and

$$\widehat{\lambda}_{sk} := \widehat{\lambda}_s + \widehat{\lambda}_k \xrightarrow{d} \chi^2(2K). \tag{4.5.18}$$

Thus, all three statistics may be used for testing nonnormality.

As we have seen, the results hold for any matrix satisfying $\widehat{P}\widehat{P}' = \widehat{\Sigma}_u$. For example, $\widehat{P}$ may be a lower triangular matrix with positive diagonal obtained by a Choleski decomposition of $\widehat{\Sigma}_u$. Clearly, in this case $\widehat{P}$ is a consistent estimator of the corresponding matrix P (see Proposition 3.6). Doornik & Hansen (1994) point out that with this choice the test results will depend on the ordering of the variables. Therefore they suggest using a matrix based on the square root of the correlation matrix corresponding to $\widehat{\Sigma}_u$ instead. In any case, the matrix $\widehat{P}$ is not unique and, hence, the tests will depend to some extent on its choice. Strictly speaking, if one particular $\widehat{P}$ is found for which the null hypothesis can be rejected, this result provides evidence against the normality of the process. Thus, different $\widehat{P}$ matrices could be applied in principle.

For illustrative purposes we consider our standard investment/income/consumption example from Section 3.2.3. Using the least squares residuals from the VAR(2) model with intercepts and a Choleski decomposition of $\widehat{\Sigma}_u$ yields

$$\widehat{\lambda}_s = 4.26 \quad \text{and} \quad \widehat{\lambda}_k = 17.70$$

which may be compared to $\chi^2(3)_{.95} = 7.81$, the critical value of an asymptotic 5% level test. Also

$$\widehat{\lambda}_{sk} = 21.96 > \chi^2(6)_{.95} = 12.59.$$

Thus, based on these asymptotic tests we reject the null hypothesis of a Gaussian data generation process.

It was pointed out by Kilian & Demiroglu (2000) that the small sample distributions of the test statistics may differ substantially from their asymptotic approximations. Thus, the tests may not be very reliable in practice. Kilian & Demiroglu (2000) proposed bootstrap versions to alleviate the problem.

4.6 Tests for Structural Change

Time invariance or stationarity of the data generation process is an important condition that was used in deriving the properties of estimators and in computing forecasts and forecast intervals. Recall that stationarity is a property that ensures constant means, variances, and autocovariances of the process through time. As we have seen in the investment/income/consumption example, economic time series often have characteristics that do not conform with the assumption of stationarity of the underlying data generation process. For instance, economic time series often have trends or pronounced seasonal components and time varying variances. While these components can sometimes