+1}(k_{t+1}, y_{t+1})}{D^t(k_t, y_t)},\tag{12}$$

$$\left(\frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^{t}(\tilde{k}_{t+1})}\frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t})}{\tilde{\mathbf{y}}^{t}(\tilde{k}_{t})}\right)^{1/2} = \left(\frac{D^{t}(k_{t+1}, y_{t+1})}{D^{t+1}(k_{t+1}, y_{t+1})}\frac{D^{t}(k_{t}, y_{t})}{D^{t+1}(k_{t}, y_{t})}\right)^{\frac{1}{2}}, \text{ and}$$
(13)

$$\left(\frac{\tilde{\mathbf{y}}^{t}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^{t}(\tilde{k}_{t})}\frac{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t+1})}{\tilde{\mathbf{y}}^{t+1}(\tilde{k}_{t})}\right)^{1/2} = \left(\frac{D^{t}(k_{t}, y_{t})}{D^{t}(k_{t+1}, y_{t+1})}\frac{D^{t+1}(k_{t}, y_{t})}{D^{t+1}(k_{t+1}, y_{t+1})}\right)^{\frac{1}{2}}\left(\frac{y_{c}}{y_{b}}\right)$$
(14)