The condition (2.1.12) provides an easy tool for checking the stability of a VAR process. Consider, for instance, the three-dimensional VAR(1) process

$$y_t = \nu + \begin{bmatrix} .5 & 0 & 0 \\ .1 & .1 & .3 \\ 0 & .2 & .3 \end{bmatrix} y_{t-1} + u_t. \tag{2.1.14}$$

For this process the reverse characteristic polynomial is

$$\begin{aligned} &\det\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .5 & 0 & 0 \\ .1 & .1 & .3 \\ 0 & .2 & .3 \end{bmatrix} z \right) \\ &\quad = \det \begin{bmatrix} 1 - .5z & 0 & 0 \\ -.1z & 1 - .1z & -.3z \\ 0 & -.2z & 1 - .3z \end{bmatrix} \\ &\quad = (1 - .5z)(1 - .4z - .03z^2). \end{aligned}$$

The roots of this polynomial are easily seen to be

$$z_1 = 2, \qquad z_2 = 2.1525, \qquad z_3 = -15.4858.$$

They are obviously all greater than 1 in absolute value. Therefore the process (2.1.14) is stable.

As another example consider the bivariate (two-dimensional) VAR(2) process

$$y_t = \nu + \begin{bmatrix} .5 & .1 \\ .4 & .5 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0 & 0 \\ .25 & 0 \end{bmatrix} y_{t-2} + u_t. \tag{2.1.15}$$

Its reverse characteristic polynomial is

$$\det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .5 & .1 \\ .4 & .5 \end{bmatrix} z - \begin{bmatrix} 0 & 0 \\ .25 & 0 \end{bmatrix} z^2 \right) = 1 - z + .21z^2 - .025z^3.$$

The roots of this polynomial are

$$z_1 = 1.3, \qquad z_2 = 3.55 + 4.26i, \qquad \text{and} \qquad z_3 = 3.55 - 4.26i.$$

Here i denotes the imaginary unit ($i^2 = -1$). Note that the modulus of z_2 and z_3 is $|z_2| = |z_3| = \sqrt{3.55^2 + 4.26^2} = 5.545$. Thus, the process (2.1.15) satisfies the stability condition (2.1.12) because all roots are outside the unit circle. Although the roots for higher dimensional and higher order processes are often difficult to compute by hand, efficient computer programs exist that do the job.

To understand the implications of the stability assumption, it may be helpful to visualize time series generated by stable processes and contrast them with realizations from unstable VAR processes. In Figure 2.1 three pairs