Why Governments Should Tax Mobile Capital in the Presence of Unemployment

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Abstract

This paper shows that a small open economy that suffers from involuntary unemployment should levy a positive source-based tax on capital income. A revenue-neutral tax reform that increases the capital tax rate and reduces the labour tax rate will induce firms to substitute labour for capital. Such a tax reform will lower the marginal cost of production, increase output, reduce unemployment, and increase domestic welfare as long as the labour tax rate exceeds the capital tax rate. The result holds even though trade unions might succeed in subsequently increasing the net-of-tax wage rate, if the elasticity of substitution between capital and labour is above a critical value (which is itself below one). Finally, and importantly, independent of the size of the elasticity of substitution, the government can promote wage moderation and reduce unemployment by increasing the personal tax credit of employed workers instead of reducing the labour tax rate.
1. Introduction

Tax policy in countries facing persistently high unemployment rates is in a catch-22 situation. In many European countries trade unions succeed in keeping the wage rates above the market-clearing level, thus causing unemployment. The resulting labour market imperfections are exaggerated by government interventions: high tax rates on labour income and high social insurance contributions, combined with generous unemployment benefits, distort labour supply, increase wage pressure in the wage negotiations between trade unions and firms and, consequently, increase unemployment. Reducing the share of the tax burden borne by labour in order to counter unemployment is therefore commonly suggested (cf. e.g. Lockwood and Manning 1993, OECD 1995, Pissarides 1998, Nickell and Layard 1999). Governments, however, find it relatively difficult to reduce public spending or, because of the European stability and growth pact, to increase debt. The question therefore arises of how the reduction of the overall tax burden on labour should be financed.

One possibility would be to raise source-based capital taxes. However, increasing economic integration and the removal of economic borders between countries in recent years has led to increased international mobility of capital that has made it more difficult for national tax authorities to tax capital income at source without causing capital flight (cf. Commission of the European Communities 1996). Reducing source-based capital taxes rather than increasing them has become the favoured policy, backed up by the theoretical literature on taxing internationally-mobile capital. This literature argues that abolishing source-based capital taxes is beneficial from the viewpoint of a small open economy. Since capital supply is perfectly elastic, the whole tax burden of a capital tax falls on immobile labour. Thus, to avoid the excess burden of capital taxation it is better to tax labour only (cf. e.g. MacDougall 1960, Diamond and Mirrlees 1971, Gordon 1986, Razin and Sadka 1991, Bucovetsky and Wilson 1991 and Eggert and Hauffer 1999 for a synthesis of the literature). Accepting this result would thus rule out the raising of source-based capital taxes in order to finance cuts in labour taxes.

However, these results on capital income taxation are driven by the assumption that domestic labour markets are sufficiently flexible for the wage rate to adjust to changes in labour demand and supply so that full employment is sustained. This is definitely not true for countries suffering from persistently high unemployment. Given the labour market imperfections in most European countries, it is important to analyse whether the standard result that internationally-mobile capital should not be taxed also applies to those economies.

This paper focuses on the question of whether a policy that increases source-based capital taxes and reduces labour taxes can be an efficient instrument for alleviating unemployment and increasing social welfare. It is complementary in four respects to a recent paper by Koskela and Schöb (2000), who study the optimal tax structure of factor income taxation. Firstly, it abstracts from product market imperfections. Secondly, it focuses on tax reform rather than on tax design. Thirdly, it considers a more general tax system than Koskela and Schöb (2000) as it considers a progressive labour tax system and thus allows for the analysis of different types of tax reforms. Fourthly, it focuses not only on social welfare but also on other policy variables such as unemployment, domestic output and income.
We proceed as follows: after presenting the basic model in Section 2, we first consider a benchmark case where unemployment is caused by a constant net-of-tax wage rate that exceeds the market-clearing wage rate (Section 3). Given net-of-tax factor prices, any shift in the tax burden from labour to capital will lead firms to substitute labour for capital. This will affect the cost of production, and thus output supply and input demands will change. Our analysis shows that, as long as the labour tax rate exceeds the capital tax rate, a marginal revenue-neutral tax reform towards higher capital tax rates is Pareto-improving and will increase domestic output, domestic profits and employment. We then extend the analysis in Sections 4 and 5 by allowing wages to be determined endogenously in a bargaining process between a trade union and a firm that produces with decreasing returns to scale with respect to capital and labour input. The wage negotiations are analysed using a ‘right-to-manage’ model where the trade union and the firm bargain over wages and the firm then unilaterally chooses the profit-maximizing employment level.

The analysis of wage negotiations suggests that if the elasticity of substitution between labour and capital exceeds unity, then the net-of-tax wage rate will fall and the employment effect will be stronger than in the case of a constant net-of-tax wage rate. If the elasticity of substitution is smaller than one, however, the net-of-tax wage rate will increase and the net effect on employment becomes ambiguous. But even in the case of a low substitutability between labour and capital, we show by using numerical calculations that positive employment effects will still occur as long as the elasticity of substitution is not too low. Section 6 demonstrates that, independent of the value of the elasticity of substitution, wage moderation can be promoted if the government increases the employed workers’ personal tax credit instead of lowering the labour tax rate.

Our results suggest that the predominant view in the literature on capital taxation that internationally perfectly mobile capital should not be taxed is valid only when labour markets are sufficiently flexible. With involuntary unemployment due to excessively high wages, however, it may be beneficial for a small open economy to raise a positive tax rate on capital income. This is because in the presence of involuntary unemployment, at least locally, labour supply is also perfectly elastic, which provides no reason to discriminate between labour income and capital income. On the contrary, as the social marginal cost of labour falls short of private marginal cost, labour income should be taxed at a lower rate than capital income. Finally Section 7 relates our findings to the existing literature on factor taxation and offers a brief conclusion.

2. The model

We consider a small open economy with many firms each producing the same good $Y$, which is not consumed domestically but only sold on the world market at given world market prices. The price for good $Y$ is normalized to unity. For analytical convenience, we assume that domestic production can be represented by a single domestic firm that produces good $Y$ with capital $K$ and labour $L$ as inputs. To focus on the effects of tax reforms on the cost side of production, we assume a constant profit share. The production function exhibits decreasing returns to scale,

$$Y = f(L, K)^{1 \over \varepsilon}$$

(1)
where \( f(L,K) \) is linear-homogenous, and \( \varepsilon > 1 \) denotes the degree of decreasing returns to scale. Capital is assumed to be perfectly mobile between countries and labour to be internationally immobile. The firm maximizes profits whereby it considers the factor prices \( \tilde{r} \) and \( \tilde{w} \) as given. The gross interest rate \( \tilde{r} \) is the net-of-tax interest rate plus a source-based capital tax, i.e.
\[
\tilde{r} = (1 + t_r) r
\]
with \( t_r \) denoting the capital tax rate. The gross wage \( \tilde{w} \) is the net-of-tax wage plus the labour tax, i.e.
\[
\tilde{w} = (1 + t_w) w,
\]
with \( t_w \) denoting the labour tax rate. The net-of-tax wage \( w \) is negotiated between a trade union and the firm (see Section 4). Profit is then given by
\[
\pi = Y - C(\tilde{w}, \tilde{r}, Y), \tag{2}
\]
where \( C(\tilde{w}, \tilde{r}, Y) \) denotes the cost function. These profits accrue to a fixed but unspecified third factor of production.

