Remark 2 Although we only give the limiting distributions of the LR statistics under the null hypothesis in the proposition, the asymptotic distributions under local alternatives of the form

$$\mathbf{\Pi} = \alpha\beta' + \frac{1}{T}\alpha_1\beta_1'$$

were also derived (see Johansen (1995) and Saikkonen & Lütkepohl (1999, 2000b)). Here α and β are fixed $(K \times r_0)$ matrices of rank r_0 and α_1 and β_1 are fixed $(K \times (r - r_0))$ matrices of rank $r - r_0$ and such that the matrices $[\alpha : \alpha_1]$ and $[\beta : \beta_1]$ have full column rank r. Thus, in this setup, the matrix $\mathbf{\Pi}$ is assumed to depend on the sample size. Local power studies have been performed to shed light on the power properties of the LR tests when the alternative is true but the corresponding parameter values are close to the region where the null hypothesis holds. ■

Remark 3 Power comparisons between the alternative test versions can help in deciding whether to use trace or maximum eigenvalue tests. Lütkepohl, Saikkonen & Trenkler (2001) performed a detailed small sample and local power comparison of several test versions and concluded that trace and maximum eigenvalue tests have very similar local power in many situations, whereas each test version has its relative advantages in small samples, depending on the criterion for comparison. Thus, neither of the tests is generally preferable in practice. ■

Remark 4 It is also possible to derive the asymptotic properties of the LR tests for other deterministic terms. For example, higher order polynomial trends may be considered. Such terms lead to changes in the null distributions of the test statistics. We do not consider them here because they seem to be of lesser importance from a practical point of view. ■

Remark 5 Seasonal dummy variables are another type of deterministic terms which are of practical importance. They are often used to account for seasonal fluctuations in the variables (see, e.g., the example in Section 7.2.6). If seasonal dummies are added in addition to an unrestricted intercept term, they do not affect the asymptotic distributions of the LR statistics for the cointegration rank. We have considered two models, however, where no unrestricted intercept term was included. The first one was the model of Section 8.2.1 without any deterministic terms at all. As this model is of limited practical use anyway, we do not consider the implications of adding seasonal dummy variables. The other model without an unrestricted intercept term was the one with a nonzero mean discussed in Section 8.2.2. It is of more use in practice and it is therefore of interest to consider the possibility of adding seasonal dummies.

Suppose there are q seasons and the deterministic term is of the form

$$\mu_t = \mu_0 + \sum_{i=1}^{q-1} \delta_i s_{it},$$