Notice that these estimators are, of course, identical to those based on the levels VAR model (7.5.1) because we have just reparameterized the model. Hence, the following proposition from Dolado & Lütkepohl (1996, Theorem 1) is obtained.

Proposition 7.8 (*Asymptotic Distribution of the Wald Statistic*)
Let y_t be a K-dimensional $I(1)$ process generated by the VAR(p) process in (7.5.1) and denote the LS estimator of A_i by $\widehat{A}_i$ ($i = 1, \dots, p$). Moreover, let $\boldsymbol{\alpha}_{(-i)}$ be a $K^2(p-1)$-dimensional vector obtained by deleting A_i from $[A_1, \dots, A_p]$ and vectorizing the remaining matrix. Analogously, let $\widehat{\boldsymbol{\alpha}}_{(-i)}$ be a $K^2(p-1)$-dimensional vector obtained by deleting $\widehat{A}_i$ from $[\widehat{A}_1, \dots, \widehat{A}_p]$ and vectorizing the remainder. Then

$$\sqrt{T}(\widehat{\boldsymbol{\alpha}}_{(-i)} - \boldsymbol{\alpha}_{(-i)}) \xrightarrow{d} \mathcal{N}(0, \Sigma_{\boldsymbol{\alpha}_{(-i)}}), \tag{7.6.7}$$

where the $(K^2(p-1) \times K^2(p-1))$ covariance matrix $\Sigma_{\boldsymbol{\alpha}_{(-i)}}$ is nonsingular and the Wald statistic λ_W for testing $H_0 : C\boldsymbol{\alpha}_{(-i)} = 0$ has a limiting $\chi^2(N)$-distribution, that is,

$$\lambda_W = T\widehat{\boldsymbol{\alpha}}'_{(-i)}C'(C\widehat{\Sigma}_{\boldsymbol{\alpha}_{(-i)}}C')^{-1}C\widehat{\boldsymbol{\alpha}}_{(-i)} \xrightarrow{d} \chi^2(N)$$

under H_0. Here C is an $(N \times K^2(p-1))$ matrix with $\text{rk}(C) = N$ and $\widehat{\Sigma}_{\boldsymbol{\alpha}_{(-i)}}$ is a consistent estimator of $\Sigma_{\boldsymbol{\alpha}_{(-i)}}$. ■

Note that

$$\Sigma_{\boldsymbol{\alpha}_{(-i)}} = \text{plim } T(X_{(-i)}X'_{(-i)})^{11} \otimes \Sigma_u,$$

where $X_{(-i)} = [X_0^{(-i)}, \dots, X_{T-1}^{(-i)}]$ with

$$X_{t-1}^{(-i)} = \begin{bmatrix} \Delta_{i-1}y_{t-1} \\ \vdots \\ \Delta_{i-p}y_{t-p} \\ y_{t-i} \end{bmatrix} \quad (Kp \times 1)$$

and $(X_{(-i)}X'_{(-i)})^{11}$ denotes the upper left-hand $(K(p-1) \times K(p-1))$ dimensional submatrix of $(X_{(-i)}X'_{(-i)})^{-1}$. Thereby a consistent estimator of $\Sigma_{\boldsymbol{\alpha}_{(-i)}}$ is obtained as

$$\widehat{\Sigma}_{\boldsymbol{\alpha}_{(-i)}} = T(X_{(-i)}X'_{(-i)})^{11} \otimes \widehat{\Sigma}_u,$$

where $\widehat{\Sigma}_u$ is the residual covariance matrix obtained from the LS residuals.

Proposition 7.8 shows that, whenever the elements in at least one of the complete coefficient matrices A_i are not restricted under H_0, the Wald statistic has its usual asymptotic χ^2-distribution. In other words, if restrictions are