The government requires a fixed amount of tax revenue to finance the public good \( G \) and, in addition, it has to pay unemployment benefits \( b \) to all unemployed workers. Denoting the total number of workers by \( N \), the number of unemployed workers is given by \( N - L \). The government levies the labour tax \( t_w \) on wage income and grants a personal tax credit \( a \) to each employed worker. The revenues from taxing labour are thus given by \((t_w w - a)L\). In addition, the government levies the source-based tax on domestic capital input \( t_r \), so that the government budget constraint is given by
\[
(t_w w - a)L + t_r K = G + b(N - L). \tag{3}
\]
The government is interested in pursuing Pareto-improving tax reforms. Assuming involuntary unemployment implies that all \( N \) workers prefer working, since the wage rate exceeds the disutility of labour. To see this, consider the representative worker’s preferences that can be described by a utility function
\[
U = u(X^i, L^i, G), \quad X^i = wL^i
\]
indicates the worker’s consumption of the private (imported) good \( X \) and \( u_i < 0, \ u_{iL} < 0 \). The term \( u_X > 0 \) indicates the marginal utility of consumption, and the term \(-u_{iX}\) indicates the marginal utility of leisure lost when working. The marginal rate of substitution between leisure and consumption,
\[
-\frac{u_{iL}}{u_X},
\]
is decreasing. While a cleared labour market is characterized by \( u_i w = -u_{iX} \), involuntary unemployment is present in the economy if \( u_i w > -u_{iX} \). This can happen when the net-of-tax wage rate is exogenously fixed at a too high level or trade unions can increase the wage rate above the market clearing level. Throughout the analysis we assume that involuntary unemployment exists. Thus, the group of \( N \) workers is better off at a given net-of-tax wage rate the more workers are employed. Furthermore, increases in profits raise the utility of the firm’s shareholders. A tax reform is thus Pareto-improving if neither the net-of-tax wage, nor employment, nor profits fall.

The resource constraint of an open economy that exports all of its domestic production \( Y \) requires that domestic income \( I \) equals private plus public consumption \( X + G \), plus exports \( Y \), minus imports that consist of total domestic private consumption \( X \) and domestic public consumption \( G \), minus the interest payments to foreign owners for domestically used capital \((rK)^1\), i.e.
\[
I = X + G + Y - (X + G + rK). \tag{4}
\]
It follows from equations (2) and (3) that national income equals net-of-tax labour income plus profits plus the government tax revenue, which is equal to public consumption, so
\[
I = (w + a)L + \pi + b(N - L) + G.
\]

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1 Without loss of generality, we abstract from domestically-owned capital.
3. Labour tax system vs. capital tax system

In a small open economy with free capital mobility, changes in the source-based capital tax rate only affect the gross interest rate \( r \), domestic firms have to pay for capital but leave the world net interest rate \( r \) constant. In this section, we assume that the net-of-tax wage \( w \) is also fixed such that unemployment exists (though we defer the discussion of the reason for this failure of the market to clear). This serves as a benchmark case, as this assumption means that the whole labour tax burden falls on the firm. It will be relaxed in Section 4 when wage negotiations between trade unions and the firm are incorporated into the analysis.

When analysing a marginal reform of factor taxation it is important to know what type of tax system is to be reformed. In the following we therefore distinguish between a labour tax system and a capital tax system. The initial tax system is labeled a \textit{labour tax system} if the labour tax rate exceeds the capital tax rate, i.e. \( t_w > t_r \), and a \textit{capital tax system} if the capital tax rate exceeds the labour tax rate, i.e. \( t_w < t_r \).

3.1 A marginal revenue-neutral tax reform

Using this model with unemployment, we analyse the employment and output effects of a marginal tax reform that leaves the public expenditures for the public good \( G \) unaffected, \( dG = 0 \), while increasing the capital tax rate and lowering the labour tax rate accordingly. In what follows we refer to such a tax reform as a revenue-neutral increase in the capital tax rate.

To interpret the results, it is appropriate to split this tax reform analytically into two separate steps. First, we consider a reform that keeps output constant, i.e. \( dY = 0 \). This implies a movement along the isoquant that guarantees an increase in labour input while leaving marginal cost constant. If such a reform generates excess tax revenues \( dG > 0 \), the surplus in tax revenues will be rebated in a second step by reducing the two tax rates equiproportionately so that \( dG = 0 \) is satisfied. An equiproportional tax rate cut reduces marginal cost and increases both output and factor demands. Hence, the whole tax reform will unambiguously increase employment while the effect on capital is \textit{a priori} ambiguous.

To determine the output-neutral tax reform, we have to differentiate the production function with respect to the tax rates \( t_w \) and \( t_r \):

\[
dY = df(L,K) = 0 = \left[ \frac{f_L(K,L)L_w w + f_K(K,L)K_w w}{dt_w} + \frac{f_L(K,L)L_r r + f_K(K,L)K_r r}{dt_r} \right] dt_w + \frac{f_L(K,L)L_r r + f_K(K,L)K_r r}{dt_r} dt_r ,
\]

where subscripts on \( f, L, \) and \( K \) represent partial derivatives. Solving for \( dt_w \) yields the condition for the output-neutral tax reform (see Appendix 1 and 2):

\[
\left. \frac{dt_w}{dt_r} \right|_{dt_r=0} = -\frac{(1-s)(1+t_w)}{s(1+t_r)},
\]

(4)

where \( s \equiv \bar{w}L/C \) denotes the cost share of labour, \( (1-s) \equiv 1 - \bar{w}L/C = \bar{r}K/C \) is the cost share of capital, and \( C \) is the total cost of production. The impact such an output-neutral tax reform has on the government budget is given by:

\[
dG = \left[ \frac{wL + [t_w w + (b-a)]L_w w + t_r K_w w}{dt_w} + \frac{rK + [t_w w + (b-a)]L_r r + t_r K_r r}{dt_r} \right] dt_r .
\]

(5)

As shown in Appendix 2, substituting the condition (4) into (5) yields
Under the reasonable assumption that the unemployment benefit exceeds the tax credit, i.e. \( b > a \), condition (6) shows that in a labour tax system (where \( t_w > t_r \)), the first step of the tax reform always yields a budget surplus. We have two reasons for this. First, a move towards more equal tax rates on factor incomes reduces the factor price distortion. For a given output level and hence constant total private cost, this implies higher tax revenues. Second, the output-neutral tax reform unambiguously increases employment as labour is substituted for capital. Although more workers become eligible for a tax credit \( a \), public expenditures decrease by \( b - a \) for each additional employee.

