$$\sqrt{T}\,\mathrm{vec}(\widehat{\Phi}_i - \Phi_i) \xrightarrow{d} \mathcal{N}(0, G_i \Sigma_{\widehat{\boldsymbol{\alpha}}} G_i'), \quad i = 1, 2, \dots, \tag{3.7.5}$$

where

$$G_i := \frac{\partial\,\mathrm{vec}(\Phi_i)}{\partial \boldsymbol{\alpha}'} = \sum_{m=0}^{i-1} J(\mathbf{A}')^{i-1-m} \otimes \Phi_m.$$

$$\sqrt{T}\,\mathrm{vec}(\widehat{\Psi}_n - \Psi_n) \xrightarrow{d} \mathcal{N}(0, F_n \Sigma_{\widehat{\boldsymbol{\alpha}}} F_n'), \quad n = 1, 2, \dots, \tag{3.7.6}$$

where $F_n := G_1 + \cdots + G_n$.

If $(I_K - A_1 - \cdots - A_p)$ is nonsingular,

$$\sqrt{T}\,\mathrm{vec}(\widehat{\Psi}_\infty - \Psi_\infty) \xrightarrow{d} \mathcal{N}(0, F_\infty \Sigma_{\widehat{\boldsymbol{\alpha}}} F_\infty'), \tag{3.7.7}$$

where $F_\infty := \underbrace{(\Psi_\infty', \dots, \Psi_\infty')}_{p \text{ times}} \otimes \Psi_\infty$.

$$\sqrt{T}\,\mathrm{vec}(\widehat{\Theta}_i - \Theta_i) \xrightarrow{d} \mathcal{N}(0, C_i \Sigma_{\widehat{\boldsymbol{\alpha}}} C_i' + \bar{C}_i \Sigma_{\widehat{\boldsymbol{\sigma}}} \bar{C}_i'), \quad i = 0, 1, 2, \dots, \tag{3.7.8}$$

where

$$C_0 := 0,\ C_i := (P' \otimes I_K) G_i, i = 1, 2, \dots,\ \bar{C}_i := (I_K \otimes \Phi_i) H,\ i = 0, 1, \dots,$$

and

$$\begin{aligned} H &:= \frac{\partial\,\mathrm{vec}(P)}{\partial \boldsymbol{\sigma}'} = \mathbf{L}_K' \{\mathbf{L}_K[(I_K \otimes P)\mathbf{K}_{KK} + (P \otimes I_K)]\mathbf{L}_K'\}^{-1} \\ &= \mathbf{L}_K' \{\mathbf{L}_K(I_{K^2} + \mathbf{K}_{KK})(P \otimes I_K)\mathbf{L}_K'\}^{-1}. \end{aligned}$$

$$\sqrt{T}\ \mathrm{vec}(\widehat{\Xi}_n - \Xi_n) \xrightarrow{d} \mathcal{N}(0, B_n \Sigma_{\widehat{\boldsymbol{\alpha}}} B_n' + \bar{B}_n \Sigma_{\widehat{\boldsymbol{\sigma}}} \bar{B}_n'), \tag{3.7.9}$$

where $B_n := (P' \otimes I_K) F_n$ and $\bar{B}_n := (I_K \otimes \Psi_n) H$.

If $(I_K - A_1 - \cdots - A_p)$ is nonsingular,

$$\sqrt{T}\ \mathrm{vec}(\widehat{\Xi}_\infty - \Xi_\infty) \xrightarrow{d} \mathcal{N}(0, B_\infty \Sigma_{\widehat{\boldsymbol{\alpha}}} B_\infty' + \bar{B}_\infty \Sigma_{\widehat{\boldsymbol{\sigma}}} \bar{B}_\infty'), \tag{3.7.10}$$

where $B_\infty := (P' \otimes I_K) F_\infty$ and $\bar{B}_\infty := (I_K \otimes \Psi_\infty) H$.

Finally,

$$\begin{aligned} \sqrt{T}(\widehat{\omega}_{jk,h} - \omega_{jk,h}) &\xrightarrow{d} \mathcal{N}(0, d_{jk,h} \Sigma_{\widehat{\boldsymbol{\alpha}}} d_{jk,h}' + \overline{d}_{jk,h} \Sigma_{\widehat{\boldsymbol{\sigma}}} \overline{d}_{jk,h}') \\ &\qquad j, k = 1, \dots, K, \quad h = 1, 2, \dots, \end{aligned} \tag{3.7.11}$$

where

$$\begin{aligned} d_{jk,h} &:= \frac{2}{\mathrm{MSE}_j(h)^2} \sum_{i=0}^{h-1} \Big[\mathrm{MSE}_j(h)(e_j' \Phi_i P e_k)(e_k' P' \otimes e_j') G_i \\ &\qquad - (e_j' \Phi_i P e_k)^2 \sum_{m=0}^{h-1} (e_j' \Phi_m \Sigma_u \otimes e_j') G_m \Big] \end{aligned}$$