$$\sqrt{T}\begin{bmatrix} \widetilde{\boldsymbol{\beta}} - \boldsymbol{\beta} \\ \widetilde{\boldsymbol{\sigma}} - \boldsymbol{\sigma} \end{bmatrix} \xrightarrow{d} \mathcal{N}\left(0, \begin{bmatrix} R[R'(\Gamma \otimes \Sigma_u^{-1})R]^{-1}R' & 0 \\ 0 & 2\mathbf{D}_K^+(\Sigma_u \otimes \Sigma_u)\mathbf{D}_K^{+\prime} \end{bmatrix}\right),$$

where $\mathbf{D}_K^+ = (\mathbf{D}_K'\mathbf{D}_K)^{-1}\mathbf{D}_K'$ is, as usual, the Moore-Penrose inverse of the $(K^2 \times K(K+1)/2)$ duplication matrix $\mathbf{D}_K$. ■

Of course, we could have stated the proposition in terms of the joint distribution of $\widehat{\boldsymbol{\gamma}}$ and $\widetilde{\boldsymbol{\sigma}}$ instead. In the following, the distribution given in the proposition will turn out to be more useful, though.

Both EGLS and ML estimation can be discussed in terms of the mean-adjusted model considered in Section 3.3. However, the present discussion includes restrictions for the intercept terms in a convenient way. If the restrictions are equivalent in the different versions of the model, the asymptotic properties of the estimators of $\boldsymbol{\alpha} := \text{vec}(A_1, \dots, A_p)$ will not be affected. For instance, the asymptotic covariance matrix of $\sqrt{T}(\widetilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha})$, where $\widetilde{\boldsymbol{\alpha}}$ is the ML estimator, is just the lower right-hand $(K^2p \times K^2p)$ block of $R[R'(\Gamma \otimes \Sigma_u^{-1})R]^{-1}R'$ from Proposition 5.5. If the sample means are subtracted from all variables and the constraints are given in the form $\boldsymbol{\alpha} = R\boldsymbol{\gamma} + r$ for a suitable matrix R and vectors $\boldsymbol{\gamma}$ and r, the covariance matrix of the asymptotic distribution of $\sqrt{T}(\widetilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha})$ can be written as

$$R[R'(\Gamma_Y(0) \otimes \Sigma_u^{-1})R]^{-1}R', \tag{5.2.20}$$

where $\Gamma_Y(0) := \Sigma_Y = \text{Cov}(Y_t)$ with $Y_t := (y_t', \dots, y_{t-p+1}')'$.

5.2.4 Constraints for Individual Equations

In practice, parameter restrictions are often formulated for the K equations of the system (5.1.1) separately. In that case, it may be easier to write the restrictions in terms of the vector $\mathbf{b} := \text{vec}(B')$ which contains the parameters of the first equation in the first $Kp + 1$ positions and those of the second equation in the second $Kp + 1$ positions etc. If the constraints are expressed as

$$\mathbf{b} = \bar{R}\mathbf{c} + \bar{r}, \tag{5.2.21}$$

where $\bar{R}$ is a known $((K^2p + K) \times M)$ matrix of rank M, $\mathbf{c}$ is an unknown $(M \times 1)$ parameter vector, and $\bar{r}$ is a known $(K^2p + K)$-dimensional vector, the restricted EGLS and ML estimators of $\mathbf{b}$ and their properties are easily derived. We get the following proposition:

Proposition 5.6 (*EGLS Estimator of Parameters Arranged Equationwise*)
Under the conditions of Proposition 5.2, if $\mathbf{b} = \text{vec}(B')$ satisfies (5.2.21), the EGLS estimator of $\mathbf{c}$ is

$$\widehat{\widehat{\mathbf{c}}} = [\bar{R}'(\bar{\Sigma}_u^{-1} \otimes ZZ')\bar{R}]^{-1}\bar{R}'(\bar{\Sigma}_u^{-1} \otimes Z)[\text{vec}(Y') - (I_K \otimes Z')\bar{r}], \tag{5.2.22}$$