Rebating this budget surplus reduces marginal cost and consequently increases output and factor demands. Figure 1 shows a path for consecutive marginal tax reforms. The line through the origin indicates the labour-capital ratio for non-distorted factor prices (where \( t_w = t_r \)). All labour tax systems are located on the right-hand side of this line, because the labour-capital ratio is smaller with a higher factor price ratio \( \tilde{w}/\tilde{r} \). All capital tax systems are to the left of the path through the origin. Point A indicates the equilibrium for an initial labour tax system. Starting from A, both employment and output will increase by a marginal increase in the capital tax rate and a revenue-neutral reduction of the labour tax rate. The same is true as long as we consider a marginal reform of a labour tax system. But even at \( t_w = t_r \), an output-neutral tax reform generates a budget surplus because the positive employment effect reduces unemployment benefit payments more than it increases tax credits. Hence, the output maximum can only be reached with a capital tax system, i.e. a tax system where the capital tax rate exceeds the labour tax rate [cf. equation (6)].

**Figure 1: Consecutive marginal tax reforms**

The maximum output level is indicated by the point C on the left-hand side of the \( t_w = t_r \) line. A further increase in \( t_r \) will result in output reductions, and this negative output effect will
countervail the substitution effect of moving along the isoquant (due to a budget deficit resulting from an output-neutral tax reform). In Figure 1, we consider the case where the fall in output is small and the substitution effect dominates the output effect. A movement from C to B then increases employment but reduces output. The point B indicates a tax system that, for a given level of the public good $G$, yields the same output as the existing tax system A but generates higher employment. The following proposition summarizes:

PROPOSITION 1: In a small open economy with a labour tax system and involuntary unemployment, a marginal revenue-neutral increase in the capital tax rate that leaves the net-of-tax wage rate unaffected will increase both output and employment.

With respect to output we have shown that, as $dG > 0$ for $t_w = t_r$ and $b > a$, the following applies:

PROPOSITION 2: In a small open economy with involuntary unemployment due to a fixed net-of-tax wage rate that is too high, the output maximizing tax system is a capital tax system.

Note that the presence of factor taxes in existing tax systems means that the government does not apply unrestricted profit taxes, because in our framework, the maximum tax revenues in this market with fixed net-of-tax factor prices are equal to the rent of the unspecified fixed third factor if no factor taxes are levied. A 100% profit tax could thus extract all tax revenues, and it would be beneficial to tax profits only and to abandon taxes on factor incomes altogether. For several reasons, however, it may not be possible for governments to tax away profits completely (see Huizinga and Nielsen 1997). Therefore, to analyze tax reforms, we abstract from taxes on profits and on the interaction between factor taxation and profit taxation. It should be mentioned, however, that even with zero tax rates on both labour and capital income, a tax reform as described above would increase both employment and output.

Throughout the analysis we have assumed that unemployment still prevails at point C in Figure 1. If full-employment (which is characterized by equality of the net-of-tax wage rate and the marginal willingness to sell labour) is reached at a level below point C, it can be shown that a further increase in the capital tax rate would result in a reduction in output, capital demand, and profits, without having a positive effect on employment.

3.2 Domestic income and Pareto-improvement

Domestic income consists of labour income, unemployment benefits, profits that accrue to domestic shareholders, and tax revenue for public good provision. If a tax reform increases employment, which is the case when moving from A to C in Figure 1, the sum of net-of-tax labour income and unemployment benefit payments is increasing with employment as the net-of-tax wage rate remains constant. Furthermore, as long as output increases, domestic profits

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2 If the cost share becomes very large, however, it might be that in some interval between C and B, both output and employment fall simultaneously.

3 For a thorough analysis of the interaction of profit taxes and factor income taxes in an optimal taxation framework see Koskela and Schöb (2000).

4 A complete set of results for this case is available on request.
will also rise and domestic shareholders will share in the gain. As the production function (1) ensures a constant profit share, \( \pi/Y = 1/\varepsilon \), profits are increasing in \( Y \). Hence, increasing the capital tax rate in a labour tax system and lowering the labour tax rate accordingly always increases both components of domestic income. Note that domestic capital owners always obtain \( r \), regardless of whether they invest in the home country or abroad.

A movement from a labour tax system towards a capital tax system increases employment and all new workers are strictly better off, as the net-of-tax wage rate workers receive exceeds their marginal willingness to sell labour. Since profits are increasing in output, both producer and worker surplus are boosted as long as the tax reform increases both employment and output. This leads to

**PROPOSITION 3:** In a small open economy with a labour tax system and involuntary unemployment, a marginal revenue-neutral increase in the capital tax rate that leaves the net-of-tax wage rate unaffected is Pareto-improving.

The standard literature on capital income taxation considers the optimal capital tax to be zero for an economy with labour market clearing. Yet Proposition 3 applies for the case where a revenue-neutral increase in the capital tax rate starts from a point where the capital tax rate is zero and the labour tax rate is positive. Thus, in the case of involuntary unemployment, it is beneficial to have a positive capital tax rate.5

### 4. Trade unions and wage bargaining

Thus far we have assumed that the net-of-tax wage rate is not affected by changes in the structure of factor taxation. It is now time to relax this assumption and consider the case where the wage level is determined in wage negotiations between a small trade union and the firm, to see how the results derived in Section 3 have to be modified.

#### 4.1 Wage negotiations between trade union and firm

We consider a small trade union that acts at the level of the individual firm. The objective of the trade union is to maximize its \( N \) members’ net-of-tax income.6 Each member supplies one unit of labour if employed, or zero labour if unemployed. The net-of-tax income of a working member depends on the net-of-tax wage rate \( w \) and the personal tax credit \( a \), so net-of-tax income is given by \( w + a \). If a trade union member becomes unemployed, she is entitled to unemployment benefits \( b \). The objective function of the trade union can then be written as

\[
V^* = (w + a)L + b(N - L).
\]

---

5 It may be worth mentioning that a tax system that maximizes the sum of producer and worker surplus does not maximize output. A marginal revenue-neutral tax reform that increases the capital tax rate at the output maximum leaves profit unaffected but raises employment. Thus the worker surplus increases while profits do not decrease. Although going beyond the output maximum is not Pareto-improving in the strict sense, it is welfare improving as long as workers could compensate share owners without being made worse off (cf. Koskela and Schöb 2000).

