Gains from Migration in a New-Keynesian Framework

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Abstract

This paper presents a simple New-Keynesian small open economy model allowing for labor to be supplied both domestically and abroad. From this small change in the otherwise standard setup follows an important implication for the Phillips-curve: The introduction of migration reduces the sensitivity of inflation to changes in the output gap, that is, the Phillips-curve becomes flatter. The theoretic intuition is simple: When the home economy booms due to high productivity or demand, workers migrate back from abroad because real wages improve relative to those in the rest of the world. This additional labor supply at home relieves the pressure on home wages such that marginal costs, and consequently prices, increase less. A welfare function is derived to show the welfare losses implied by a deviation from the optimal policy rule. These losses change when migration is allowed in that the weight of output gap volatility falls relative to inflation volatility. I show that demand shocks result in bigger output increases while the effects of productivity shocks depend on the choice of parameters. The flip side of the integration of labor markets is that shocks affecting labor markets abroad have spillover effects to the domestic economy.

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1 Introduction

In recent years researchers have become increasingly aware of the fact that labor migration is not just a once-and-for-all move from one country to another:

> "[.] it has now become a reality that circular, repeat, recurrent, revolving door, multiple, frequent, repetitive, intermittent, seasonal, sojourning, cyclical, recycling, chronic or shuttling migration is a salient trait of migration." (Constant and Zimmermann, 2003a)

Despite a lack of data for a broad evaluation of the phenomenon,[2] some evidence has been brought forward in support of it. For Germany, Constant and Zimmermann (2003a,b) found that more than 60% of immigrants have exited Germany in the sample from 1984-1997 at least once and stayed in the country of origin for at least one year, using a representative GSOEP data set.[3] [4] Major reasons for an increased importance of non-permanent forms of migration relative to permanent migration are improved communication technologies, allowing intensified ties of migrants with source countries, and cheap transportation, making frequent return visits or circular migration patterns easier (O'Neil, 2003).

What could the macroeconomic consequences be when a significant share of a country's labor force emigrates temporarily? This paper will argue that, to the extent that workers' labor supply decisions are guided by working opportunities inside *and* beyond national borders because they allocate their labor supply both abroad and domestically, firms' supply conditions are different than in a purely closed-labor-market economy.[5] In particular, the paper will show that this labor market structure flattens the slope of the Phillips curve, that is, the responsiveness of inflation with respect to changes in the output gap, relative to a setting without migration. The intuition behind this phenomenon is simple: When an economy booms due to productivity or demand shocks, workers will return home from abroad because of improving domestic real wages relative to real wages abroad. All else being equal, this additional labor supply relieves the pressure on wages at home and, therefore, on marginal costs and prices.

The model I employ uses the basic set up of Galí and Monacelli (2005) for a small open economy. The novelty in my framework is that workers are allowed to choose between work at home and abroad, hence migration is analyzed from the perspective

[2] Current data on migration flows are mainly based on census and administrative sources not able to capture the repetitive nature of a great proportion of current migration flows (O'Neil, 2003). Therefore, compilation of more longitudinal data sets will be necessary for an appropriate assessment.

[3] Furthermore, they found that the frequency of returns depends on the degree to which moving back and forth is restricted or not. Migrants more frequently returned to those countries of origin from which re-entry is easier. This is interpreted as being evidence in favor of a lock-in argument: The risk of not being able to continue to benefit from the higher wages abroad results in less frequent visits and a potentially reduced attachment to the host country. This could be the reason why *"Gastarbeiter"* from Turkey and former Yugoslavia had a much lower return rate than EU-nationals.

[4] For evidence for the UK, see Dustmann and Weiss (2007).

[5] This indicates that I focus on purely economically driven migration and abstract from other motives.

of the sending country. I incorporate differing steady state output levels in that the small economy is poorer than the rest of the world on average. In the steady state, this establishes the incentive to migrate and supply labor in two labor markets, those of the domestic and the world economy.

Furthermore, changing cyclical output developments induce cyclical movements of workers across borders. I analyze the differential effects of productivity and demand shocks due to the open labor market setting with a calibration for the Polish economy. This country provides a good example, with its large diaspora and highly mobile workforce. The main result from this exercise is that the increase in domestic output due to a demand shock is substantially higher when workers are allowed to migrate, while the effects of a productivity shock depend largely on the choice of parameters.

Monetary policy plays an important role in this process. If migration is fairly elastic, the central bank can react much less restrictively for a given rate of inflation, that is, more accommodative, to a positive demand shock. This is because firms can tap the pool of returning workers rather than having to compensate workers for reduced leisure. Hence, there is a substitution effect at play between work effort abroad and work effort at home in addition to the closed labor market substitution between leisure and work effort at home. This reduces wage and inflationary pressures and the central bank can allow output to increase by more for a given inflation target. There are therefore significant gains from migration from the perspective of the sending country in the form of output increases compared to a scenario without migration.[6]

However, if migration is symmetric, then the opposite is true as well: When negative demand shocks are associated with large emigration, then the fall in output will be larger and the inflation response smaller than in the scenario without migration.

The welfare implications of open labor markets are analyzed by means of a welfare function that is derived through a second order approximation of the representative household's utility function and by an optimal monetary policy rule for a restricted parameterization. This rule turns out to be identical to the one without migration in that in both cases the central bank perfectly stabilizes the output gap and domestic inflation. Since the welfare function is written in terms of the output gap and domestic inflation, there are no welfare implications if the central bank follows this optimal rule. The positive implications of return migration after a positive demand shock described above could thus not be shown to be welfare relevant.

If, however, for some reason that may be exogenous to the model, the central bank follows a non-optimal rule (e.g., a currency peg), then migration can have significant welfare implications. In particular, I show that the weight of domestic inflation relative to the output gap in the welfare function increases when migration is allowed.

How does this relate to the existing literature on the relationship between openness and the Phillips-curve? Loungani et al. (2001) observed that Phillips curves tend to be the flatter the more open an economy is. Razin and Yuen (2002) explained this phenomenon in an open-economy version of the model in Woodford (2003, ch.3) in which the

[6] Since the analysis is within a general equlibrium framework, feedback effects are at play as well. For relatively high goods demand elastcities, the output effect can be so much larger when migration is allowed, that the central bank will have to increase interest rates by more than in the case without migration, given the rule used in the analysis.

opening of the trade balance and the capital account both flattened the Phillips-curve.[7] Whether the additional opening of the "labor account" works in the same direction, had not been considered in this literature until recently. This work, therefore, contributes to this more general literature on the microfoundations of openness and the Phillips curve. The approach developed in this paper has been adopted by Binyamini and Razin (2007), who analyze migration from the perspective of a receiving country and show that migration also flattens the Phillips curve in that case. Furthermore, Bentolilla et al. (2008) show, in an empirical analysis, that immigration indeed lowered the trade-off in the case of Spain.

The basic structure draws heavily on much older microeconomic literature on temporary migration. A common assumption is location specific preferences (e.g. in Hill, 1987, Djajic and Milbourne, 1988, Raffelhüschen, 1992). Migrants emigrate only when they are compensated for being away from home, therefore, they have a higher relative preference to work at home rather than abroad. Furthermore, the usually much higher purchasing power of the host country's currency in the home country's economy is another driving force for emigration, remittances and return migration (Dustmann, 1995,1997).[8] In the present analysis, I introduce only the location specific preferences and simply assume that the representative household only consumes the domestic consumption bundle.

This paper fills a gap in the literature between the microeconomic literature, focused on the partial equilibrium, and macroeconomic open economy approaches that generate insights in a general equilibrium framework, in particular, on the implications for the central bank and interest rates. The latter effects are far-reaching, in particular for receiving countries, because they put into question political conclusions derived from observations of falling wages due to immigration.

What is crucial for my model to be empirically valid is a link between migration flows and wages at business cycle frequencies. If migration is supposed to have any significant impact, this will be reflected in real wage changes. A link between emigration and wages has indeed been shown for Mexico and Poland, two important sending countries (Hanson, 2006; Aydemir and Borjas, 2006; Mishra, 2007 Budnik, 2008) where emigration increases wages of those staying behind. Whether a cyclical spell of return migration or immigration has an effect on wages, has not yet been shown for sending countries, to my knowledge.

This paper is structured as follows. Section 2 presents the representative household's decision problem, thereby first introducing the standard model and then presenting the way migration can be incorporated into this decision problem. Section 3 turns

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[7] Trade openness reduces the effect of domestic output fluctuations on CPI-inflation relative to an economy that is closed to trade because of a composition effect. The consumption basket in an open economy includes imported goods whose prices are, by construction, not affected by domestic output changes because marginal costs for their production are not affected. The CPI therefore reacts less. Open capital markets affect the Phillips-curve because risk sharing breaks up the link between production and consumption in the closed economy. When output changes, consumption does not automatically change. For that reason, the marginal rate of substitution between consumption and leisure, and thereby the real wage, marginal costs and inflation are less affected.

[8] Further motives put forward but unrelated to the present analysis are credit market rationing in sending countries (Mesnard, 2004), higher returns to human capital, acquired in the host country, in the sending country (Dustmann, 1995, 1997).

to the decision of the firm, where I again formulate the standard case and then introduce the way migration and production are related. Sections 4 and 5 describe the aggregate demand and aggregate supply sides respectively. The discussion of the aggregate supply relation, that is, the Phillips-curve, provides the core of the analysis and the main results. Section 6 derives the optimal monetary policy and the welfare, while Section 7 discusses the simulations of demand and productivity shocks. Section 8 concludes.

2 The Representative Household

The representative household maximizes his utility function, taking account of his budget constraint. He works both at home and abroad with a relative preference to work at home. That means he "suffers" more when working abroad than at home, or can be described as "homesick". This is reflected in a wedge between the contribution to his disutility of labor of an hour worked for a domestic firm relative to an hour worked for a firm in the rest of the world.

He is assumed to consume only domestically, that is, he remits all his earnings from abroad back home indicating his preference to consume at home, a standard assumption in the return migration literature. The representative agent's Euler equation is the basis of a dynamic IS-equation.

Section 2.1 first derives goods demand functions and relative prices in a standard New-Keynesian open economy model. Then, Section 2.2 introduces the specifics of a migrant household and the resulting first order conditions.

2.1 The Standard Model

Utility function and budget constraint The representative agent in the small open economy maximizes the following utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(C_t\right) - f\left(N_{H,t}, N_{M,t}\right) \right\} \tag{1}$$

where E_0 are the household's rational expectations in $t = 0$, β is the discount factor with $\beta < 1$, $u(.)$ and $f(.)$ are the additively separable utility functions of consumption and labor respectively. $N_{H,t}$ and $N_{M,t}$ are the household's labor inputs in the domestic economy and abroad and are explained in more detail below. The domestic consumption index is C_t,

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

with $C_{H,t}$ as an index for domestic consumption of domestically produced goods $C_{H,t}(j)$ with $j \in [0, 1]$:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$C_{F,t}$ is an index for domestic consumption of foreign goods produced in country $i \in [0,1]$, $C_{i,t}$:

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

$C_{i,t}$, in turn, is an index of goods produced in country i, $C_{i,t}(j)$ with $j \in [0,1]$:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon^*-1}{\varepsilon^*}} dj \right)^{\frac{\varepsilon^*}{\varepsilon^*-1}}$$

α is an indicator of the degree of openness of the domestic economy and indicates (inversely) the degree of home-bias in consumption preferences, ε, ε^*, η and γ are the elasticities of substitution within the respective indices.

The consumer faces the period budget constraint

$$\begin{aligned} W_t N_{H,t} + \epsilon_t W_{M,t} N_{M,t} + D_t \quad \geq \quad & \int_0^1 P_{H,t}(j) C_{H,t}(j) dj \\ & + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \{ Q_{t,t+1} D_{t+1} \} \end{aligned} \tag{2}$$

where W_t is the domestic nominal hourly wage; $W_{M,t}$ is the world average of the wages the migrant faces in the rest of the world (discussed below); ϵ_t is the nominal effective exchange rate; D_t is the nominal pay-off of the portfolio in period t; $Q_{t,t+1}$ is a stochastic discount factor; $P_{H,t}(j)$ is the price of domestically produced good j; and $P_{i,t}(j)$ is the price of good j produced in country i.

Let us now turn to the goods demand functions and the respective price indexes.

Demand functions and price indexes From the consumption indexes, the demand functions for the individual goods can be derived. They are as follows:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad \text{and} \quad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon^*} C_{i,t} \tag{3}$$

where

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon^*} dj \right)^{\frac{1}{1-\varepsilon^*}}$$

Aggregation of these demand functions over all goods j delivers

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}$$

and

$$\int_0^1 P_{i,t}(j) C_{i,t}(j) dj \quad = \quad P_{i,t} C_{i,t}$$

respectively. Furthermore, expressed as functions of the domestic and foreign indexes, the demand functions are

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{4}$$

$$\text{with } P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$$

Aggregating these over all countries i, gives $\int_0^1 P_{i,t}C_{i,t}di = P_{F,t}C_{F,t}$. Finally, because

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{5}$$

$$\text{with } P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

gives $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$, allowing the period budget constraints to be rewritten as

$$W_tN_{N,t} + \epsilon_t W_{M,t}N_{M,t} + D_t \geq P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \tag{6}$$

In Section 4, the demand functions for individual goods expressed in terms of total consumption are needed. They can be derived by combining (3), (4) and (5):

$$C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{7}$$

$$C_{i,t}(j) = \alpha\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon^*}\left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{8}$$

Next, let us turn to the relationship between the prices, inflation, the terms of trade and the real exchange rate.

