where $A_0^* := I_K - \mathsf{A}$. Because W is lower triangular with unit diagonal, the same is true for A. Hence,

$$A_0^* = I_K - \mathsf{A} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ \beta_{21} & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ \beta_{K1} & \beta_{K2} & \dots & \beta_{K,K-1} & 0 \end{bmatrix}$$

is a lower triangular matrix with zero diagonal and, thus, in the representation (2.3.30) of our VAR(p) process, the first equation contains no instantaneous y's on the right-hand side. The second equation may contain y_{1t} and otherwise lagged y's on the right-hand side. More generally, the k-th equation may contain $y_{1t}, \dots, y_{k-1,t}$ and not $y_{kt}, \dots, y_{Kt}$ on the right-hand side. Thus, if (2.3.30) reflects the actual ongoings in the system, y_{st} cannot have an instantaneous impact on y_{kt} for $k < s$. In the econometrics literature such a system is called a *recursive model* (see Theil (1971, Section 9.6)). Herman Wold has advocated these models where the researcher has to specify the instantaneous "causal" ordering of the variables. This type of causality is therefore sometimes referred to as *Wold-causality*. If we trace ε_{it} innovations of size one standard error through the system (2.3.30), we just get the Θ impulse responses. This can be seen by solving the system (2.3.30) for y_t,

$$y_t = (I_K - A_0^*)^{-1} A_1^* y_{t-1} + \dots + (I_K - A_0^*)^{-1} A_p^* y_{t-p} + (I_K - A_0^*)^{-1} \varepsilon_t.$$

Noting that $(I_K - A_0^*)^{-1} = W = PD^{-1}$ shows that the instantaneous effects of one-standard deviation shocks (ε_{it}'s of size one standard deviation) to the system are represented by the elements of $WD = P = \Theta_0$ because the diagonal elements of D are just standard deviations of the components of ε_t. The Θ_i may then be obtained by tracing these effects through the system.

The Θ_i's may provide response functions that are quite different from the Φ_i responses. For the example VAR(1) system (2.3.25) with Σ_u as in (2.1.33) we get

$$\Theta_0 = P = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & .5 & .7 \end{bmatrix},$$

$$\Theta_1 = \Phi_1 P = \begin{bmatrix} .75 & 0 & 0 \\ .15 & .25 & .21 \\ 0 & .35 & .21 \end{bmatrix}, \qquad (2.3.31)$$

$$\Theta_2 = \Phi_2 P = \begin{bmatrix} .375 & 0 & 0 \\ .090 & .130 & .084 \\ .030 & .155 & .105 \end{bmatrix},$$