6 Insofar as small-scale wage negotiations do not affect the consumer price level, it does not matter whether the trade union maximizes nominal or real income of its members.
The firm maximizes profits, which are defined by equation (2). The wage rate is determined in a bargaining process between the trade union and the firm subject to the condition that the firm unilaterally determines employment. This is modeled by asymmetric Nash bargaining.\footnote{This approach can be justified either axiomatically (cf. Nash 1950), or strategically (cf. Binmore, Rubinstein and Wolinsky 1986). It reflects the observation that in most European countries, over three-quarters of the workforce earn wages that are covered by collective bargaining in which trade unions and employer organisations agree upon wages only (cf. Layard and Nickell and Jackman 1991, Oswald 1993 and Nickell and Layard 1999).} The fall-back position of the trade union is given by
\[ V_b = 0, \]
and if the negotiations break down, all members receive their reservation wage equal to the unemployment benefit payments. The fall-back position of the firm is given by zero profits, i.e. \( \pi^0 = 0 \). Hence, using \( V = V^* - V^0 \), the Nash bargaining maximand can be written as
\[
\Omega = V^\beta \pi^{1-\beta},
\]
with \( \beta \) representing the bargaining power of the trade union. The first-order condition with respect to the net-of-tax wage rate is
\[
\Omega_w = 0 \iff \beta \frac{V_w}{V} + (1-\beta) \frac{\pi_w}{\pi} = 0, \tag{8}
\]
where the subscript \( w \) represents a partial derivative with respect to \( w \). In the following, we assume a CES production technology. This allows us to use an explicit formulation of the wage elasticity of labour demand \( \eta_{L,w} \equiv L_w \tilde{w} / L, \) which is useful in understanding the comparative statics. The wage elasticity can be written as [see Appendix 1]
\[
\eta_{L,w} = -\sigma + s(\sigma - \varepsilon), \tag{9}
\]
where \( s \equiv \tilde{w}L / C \) denotes the cost share of labour, and \( \sigma \) is the elasticity of substitution between labour and capital. The output effect is represented by \( \varepsilon \), the returns-to-scale parameter. The higher the returns to scale, the more elastic is labour demand. By looking at the cross-price elasticities, we can infer that if \( \sigma > \varepsilon \), factors are price substitutes and they are price complements if the reverse is true. In the following, we focus on the case where labour and capital are price complements (\( \sigma < \varepsilon \)). By using equation (9), equation (8) can be rewritten as
\[
\Omega_w = 0 \iff (w + a - b)(\beta \eta_{L,w} + (1-\beta)s(1-\varepsilon)) + w\beta = 0. \tag{10}
\]
The second-order condition is assumed to hold throughout, i.e. \( \Omega_{www} = y + xz < 0 \), with
\[ y = \beta(1 + \eta_{L,w}) + (1-\beta)(1-\varepsilon)s, \quad z = [\beta(\sigma - \varepsilon) + (1-\beta)(1-\varepsilon)]s_z (1 + t_w), \] and \( x = w + a - b \). Equation (10) defines the negotiated net-of-tax wage from Nash bargaining as a function of the tax policy parameters \( a, b, t_w \), and \( t_r \). Therefore, we have \( w = w(a, b, t_w, t_r) \).

### 4.2 Comparative statics

In the following, we study how changes in factor taxes affect the negotiated wage rate. The impact of the labour tax rate on the net-of-tax wage rate can be derived by total differentiation of equation (10)
\[
w_{t_w} = -(y + xz)^{-1} x [\beta(\sigma - \varepsilon) + (1-\beta)(1-\varepsilon)]s_z. \tag{11}
\]
Assuming \( f(L,K) \) to have a constant elasticity of substitution \( \sigma \) between labour and capital\(^8\), the partial derivative of the cost share of labour with respect to the labour tax rate is given by

\[
 s_{t_w} = s_w = \frac{s}{(1+t_w)(1-\sigma)} \begin{cases} > & \sigma > 1 \\ < & \sigma < 1 \\ = & \sigma = 1 \end{cases} \quad 0 \Leftrightarrow \sigma \begin{cases} > & < \\ < & > \\ = & = \end{cases} 1. \quad (12)
\]

Given a constant elasticity of substitution \( \sigma \), the partial derivative of the wage elasticity of labour demand for \( \sigma < \varepsilon \) is given by

\[
 \frac{\partial \eta_{L,w}}{\partial t_w} = s_{t_w} (\sigma - \varepsilon) \begin{cases} > & \sigma > 1 \\ = & \sigma = 1 \\ < & \sigma < 1 \end{cases} \quad 0 \Leftrightarrow \sigma \begin{cases} > & < \\ < & > \\ = & = \end{cases} 1. \quad (13)
\]

If substitutability is low, i.e. \( \sigma < 1 \), the cost share of labour \( s \) increases with the labour tax rate. A larger share \( s \) implies that a one percent change in the wage rate induces a larger increase in total cost and hence a lower output. Hence, if \( s \) increases, labour demand becomes more elastic. This weakens the relative bargaining position of the trade union as it increases the potential losses of a wage increase. Substituting (12) into (11) shows that for \( \sigma < \varepsilon \), we have

\[
 s_{t_w} \begin{cases} < 0 & \sigma < 1 \\ = 0 & \sigma = 1 \\ > 0 & \sigma > 1 \end{cases} \quad (14)
\]

When the elasticity of substitution is less than one, a rise in the labour tax rate increases the wage elasticity of labour demand (cf. equation (13)) and thereby weakens the ability of the trade union to extract rent in the wage negotiations (the net-of-tax wage rate falls in equation (14)). In the case of unit elasticity of substitution, the wage elasticity remains unchanged, and the labour tax rate has no effect on the net-of-tax wage rate.\(^9\)

Comparative statics for a capital tax rate change are opposite in sign. The trade union’s ability to extract rent again depends on how a change in the capital tax affects the wage elasticity of labour demand. If the elasticity of substitution is constant, the labour demand elasticity changes only if the cost share of labour changes. Using the following condition

\[
 s_{t_r} = -\frac{(1+t_r)}{(1+\sigma)} s_{t_r},
\]

it follows immediately that an exogenous increase in the capital tax rate has an effect on the cost share of labour opposite to that of the increase in the labour tax rate. Depending on the elasticity of substitution, we can summarize the total effect of an increase in \( t_r \) as:

\[
 w_{t_r} \begin{cases} > 0 & \sigma < 1 \\ = 0 & \sigma = 1 \\ < 0 & \sigma > 1 \end{cases} \quad (15)
\]

\(^8\) It is convenient to assume a CES production function in order to simplify calculations for marginal changes of tax rates. As we do not study non-marginal tax reforms, the elasticity of substitution need not be globally constant.

\(^9\) If the factors were substitutes, the effects would work into the opposite direction and it would not be possible to \textit{a priori} sign the effect of a labour tax rate increase on the wage negotiations.
5. Substituting the capital tax for the labour tax with endogenous wage

The comparative statics results have demonstrated that, with the exception of the case of a Cobb-Douglas production function with respect to capital and labour, i.e. \( \sigma = 1 \), it is necessary to take account of the effects of tax rate changes on the negotiated wage rate to determine the employment effect of a revenue-neutral tax reform. The condition for a revenue-neutral change in the structure of factor taxation is given by

\[
dG = G_t \, dt_w + G_t \, dt_r = 0, \tag{16}
\]

where the effects of tax rate changes on the net-of-tax wage rate have already been taken into account. Define the tax revenue elasticity with respect to the tax rate \( \tau \) as

\[
\tau = \frac{\tau}{\tau} \frac{dG}{G} = \frac{dt_w}{(1 + t_w)} + \frac{dt_r}{(1 + t_r)}.
\]