Relative prices and inflation The *bilateral terms of trade* $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$ are defined as the relative price of country i´s and the domestic economy´s goods. The *effective terms of trade* S_t are given by

$$\begin{aligned} S_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{9}$$

which, in the symmetric case, is approximately

$$\widehat{s}_t = \int_0^1 \widehat{s}_{i,t} di$$

where lower case letters indicate logarithms, and hats over lower case letters indicate percent deviations from the respective steady state values. Symmetry is a reasonable and simplifying assumption as long as the focus of the analysis is not structural differences between countries. With rich and poor countries and asymmetric incentives to move across borders (i.e., no incentive for rich countries' workers and a strong incentive for poor countries' workers, as is introduced below), this is no longer the case. For simplicity, I assume two different types of countries, rich and poor, with each group having a common average steady state level for the terms of trade. If, in addition, the weight of the poor group approaches zero in the calculation of the effective terms of trade, then the above approximation remains valid. This may be a reasonable approach for poor countries trading mainly with rich countries and little with other poor countries. It will be shown in the Appendix B that the terms of trade are not necessarily equal to one in the steady state. The precise value, rather, depends on the choice of parameters.

The consumer price index P_t can be approximated by

$$\begin{aligned} \widehat{p}_t &= \frac{1-\alpha}{1-\alpha+\alpha S^{1-\eta}}\widehat{p}_{H,t} + \frac{\alpha}{(1-\alpha)S^{\eta-1}+\alpha}\widehat{p}_{F,t} \\ \widehat{p}_t &= \widehat{p}_{H,t} + \alpha_S \widehat{s}_t \end{aligned} \quad (10)$$

where $\alpha_S \equiv \frac{\alpha}{(1-\alpha)S^{\eta-1}+\alpha}$ and where variables with capital letters and without indexes are steady state values. This implies that the level of the steady state terms of trade influence the reaction of the domestic price level on terms of trade fluctuations. For example, if $S > 1$ and if the elasticity of substitution between domestic and foreign goods is smaller than one, any terms of trade change has a larger effect on the price index. For the symmetric case of $S = 1$ and for $\eta = 1$, however, $\alpha_S = \alpha$.

From (10), it follows that *domestic inflation*, defined as the rate of change of the domestic goods prices, $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, is related to *CPI inflation* according to

$$\pi_t = \pi_{H,t} + \alpha_S \Delta s_t \quad (11)$$

The *law of one price* (LOOP) holds at all times, that is, $P_{i,t}(j) = \epsilon_{i,t} P^i_{i,t}(j)$ for all $i, j \epsilon [0,1]$ where $\epsilon_{i,t}$ is the bilateral nominal exchange rate between country i and the domestic country, defined as the price of country i´s currency in terms of the domestic currency, and $P^i_{i,t}(j)$ is the price of country i´s good j expressed in country i´s, the producer´s currency. With this assumption and the definition of $P_{i,t}$ one obtains $P_{i,t} = \epsilon_{i,t} P^i_{i,t}$, where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$. Substituting the definition of the LOOP into the definition of $P_{F,t}$, assuming again two types of countries (rich and poor) with a common average value of P_i in the steady state, and neglecting the values of the poor countries one obtains through log-linearization:

$$
\begin{aligned}
p_{F,t} &= \int_0^1 p_{i,t} di \\
&= \int_0^1 (e_{i,t} + p_{i,t}^i) di \\
&= e_t + p_t^* \qquad (12)
\end{aligned}
$$

where $e_t \equiv \int_0^1 e_{i,t} di$ is the log *nominal effective exchange rate,* $p_{i,t}^i \equiv \int_0^1 p_{i,t}^i(j) dj$ is the log domestic price index of country i in its own currency, and $p_t^* \equiv \int_0^1 p_{i,t}^i di$ is the log *world price index*. Note that for the rest of the world as a whole, the distinction between the domestic and the consumer price index fades because $\alpha \to 0$.

From (9) and (12), I derive a relationship between the terms of trade and the nominal exchange rate:

$$
s_t = e_t + p_t^* - p_{H,t} \qquad (13)
$$

Defining the *bilateral real exchange rate with country* i as $RER_{i,t} \equiv \frac{\epsilon_{i,t} P_t^i}{P_t}$ and the (log) real effective exchange rate $reer_t \equiv \log REER_{i,t} \equiv \int_0^1 rer_{i,t} di$, and using (13) and (10) gives a relationship between the real effective exchange rate and the terms of trade:

$$
\begin{aligned}
reer_t &= \int_0^1 (e_{i,t} + p_{i,t}^i - p_t) di \\
&= e_t + p_t^* - p_t \\
&= s_t + p_{H,t} - p_t \\
&= (1 - \alpha_S) s_t \qquad (14)
\end{aligned}
$$

Having introduced goods demand functions and relative prices which were, so far, unaffected by the migration decision, I now turn to the specifics of the migrant household.

2.2 The Representative Household and Migration

In order to derive the household's optimal allocation of consumption, hours abroad and hours at home, I employ the following period utility function:

$$
u(C_t) - f(N_{H,t}, N_{M,t}) \equiv \frac{e^{d_t} C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^H + \phi N_t^M\right)^{1+\varphi}}{1+\varphi} \qquad (15)
$$

where e^{d_t} is a domestic demand shock, assumed to follow an AR-process in logs, that is, $d_t = \rho_d d_{t-1} + \varepsilon_t^d$ with $E_t \left\{\varepsilon_{t+1}^d\right\} = 0$. The disutility of labor function has two arguments: The labor supplied domestically, $N_{H,t}$, and the labor supplied abroad, $N_{M,t}$. The latter of the two is multiplied by the factor $\phi > 1$, indicating his relative preference for working at home rather than working abroad, or "home sickness". $N_{H,t}$

and $N_{M,t}$ are indexes explained in more detail in Section 3 and are constrained to be non-negative.

Foreign labor is not allowed to migrate to the small open economy, hence the utility functions of households abroad assign a value of either infinity or zero to ϕ. In the first case ($\phi = 0$ in the rest of the world), one would simply assume that it is not possible for households to migrate to the domestic economy. In the second case ($\phi = \infty$ in the rest of the world), one would assume that it is not regarded as desirable to migrate.

The world average nominal wage $W_{M,t}$ that the household faces when working abroad will be assumed to be exogenous. This may be an appropriate assumption for a small open economy where the migrants are price takers in the labor market of their host country. For a large host country with several small countries jointly providing a large group of imigrants, this assumption would certainly have to be relaxed in order to allow for changes of the foreign wage due to changes in supplied hours.

Maximizing (1) w.r.t. C, N_H and N_M, subject to (6) and taking account of (15), one gets the marginal rates of substitution equated to the respective real wages and, in case of the foreign wage in terms of the domestic goods prices, adjusted for the home-sickness coefficient ϕ:

$$e^{-d_t} C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{16}$$

$$e^{-d_t} C_t^{\sigma} N_t^{\varphi} \phi = \frac{\epsilon_t W_{M,t}}{P_t} \tag{17}$$

where $N_t = N_{H,t} + \phi N_{M,t}$is the argument of the disutility of labor function. From this, it follows that $\frac{W_t}{P_t} \leq \frac{\epsilon_t W_{M,t}}{P_t}$, that is, the purchasing power of the foreign wages in the domestic economy is larger than the one of domestic wages. To be more precise, the wedge in domestic earnings and those from abroad is $\phi = \frac{\epsilon_t W_{M,t}}{W_t}$. Therefore, with the world nominal wage assumed to be exogenous for the domestic economy, the exchange rate and the domestic nominal wage rate are endogenously determined to keep the wedge constant for a given value of ϕ.

From the first order condition w.r.t. D_t, one obtains the intertemporal optimality condition

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) e^{d_{t+1} - d_t} = Q_{t+1} \tag{18}$$

which, when taking conditional expectations on both sides, yields the standard stochastic Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} e^{d_{t+1} - d_t} \right\} = 1$$

where $R_t^{-1} = E_t \{Q_{t,t+1}\}$ is the domestic currency price of a one-period riskless bond.

An international risk sharing relationship can be derived by relating the domestic and foreign Euler equations. Assuming perfect securities markets, an intertemporal equilibrium condition for country i, analogous to equation (18) of the form

$$\beta E_t \left\{ \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \frac{P^i_t}{P^i_{t+1}} \frac{\epsilon^i_t}{\epsilon^i_{t+1}} e^{d^*_{t+1}-d^*_t} \right\} = Q_{t,t+1} \tag{19}$$

has to hold, where ϵ^i_t is the nominal effective exchange rate of country i and where d^*_t is a demand shock that, for simplicity, is assumed to be the same for all countries except the domestic economy. Equating (18) and (19), gives a relationship linking domestic and country i's consumption,

$$C_t = \vartheta_i C^i_t \left(RER_{i,t}\right)^{\frac{1}{\sigma}} \left(e^{d_t - d^*_t} \right)^{\frac{1}{\sigma}} \tag{20}$$

where $\vartheta_i \equiv \frac{C_0}{C^i_0} RER_{i,0}^{-\frac{1}{\sigma}}$, under the assumption that $d_0 = d^*_0 = 0$. The term ϑ_i is an initial condition for relative consumption at time zero in the absence of shocks. Assuming symmetric initial conditions across countries, $\vartheta_i = 1$, Galí and Monacelli (2005) showed that this would lead to a symmetric steady state where $C = C^i = C^*$, and where C^* is an index of world consumption, and $REER_t = S_t = 1$.

Here, however, I deviate from this assumption and allow that on average $\vartheta_i < 1$, that is, the domestic economy started with below country i's level of consumption. The asymmetric initial condition can be interpreted as an unsymmetric distribution of wealth across countries that suppresses domestic relative to other countries' consumption even today. For simplicity I assume that $\vartheta_i = \vartheta$ for all i. In Appendix B, I show that this assumption affects the steady state of the terms of trade.

Taking logs on both sides of (20) and integrating over i gives

$$\begin{aligned} c_t &= \log \vartheta + c^i_t + \frac{1}{\sigma} rer_{i,t} + \frac{1}{\sigma}(d_t - d^*_t) \\ &= \log \vartheta + c^*_t + \frac{1}{\sigma} reer_t + \frac{1}{\sigma}(d_t - d^*_t) \\ &= \log \vartheta + c^*_t + \frac{1-\alpha_s}{\sigma} s_t + \frac{1}{\sigma}(d_t - d^*_t) \end{aligned}$$

where $c^*_t = \int_0^1 c_{i,t} di$. This is an international risk sharing condition that relates domestic and foreign consumption. A wedge is created between the two by the initial condition, the terms of trade and asymmetric demand shocks, that is, $d_t - d^*_t \neq 0$.[9]

In summary, the optimality conditions (apart from the budget constraints) take the following log-linearized form:

[9] Alternatively, the uncovered interest parity could be derived as a risk sharing condition. In any country i, an analogous relationship to $R_t^{-1} = E_t \{Q_{t,t+1}\}$ has to hold and because of complete international securities markets, one has $\epsilon_{i,t} \left(R^i_t\right)^{-1} = E_t \{Q_{t,t+1}\epsilon_{i,t+1}\}$. Combining these two equations, log-linearizing and aggregating over all i, gives $r_t - r^*_t = E_t \{\Delta e_{t+1}\}$, the uncovered interest parity condition.

$$w_t - p_t = \sigma c_t + \varphi n_t - d_t \quad (21)$$

$$w_{M,t} - p_t + e_t = \sigma c_t + \varphi n_t + \log \phi - d_t \quad (22)$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\} - \rho) + (1-\rho_d)d_t \quad (23)$$

$$c_t = \log \vartheta + c_t^* + \frac{1-\alpha_s}{\sigma} s_t + \frac{1}{\sigma}(d_t - d_t^*) \quad (24)$$

where $e_t \equiv \log \epsilon_t$, $\rho \equiv \beta^{-1} - 1$ is the subjective rate of time preference and $\pi_t \equiv p_t - p_{t-1}$ is defined as the CPI inflation rate.

3 Firms

Domestic firms employ domestic labor and set prices in a forward-looking way with price staggering à la Calvo (1983), allowing the derivation of a New-Keynesian Phillips-curve in Section 5. The rest of the world is modeled in a mainly analogous manner, with the main difference that workers from the rest of the world are not assumed to work in the domestic small economy. Section 3.1 derives the standard model of firms in the open economy, while Section 3.2 introduces migration.

3.1 The Standard Model

The domestic firm $j \epsilon [0,1]$ produces with the linear production function

$$Y_t(j) = A_t N_{H,t}(j)$$

where $a_t \equiv \log A_t$ follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$ with $E_t\{\varepsilon_{t+1}\} = 0$. From this, and the definitions $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ and $N_{H,t} \equiv \int_0^1 N_{H,t}(j)dj$, the following first order approximation to an aggregate production relationship can be derived:

$$N_{H,t} = \frac{Y_t Z_t}{A_t} \quad (25)$$

where $Z_t \equiv \left(\int_0^1 \frac{Y_t(j)}{Y_t} dj\right)$. Galí and Monacelli (2005) further show that equilibrium variations in $z_t \equiv \log Z_t$ around the perfect foresight steady state are of second order. Therefore, up to a first order approximation

$$y_t = a_t + n_{H,t} \quad (26)$$

can be written for aggregate production.

