[Hint: Note that

$$\begin{aligned}(y_{t-1} - \overline{y}) \otimes (u_t - \overline{u}) \otimes (u_t - \overline{u}) &= (y_{t-1} - \mu) \otimes u_t \otimes u_t \\ &\quad -(y_{t-1} - \mu) \otimes u_t \otimes \overline{u} + \cdots ,\end{aligned}$$

define new variables of the type

$$z_t = (y_{t-1} - \mu) \otimes u_t \otimes u_t$$

and use that

$$\operatorname{plim} \frac{1}{T} \sum_t z_t = E(z_t) = 0.\Big]$$

Problem 4.5
Using the notation and assumptions from Proposition 4.1, show that

$$\frac{\partial \ln l(\widetilde{\boldsymbol{\beta}}_r)}{\partial \boldsymbol{\beta}'} \frac{\partial^2 \ln l(\widetilde{\boldsymbol{\beta}}_r)}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'} \frac{\partial \ln l(\widetilde{\boldsymbol{\beta}}_r)}{\partial \boldsymbol{\beta}} = (\widetilde{\boldsymbol{\beta}}_r - \widetilde{\boldsymbol{\beta}})'(ZZ' \otimes (\widetilde{\Sigma}_u^r)^{-1})(\widetilde{\boldsymbol{\beta}}_r - \widetilde{\boldsymbol{\beta}}).$$

4.7.2 Numerical Problems

The following problems require the use of a computer. They refer to the bivariate series $y_t = (y_{1t}, y_{2t})'$ of first differences of the U.S. investment data in File E2, available from the website www.jmulti.de.

Problem 4.6
Set up a sequence of tests for the correct VAR order of the data generating process using a maximum order of $M = 4$. Compute the required χ^2 and F likelihood ratio statistics. Which order would you choose?

Problem 4.7
Determine VAR order estimates on the basis of the four criteria FPE, AIC, HQ, and SC. Use a maximum VAR order of $M = 4$ in a first estimation round and $M = 8$ in a second estimation round. Compare the results.

Problem 4.8
Compute the residual autocorrelations $\widehat{R}_1, \ldots, \widehat{R}_{12}$ and estimate their standard errors using the VAR(1) model obtained in Problem 3.12. Interpret your results.

Problem 4.9
Compute LM test values $\lambda_{LM}(1)$, $\lambda_{LM}(2)$, and $\lambda_{LM}(4)$ and portmanteau test values Q_h and $\bar{Q}_h$ for $h = 10$ and 12 for the VAR(1) model of the previous problem. Test the whiteness of the residuals.

Problem 4.10
On the basis of a VAR(1) model, perform a test for nonnormality of the example data.