The change in employment is given by

\[
dL = \frac{L}{(1 + t_w)} \eta_{L, \omega}(1 + \omega_{L, \omega}) dt_w + \frac{L}{(1 + t_r)} \left[ \eta_{L, \omega} \omega_{L, \omega} + \eta_{L, \omega} \right] dt_r, \tag{18}
\]

where \( \eta_{L, \omega} = \frac{L \tilde{r}}{L} \) denotes the interest rate elasticity of labour demand and \( \omega_{L, \omega} = \frac{w_{L, \omega} \cdot (1 + t_w)}{w} \) and \( \omega_{L, \omega} = \frac{w_{L, \omega} \cdot (1 + t_r)}{w} \) describe the net-of-tax wage elasticities with respect to \( t_w \) and \( t_r \), respectively. Substituting the condition (17) into (18) and rearranging yields the following condition for the change in employment:

\[
\frac{dL}{dt_r} \bigg{|}_{\omega = 0} \begin{cases} > & \iff \tau \geq \tau \geq \frac{\eta_{L, \omega} \omega_{L, \omega} + \eta_{L, \omega}} {\eta_{L, \omega} (1 + \omega_{L, \omega})} \end{cases}. \tag{19}
\]

If a tax reform increases the gross capital price \( \tilde{r} \) by one percent, the ratio of the left-hand side indicates the percentage by which the gross wage \( \tilde{w} \) has to decrease because of a cut in the labour tax rate in order to keep the public good provision \( G \) constant. The ratio of the right-hand side denotes the percentage the gross wage has to decline to keep the employment level constant. If the revenue-neutrality requirement allows the government to cut the labour tax rate by a larger amount than is necessary to sustain the employment level, wage negotiations lead to lower wages and increase employment accordingly. Three different cases can be distinguished depending on the reaction of the net-of-tax wage rate.

Cobb-Douglas production technology

For the case of a Cobb-Douglas production technology with \( f(L,K) = AL^\alpha K^{1-\alpha} \), it can be seen from conditions (13) and (15) that wage negotiations are unaffected by changes in the factor tax rates. Thus, equation (16) is the same as equation (5), and the analysis of Section 3 can be applied even when the wage rate is negotiated between the trade union and the firm: a revenue-neutral increase in the capital tax rate leaves the net-of-tax wage rate unaffected and increases output and employment if the labour tax rate exceeds the capital tax rate. Such a reform is Pareto-improving, because unemployed workers and the shareholders benefit while no incumbent worker is worse off.
The elasticity of substitution between labour and capital exceeds unity

If the elasticity of substitution exceeds unity, the net-of-tax wage elasticity with respect to $t_w$, is positive, $\omega_{w,t} > 0$, so the net-of-tax wage rate is reduced by a cut in the labour tax rate. This effect increases labour demand. As a fall in the net-of-tax wage rate also increases tax revenues via higher employment, i.e. $G_w < 0$, and therefore allows for a larger cut of labour taxes, the total employment effect is larger than in the case of a constant net-of-tax wage rate.\(^{10}\) Formally, Appendix 3 shows that the left-hand side of the right part of condition (19) is increasing in $\omega_{w,t}$:

$$\frac{\partial}{\partial \omega_{w,t}} \tau_t > 0.$$  

Applying the symmetry condition, $\omega_{t,w} = -\omega_{w,t}$ (see Appendix 3), and differentiating shows that the right-hand side of the right part of (19) is decreasing in $\omega_{w,t}$:

$$-\frac{\partial}{\partial \omega_{w,t}} \frac{\eta_{L,w} \omega_{t,w} + \eta_{L,t}}{\eta_{L,w}(1 + \omega_{w,t})} = -\frac{\eta_{L,w} + \eta_{L,t}}{\eta_{L,w}(1 + \omega_{w,t})^2} < 0.$$  

If employment is increasing when the net-of-tax wage rate is unaffected, these two facts establish that employment is also boosted when the negotiated wage falls due to the revenue-neutral tax reform. This can be summarized in

**PROPOSITION 4:** If the net-of tax wage rate is determined by asymmetric Nash bargaining between a firm and a trade union, and the elasticity of substitution between labour and capital is equal to or larger than unity, then starting in a labour tax system, a revenue-neutral increase in the capital tax rate is Pareto-improving and will increase both output and employment.

The elasticity of substitution between labour and capital is less than unity

If substitutability of factors is less than unity with this tax reform, then the trade union will succeed in increasing the net-of-tax wage rate. Furthermore, the rise in the net-of-tax wage rate reduces tax revenues and, therefore, allows for smaller tax rate cuts.\(^{11}\) Both effects reduce employment, so the total effect on employment becomes ambiguous.

Clearly, if the elasticity of substitution is sufficiently close to unity, the employment effect will still be positive. The argument goes as follows. Starting in a labour tax system, a revenue-neutral increase of the capital tax rate increases employment when $\sigma = 1$. Furthermore, it can be shown that the positive employment effect is increasing with the elasticity of substitution at $\sigma = 1$, so the employment effect is still positive for some values of $\sigma < 1$.\(^{12}\) This result can be summarized in

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\(^{10}\) As $\varepsilon > \sigma > 1$, it follows that $\eta_{L,w} \leq -1$. This is a sufficient condition for $G_w < 0$ to hold.

\(^{11}\) A sufficient condition is $\eta_{L,w} \leq -1$, which is guaranteed if $\varepsilon \geq 1/s$.

\(^{12}\) A proof is available on request.
PROPOSITION 5: If the net-of tax wage rate is determined by asymmetric Nash bargaining between a firm and a trade union, and the elasticity of substitution between capital and labour is above a critical value $\sigma^*$, which is itself less than one, then starting from a labour tax system, a revenue-neutral increase in the capital tax rate increases employment.

Table 1: Factor income tax rates for some European countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Capital tax rate %</th>
<th>Labour tax rate %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>7.4</td>
<td>52.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>15.3</td>
<td>52.1</td>
</tr>
<tr>
<td>Finland</td>
<td>10.7</td>
<td>47.7</td>
</tr>
<tr>
<td>France</td>
<td>7.4</td>
<td>46.6</td>
</tr>
<tr>
<td>Germany</td>
<td>10.7</td>
<td>48.1</td>
</tr>
<tr>
<td>Italy</td>
<td>15.3</td>
<td>52.8</td>
</tr>
<tr>
<td>Netherlands</td>
<td>10.7</td>
<td>51.6</td>
</tr>
<tr>
<td>Spain</td>
<td>19.4</td>
<td>46.2</td>
</tr>
<tr>
<td>Sweden</td>
<td>0</td>
<td>49.7</td>
</tr>
<tr>
<td>U.K.</td>
<td>15.3</td>
<td>40.2</td>
</tr>
</tbody>
</table>


Legend: The marginal effective capital tax rate is given by the formula: cost of capital minus real interest rate, divided by the cost of capital (1991). The labour tax rate measures the marginal labour tax (including employers’ and employees’ social insurance contributions) on gross wages for a one-earner couple with two children whose wage equals that of an average productive worker (1992).

For the case of a low substitututability, we provide some numerical results for the worst scenario (with respect to the employment effect) of a monopoly trade union. We show how the sign of the employment effect depends on the initial tax system and the elasticity of substitution. Table 1 provides a comparison of marginal labour tax rates and marginal effective capital tax rates for ten European countries. It turns out that all countries have a labour tax system.