Variable costs, in terms of domestic prices, are common across domestic firms and given by $w_t - p_{H,t} + y_t - a_t$. Domestic real marginal costs are thus given by

$$mc_t = w_t - p_{H,t} - a_t \tag{27}$$

With Calvo-type price-setting (Calvo, 1983), a measure $1-\theta$ of randomly-selected firms sets new prices every period with the probability of being selected independent of the time elapsed since prices were last adjusted. The optimal price-setting rule can then be approximated by

$$\overline{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(mc_{t+k} + p_{H,t}\right)$$

where $\overline{p}_{H,t}$ denotes the optimal newly-adjusted log price and $\mu \equiv \log(\varepsilon/(\varepsilon-1))$ denotes the optimal mark-up in the steady state. Therefore, firms set their prices in a forward-looking manner equal to a weighted average of the expected discounted marginal costs, plus a mark-up.

In an analogous way, world output is produced: Individual country i produces with production function $y_t^i = a_t^i + n_t^i$ where $a_t^i = \rho_{a^i} a_{t-1}^i + \varepsilon_t^i$ with $E_t\left\{\varepsilon_{t+1}^i\right\} = 0$ and $E_t\left\{\varepsilon_{t+1}^i \varepsilon_{t+1}^j\right\} = 0 \; \forall \; i,j$, implying marginal costs of $mc_t^i = w_t^i - p_{H,t}^i - a_t^i$. Integrating these relationships over all countries, results in the world production and marginal cost functions:

$$\begin{aligned} y_t^* &\equiv \int_0^1 y_t^i di = \int_0^1 a_t^i di + \int_0^1 n_t^i di \equiv a_t^* + n_t^* \\ mc^* &\equiv \int_0^1 mc_t^i di = \int_0^1 w_t^i di - \int_0^1 p_{H,t}^i di - \int_0^1 a_t^i di \equiv w_t^* - p_t^* - a_t^* \end{aligned}$$

where asterisks indicate world averages.

3.2 Production and Migrant Labor

Having established the standard decision of firms, I now show how labor migration can be introduced into this framework in a very general way. To that end, I show how migrant labor is related to the production side of the economy and how this, in turn, is related to the household's preferences.

I assume that the migrant's labor input does not appear in the world production function. This means that changes in N_M will not result in measurable changes of world output. This is reasonable for the small country/rest-of-the-world setting. However, I assume that changes in world output and in world productivity have non-negligible effects on N_M through an assumed correlation between output and productivity of the world economy, and the sectors employing migrant workers. Rather than modelling N_M explicitly in the world production function, I assume that the first effect increases the migrant's labor input abroad, while the second decreases it. Furthermore, I assume that migrant labor hours are a negative function of domestic output, that is, domestic output growth induces migrants to return, while they are a positive function of domestic productivity shocks, indicating that when set free domestically, labor partly moves abroad.

Therefore, I can describe N_M as a function of four factors:

$$N_M = N_M(\overset{-}{Y}, \overset{+}{A}, \overset{+}{Y^*}, \overset{-}{A^*}) \tag{28}$$

The plus and minus signs indicate the signs of the first derivatives of N_M with respect to the respective variables. I further assume that these partial derivatives are additively separable.

These assumptions are necessary in order to establish a link between domestic and foreign business cycles due to exogenous shocks on the one hand and the migrant's decision with respect to his place of work on the other hand. The reason for choosing this general rather than a specific functional form is that I want to illustrate in a transparent way which relationships between these variables are required in order to derive the conclusions below. In an empirical analysis, one could then directly check whether these relationships can be verified. In a future theoretical analysis with more specific functional forms, one would then need to make sure that these relationships are indeed fulfilled.

Applying a Taylor expansion to this function around the steady state, gives

$$\widehat{n}_{M,t} = \frac{\partial N_{M,t}}{\partial Y_t}\frac{Y}{N_M}\widehat{y}_t + \frac{\partial N_{M,t}}{\partial A_t}\frac{A}{N_M}a_t + \frac{\partial N_{M,t}}{\partial Y_t^*}\frac{Y^*}{N_M}\widehat{y}_t^* + \frac{\partial N_{M,t}}{\partial A_t^*}\frac{A^*}{N_M}a_t^*$$

What is of ultimate interest here, however, is the relationship between output and productivity fluctuations on the one hand and the overall disutility of labor on the other hand. Approximated around the steady state, the argument in the disutility of labor function $N_t = N_{H,t} + \phi N_{M,t}$ is

$$\widehat{n}_t = \nu \widehat{n}_{Ht} + (1-\nu)\widehat{n}_{M,t}$$

where $\nu = \frac{N_H}{N} < 1$. Plugging the above result and equation (26) into this approximation, I can relate the disutility of labor to domestic and foreign output fluctuations, and to domestic and foreign productivity shocks as follows:

$$\begin{aligned} \widehat{n}_t &= \nu\widehat{y}_t - \nu a_t + \\ &\quad (1-\nu)\left[\frac{\partial N_{M,t}}{\partial Y_t}\frac{Y}{N_M}\widehat{y}_t + \frac{\partial N_{M,t}}{\partial A_t}\frac{A}{N_M}a_t + \frac{\partial N_{M,t}}{\partial Y_t^*}\frac{Y^*}{N_M}\widehat{y}_t^* + \frac{\partial N_{M,t}}{\partial A_t^*}\frac{A^*}{N_M}a_t^*\right] \\ \widehat{n}_t &= (\nu - \zeta_Y)\,\widehat{y}_t - (\nu - \zeta_A)\,a_t + \zeta_{Y^*}\widehat{y}_t^* - \zeta_{A^*}a_t^* \end{aligned} \tag{29}$$

where $\zeta_X \equiv \left|\frac{\partial N_{M,t}}{\partial X_t}\frac{X}{N_M}\frac{\phi N_M}{N}\right|$ is defined as the elasticity of the argument of the disutility of labor function, N_t, with respect to changes in variable X. For example, ζ_Y is the elasticity by which N_t reacts to changes in domestic output due to the reduced labor input abroad. The term $(\nu - \zeta_Y)$ needs to be restricted to positive values to avoid N_t to become negative (see Appendix C).

In a closed labor market setting, where the $\zeta_X = 0$ for $X = Y, A, Y^*$ and A^* and $\nu = 1$, this equation reduces to the relationship between labor input and output derived from the aggregate production function, $\widehat{n}_t = \widehat{y}_t - a_t$. Here, however, purely domestic variations of economic activity and productivity only partially affect the disutility of labor when the "labor account" is open because $(\nu - \zeta_Y) < 1$ and $(\nu - \zeta_A) < 1$.

Two effects contribute to weaken the link between domestic output and productivity on the one hand and the disutility of labor on the other hand. First, in the deterministic steady state, a fraction $1 - \nu$ of hours worked constitute a factor input abroad. This fraction is unaffected by fluctuations of the domestic variables y_t and a_t. Only the fraction ν is affected. Second, when favorable domestic conditions, that is, positive deviations of output from the steady state, result in return migration, this return migration constitutes a substitution of hours worked domestically for hours worked abroad. To that extent an increased domestic labor input leaves the disutility of labor unaffected. In the model, this is introduced through the elasticity ζ_Y. These two effects are the reason for the reduction of the slope of the Phillips-curve as shown below.

Furthermore, world output and productivity affect the disutility of labor because of their impact on the migrant's hours worked.

4 Aggregate Demand

I now derive the domestic equilibrium by establishing a goods market clearing condition that relates domestic output to domestic and foreign consumption. In this relationship, a wedge between domestic output and consumption is created by fluctuations in the terms of trade and a relative demand shock that result in changing relative demand for domestically produced goods. This condition will be used to derive an intertemporal Euler equation in terms of the output gap. The external equilibrium is determined by the net export equation. It turns out that migration affects the demand side of the model only through the steady state's impact on the dynamics.

In a goods market equilibrium, domestic supply has to equal domestic and foreign demand. In Appendix A, this is explicitly derived and approximated around the steady state. The resulting equilibrium is

$$\widehat{y}_t = \widehat{c}_t + \frac{\alpha \varpi_S}{\sigma} \widehat{s}_t - \frac{\alpha}{\sigma} l(S) \left[d_t - d_t^* \right] \tag{30}$$

where I made use of the substitutions

$$\begin{aligned} l(S) &\equiv \left[(1-\alpha)\vartheta S^{\eta-\gamma} reer(S)^{\frac{1}{\sigma}-\eta} + \alpha \right]^{-1} \\ reer(S) &\equiv REER \\ \varpi_S &\equiv \left[\sigma\gamma + (1-\alpha_S)(\sigma\eta - 1) \right] l(S) + \sigma\eta \left[\frac{\alpha_s}{\alpha} - l(S) \right] \end{aligned}$$

Therefore, the terms of trade and asymmetric demand shocks $d_t - d_t^* \neq 0$ cause wedges between output and consumption in a small open economy. The size of these effects depends on parameters, the steady state and the initial condition ϑ.[10]

[10] Note that $l(S) = 1$ and $\varpi_S = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$ for $S = \vartheta = 1$, which is the solution for the symmetric case.

Condition (30) holds for every country i, therefore $\widehat{y}_t^i = \widehat{c}_t^i + \frac{\alpha\varpi_S}{\sigma}\widehat{s}_t^i$. Aggregation over all countries results in the world market clearing condition:

$$y_t^* \equiv \int_0^1 y_t^i di = \int_0^1 c_t^i di \equiv c_t^* \tag{31}$$

where y_t^* and c_t^* are indexes for log world output and consumption. Combining this with (30) and (24) gives

$$\widehat{y}_t = \widehat{y}_t^* + \sigma_{\alpha,s}^{-1}\widehat{s}_t + \frac{1-\alpha l(S)}{\sigma}(d_t - d_t^*) \tag{32}$$

where $\sigma_{\alpha,s}^{-1} = \frac{(1-\alpha_S)+\alpha\varpi_S}{\sigma}$.

Finally, combining (30) with the domestic Euler equation (23) and the conditional expectations of both sides of equation (11) gives the Euler equation in terms of domestic output,

$$\begin{aligned}\widehat{y}_t &= E_t\left\{\widehat{y}_{t+1}\right\} - \frac{1}{\sigma_{\alpha,s}}(\widehat{r}_t - E_t\left\{\pi_{H,t+1}\right\}) + \alpha\Theta_{y^*}E_t\left\{\Delta\widehat{y}_{t+1}^*\right\} \\ &\quad -\Theta_d d_t + \Theta_{d^*}d_t^*\end{aligned}$$

where I made use of the substitutions

$$\begin{aligned}\Theta_{y^*} &\equiv (\omega_S - \frac{\alpha_S}{\alpha}) \\ \Theta_d &\equiv \left[\frac{\alpha\Theta_{y^*}}{\sigma}\sigma_{\alpha,s}\left(1-\alpha l(S)\right) + \alpha l(S) - \sigma\right]\frac{1-\rho_c}{\sigma_{\alpha,s}} \\ \Theta_{d^*} &\equiv \left[\frac{\alpha\Theta_{y^*}}{\sigma}\sigma_{\alpha,s}\left(1-\alpha l(S)\right) + \alpha l(S)\right]\frac{1-\rho_c^*}{\sigma_{\alpha,s}}\end{aligned}$$

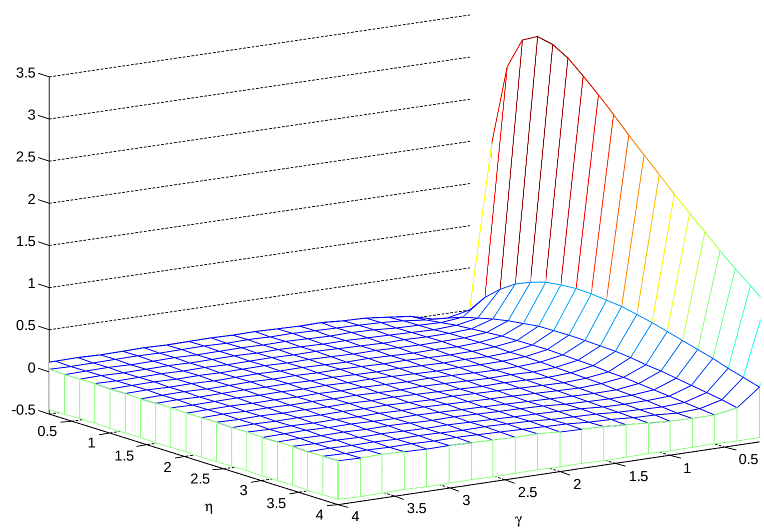
$\sigma_{\alpha,s}^{-1}$ is the intertemporal elasticity of substitution.[11] The inflation rate that matters for the Euler-equation expressed in terms of output in an open economy setting is the domestic inflation rate rather than the CPI rate.

How does migration affect the steady state and thereby the output Euler equation? In Appendix B, I show how emigration affects the unique steady state for output and the terms of trade. The intuition is that, ceteris paribus, an increase in hours worked abroad (i.e., an increase of emigration) increases the marginal disutility from labor, driving up the real wage and thereby reducing S. This deteriorates firms' international competitiveness and reduces output. Therefore, in an analysis of the model's dynamic behavior in the migration case relative to the no-migration benchmark, like the one presented below, output is clearly smaller while the terms of trade improved, that is, S is smaller.

[11] This equation nests the symmetric benchmark for $S = 1$, where $\Theta_{y^*} \equiv \omega_S - 1$.

