$$\mathbf{\Gamma}(z) := I_K - \sum_{i=1}^{p-1} \mathbf{\Gamma}_i z^i,$$

$$B_*(z) := Q[\mathbf{\Gamma}(z)\bar{\beta}(1-z) - \alpha z : \mathbf{\Gamma}(z)\beta_\perp],$$

$$B(z) = I_K - \sum_{i=1}^{p} B_i z^i := Q^{-1} B_*(z) Q, \tag{6.3.13}$$

and

$$\mathbf{\Theta}(z) := B(z)^{-1} = \sum_{j=0}^{\infty} \mathbf{\Theta}_j z^j.$$

Notice that $B(0) = Q^{-1}B_*(0)Q = [\bar{\beta} : \beta_\perp]Q = I_K$. Hence, $B(z)$ has the representation $I_K - \sum_{i=1}^{p} B_i z^i$ stated in (6.3.13). Moreover, the matrix operator $\mathbf{\Theta}(z)$ can be decomposed as

$$\mathbf{\Theta}(z) = \mathbf{\Theta}(1) + (1-z)\mathbf{\Theta}^*(z),$$

where expressions for the $\mathbf{\Theta}_j^*$'s can be found by comparing coefficients in $\mathbf{\Theta}(z) = \sum_{j=0}^{\infty} \mathbf{\Theta}_j z^j$ and

$$\begin{aligned}
\mathbf{\Theta}(1) + (1-z)\mathbf{\Theta}^*(z) &= \mathbf{\Theta}(1) + \sum_{j=0}^{\infty} \mathbf{\Theta}_j^* z^j (1-z) \\
&= (\mathbf{\Theta}(1) + \mathbf{\Theta}_0^*) + \sum_{j=1}^{\infty} (\mathbf{\Theta}_j^* - \mathbf{\Theta}_{j-1}^*) z^j.
\end{aligned}$$

Hence,

$$\mathbf{\Theta}_0 = \mathbf{\Theta}(1) + \mathbf{\Theta}_0^*$$

and

$$\mathbf{\Theta}_i = \mathbf{\Theta}_i^* - \mathbf{\Theta}_{i-1}^*, \quad i = 1, 2, \ldots.$$

Using the last expression, we get by successive substitution,

$$\begin{aligned}
\mathbf{\Theta}_i^* &= \mathbf{\Theta}_i + \mathbf{\Theta}_{i-1}^* = \sum_{j=1}^{i} \mathbf{\Theta}_{i-j} + \mathbf{\Theta}_0^* \\
&= \sum_{j=1}^{i} \mathbf{\Theta}_{i-j} + \mathbf{\Theta}_0 - \mathbf{\Theta}(1) = -\sum_{j=i+1}^{\infty} \mathbf{\Theta}_j, \quad i = 1, 2, \ldots.
\end{aligned} \tag{6.3.14}$$

From these quantities the operator $\mathbf{\Xi}^*(z)$ in (6.3.11) can be obtained as

$$\mathbf{\Xi}^*(z) = [\beta_\perp \bar{\beta}_\perp' \mathbf{\Theta}^*(z) + \bar{\beta}\beta' B(z)^{-1}] \tag{6.3.15}$$

(see the proof of Proposition 6.1). The representation (6.3.11) will turn out to be useful, for example, in Chapter 9, where structural VECMs are discussed. The coefficient matrices $\mathbf{\Xi}_j^*$ of the operator $\mathbf{\Xi}^*(z)$ will then play an important role as specific impulse response coefficients. ■