Using the figures presented in Table 1, we focus on the case for Spain with the highest capital tax rate and Sweden with the lowest capital tax rate, and we calculate the critical values of the elasticity of substitution that ensure that the marginal tax reform is revenue-neutral. The bold lines in Figure 2 show the combinations of parameter values for the elasticity of substitution $\sigma<1$ and the initial labour tax $\theta^*_w = t^*_w / (1 + t^*_w) < 1$ where the employment effect is zero. The lower line represents Sweden, and the upper line represents the geometric loci for Spain. The horizontal lines indicate the present labour tax rates in

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13 A proof of why the monopoly trade union case is the worst case scenario in terms of the employment effect is available on request. It results from the notion that for given wage elasticity of labour demand, the range for positive employment effect increases with the relative bargaining power of the firm.
Sweden and Spain, respectively (which are very similar). For both countries, we use the same returns to scale parameter ($\varepsilon = 5$) and cost share for labour ($s = 0.67$).

In the case of Sweden, any elasticity of substitution above $\sigma^* = 0.38$ would guarantee a positive employment effect. In Spain, where the initial capital tax rate is much higher, only an elasticity of substitution above $\sigma^* = 0.75$ would be sufficient to guarantee a positive employment effect. Further calculations indicate that, given the parameter values of the other countries, the critical values of $\sigma^*$ for all examples are in the range of $[0.46; 0.65]$.

To conclude: in the presence of unemployment, the positive employment effects of a tax-revenue-neutral reform that raises the capital tax rate and lowers the labour tax rate seems to hold for relatively low elasticities of substitution between labour and capital.

Figure 2: The employment-neutral labour tax rates for Sweden and Spain

6. Employment can be improved even for low elasticities of substitution

It has been argued in the literature on union bargaining that an increase in progressivity of the tax levied on the members of the trade union moderates the net-of-tax wage, ceteris paribus, and thereby boosts employment.\textsuperscript{14} This paper has not considered taxes levied on the members of the trade union, so one might ask whether the employment-boosting effect of tax progressivity holds in the case of the labour tax levied on firms. If that is the case, then government can promote wage moderation, and thereby employment, even when the elasticity of substitution between labour and capital is too low for the revenue-neutral tax reform to boost employment.

\textsuperscript{14} See e.g. Lockwood and Manning (1993) and Koskela and Vilmunen (1996).
To consider the revenue-neutral increase in tax progressivity, where both the labour tax rate and the employed workers’ tax credit increase, assume that the government requires a fixed amount of tax revenues to finance public good $G$.\(^\text{15}\) Furthermore, abstract from changes in the government budget constraint due to changes in unemployment benefit payments and tax revenues from capital income taxation (because the net effect on $G$ is positive if employment increases and negative if employment falls). These simplifications yield the following budget constraint

$$G = (t_w w - a)L,$$  \hspace{1cm} (20)

where $a$ is an employed worker’s tax credit that can be interpreted as an employment subsidy. The condition for a revenue-neutral change in tax progressivity is given by $dG = 0 = G_a da + G_{t_w} dt_w$. Differentiating (20) with respect to the tax credit $a$ and the labour tax rate $t_w$ and taking account of their direct and indirect effects via the net-of-tax wage and employment, after some manipulations yields

$$G_a = -L(1 - uw),$$  \hspace{1cm} (21)

$$G_{t_w} = wL(1 + t_w)^{-1}\left[1 + u(1 + \omega_{t_w})\right].$$ \hspace{1cm} (22)

where $u = t_w(1 + (1-a/t_w)\eta_{L,w})$. The impact of the tax credit $a$ on the wage rate is

$$w_a = -(y + xz)^{-1}(y - \beta) < 0.$$ \hspace{1cm} (23)

A higher personal tax credit for employed workers leads the trade union to accept a lower net-of-tax wage rate, as the gains for new workers from starting to work increase while the losses for those already employed remain constant.

We are now in the position to derive the total effect of a combined change in $a$ and $t_w$ on the gross wage, for the case when the Laffer curve is upward-sloping (i.e. $G_a < 0$, $G_{t_w} > 0$). The total differential of the gross wage $\tilde{w} = w(1 + t_w)$ with respect to $t_w$ and $a$ can be written as

$$d\tilde{w} = w dt_w + (1 + t_w)w_{t_w} dt_w + (1 + t_w)w_a da = w(1 + \omega_{t_w}) dt_w + (1 + t_w)w_a da.$$ \hspace{1cm} (24)

Substituting the tax-revenue-neutrality condition $da = -G_a^{-1}G_{t_w} dt_w$ for $da$ in equation (24) yields

$$\frac{d\tilde{w}}{dt_w} \bigg|_{G=0} = G_a^{-1}\left[G_a w(1 + \omega_{t_w}) - G_{t_w} (1 + t_w)w_a\right].$$

Now the straightforward substitutions from the equations (22) and (23) yield

$$\left[G_a w(1 + \omega_{t_w}) - G_{t_w} (1 + t_w)w_a\right] = -wL(1 + \omega_{t_w} + w_a) = -wL[y + xz]^{-1}\beta > 0.$$ \hspace{1cm} (25)

As $G_a < 0$, increasing tax progressivity lowers the gross wage and boosts employment regardless of the value of the elasticity of substitution.

**PROPOSITION 6:** If the net-of tax wage rate is determined by asymmetric Nash bargaining between a firm and a trade union, the employment-neutral value of the

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\(^{15}\) We do not consider the change in overall progressivity, but focus on the tax progressivity for workers. That is, we analyse only revenue-neutral changes in the labour tax rate and the tax credit for employed workers.
elasticity of substitution is lower if a revenue-neutral increase in the capital tax rate is combined with an increase in tax progression.

This result is particularly important in the case of $\sigma < 1$, as it implies that the employment-neutral value of the elasticity of substitution, $\sigma^*$, will be lower if the government increases the employment tax credit instead of reducing the labour tax rate.

7. Concluding remarks

In a model where a small open economy suffers from persistently high unemployment due to excessively high wages, the preceding analysis has shown that a revenue-neutral shift in factor taxation that increases capital tax rates and cuts labour tax rates will boost production and alleviate unemployment as long as the initial labour tax rate exceeds the capital tax rate (and the net-of-tax wage rate is not increased by subsequent wage negotiations between a trade union and a firm). If the negotiated net-of-tax wage rate increases as a consequence of a revenue-neutral increase in capital taxation, however, then tax policies to alleviate unemployment are less effective. Nevertheless, shifting the labour tax system towards a capital tax system may boost employment provided that substitutability between labour and capital is not too low. But even if this tax reform fails to boost employment, the government can promote wage moderation by increasing the employed workers’ personal tax exemption instead of reducing the labour tax rate.

The results derived in this paper are complementary to the conclusions usually found in the literature on capital income taxation in open economies. If labour markets clear, the standard result is that capital should be exempted from source-based taxes (cf. e.g. MacDougall 1961, Gordon 1986, Razin and Sadka 1991, Bucovetsky and Wilson 1991, Eggert and Haufler 1999). Sometimes capital should even be subsidized (cf. Gordon and Bovenberg 1996). In economies with involuntary unemployment due to excessively high wages, our analysis suggests that the capital tax should be positive and should not be lower than the labour tax rate.