Figure 1 sheds light on the magnitude of the impact of the change in the steady state terms of trade due to migration on the Euler equation through its effect on the intertemporal elasticity of substitution . The figure shows the difference in $\sigma_{\alpha,s}$ when S is set to the arbitrary number of 1 and when it is arbitrarily set to 2 for a wide range of the elasticities η and γ. This change could be due to emigration, which reduces S

Figure 1: Change in $\sigma_{\alpha,s}$ when S falls from 2 to 1 ($\sigma = 1$, $\alpha = 0.4$, $\vartheta = 0.1$)



as mentioned above. The other parameters are chosen as in the calibration presented below for the Polish economy. The figure shows that for a large fraction of the $\eta-\gamma-$space there is no large difference. For the special case when $\eta = \gamma = \sigma = 1$ (for which I will compute the optimal policy below), there is no difference at all because $\varpi_S = \sigma_{\alpha,s} = 1$, $\alpha_S = \alpha$ and $l(S)$ is independent from S, so that the steady state does not affect the Euler equation in this case. Only for values of γ close to zero is there a large impact on $\sigma_{\alpha,s}$. But in today's globally integrated goods markets, it is unrealistic to assume a price elasticity of almost zero between goods from different countries. I thus regard the figure as an indication that, in this model, with this parameterization for a country like Poland, migration has at most a minor impact on the intertemporal elasticity of substitution.[12]

Below, the aggregate supply side is expressed in terms of inflation and output gap fluctuations x_t, that is, the difference between actual and the natural rate of output,

[12] Note also, that the graph displays a very large change in the steady state terms of trade, which might be far from realistic. In reality, the actual impact should be even smaller than displayed here.

rather than output fluctuations $\widehat{y}_t$. The output gap is derived in detail below and it can be shown that, in turn, the Euler equation can be rewritten as

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_{\alpha,s}}(\widehat{r}_t - E_t\{\pi_{H,t+1}\} - \widehat{r}_t^n) \tag{33}$$

with the natural rate of interest

$$\widehat{r}_t^n = -\Gamma_a a_t + \Gamma_{\Delta y^*} E\left\{\Delta \widehat{y}_{t+1}^*\right\} - \Gamma_{a^*} a_t^* + \Gamma_d d_t + \Gamma_{d^*} d_t^*$$

and where

$$\begin{aligned}
\Gamma_a &= \left(\frac{1+\varphi(\nu-\zeta_A)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\sigma_{\alpha,s}\right)(1-\rho_a) \\
\Gamma_{\Delta y^*} &= \left(\alpha\Theta_{y^*}\sigma_{\alpha,s} - \frac{\sigma-\sigma_{\alpha,s}+\varphi\zeta_{Y^*}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\sigma_{\alpha,s}\right) \\
\Gamma_{a^*} &= \left(\frac{\varphi\zeta_{A^*}\sigma_{\alpha,s}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)(1-\rho_{a^*}) \\
\Gamma_d &= \left[\sigma + \frac{\sigma_{\alpha,s}}{\sigma}(1-\alpha l(S))\frac{\varphi(\nu-\zeta_Y)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)} - 1\right](1-\rho_d) \\
\Gamma_{d^*} &= \left[\left(1-\frac{\sigma_{\alpha,s}}{\sigma}(1-\alpha l(S))\right)\left(\frac{\varphi(\nu-\zeta_Y)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)\right](1-\rho_{d^*})
\end{aligned}$$

The open labor market structure thus affects the real rate of interest through multiple channels. It is affected by demand and productivity shocks and fluctuations in world output.

Net exports are determined by output, consumption and the terms of trade, too. Expressed in terms of domestic output in the steady state, net exports are

$$nx_t \equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$$

Up to a first order approximation, this is

$$\widehat{nx}_t = \widehat{y}_t - (1-nx)\widehat{c}_t - (1-nx)\alpha_S\widehat{s}_t$$

5 Aggregate Supply

5.1 The Standard Model

As shown by Galí and Monacelli (2005), the *domestic* inflation dynamics in this model are analogous to CPI-inflation dynamics in a closed-economy:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda\widehat{mc}_t \tag{34}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Marginal costs, in turn, are proportional to the output gap x_t which is defined as the difference between (log) domestic output y_t and its natural level y_t^n, that is, the equilibrium output in the absence of nominal rigidities. The standard aggregate supply relation, the Phillips-curve, is therefore

$$\pi_{H,t} \quad = \quad \beta E_t \left\{ \pi_{H,t+1} \right\} + \kappa x_t$$

The slope of the Phillips-curve, κ, is thus the crucial parameter for the effects of output gap fluctuations on inflation. How migration affects this trade-off will be explained in the next Section. To that end, the marginal cost function and the Phillips-curve are derived in detail.

5.2 Aggregate Supply and Migration

The channel through which migration affects domestic inflation is its effect on marginal costs. If output expands and migrants are attracted back to the domestic economy, then marginal costs and domestic inflation react less. The formal argument is summarized and proven in the following two Propositions and the intuition explained thereafter.

Proposition 1 *The open labor market structure reduces the effects of output expansions on domestic marginal cost fluctuations $\widehat{mc}_t$ relative to the closed labor market setup for reasonable parameterizations.*

Proof. *From (27), (21) and (10) I get*

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= \sigma c_t + \varphi n_t + \alpha_S s_t - a_t - d_t \end{aligned} \tag{35}$$

which, when evaluated in the neighborhood of the steady state, is

$$\begin{aligned} \widehat{mc}_t &= \sigma \widehat{c}_t + \varphi \widehat{n}_t + \alpha_S \widehat{s}_t - a_t - d_t \\ &= \sigma \widehat{y}_t^* + (1-\alpha_S)\widehat{s}_t + d_t - d_t^* + \varphi \widehat{n}_t + \alpha_S \widehat{s}_t - a_t - d_t \\ &= \sigma \widehat{y}_t^* + \widehat{s}_t + \varphi \left[(\nu - \zeta_Y)\, \widehat{y}_t - (\nu - \zeta_A)\, a_t + \zeta_{Y^*} \widehat{y}_t^* - \zeta_{A^*} a_t^* \right] - a_t - d_t^* \\ &= (\sigma_{\alpha,s} + \varphi\, (\nu - \zeta_Y))\, \widehat{y}_t - (1 + \varphi\, (\nu - \zeta_A))\, a_t + (\sigma - \sigma_{\alpha,s} + \varphi \zeta_{Y^*}) \widehat{y}_t^* \\ &\quad - \varphi \zeta_{A^*} a_t^* - d_t^* - (1 - \alpha l(S))\, \frac{\sigma_{\alpha,s}}{\sigma}\, (d_t - d_t^*) \\ &= (\sigma_{\alpha,s} + \varphi\, (\nu - \zeta_Y))\, \widehat{y}_t - (1 + \varphi\, (\nu - \zeta_A))\, a_t + (\sigma - \sigma_{\alpha,s} + \varphi \zeta_{Y^*}) \widehat{y}_t^* \\ &\quad - \varphi \zeta_{A^*} a_t^* - (1 - \alpha l(S)) \frac{\sigma_{\alpha,s}}{\sigma} d_t + \left((1 - \alpha l(S)) \frac{\sigma_{\alpha,s}}{\sigma} - 1 \right) d_t^* \end{aligned} \tag{36}$$

where I inserted (31), (24), (32) and (29). In contrast, in the closed labor market setting I have

$$\widehat{mc}_t = (\sigma_{\alpha,s} + \varphi)\,\widehat{y}_t - (1+\varphi)a_t + (\sigma - \sigma_{\alpha,s})\widehat{y}_t^* \\ -(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}d_t + \left((1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma} - 1\right)d_t^* \tag{37}$$

Comparing coefficients on output fluctuations $\widehat{y}_t$, one can see that in the migration case, the coefficient is smaller because $\nu - \zeta_Y < 1$ and because, as shown above, $\sigma_{\alpha,s}$ changes little for a reasonable parameter choice in the presence of migration. Therefore, the impact of output expansions on marginal costs in the open labor market setting is smaller than in the case of no migration. ■

The intuition behind this result is as follows: In a scenario WITHOUT migration, given productivity, an increase in output requires an increase in labor input. This incurs an increased disutility from labor because leisure needs to be reduced. Consequently, the real wage and real marginal costs increase, the magnitude of this effect is $\varphi\widehat{y}_t$. Migration reduces this effect because in this case there is a substitution out of labor abroad, rather than out of leisure. Workers skip one job (abroad) for another (at home). Therefore, there is a reduced impact on the disutility of labor when output expands at home. The term $-\varphi\zeta_Y\widehat{y}_t$ takes account of the disutility of labor reducing effect due to return migration, while $\varphi\nu\widehat{y}_t$ takes account of the fact that only a fraction of the disutility of labor argument is affected by domestic labor input.

I now turn to the New-Keynesian Phillips-curve. In order to derive it, I need equations (34) and (36), and an expression for the output gap x_t. This is derived by setting marginal costs equal to its flexible price value $-\mu$ so that $\widehat{mc}_t = 0$ in (36), and solving for output:

$$\widehat{y}_t^n = \left(\frac{1+\varphi(\nu-\zeta_A)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)a_t - \left(\frac{\sigma-\sigma_{\alpha,s}+\varphi\zeta_{Y^*}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)\widehat{y}_t^* \\ +\left(\frac{\varphi\zeta_{A^*}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)a_t^* \\ +\left(\frac{(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)d_t - \left(\frac{(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}-1}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)d_t^* \tag{38}$$

With this, and because

$$\widehat{y}_t = \left(\frac{1}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)\widehat{mc}_t + \widehat{y}_t^n$$

the relationship between marginal costs and the output gap is approximated by:

$$\widehat{mc}_t = (\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y))\,x_t \tag{39}$$

Proposition 2 extends the reasoning of Proposition 1 to the Phillips-curve:

Proposition 2 *The open labor market structure reduces the effect of output gap changes on increases of domestic inflation relative to a closed labor market structure. In other words, the Phillips-curve becomes flatter.*

Proof. *Combining (*39*) with (*34*), gives the open economy New Keynesian Phillips-curve in terms of the output gap,*

$$\pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + \kappa^{open} x_t \tag{40}$$

where

$$\kappa^{open} = \lambda \left(\sigma_{\alpha,s} + \varphi \left(\nu - \zeta_Y \right) \right)$$

is the slope factor of the Phillips-curve in the open labor market setting. In contrast, in the closed labor-market, this same calculation yields

$$\kappa^{closed} = \lambda \left(\sigma_{\alpha,s} + \varphi \right)$$

Because $(\nu - \zeta_Y) < 1$ and because $\sigma_{\alpha,s}$ is basically unchanged for reasonable parameterizations, I have

$$\kappa^{open} < \kappa^{closed}$$

Therefore, the effect of output gap variations on the domestic inflation rate is smaller when the labor market is open, that is, the Phillips-curve becomes flatter. ■

What's the mechanism behind this phenomenon? As pointed out above, when output expands in the domestic economy, workers return from abroad and thereby serve as an extra, "cheap" pool for the additional labor, which is needed for the expansion as an input to production. This pool is "cheap" in the sense that the alternative in the closed labor market setting is a reduction of leisure, while returning emigrants substitute labor at home for labor abroad. To the extent that output expansions at home are fed by this substitution effect, the disutility of labor does not increase and the real wage, marginal costs, prices and inflation increase less as well.

Tables 1 and 2 shed light on the dimension of this effect for various parameterizations of κ^{open}. The columns in Table 1 indicate possible fractions of the labor force that work abroad in the steady state, that is, $\frac{N_M}{N_M+N_H} * 100$. The rows indicate various values for the wage gap between the home country and abroad ϕ. Results are shown for an elasticity with that the stock of migrants abroad reacts to domestic output changes, $\frac{\partial N_M}{\partial Y} \frac{Y}{N_M}$, equal to 0.1. Further assumptions are as follows: $\eta = \gamma = \sigma = 1$ is the parameterization for which a welfare function is derived analytically below; $\alpha = 0.4$ is the degree of trade openness of Poland, the country for which the model is calibrated below; $\theta = 0.75$ is consistent with an average duration of prices of one year; $\beta = 0.99$ and $S = 1$. ϑ, the initial condition, does not affect the slope of the Phillips-curve for this particular parameter choice because $\sigma_{\alpha,s} = \sigma = 1$. The benchmark κ^{closed} is 0.34.

One can see in the table that the larger the share of migrant labor hours in total hours and the larger the wage differential, the lower is the impact of output gap fluctuations on domestic inflation.[13] For $\phi = 5$ and a fraction of the labor force abroad of 10%, the

[13] Increasing the return elasticity $\frac{\partial N^M}{\partial Y} \frac{Y}{N^M}$ further reduces the slope coefficient as well. However, $\nu - \zeta_Y$

Table 1: Phillips-curve slope coefficient

	Share of labor force abroad			
ϕ	*5%*	*10%*	*20%*	*30%*
3	0.32	0.29	0.25	0.21
5	0.28	0.24	0.19	0.15
10	0.25	0.19	0.14	0.11
20	0.20	0.15	0.11	0.09

Note: $\frac{\partial N_M}{\partial Y}\frac{Y}{N_M} = 0.1,\ \eta = \gamma = \sigma = 1$, $\alpha = 0.4$, $\beta = 0.99$, $\theta = 0.75$ *and* $S = 1$
Benchmark without migration: $\kappa^{closed} = 0.34$

slope coefficient falls by almost a quarter to 0.24. An output gap of 1% would imply a response of domestic inflation of 0.24% rather than 0.34% in this case.

Table 2 shows the same calculation with the only difference that the elasticities of substitution between goods, γ and η, are increased to 2. The benchmark value of κ^{closed} is now 0.30. The reduction of the slope of the Phillips curve is even more severe in this case. The return migration interacts with the degree of substitutability of the demand for goods.

