Here F_T and F are the joint distribution functions of x_T and x, respectively. Almost sure convergence and convergence in probability can be defined for matrices in the same way in terms of convergence of the individual elements. Convergence in quadratic mean and in distribution is easily extended to sequences of random matrices by vectorizing them. In the following proposition, the relationships between the different modes of convergence are given.

Proposition C.1 (*Convergence Properties of Sequences of Random Variables*)
Suppose $\{x_T\}$ is a sequence of K-dimensional random variables. Then the following relations hold:

(1) $x_T \stackrel{a.s.}{\to} x \Rightarrow x_T \stackrel{p}{\to} x \Rightarrow x_T \stackrel{d}{\to} x$.
(2) $x_T \stackrel{q.m.}{\to} x \Rightarrow x_T \stackrel{p}{\to} x \Rightarrow x_T \stackrel{d}{\to} x$.
(3) If x is a fixed, nonstochastic vector, then

$$x_T \stackrel{q.m.}{\to} x \quad \Leftrightarrow \quad [\lim E(x_T) = x \text{ and } \lim E\{(x_T - Ex_T)'(x_T - Ex_T)\} = 0].$$

(4) If x is a fixed, nonstochastic vector, then

$$x_T \stackrel{p}{\to} x \quad \Leftrightarrow \quad x_T \stackrel{d}{\to} x.$$

(5) (Slutsky's Theorem) If $g : \mathbb{R}^K \to \mathbb{R}^m$ is a continuous function, then

$$x_T \stackrel{p}{\to} x \quad \Rightarrow \quad g(x_T) \stackrel{p}{\to} g(x) \quad [\text{plim } g(x_T) = g(\text{plim } x_T)],$$

$$x_T \stackrel{d}{\to} x \quad \Rightarrow \quad g(x_T) \stackrel{d}{\to} g(x),$$

and

$$x_T \stackrel{a.s.}{\to} x \quad \Rightarrow \quad g(x_T) \stackrel{a.s.}{\to} g(x).$$

■

Proposition C.2 (*Properties of Convergence in Probability and in Distribution*)
Suppose $\{x_T\}$ and $\{y_T\}$ are sequences of $(K \times 1)$ random vectors, $\{A_T\}$ is a sequence of $(K \times K)$ random matrices, x is a $(K \times 1)$ random vector, c is a fixed $(K \times 1)$ vector, and A is a fixed $(K \times K)$ matrix.

(1) If plim x_T, plim y_T, and plim A_T exist, then
  (a) $\text{plim}\,(x_T \pm y_T) = \text{plim } x_T \pm \text{plim } y_T$;
  (b) $\text{plim}\,(c'x_T) = c'(\text{plim } x_T)$;
  (c) $\text{plim } x_T'y_T = (\text{plim } x_T)'(\text{plim } y_T)$;
  (d) $\text{plim } A_T x_T = \text{plim}\,(A_T)\text{plim}\,(x_T)$.
(2) If $x_T \stackrel{d}{\to} x$ and $\text{plim}\,(x_T - y_T) = 0$, then $y_T \stackrel{d}{\to} x$.
(3) If $x_T \stackrel{d}{\to} x$ and $\text{plim } y_T = c$, then