From a purely theoretical perspective, one might be inclined to argue that this result holds only for the extreme case when both labour and capital supply are perfectly elastic. Although this argument is correct, it neglects the fundamental fact that involuntary unemployment implies that labour supply is – at least locally – infinitely elastic. Hence, in the presence of involuntary unemployment, we have no reason to discriminate between labour and capital. When the whole tax burden falls on domestic owners of a country-specific resource (shareholders), factor prices should not be distorted, so labour tax rates and capital tax rate should be equal. The positive capital tax is thus a direct implication of the elasticity rule of optimal taxation: if the government has to apply factor taxes, it should not discriminate between factors having the same supply elasticity. Furthermore, because the marginal social cost of labour falls short of the net-of-tax market price while the marginal social cost of capital for a small open economy is equal to the interest rate at which the economy can borrow capital, it is beneficial to substitute labour for capital further by going beyond equi-proportional factor tax rates.

However, if the elasticity of substitution between labour and capital is low, we have an opposite effect of taxes: ceteris paribus, increasing the labour tax and reducing the capital tax
rate serves as an indirect tool to reduce the labour market distortion, as these tax rate changes increase the labour demand elasticity and hence reduce the possibility of the trade union to extract rents. This result is in line with Richter and Schneider (2001) who showed this effect in a monopoly union model.

If the government could tax profits at 100 percent, then it is not necessary in our framework to tax factors at all (because the maximum tax revenues in a market with fixed net-of-tax factor prices are equal to the rent of some fixed factor when no factor taxes are levied). It would then be optimal to set the capital tax rate to zero. In this case, our analysis confirms recent results that the optimal capital tax rate is zero in the presence of labour market imperfections if profits are fully taxed away. It also confirms the result that labour should be subsidized if the labour market is monopolized, because the social marginal cost of labour falls short of private marginal cost of labour (cf. Boeters and Schneider 1999 and Koskela and Schöb 2000).

When profits are not fully taxed, however, increasing capital tax rates actually increases profits as long as the tax reform starts from a labour tax system. This positive effect on profit can be of great importance for the location decisions of firms, decisions that have not been considered in our framework. Our results would be strengthened if location decisions of firms were taken into account, as increasing the capital tax up to the level of the labour tax rate increases profits and therefore the incentive to move into the country.

Contact information and acknowledgements

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Appendix 1: factor demand elasticities

The function \( f(L,K) \) in (1) is CES:

\[
Y = \left[ \frac{K^{-\sigma} + L^{-\sigma}}{\sigma} \right]^{\frac{\sigma}{\sigma-1}}, \tag{A1}
\]

where \( \sigma \) denotes the elasticity of substitution between labour and capital. Denoting \( \alpha = 1 - 1/\varepsilon \) as the decreasing returns to scale parameter, the first-order conditions are

\[
\tilde{w} = Y_L = \alpha Y^{-\frac{\sigma-1}{\sigma\alpha}} L^{-\frac{1}{\sigma}}, \quad \tilde{r} = Y_K = \alpha Y^{-\frac{\sigma-1}{\sigma\alpha}} K^{-\frac{1}{\sigma}}. \tag{A2}
\]

Substituting (A2) in (A1) yields \( \tilde{w}^{-\frac{\sigma-1}{\sigma}} \tilde{r}^{-\frac{1}{\sigma}} K = L \). Rearranging (A1),

\[
K^{-\frac{\sigma-1}{\sigma}} = Y^{-\frac{\sigma-1}{\sigma\alpha}} L^{-\frac{1}{\sigma}},
\]

and substituting in (A2) yields:
£\frac{\sigma}{(1-\sigma)}R^\sigma \left( \frac{\sigma - 1}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}} \right) = L^{1-\sigma}.

This equation allows us to derive the conditional labour and capital demands:

\begin{equation}
L = \frac{\tilde{\sigma}}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}} \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}} \quad \text{and} \quad K = \tilde{R}^{1-\sigma} \left( \frac{\tilde{\sigma}}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}} \right) \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}}.
\end{equation}

From \( C = \tilde{W}L + \tilde{R}K \) we can then derive the cost function by substituting in (A2) and (A3):

\begin{equation}
C(\tilde{W}, \tilde{R}, Y) = \frac{1}{\sigma} \left[ \frac{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}}{\sigma} \right] \frac{1}{\tilde{Y}^{\frac{1}{\sigma}}} \equiv \frac{1}{\sigma} c(\tilde{W}, \tilde{R}).
\end{equation}

Using Shephard's lemma, we have

\begin{equation}
L = C_{\tilde{W}}(\tilde{W}, \tilde{R}, Y) = \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}}, \quad \text{with}
\end{equation}

\begin{equation}
c_{\tilde{W}}(\tilde{W}, \tilde{R}) = \frac{\tilde{W}}{\sigma} \left[ \frac{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}}{\tilde{Y}^{\frac{1}{\sigma}}} \right].
\end{equation}

The cost share of labour and capital are

\begin{equation}
s \equiv \frac{\tilde{W}L}{\sigma} = \frac{\tilde{W}^{1-\sigma}}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}}, \quad (1-s) \equiv \frac{1-\tilde{W}L}{\sigma} = \frac{\tilde{R}K}{\sigma}.
\end{equation}

Using the cost share of labour, we have from (A2):

\begin{align}
L_{\tilde{W}} &= -\sigma \tilde{W}^{1-\sigma} \left[ \frac{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}}{\tilde{Y}^{\frac{1}{\sigma}}} \right] \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}} + \tilde{W}^{1-\sigma} \cdot \frac{1-\sigma}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}} \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}}
\end{align}

\begin{align}
&+ \tilde{W}^{1-\sigma} \left[ \frac{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}}{\tilde{Y}^{\frac{1}{\sigma}}} \right] \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}} Y_{\tilde{W}} = -\frac{\sigma L}{\tilde{W}} + \frac{\sigma L}{\tilde{W}^{1-\sigma} + \tilde{R}^{1-\sigma}} \frac{1}{\sigma} \tilde{Y}^{\frac{1}{\sigma}} Y_{\tilde{W}} + \frac{1}{\sigma} \tilde{W}^{1-\sigma} Y_{\tilde{W}}.
\end{align}

Substituting in (A5) and the first-order condition for a profit maximum,

\begin{equation}
1 = MC(\tilde{W}, \tilde{R}, Y) = \frac{1}{\sigma} \frac{1}{\tilde{Y}^{\frac{1}{\sigma}}} c(\tilde{W}, \tilde{R}) = \frac{1}{\sigma} \frac{1}{\tilde{Y}^{\frac{1}{\sigma}}} c(\tilde{W}, \tilde{R}),
\end{equation}

we obtain

\begin{equation}
\eta_{L,\tilde{W}} \equiv \frac{L_{\tilde{W}}}{L} = -\sigma(1-s) - s \frac{1}{1-\sigma} = -\sigma(1-s) - s \varepsilon.
\end{equation}