Table 2: Phillips-curve slope coefficient with high demand elasticity

	Share of labor force abroad			
ϕ	*5%*	*10%*	*20%*	*30%*
3	0.28	0.25	0.21	0.17
5	0.24	0.20	0.14	0.11
10	0.20	0.15	0.10	0.07
20	0.16	0.11	0.07	0.05

Note: $\frac{\partial N_M}{\partial Y}\frac{Y}{N_M} = 0.1,\ \eta = \gamma = 2$, $\sigma = 1$, $\vartheta = 0.1$, $\alpha = 0.4$, $\beta = 0.99$, $\theta = 0.75$ *and* $S = 1$
Benchmark without migration: $\kappa^{closed} = 0.30$

quickly becomes negative, which I excluded by assumption in order to prevent the argument in the disutility of labor function to turn negative. Therefore, I do not present results for higher return elasticities.

6 Welfare Analysis

In order to derive implications of migration for monetary policy and social welfare, the optimal monetary policy and a utility based welfare function are now derived. This analysis is analytically possible only for the restricted parameterization of $\sigma = \eta = \gamma = 1$.

First, the optimal allocation from the social planner's perspective is derived. This result is used to determine an optimal subsidy that makes the flexible price allocation the optimal one as is a common approach in the literature. The subsidy eliminates the distortion of the monopolistic market structure while taking into account the open economy characteristics.

With the optimality of the flexible price allocation, the only distortion remaining is the stickyness of prices. Consequently, the optimal monetary policy is shown to be the one that replicates this optimal allocation. As it turns out, this implies full stabilization of both the ouput gap and the domestic rate of inflation. Furthermore, the optimal rule that accomplishes this optimum is the same as the one without migration.

Lastly, I derive a welfare function that allows the determination of welfare losses that are incurred by non-optimal monetary policies. These rules might be pursued because of other policy goals that are exogenous to this model. Since the opening of the "labor account" does not change the optimal monetary policy rule, there are no welfare implications of migration under the optimal policy. However, under non-optimal rules, migration matters in that the weight of the output gap falls relative to domestic inflation in the welfare function. This last result corresponds to the finding of Binyamini and Razin (2007).

6.1 Optimal Allocation and Policies

The efficient allocation from the social planner's perspective is derived by maximizing the representative household's utility function $U(C_t, N_t)$ under the following constraints:

1. technological constraint $Y_t = A_t N_{H,t}$
2. the relationship $N_{M,t} = N_M(Y_t, ., .)$
3. risk sharing condition (24) in combination with (31), i.e., $C_t = \vartheta Y_t^* S_t^{1-\alpha} e^{(d_t - d_t^*)}$
4. market clearing condition (A-2)

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t \left[(1-\alpha) + \alpha \left(\vartheta e^{d_t - d_t^*}\right)^{-1}\right] \\ &= S^\alpha C_t \left[(1-\alpha) + \alpha \left(\vartheta e^{d_t - d_t^*}\right)^{-1}\right] \end{aligned}$$

Note that for the last constraint, the CPI for $\eta = 1$ is $P_t = (P_{H,t})^{1-\alpha} (P_{F,t})^\alpha$ implying $\frac{P_t}{P_{H,t}} = S^\alpha$.

In Appendix C, I show that in the optimal allocation

$$N_t = \left(\frac{1-\alpha}{\nu - \zeta_Y}\right)^{\frac{1}{1+\varphi}}$$

under the assumption that the term $\nu - \zeta_Y$ is constant. This implies that in the optimum, the argument of the disutility of labor function N_t is constant rather than $N_{H,t}$, as in Galí and Monacelli (2005). Note that for N_t to be non-negative, $\nu - \zeta_Y$ needs to be restricted to positive values.

This result will now be used to determine the optimal allocation under flexible prices. Under flexible prices, the equilibrium satisfies

$$1 - \frac{1}{\varepsilon} = MC_t^n.$$

In Appendix C, I further show that this can be rewritten as

$$1 - \frac{1}{\varepsilon} = (1-\tau)\left(N_t^n\right)^{\varphi} N_{H,t}^n \left[(1-\alpha)e^{d_t} + \alpha\vartheta^{-1}e^{d_t^*}\right]^{-1}$$

where I introduced the production subsidy τ, which the social planner can use to implement the optimal allocation in the flexible price equilibrium.

Using the optimal argument of the disutility of labor function, gives

$$1 - \frac{1}{\varepsilon} = (1-\tau)\left(\frac{1-\alpha}{\nu - \zeta_Y}\right)^{\frac{\varphi}{1+\varphi}} N_{H,t}^n \left[(1-\alpha)e^{d_t} + \alpha\vartheta^{-1}e^{d_t^*}\right]^{-1}$$

As discussed above, domestic output in the flexible price equilibrium is determined by the exogenous shocks and foreign output. Thereby, the domestic employment level $N_{H,t}^n$ is uniquely pinned down as well (and, because of the constant optimal value of N_t, also $N_{M,t}^n$). The subsidy can therefore be set such that employment $N_{H,t}$ is at its optimum for $d_t = d_t^* = 0$. Therefore, the social planner needs to set τ so that the equation

$$N_{H,t}^n \quad = \quad \frac{\left(1-\frac{1}{\varepsilon}\right)\left(\frac{1-\alpha}{\nu-\zeta_Y}\right)^{\frac{-\varphi}{1+\varphi}}\left[(1-\alpha)+\alpha\vartheta^{-1}\right]}{1-\tau}$$

is fulfilled for the optimal $N_{H,t}^n$. Thereby, the planner guarantees the efficiency of the flexible price allocation in the absence of shocks.

With the flexible price equilibrium being optimal, the optimal output gap is zero, that is, $x_t = 0$ at all times. Given the Phillips curve, the optimal monetary policy is then one that perfectly stabilizes domestic inflation, that is, $\pi_{H,t} = 0$. The optimal monetary policy is thus the same as in Galí and Monacelli (2005). These authors show that a unique equilibrium of that kind can be achieved through the policy rule

$$r_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t \tag{41}$$

under the condition

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_x \quad > \quad 0$$

for non-negative values of $\phi_\pi$and ϕ_x as shown by Bullard and Mitra (2002).

The optimal monetary policy rule is thus not affected by the open labor market structure. From this perspective, the social planner does not need to change monetary policy when labor migration is allowed. The only thing that is differrent is the optimal subsidy τ.

6.2 The Welfare Function

From a welfare perspective, how costly is a deviation from the optimal policy just described? One could imagine a scenario in which the social planner has other policy goals that are exogenous from the perspective of this model. This could be an exchange rate peg that is introduced in preparation to the introduction of a foreign currency such as the euro. A welfare function that allows to assess the costs of such a policy would therefore be desirable for a policy maker.

In Appendix D, I derive the following welfare function as a second order Taylor approximation of the representative household's utility function around the flexible price equilibrium:

$$W = -\frac{1}{2}(1-\alpha)\sum_{t=0}^{\infty}\beta^t \left\{ \left(\frac{\nu}{\nu - \zeta_Y}\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + (1+\varphi)(\nu - \zeta_Y)x_t^2 \right) \right\}$$

Taking unconditional expectations on both sides and assuming $\beta \to 1$, the expected welfare loss becomes a function of the volatilities of domestic inflation and the output gap:

$$EW = -\frac{1}{2}(1-\alpha)\left(\frac{\nu}{\nu - \zeta_Y}\frac{\varepsilon}{\lambda} var(\pi_{H,t}) + (1+\varphi)(\nu - \zeta_Y) var(x_t) \right)$$

The open labor market terms are ν and ζ_Y. The following consequences for this welfare metric stand out for the open labor market setting compared to a closed labor market environment. Because

1. $\nu - \zeta_Y < 1$, output gap fluctuations reduce welfare by less and because
2. $\frac{\nu}{\nu - \zeta_Y} > 1$, inflation fluctuations affect welfare more negatively.

The intuition for the first result derives from the risk aversion with respect to employment fluctuations. A positive output gap implies an increase in domestic employment, which reduces utility. With return migration, the household is able to smooth this effect on disutility through a substitution of a job abroad for a job at home.

The second result states that the negative effect of inflation on welfare is amplified by migration. Inflation reduces welfare because of the inefficiency that results from output dispersion across goods and the corresponding price dispersion across goods due to price stickyness (Woodford, 2003, ch. 6). When workers are allowed to migrate, the link between the output fluctuations and prices becomes weaker: Any given inflation rate is associated with a larger output dispersion. This larger output dispersion decreases welfare at a given inflation rate, the costs of inflation increase.

From this analysis, it follows that when the central bank follows the optimal rule and thereby perfectly stabilizes both domestic inflation and the output gap, there are no welfare implications of migration. In the simulation exercises presented in the next Section, there are thus no welfare consequences in the respective specifications. However, the dynamic behavior of output, employment, etc., are affected by migration and might therefore be of interest from a policy perspective.

7 Simulation

In order to illustrate the dynamics of the model, I present a simulation exercise for the differential impact of demand and supply shocks in the closed and the open labor market setting. A quite robust result is that output expands more after a positive demand shock in a scenario with migration, compared to the scenario of closed labor markets. Whether or not there is a difference for productivity shocks depends on the parameter choice.

Parameters are chosen to mimic the structure of the Polish economy. Poland has several characteristics that make it a candidate country for which the mechanics underlying this model may apply, in particular after joining the European Union and the opening of the British, Irish and Swedish labor markets for Polish workers. I assume ϕ to be 5, proxying the wage differential between Poland and the EU15. The share of emigrant hours in total hours is assumed to be 15%, a conservative estimate of the large Polish diaspora, while the elasticity by which N_M reacts to changes in domestic output is set to 0.1. This last figure is a guess because of a lack of empirical estimates. The curvature parameter of the disutility of labor function, φ, is assumed to be 3, as in Galí and Monacelli (2005), in order to make results comparable.

Setting α to 0.4 is roughly in line with the country's imports to GDP ratio. The initial condition ϑ is assumed to be 0.1, somewhat below the ratio of Poland's real GDP per capita to the EU15's at the beginning of the transition period in 1989, and is increased to 1 in a robustness check to illustrate the impact of this asymmetry. As above, θ is set to 0.75, which is consistent with an average duration of prices of one year, and $\beta = 0.99$, implying a real rate of interest of about 4% in the steady state.

In the baseline specification for which the optimal monetary policy was derived above, η, γ and σ are all set to 1. In a robustness check, η and γ are increased to 2 to check the role of these elasticities of substitution in the adjustment process. This range of values are in line with Obstfeld and Rogoff's (2005) discussion of the literature on trade elasticities. For trade equations with aggregate data and for calibrated dynamic general equilibrium models, typical estimates for trade elasticities are 1 or even less, while for estimates with disaggregated data, 2 is a rather conservative value. Choosing 1 and 2 can therefore be regarded as a compromise.

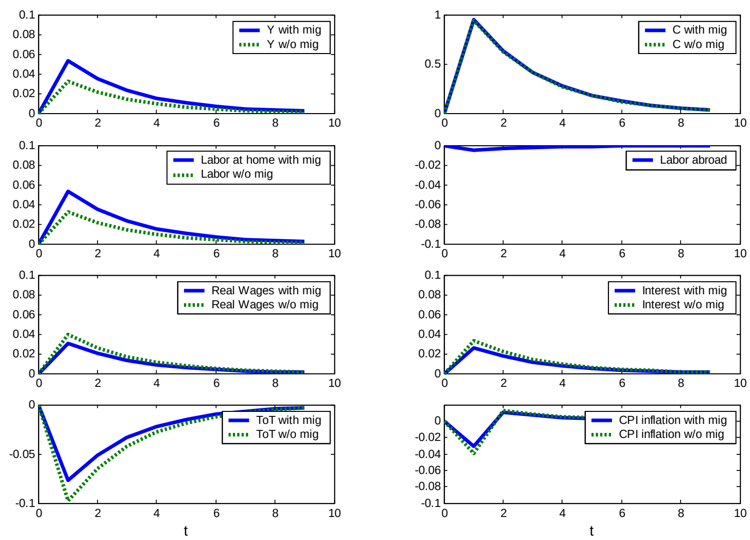
The steady state terms of trade are set to 2 for the closed labor market setting and to 1 for open labor markets. The precise figures are somewhat arbitrary, but without further assumptions, realistic numbers cannot be obtained. However, in the baseline specification with $\eta = \gamma = \sigma = 1$, the steady state is not affected at all. But for $\eta = \gamma = 2$, this is not the case and both the steady state and the dynamics are affected. In order to assess the impact of the change in the steady state on the dynamics, I also present a simulation with an unchanged steady state in a robustness check.

These assumptions imply Phillips-curve slope coefficients κ^{open} of 0.21 and 0.17 for η and γ equal to 1 and 2 respectively, which is 38% and 43% less than the benchmarks without migration κ^{closed} of 0.34 and 0.30 respectively. In all the specifications below, I assume that the central bank follows the rule (41), which is the optimal rule for the parameter choice $\eta = \gamma = \sigma = 1$.