Analogously, we have

\begin{equation}
\eta_{K,\tilde{R}} = -s\sigma - (\varepsilon - s), \quad \eta_{L,\tilde{R}} = (1-s)(\sigma - \varepsilon), \quad \eta_{K,\tilde{W}} = s(\sigma - \varepsilon).
\end{equation}

Appendix 2: Derivation of equations (4) and (6)

Differentiation of the decreasing returns to scale production function with respect to the factor taxes for given output yields

\begin{equation}
dY = 0 = \left[ \frac{Y_L}{L} \frac{L_{\tilde{W}}}{\tilde{W}} \frac{Y_K}{K} \frac{K_{\tilde{W}}}{\tilde{W}} \right] dt_{\tilde{W}} + \left[ \frac{Y_L}{L} \frac{L_{\tilde{R}}}{\tilde{R}} \frac{Y_K}{K} \frac{K_{\tilde{R}}}{\tilde{R}} \right] dt_{\tilde{R}},
\end{equation}

which can be rewritten as

\begin{equation}
dY = 0 = \left[ \frac{L_{\tilde{W}}}{L} \frac{L_{\tilde{R}}}{\tilde{R}} \right] dt_{\tilde{W}} + \left[ \frac{Y_L}{L} \frac{L_{\tilde{R}}}{\tilde{R}} + \frac{Y_K}{K} \frac{K_{\tilde{R}}}{\tilde{R}} \right] dt_{\tilde{R}}.
\end{equation}

Moreover we have
where $\sigma$ is the decreasing returns to scale parameter as defined in Appendix 1. Using the conditional labour and capital demand functions (A3) and the definition of the cost shares (A5), it is straightforward to show that

$$\frac{Y_r L}{Y} = \alpha s \quad \text{and} \quad \frac{Y_r K}{Y} = \alpha (1-s).$$

(A7)

Using the definitions for the factor demand elasticities, (A1) can be written as

$$dY = 0 \Leftrightarrow \left[ s \eta_{L,\bar{w}} + (1-s) \eta_{K,\bar{w}} \right] \frac{dt_w}{1+t_w} + \left[ s \eta_{L,\bar{r}} + (1-s) \eta_{K,\bar{r}} \right] \frac{dt_r}{1+t_r} = 0. \quad \text{(A8)}$$

Substituting the factor demand elasticities into (A8) leads to equation (4) in the text. Applying the factor demand elasticities, equation (5) in the text can be rewritten as:

$$dG = wL \left[ 1 + \frac{t_w}{(1+t_w)} \eta_{L,\bar{w}} + \frac{t_r}{(1+t_r)} \eta_{K,\bar{r}} \right] dt_w + \left[ 1 + \frac{t_w}{(1+t_w)} \eta_{L,\bar{r}} + \frac{t_r}{(1+t_r)} \eta_{K,\bar{r}} \right] dt_r. \quad \text{(A9)}$$

Using the definitions of the cost share of labour and capital (A5) yields

$$dG = wL \left[ 1 + \frac{t_w}{(1+t_w)} \eta_{L,\bar{w}} + \frac{t_r}{(1+t_r)} \eta_{L,\bar{r}} \right] dt_w + \left[ 1 + \frac{t_w}{(1+t_w)} \eta_{K,\bar{w}} + \frac{t_r}{(1+t_r)} \eta_{K,\bar{r}} \right] dt_r. \quad \text{(A10)}$$

Substituting equation (4) in (A10) yields

$$dG \bigg|_{dY=0} \begin{cases} > 0 \Leftrightarrow t_w (\eta_{L,\bar{w}} + \eta_{K,\bar{w}}) > t_r (\eta_{K,\bar{r}} + \eta_{L,\bar{r}}) \quad \text{if} \quad t_w > 0, \\ < 0 \Leftrightarrow t_w (\eta_{L,\bar{w}} + \eta_{K,\bar{w}}) < t_r (\eta_{K,\bar{r}} + \eta_{L,\bar{r}}) \quad \text{if} \quad t_w < 0. \end{cases} \quad \text{(A11)}$$

Substituting the factor demand elasticities in (A11) finally yields condition (6).

### Appendix 3: Net-of-tax wage elasticities

The signs of the net-of-tax wage elasticities are determined by $w_{t_w} = -\Omega_{w_t}^{-1} \Omega_{w_r}$, $w_{t_r} = -\Omega_{w_r}^{-1} \Omega_{w_t}$. Using condition (8) for the labour tax and a similar condition for the capital tax, we have:

$$\Omega_{w_r} = \frac{\beta}{V^2} (VV_{w_r} - V_w V_{t_r}) + \frac{(1-\beta)}{\pi^2} (\pi \pi_{w_r} - \pi_w \pi_{t_r})$$

$$= -\frac{(1+t_r)}{(1+t_w)} \left[ \frac{\beta}{V^2} (VV_{w_r} - V_w V_{t_r}) + \frac{(1-\beta)}{\pi^2} (\pi \pi_{w_r} - \pi_w \pi_{t_r}) \right] = -\frac{(1+t_r)}{(1+t_w)} \Omega_{w_t}. \quad \text{(A12)}$$

It is straightforward to derive the symmetry condition $\omega_{t_r} = -\omega_{t_w}$ from (A12).

To determine the left-hand side of condition (19) we make use of the explicit partial derivatives in (19):

$$G_{t_r} = \frac{wL}{(1+t_w)} \left[ 1 + \left( t_w (1+\eta_{L,\bar{w}}) + \frac{b-a}{w} \eta_{L,\bar{w}} + t_r \frac{rK}{wL} \eta_{K,\bar{w}} \right) (1+\omega_{t_r}) \right]. \quad \text{(A13)}$$
and

\[ G_t = \frac{rK}{1+t_w} \left[ 1 + t_t (1 + \eta_{L,w}) + \left( t_w + \frac{b-a}{w} \right) \frac{wL}{rK} \eta_{L,w} \right] - \omega_t \left( \left( t_w + \frac{b-a}{w} \right) \frac{wL}{rK} + t_t \eta_{K,w} \right) \]  \hspace{1cm} (A14)

The partial derivatives of equations (A13) and (A14) with respect to the net-of-tax wage elasticity are given by:

\[ \frac{\partial G_t}{\partial \omega_{t,w}} = \frac{wL}{(1+t_w)} \left( t_w (1 + \eta_{L,w}) + \frac{b-a}{w} \eta_{L,w} + t_t \frac{rK}{wL} \eta_{K,w} \right) < 0 , \hspace{1cm} (A15) \]

\[ \frac{\partial G_t}{\partial \omega_{t,w}} = -\frac{rK}{(1+t_w)} \left( \left( t_w (1 + \eta_{L,w}) + \frac{b-a}{w} \eta_{L,w} \right) \frac{wL}{rK} + t_t \eta_{K,w} \right) > 0 , \hspace{1cm} (A16) \]

where the signs are unambiguously given if \( \eta_{L,w} < -1 \), which always holds if \( \sigma > 1 \). Substituting into the left-hand side of (19) shows that the left-hand side is increasing in \( \omega_{t,w} \).

References