7.1 Demand Shock

First, I analyze the effects of a domestic demand shock. Figure 2 shows the impulse

Figure 2: Demand Shock: Baseline ($\eta = \gamma = \sigma = 1, \vartheta = 0.1$ *and* $S = 1$ $(S = 2)$ *with migration (w/o migration)*)



responses of key model variables to a unit demand shock d_t with $\rho_d = 0.66$ in a baseline specification. Output and domestic employment move 63% more when people are allowed to re-migrate after the demand shock, compared with the benchmark in which migration is not allowed. The exact coefficients on impact are 0.053 and 0.033 respectively. Therefore, the differential output effect is remarkably large, even though the elasticity of re-migration is assumed to be reasonably low, and consequently only a small fraction of the emigrated labor actually returns.

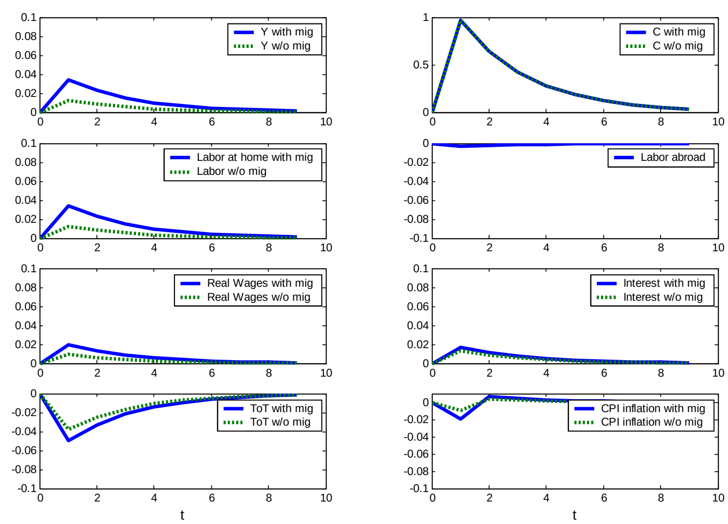
This exercise thus verifies the theoretical reasoning of Section 5 for an empirically plausible parameter choice. The re-migrating labor reduces the pressure on marginal costs, domestic prices and thereby domestic inflation, which ceteris paribus allows a greater output expansion at the zero domestic inflation rate prevailing throughout. This can be seen when comparing equations (36) and (37) under the assumption that the central bank keeps marginal costs constant: The smaller impact on marginal costs of a given output expansion allows a greater output response after the demand shock.

The demand shock increases relative demand for domestic output, thereby appreciating the terms of trade (s_t falls). This, in turn, increases the real wage (here expressed as the nominal wage relative to the CPI) because the import prices fall in line with net exports (not shown). Both the terms of trade and the real wage improve less when migration is allowed. In this specification and in most others presented below, there is no measurable differential impact on consumption after the demand shock. Consumption is thus mainly driven by the shock itself and little affected by the model inherent dynamics.

There is an important conclusion for monetary policy here. The appreciating currency corresponds to an increase in the interest rate, given that the interest parity condition holds. The differential effect through the open labor market is that the central bank reacts less restrictive after the demand shock when it follows its optimal rule. This is because the inflationary pressure is muted through the returning migrants. One interpretation of this effect is that the central bank supports and amplifies the expansionary effect of the demand shock because migrants allow a less inflationary growth of output.

A first robustness check is presented in Figure 3. Here, I increase the elasticities

Figure 3: Demand Shock: High substitution elasticity ($\eta = \gamma = 2$, $\sigma = 1$, $\vartheta = 0.1$ *and* $S = 1$ $(S = 2)$ *with migration (w/o migration)*)

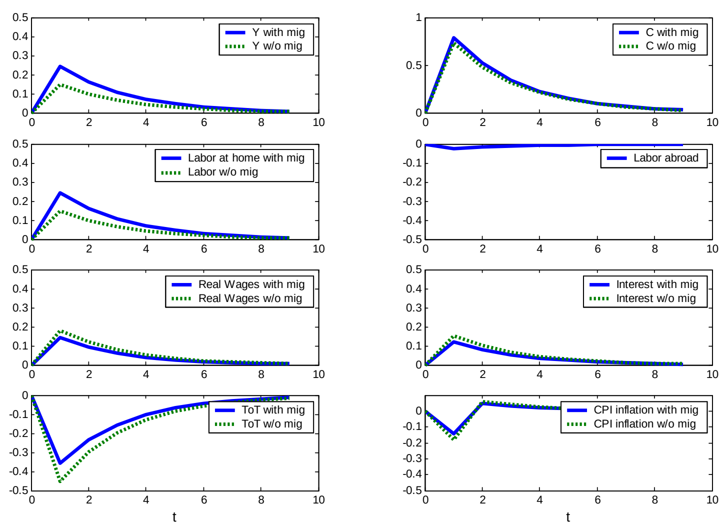


of substitution between domestic and foreign goods and between goods from different countries γ and η to 2. The output expansion is lower in both specifications, but the differential impact in the migration setting is much larger. The output expansion on impact is 173% larger than the benchmark without migration. The appreciation of the terms of trade and the real wage increase is larger when workers re-migrate, while the impact on consumption remains indistinguishable.

The conclusion for monetary policy is somewhat different here than in the baseline specification. First of all, the central bank follows the same rule as above, but in this case one cannot be sure that this is indeed the optimal rule because this could be verified in the framework above only for the parameterization of $\eta = \gamma = 1$. However, the much larger relative output expansion in the migration case results in a stronger increase in interest rates than in the case without migration. The reason for this result is that here the benign effect of migration on inflation is more than offset by the so much stronger reaction of output. The net effect is that the central bank needs to react more restrictive when migration is allowed. Consequently, the simple conclusion with respect to monetary policy from the baseline specification (i.e., that the central bank can amplify the expansionary demand effect) needs to be qualified within a general equilibrium framework.

The next robustness check (Figure 4) highlights the role of the initial condition ϑ

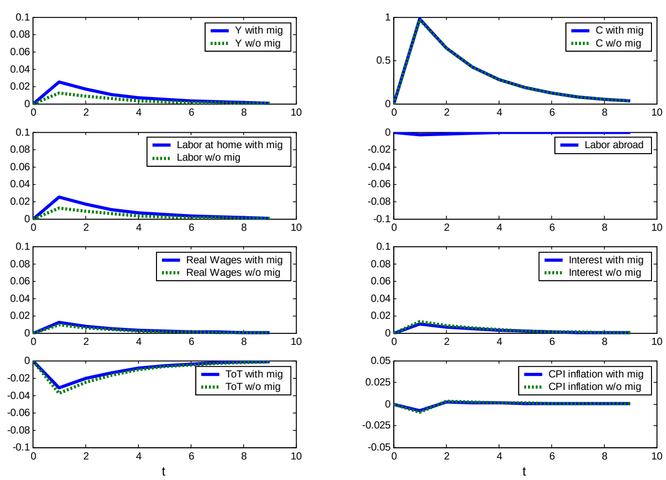
Figure 4: Demand Shock: Same initial condition ($\eta = \gamma = \sigma = 1$, $\vartheta = 1$ *and* $S = 1$ ($S = 2$) *with migration (w/o migration)*)



for the dynamics. Up to now, the value was set to 0.1, which assumed the country to have been much poorer initially. Here, instead, I assume symmetric initial conditions, that is, $\vartheta = 1$. The most important difference is the size of the fluctuations, which are much larger compared to the specification above (note the change of the scale in this Figure compared to the previous ones). Furthermore, with the open labor market, the output effect is again 63% larger than with a closed labor market. The coefficients are 0.24 in case of migration and 0.15 in the case without migration. A rich country thus benefits more from a demand shock in terms of output and has an even bigger gain in absolute terms from migration than a poor one. This is a surprising result. However, when demand elasticities are higher, this result does not prevail. Gains from migration are higher for a poor country when γ and η are set to 2.[14]

In the last robustness check (Figure 5), the elasticities of substitution η and γ are again 2, but now the steady state terms of trade are assumed to be 2 in both specifications. Hence the impulse response functions for the closed labor market are the same here as the one presented in Figure 3 while impulse responses for the migration case are now calculated for a different steady state than before. This is supposed to highlight the extent to which the different dynamics are due to the change in the steady state or to the differing labor market structures. Here the difference in the impact on output is smaller, indicating that the reduction of the size of the steady state output and the improvement in the terms of trade due to the out-migration, increases the differential reaction. The underlying mechanism of a benign effect of the returning migrants on output thus proves to be a very robust result.

Figure 5: Demand Shock: High substitution elasticity ($\eta = \gamma = 2$, $\sigma = 1$, $\vartheta = 0.1$ *and* $S = 2$ *with and w/o migration*)

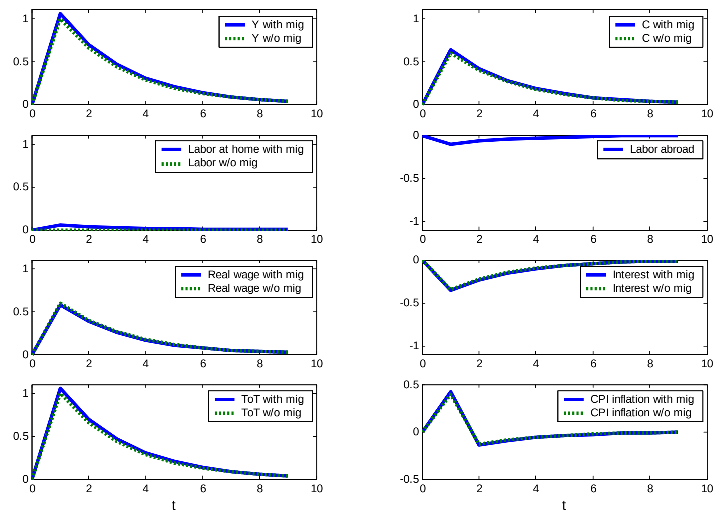


7.2 Productivity Shock

I now turn to impulse responses due to a unit productivity shock with $\rho_a = 0.66$. The benchmark parameterization is the same as for the demand shock. Furthermore $\frac{\partial N_{M,t}}{\partial A_t} \frac{A}{N_M}$, and thereby ζ_A, was set to zero. As can be seen from Figure 6, the effects on output and the other variables shown are indistinguishable between the two set ups. In both cases, the central bank accomodates the productivity increase by a reduction of the interest rate in order to stabilize marginal costs. Thereby the currency depreciates and output, consumption, the real wage and employment expand.

[14] Graphs for this specification are not shown, but are available from the author upon request.

Figure 6: Productivity Shock: Baseline ($\eta = \gamma = \sigma = 1$, $\vartheta = 0.1$ *and* $S = 1$ ($S = 2$) *with migration (w/o migration)*)



But why is there no difference? The reason is that two effects offset each other. This can be seen from equation (36) when setting all variables except output and productivity to zero. Marginal costs do not change because of the assumed monetary policy rule, therefore, this equation becomes

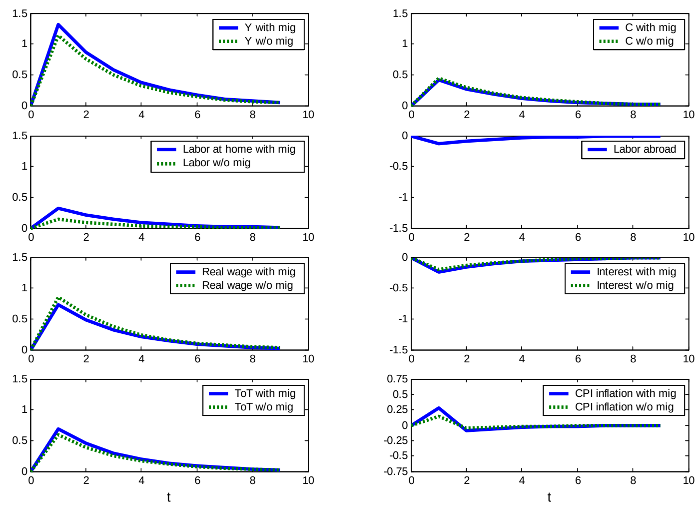
$$\widehat{mc}_t = 0 = (\sigma_{\alpha,s} + \varphi (\nu - \zeta_Y)) \widehat{y}_t - (1 + \varphi (\nu - \zeta_A)) a_t$$

In the open labor market setting both coefficients on output $\widehat{y}_t$ and productivity a_t are reduced: The latter describes the reduced effect on marginal costs after the productivity shock because only a fraction of the labor force is affected. Thereby, the disutility of labor is, ceteris paribus, only partially reduced given output. The marginal rate of substitution between labor and consumption and the real wage are less affected, and there remains less room for an output expansion at constant marginal costs. This would be reinforced if $\frac{\partial N_{M,t}}{\partial A_t} \frac{A}{N_M} > 0$ and $\zeta_A > 0$, that is, if emigration occurred due to the domestic productivity shock. The coefficient on $\widehat{y}_t$, on the other hand, is reduced too. The increased output that is made possible through the reduced interest rate and the depreciating currency has a lower effect on marginal costs in the open labor market structure. Therefore, the output expansion needs to be larger to keep marginal costs constant given productivity.

In summary, the two effects together imply that a lower output expansion is needed to keep marginal costs constant, but since any given output expansion has a lower effect on marginal costs when labor markets are open, a greater output expansion is needed to keep marginal costs constant. With this parameterization, there is obviously no output gain from migration after the productivity shock.

The analysis changes when the elasticities of substitution are increased to 2 (Figure 7). The reduced interest rate and the depreciated currency now have a much greater

Figure 7: Productivity Shock: High substitution elasticity ($\eta = \gamma = 2$, $\sigma = 1$, $\vartheta = 0.1$ *and* $S = 1$ ($S = 2$) *with migration (w/o migration)*)



effect on demand for domestic goods (note the change of the scale in this Figure compared to the previous one). Domestic employment and output expand more than in the case of low elasticities. More remarkably, there is now a difference in the two labor market settings: Output expands by almost 16% more when labor markets are open, while the domestic employment change is 230% higher. This is made possible by a stronger monetary policy response, accompanied by a bigger depreciation of the currency and return migration that dampens the real wage increase. At the same time, the consumption changes are almost the same in both specifications.

The reason for the different outcomes is that the greater response of demand for domestic goods due to the higher demand elasticities makes the increased labor demand large enough to increase the domestic labor input above the steady state level. When labor demand is strong, as it is in this case, the beneficial effect of returning labor is obviously strongest. Therefore, there is room for gains from migration in terms of bigger output expansions when elasticities of substitution between domestic and foreign goods are "high".

Robustness checks with analogous parameterizations as for the demand shocks confirm the results of the specifications above with insignificant differences. Graphs are available from the author upon request.

8 Conclusion

This paper presents a New Keynesian business cycle model that allows for labor to be supplied both domestically and abroad. This modification to an otherwise standard set up takes account of the observed labor movements across borders in many countries. Allowing migrants to cross borders in response to asymmetric business cycles has several important implications for the structure of the domestic economy.

First, the Phillips curve becomes flatter. When emigrants return when output expands, firms do not need to compensate workers for foregone leisure as they skip one job (abroad) for another job (at home). As a consequence, there is less pressure on wages, marginal costs and prices.

Second, the optimal monetary policy rule derived is the same as in the case of no migration but the welfare loss implied by deviations from the optimal rule is different. According to the optimal monetary policy rule, that is derived from the perspective of a social planner, both the output gap and domestic inflation need to be fully stabilized. However, according to the welfare function, which is derived as a second order approximation to the representative household's utility function, deviations from this optimal rule are shown to reduce welfare differently when migration is allowed. Domestic inflation volatility is penalized more while output gap volatility is penalized less.

The effect of output gap volatility on welfare is due to the property that workers are able to reduce the adverse effects of output volatility on disutility from labor by adjusting their labor input domestically and abroad. The negative effect of domestic inflation on welfare is explained by the fact that for a given volatility of inflation, there will be a higher inefficient variability of output across goods due to price stickyness when migration is allowed. The reason for this is the weakened link between output variation and inflation because of migration. The benign effect of migration on inflation thus exacerbates the adverse welfare effects of any given volatility of inflation.

Third, domestic demand shocks are shown to have a greater impact on domestic output in the set up with migration. When output expands due to the demand shock and migrants are attracted to the domestic economy, the pressure on marginal costs and inflation is lower. Consequently, output can expand more until inflation increases to the point at which the central bank no longer tolerates it.

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Appendix A.

The domestic demand was derived above and given in equation (7):

$$C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{A-1}$$

For the derivation of foreign demand, a demand function for domestic good j analogous to equation (8) needs to be derived. Because of the LOOP, I have

$$P_{H,t}(j) = \epsilon_{i,t} P^i_{H,t}(j)$$

where $P^i_{H,t}(j)$ is the price of the domestically produced good j expressed in terms of country i's currency units. Furthermore, defining $P^i_{H,t} = \left(\int_0^1 P^i_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ as the index of domestically produced goods in terms of country i's currency units, one can easily check that

$$P_{H,t} = \epsilon_{i,t} P^i_{H,t}$$

Country i's demand for good i, $C^i_{H,t}(j)$, is then:

$$C^i_{H,t}(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$$

with P^i_t and C^i_t defined as country i's consumer price and consumption indexes, the former expressed in its own currency. Integrating this over all countries, gives total foreign demand:

$$\int_0^1 C^i_{H,t}(j) di = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \int_0^1 \left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di$$

Total demand is therefore

$$\begin{aligned} Y_t(j) &= C_{H,t}(j) + \int_0^1 C^i_{H,t}(j) di \\ &= (1-\alpha) \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\ &\quad + \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \int_0^1 \left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \end{aligned}$$

Plugging this into the aggregate output relation, making use of the international risk sharing condition (20) and the definition of $RER_{i,t}$, gives

$$\begin{aligned} Y_t &\equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ &= (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{\epsilon_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} RER^{\eta}_{i,t} C^i_t di\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[1 - \alpha + \alpha \vartheta^{-1} \left(e^{d_t - d^*_t}\right)^{-\frac{1}{\sigma}} \int_0^1 \left(\frac{\epsilon_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} RER^{\eta-\frac{1}{\sigma}}_{i,t} di\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[1 - \alpha + \alpha \vartheta^{-1} \left(e^{d_t - d^*_t}\right)^{-\frac{1}{\sigma}} \int_0^1 \left(S^i_t S_{i,t}\right)^{\gamma-\eta} RER^{\eta-\frac{1}{\sigma}}_{i,t} di\right] \end{aligned} \quad \text{(A-2)}$$

making use of the fact that $\frac{\epsilon_{i,t} P^i_{F,t}}{P_{H,t}} = S^i_t S_{i,t}$ with S^i_t defined as country i´s effective terms of trade. Loglinearizing, assuming that $S^i = S^* = 1$, $\int_0^1 \widehat{s}^i_t di = 0$, $S_i = S$ and

$RER_i = REER$ in the steady state gives

$$
\begin{aligned}
Y_t - Y &= -\eta\left(\frac{P_H}{P}\right)^{-\eta-1} C\,[...]\left(\frac{1}{P}dP_H - \frac{P_H}{P^2}dP\right) + \left(\frac{P_H}{P}\right)^{-\eta}[...]\,dC_t \\
&\quad -\frac{1}{\sigma}(e^0)^{-\frac{1}{\sigma}-1}\left(\frac{P_H}{P}\right)^{-\eta} C\alpha\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}(d_t - d_t^*) \\
&\quad +(\gamma-\eta)\left(\frac{P_H}{P}\right)^{-\eta} C\alpha\vartheta^{-1}S^{\gamma-\eta-1}REER^{\eta-\frac{1}{\sigma}}\int_0^1 dS_{i,t}di \\
&\quad +\left(\eta-\frac{1}{\sigma}\right)\left(\frac{P_H}{P}\right)^{-\eta} C\alpha\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}-1}\int_0^1 dRER_{i,t}di
\end{aligned}
$$

$$
\begin{aligned}
\frac{Y_t - Y}{Y} &= -\eta\left(\widehat{p}_{H,t} - \widehat{p}_t\right) + \widehat{c}_t \\
&\quad -\frac{\frac{\alpha}{\sigma}\left(\frac{P_H}{P}\right)^{-\eta} C\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}}{\left(\frac{P_H}{P}\right)^{-\eta} C\left[(1-\alpha)+\alpha\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}\right]}(d_t - d_t^*) \\
&\quad +\frac{\alpha(\gamma-\eta)\left(\frac{P_H}{P}\right)^{-\eta} C\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}}{\left(\frac{P_H}{P}\right)^{-\eta} C\left[(1-\alpha)+\alpha\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}\right]}\int_0^1 \widehat{s}_{i,t}di \\
&\quad +\frac{\alpha\left(\eta-\frac{1}{\sigma}\right)\left(\frac{P_H}{P}\right)^{-\eta} C\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}}{\left(\frac{P_H}{P}\right)^{-\eta} C\left[(1-\alpha)+\alpha\vartheta^{-1}S^{\gamma-\eta}REER^{\eta-\frac{1}{\sigma}}\right]}\int_0^1 \widehat{rer}_{i,t}di
\end{aligned}
$$

$$
\begin{aligned}
\widehat{y}_t &= \alpha_s\eta\widehat{s}_t + \widehat{c}_t \\
&\quad -\frac{\alpha}{\sigma}\left[(1-\alpha)\vartheta S^{-\gamma+\eta}REER^{-\eta+\frac{1}{\sigma}}+\alpha\right]^{-1}(d_t - d_t^*) \\
&\quad +\alpha(\gamma-\eta)\left[(1-\alpha)\vartheta S^{-\gamma+\eta}REER^{-\eta+\frac{1}{\sigma}}+\alpha\right]^{-1}\widehat{s}_t \\
&\quad +\alpha\left(\eta-\frac{1}{\sigma}\right)\left[(1-\alpha)\vartheta S^{-\gamma+\eta}REER^{-\eta+\frac{1}{\sigma}}+\alpha\right]^{-1}\widehat{reer}_t
\end{aligned}
$$

$$
\begin{aligned}
\widehat{y}_t &= \widehat{c}_t + \frac{\alpha}{\sigma}\left[\eta\sigma k(S) + \sigma(\gamma-\eta)l(S) + (1-\alpha_S)(\sigma\eta-1)l(S)\right]\widehat{s}_t \\
&\quad -\frac{\alpha}{\sigma}l(S)\left[d_t - d_t^*\right] \\
\widehat{y}_t &= \widehat{c}_t + \frac{\alpha}{\sigma}\left[\left(\sigma\gamma + (1-\alpha_S)(\sigma\eta-1)\right)l(S) + \sigma\eta(k(S)-l(S))\right]\widehat{s}_t \\
&\quad -\frac{\alpha}{\sigma}l(S)\left[d_t - d_t^*\right] \\
\widehat{y}_t &= \widehat{c}_t + \frac{\alpha\varpi_S}{\sigma}\widehat{s}_t - \frac{\alpha}{\sigma}l(S)\left[d_t - d_t^*\right]
\end{aligned}
$$

with the substitutions

$$\begin{aligned}
k(S) &\equiv \left[(1-\alpha)S^{\eta-1}+\alpha\right]^{-1}\\
l(S) &\equiv \left[(1-\alpha)\vartheta S^{\eta-\gamma} reer(S)^{\frac{1}{\sigma}-\eta}+\alpha\right]^{-1}\\
reer(S) &\equiv REER\\
\varpi_S &\equiv \left[\sigma\gamma+(1-\alpha_S)(\sigma\eta-1)\right] l(S)+\sigma\eta\left[k(S)-l(S)\right]
\end{aligned}$$

Appendix B.

In the steady state, output and the terms of trade are uniquely pinned down by two equations with values determined by the relative labor market conditions facing the representative worker.

From equation (A-2), I derive the goods market clearing condition in the steady state,

$$Y = h(S)^{\eta} C\left[(1-\alpha)+\alpha\vartheta^{-1}S^{\gamma-\eta} reer(S)^{\eta-\frac{1}{\sigma}}\right]$$

making use of the risk sharing condition (37), the fact that $S^i = S^* = 1$, $S_i = S$ and $RER_i = REER\,\forall i$ in the steady state and the substitutions

$$\frac{P}{P_H} = \left[(1-\alpha)+\alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv h(S)$$

and $REER = \frac{S}{h(S)} \equiv reer(S)$. Note that $h(S) > 0$ and $reer(S) > 0$, $h'(S) > 0$ and $reer'(S) > 0$ and $h(1) = reer(1) = 1$.

Furthermore, in the steady state, the risk sharing condition, taking account of international goods market clearing, $C^* = Y^*$, is

$$C = \vartheta Y^* reer(S)^{\frac{1}{\sigma}}$$

Combining this with the goods market clearing condition gives

$$\begin{aligned}
Y &= h(S)^{\eta}\vartheta Y^* reer(S)^{\frac{1}{\sigma}}\left[(1-\alpha)+\alpha\vartheta^{-1}S^{\gamma-\eta} reer(S)^{\eta-\frac{1}{\sigma}}\right]\\
&= Y^*\left[(1-\alpha)\vartheta h(S)^{\eta} reer(S)^{\frac{1}{\sigma}}+\alpha S^{\gamma-\eta} h(S)^{\eta} reer(S)^{\frac{1}{\sigma}} reer(S)^{\eta-\frac{1}{\sigma}}\right]\\
&= Y^*\left[(1-\alpha)\vartheta S^{\eta} reer(S)^{\frac{1}{\sigma}-\eta}+\alpha S^{\gamma-\eta} h(S)^{\eta} reer(S)^{\eta}\right]\\
&= Y^*\left[(1-\alpha)\vartheta S^{\eta} reer(S)^{\frac{1}{\sigma}-\eta}+\alpha S^{\gamma}\right]\\
&\equiv Y^* v(S) \qquad \text{(B-1)}
\end{aligned}$$

with $v(S) > 0$ and $v'(S) > 0$. The intuition behind this equation is the following: In the steady state, a more depreciated currency results in a greater demand for domestic goods. Note that $v(1) < 1$, implying $Y < Y^*$ even if $S = 1$ when $\vartheta < 1$. This means that for a country that was initially poorer than the rest of the world, output remains smaller in the steady state because consumption and thereby domestic demand remains suppressed. Moreover, output is uniquely determined when the steady state terms of trade are known. This means that up to some upper limit, values of $S > 1$ are possible, which would be in line with typical observations of developing countries' terms of trade.

For the second equation to determine the unique steady states of output and the terms of trade, I rewrite the domestic labor market clearing condition:

$$
\begin{aligned}
C^{\sigma} N^{\varphi} &= \frac{W}{P} \\
&= A \frac{W}{P_H A} \frac{P_H}{P} \\
&= A \frac{1}{h(S)} MC
\end{aligned}
$$

In the steady state, I have $MC = 1 - \frac{1}{\varepsilon}$. Therefore, and because of the risk sharing condition, I get

$$
\begin{aligned}
(\vartheta Y^*)^{\sigma} reer(S) \left(\frac{Y}{A} + \phi N^M \right)^{\varphi} &= A \frac{1}{h(S)} \left(1 - \frac{1}{\varepsilon} \right) \\
Y &= A \left[\left(\frac{A \frac{1}{h(S)} \left(1 - \frac{1}{\varepsilon}\right)}{(\vartheta Y^*)^{\sigma} reer(S)} \right)^{\frac{1}{\varphi}} - \phi N^M \right] \\
Y &= A \left[\left(\frac{A \left(1 - \frac{1}{\varepsilon}\right)}{(\vartheta Y^*)^{\sigma} S} \right)^{\frac{1}{\varphi}} - \phi N^M \right] \\
Y &= k(S) \qquad \text{(B-2)}
\end{aligned}
$$

with $k'(S) < 0$. Output is thus negatively related to the terms of trade. The intuition behind this relationship is that an increase in S increases consumption through the risk sharing condition, thereby reducing the incentive to work. Labor input and output fall.

Jointly with (B-1), I have a system of two equations in the two unknowns Y and S, given parameters, N_M and productivity A. Because in (B-1) Y is strictly increasing while in (B-2) Y is strictly decreasing in S, there is a unique solution for Y and S. In the fully symmetric, no-migration benchmark model, this unique solution is determined by $S = 1$ and $Y = Y^*$ (Galí and Monacelli, 2005). The original asymmetry ($\vartheta < 1$) tends to reduce output in the first equation, while increasing it in the second. These shifts drive up the terms of trade in the steady state above one. The effect of emigration, that is, an increase in N_M, is to lower $k(S)$ and shift (B-2) down, that is, reducing Y and, in conjunction with equation (B-1), reduce S . Consequently, the model is flexible enough to be calibrated such that output is below the world average while the terms of trade can be allowed to be above and below 1 in the steady state.

Appendix C.

Here, I derive the optimal allocation and the flexible price equilibrium presented in Section 6.1. To that end, the third and the fourth constraints are combined. I rewrite the fourth constraint

$$Y_t = S^{\alpha} C_t \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]$$

$$\Leftrightarrow S_t = Y_t^{\frac{1}{\alpha}} C_t^{\frac{-1}{\alpha}} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{\frac{-1}{\alpha}}$$

and plug it into the third:

$$\begin{aligned}
C_t &= \vartheta Y_t^* \left(Y_t^{\frac{1-\alpha}{\alpha}} C_t^{\frac{-1+\alpha}{\alpha}} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{\frac{-1+\alpha}{\alpha}} \right) e^{(d_t - d_t^*)} \\
C_t^{1+\frac{1-\alpha}{\alpha}} &= \vartheta Y_t^* Y_t^{\frac{1-\alpha}{\alpha}} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{\frac{-1+\alpha}{\alpha}} e^{(d_t - d_t^*)} \\
C_t &= \left(\vartheta Y_t^* \right)^{\alpha} Y_t^{1-\alpha} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{-1+\alpha} \left(e^{(d_t - d_t^*)} \right)^{\alpha} \quad \text{(C-1)}
\end{aligned}$$

Next, I replace the arguments in the period utility function,

$$\begin{aligned}
U(C_t, N_t) &= e^{d_t} \log C_t - \frac{\left(N_{H,t} + \phi N_{M,t} \right)^{1+\varphi}}{1+\varphi} \\
U(Y_t, Y_t^*) &= e^{d_t} \log \left\{ \left(\vartheta Y_t^* \right)^{\alpha} Y_t^{1-\alpha} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{-1+\alpha} \left(e^{(d_t - d_t^*)} \right)^{\alpha} \right\} \\
&\quad - \frac{\left(\frac{Y_t}{A_t} + \phi N_M(Y_t, Y_t^*, A_t, A_t^*) \right)^{1+\varphi}}{1+\varphi}
\end{aligned}$$

and optimize that equation:

$$\begin{aligned}
\frac{U_N}{U_C} &= \frac{(1-\alpha) \left(\vartheta Y_t^* \right)^{\alpha} Y_t^{-\alpha} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{-1+\alpha} \left(e^{(d_t - d_t^*)} \right)^{\alpha}}{\frac{1}{A_t} + \phi \frac{\partial N_{M,t}}{\partial Y_t}} \\
N_t^{\varphi} C_t &= \frac{(1-\alpha) \left(\vartheta Y_t^* \right)^{\alpha} Y_t^{1-\alpha} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*} \right)^{-1} \right]^{-1+\alpha} \left(e^{(d_t - d_t^*)} \right)^{\alpha}}{\left(\frac{N_{H,t}}{Y_t} + \phi \frac{\partial N_{M,t}}{\partial Y_t} \right) Y_t} \\
N_t^{\varphi} C_t &= \frac{(1-\alpha) C_t}{N_{H,t} + \phi \frac{\partial N_{M,t}}{\partial Y_t} Y_t} \\
N_t^{1+\varphi} &= \frac{1-\alpha}{\frac{N_{H,t}}{N_t} + \frac{\partial N_{M,t}}{\partial Y_t} \frac{Y_t}{N_{M,t}} \frac{\phi N_{M,t}}{N_t}} \\
N_t &= \left(\frac{1-\alpha}{\nu_t - \zeta_{Y,t}} \right)^{\frac{1}{1+\varphi}}
\end{aligned}$$

This is the optimal allocation from the planner's perspective. The term $(\nu_t - \zeta_{Y,t})$ is proportional to the change in the argument in the disutility of labor function. Assuming this to be constant, that is, $\nu_t - \zeta_{Y,t} = \nu - \zeta_Y$, there is now a unique and constant optimal value of N_t, rather than a constant optimal N_H, as in Galí and Monacelli (2005).

The flexible price equilibrium satisfies

$$\begin{aligned}
1 - \frac{1}{\varepsilon} &= MC_t^n \\
&= \frac{1-\tau}{A_t} \frac{W_t^n}{P_{H,t}^n} \\
&= \frac{1-\tau}{A_t} e^{-d_t} C_t^n \left(N_t^n\right)^{\varphi} \frac{P_t^n}{P_{H,t}^n} \\
&= \frac{1-\tau}{A_t} e^{-d_t} C_t^n \left(N_t^n\right)^{\varphi} \left(S_t\right)^{\alpha} \\
&= \frac{1-\tau}{A_t} e^{-d_t} C_t^n \left(N_t^n\right)^{\varphi} Y_t^n \left(C_t^n\right)^{-1} \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*}\right)^{-1}\right]^{-1} \\
&= \frac{1-\tau}{A_t} \left(N_t^n\right)^{\varphi} Y_t^n \left[(1-\alpha)e^{d_t} + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*}\right)^{-1} e^{d_t}\right]^{-1} \\
&= (1-\tau) \left(N_t^n\right)^{\varphi} N_{H,t}^n \left[(1-\alpha)e^{d_t} + \alpha\vartheta^{-1} e^{d_t^*}\right]^{-1}
\end{aligned}$$

Appendix D.

In order to derive the welfare function in terms of the output gap and inflation, the utility function needs to be rewritten. It is approximated by a second order Taylor expansion around the flexible price equilibrium (the natural rate), and the resulting terms replaced by the output gap and domestic inflation.

For the utility of consumption with $\sigma = 1$, one can write

$$\log C_t = c_t^n + \widetilde{c}_t$$

where the tilde indicates the log deviation of this variable from the natural level. $\widetilde{c}_t$ can be replaced by an expression that is proportional to the output gap. Taking logs of equation (C-1)

$$c_t = \alpha \log \vartheta + \alpha y_t^* + (1-\alpha) y_t + (\alpha - 1) \log \left[(1-\alpha) + \alpha\vartheta^{-1} \left(e^{d_t - d_t^*}\right)^{-1}\right] + \alpha(d_t - d_t^*)$$

and approximating this around the natural level, gives

$$\widetilde{c}_t = \alpha x_t^* + (1-\alpha) x_t$$

Assuming the rest of the world to perfectly stabilize its output gap, consumption utility is

$$\begin{aligned}
\log C_t &= c_t^n + \widetilde{c}_t \\
&= c_t^n + (1-\alpha) x_t
\end{aligned} \qquad \text{(D-1)}$$

For the disutility of labor function, the approximation around the natural rate is:

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{(N^n)^{1+\varphi}}{1+\varphi} + (N^n)^{1+\varphi}\left(\widetilde{n}_t + \frac{1}{2}(1+\varphi)\widetilde{n}_t^2\right) + o\left\|a\right\|^3$$

Approximated around the natural rate, the argument in the disutility of labor function $N_t = N_t^H + \phi N_t^F$ is

$$\widetilde{n}_t = \nu\widetilde{n}_t^H + (1-\nu)\widetilde{n}_t^M$$

where $\nu = \frac{N^H}{N} < 1$. N_t^H can be approximated as

$$\widetilde{n}_t^H = x_t + z_t$$

(see Galí and Monacelli, 2005) while hours abroad are

$$\widetilde{n}_t^M = \frac{\partial N_{M,t}}{\partial Y_t}\frac{Y_t^n}{N_{M,t}^n}x_t + \frac{\partial N_{M,t}}{\partial Y_t^*}\frac{Y_t^*}{N_{M,t}^n}x_t^*$$

Assuming the rest of the world to perfectly stabilize the output gap x_t^*, this equation reduces to

$$\widetilde{n}_t^M = \frac{\partial N_{M,t}}{\partial Y_t}\frac{Y_t^n}{N_{M,t}^n}x_t$$

Combining these last three results, gives

$$\begin{aligned}
\widetilde{n}_t &= \nu\widetilde{n}_t^H + (1-\nu)\widetilde{n}_t^M \\
&= \nu\left(x_t + z_t\right) + (1-\nu)\left(\frac{\partial N_{M,t}}{\partial Y_t}\frac{Y_t^n}{N_{M,t}^n}x_t\right) \\
&= \nu\left(x_t + z_t\right) + \frac{\partial N_{M,t}}{\partial Y_t}\frac{Y_t^n}{N_{M,t}^n}\frac{\phi N_{M,t}}{N_t}x_t \\
&= \left(\nu - \zeta_Y\right)x_t + \nu z_t
\end{aligned}$$

Finally, this can be inserted into the approximation of the disutility of labor function from above:

$$\begin{aligned}
\frac{N_t^{1+\varphi}}{1+\varphi} &= \frac{(N^n)^{1+\varphi}}{1+\varphi} + (N^n)^{1+\varphi}\left(\widetilde{n}_t + \frac{1}{2}(1+\varphi)\widetilde{n}_t^2\right) + o\left\|a\right\|^3 \\
&= \frac{(N^n)^{1+\varphi}}{1+\varphi} \\
&\quad + (N^n)^{1+\varphi}\left(\left(\nu-\zeta_Y\right)x_t + \nu z_t + \frac{1}{2}(1+\varphi)\left(\nu-\zeta_Y\right)^2 x_t^2\right) + o\left\|a\right\|^3
\end{aligned} \tag{D-2}$$

where $o\left\|a\right\|^3$ are terms of order 3 or higher in the bound $\left\|a\right\|$ on the size of the relevant shocks.

The approximations (D-1) and (D-2) taken together are now

$$\begin{aligned}
U(C_t, N_t) &= \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
&= c_t^n + (1-\alpha)x_t - \frac{(N_t^n)^{1+\varphi}}{1+\varphi} \\
&\quad - (N_t^n)^{1+\varphi}\left((\nu - \zeta_Y)\, x_t + \nu z_t + \frac{1}{2}(1+\varphi)\,(\nu-\zeta_Y)^2\, x_t^2\right) + o\,\|a\|^3 \\
&= \left((1-\alpha - \frac{1-\alpha}{\nu-\zeta_Y}\,(\nu-\zeta_Y)\right) x_t \\
&\quad -\frac{1-\alpha}{\nu-\zeta_Y}\left(\nu z_t + \frac{1}{2}(1+\varphi)\,(\nu-\zeta_Y)^2\, x_t^2\right) + \text{t.i.p.} + o\,\|a\|^3 \\
&= -(1-\alpha)\left(\frac{\nu}{\nu-\zeta_Y} z_t + \frac{1}{2}(1+\varphi)\,(\nu-\zeta_Y)\, x_t^2\right) + \text{t.i.p.} + o\,\|a\|^3
\end{aligned}$$

where t.i.p. are terms that are independent from policies.

Finally, the welfare function is the discounted sum of the period utility functions:

$$\begin{aligned}
W &= -(1-\alpha)\sum_{t=0}^{\infty}\beta^t\left\{\left(\frac{\nu}{\nu-\zeta_Y} z_t + \frac{1}{2}(1+\varphi)\,(\nu-\zeta_Y)\, x_t^2\right)\right\} \\
&\quad + \text{t.i.p.} + o\,\|a\|^3
\end{aligned}$$

In order to replace the dispersion term z_t, Galí and Monacelli (2005) showed that it is proportional to the variance of domestic prices,

$$z_t = \frac{\varepsilon}{2} var_i\left\{p_{H,t}(i)\right\} + o\,\|a\|^3$$

and because

$$\sum_{t=0}^{\infty}\beta^t var_i\left\{p_{H,t}(i)\right\} = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t\left\{\pi_{H,t}^2\right\}$$

(see Woodford, 2003, ch. 6), it gives

$$\begin{aligned}
W &= -\frac{1}{2}(1-\alpha)\sum_{t=0}^{\infty}\beta^t\left\{\left(\frac{\nu}{\nu-\zeta_Y}\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + (1+\varphi)\,(\nu-\zeta_Y)\, x_t^2\right)\right\} \\
&\quad + \text{t.i.p.} + o\,\|a\|^3
\end{aligned}$$

This is the welfare function in Section 6.2.