Published in: Public Finance 51.1996, pp. 242-257

Internalizing Externalities in Second-Best Tax Systems*

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JEL classification: H21, Q28

ABSTRACT

We examine the analytical structure of optimal taxation of polluting and non-polluting goods in a second-best world where lump-sum taxes are infeasible. After deriving a *second-best internalization tax rate* which exactly internalizes the external effect, we show how to separate the analysis of second-best optimal environmental taxes from the analysis of the tax structure which minimizes the excess burden. This separation reveals that standard results of optimal taxation essentially carry over to economies with externalities. Our approach clarifies the controversy regarding the relation between the first-best Pigovian tax rate on polluting goods and the optimal tax rate in a second-best world.

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* Helpful comments by Clemens Fuest, Clemens Puppe, and Marcel Thum are gratefully acknowledged. The usual disclaimer applies.

I. Introduction

We examine the problem of the optimal taxation of polluting and non-polluting goods in a second-best world where lump-sum taxes are infeasible. The analysis is based on the derivation of the "second-best internalization tax" which in a second-best environment of distortionary taxes exactly internalizes the external effect associated with a particular good. If the distortionary taxes are replaced by lump-sum taxes, this second-best internalization tax becomes identical to the so-called Pigovian tax, which is equal to the marginal environmental damage. Moreover, as with the first-best Pigovian tax, our second-best internalization tax is a *real* concept, uniquely determined by the real allocation. It is, therefore, independent of the arbitrary normalization of the system of tax rates.

The second-best internalization tax provides a very useful concept for the analysis of optimal taxation when external effects are present. It will be shown that it allows the separation of the analysis of second-best optimal environmental taxes from the analysis of the tax structure which minimizes the excess burden. This separation reconfirms Sandmo's (1975) analysis and leads directly to a simple and intuitive generalization of standard optimal taxation results, such as the Ramsey rule and the Corlett-Hague rule, to economies with externalities.

The approach proposed in this paper also sheds light on a certain aspect of the double-dividend hypothesis. This hypothesis has recently become a very popular argument both in the academic and in the political debate. In a world with distortionary taxes green taxes are expected to improve the quality of the environment *and* to reduce the distortions of the existing tax system. It is therefore widely accepted that the existence of such a double dividend (cf. Goulder 1995) makes green taxes superior to other environmental instruments which are considered to be efficient in a world without tax distortions.

A controversy, however, has emerged in the literature with respect to the question about the magnitude of the second-best optimal tax on a polluting good. The early literature on the double dividend claims that, due to distortionary taxes, taxes on polluting goods should be higher than the Pigovian tax (cf. Nichols 1984, Lee and Misiolek 1986). More recently, though, Bovenberg and de Mooij (1994) have argued that the opposite is true. Hence, it seems to be unclear whether in second-best situations, characterized by distortionary taxes, optimal taxes on polluting goods should be higher or lower than the first-best Pigovian tax associated with the same allocation.

Schöb (1994) and Fullerton (1996) have shown that the difference in the results concerning the optimal tax rate on a polluting good is due to different normalizations of tax rates which lead to different definitions of what the tax on a polluting good actually is. Hence, a comparison between the second-best optimal tax rate on a polluting good (which depends on the chosen normalization) and the corresponding first-best Pigovian tax rate (which does not depend on the normalization) will generate a result which depends essentially on the arbitrary normalization. However, although these authors have pointed out the pivotal role of the normalization in these comparisons, they have not addressed the question of how to overcome this problem. Applying our approach to this question extends their analysis by providing a clear and intuitive solution to this problem.

As the second-best internalization tax on a polluting good depends exclusively on the real allocation, it turns out to be a suitable reference standard for what may be considered the environmental tax. Comparing this tax with the first-best Pigovian tax reveals unambiguously how optimal environmental taxation in a second-best world differs from that in a first-best world.

An indirect, but equivalent way to perform this comparison can be based on normalizing the tax system such that the second-best internalization tax coincides with the total tax on a given polluting good. Then, since the total tax is now identical with the second-best internalization tax, one can compare the *total* tax on this polluting good with the first-best Pigovian tax in a meaningful way. This is done in Bovenberg and de Mooij (1994) and our analysis confirms that they have chosen the "appropriate" normalization for the comparison.

The result of these comparisons is that the second-best internalization tax on a polluting good is always *lower* than the first-best Pigovian tax associated with the same allocation, provided the marginal cost of public funds exceeds unity. If a tax dollar is worth more than a private dollar, it needs less dollars to compensate for the environmental damage.

The paper is organized as follows. Section II introduces the model. The concept of internalizing externalities within a second-best framework is discussed in Section III. Section IV then analyses the optimal tax structure for the three good case. Section V applies our approach to the standard optimal taxation literature and Section VI to a result of Bovenberg and de Mooij (1994). In Section VII we generalize our analysis by considering the possibility of multiple externalities. Section VIII concludes. The Appendix contains a formal proof.

II. The model

Consider a closed economy with N identical households, $C + D$ private consumption goods, a public good G and labour ℓ. The private goods $c \in \{1,\ldots,C\}$ are clean (i.e. they have no external effect), whereas the private goods $d \in \{C+1,\ldots,C+D\}$ are dirty, i.e. their production or consumption create negative external effects which causes the environmental quality E to deteriorate.[1] In any equilibrium all households will consume identical consumption bundles $(x_1,\ldots,x_C,\ldots,x_{C+D})$ and supply an identical quantity x_ℓ of labour. The aggregate quantities are given by $(Nx_1,\ldots,Nx_C,\ldots,Nx_{C+D})$ and Nx_ℓ respectively. We assume that environmental quality is a decreasing function of the aggregate quantity Nx_d, produced and consumed, of each dirty good $d = C+1,\ldots,C+D$, i.e.

$$E = e(Nx_{C+1},\ldots,Nx_{C+D}), \quad e_i \equiv de/d(Nx_i) < 0. \tag{1}$$

[1] Although we concentrate on *negative* external effects, it is also obvious that our analysis generalizes readily to a situation where both positive and negative external effects are present.

There is a linear technology for the production of the private goods and the public good, with labour being the only input. Assuming perfect competition, we can normalize the wage rate to unity and choose units for all goods such that all producer prices are equal to one. Then, production possibilities are described by

$$Nx_{\ell} = N\sum_{c} x_c + N\sum_{d} x_d + G. \tag{2}$$

The government provides the public good *G*. To finance it, the government raises either (identical) lump-sum taxes *T* from each household or taxes on labour and on commodities. The government's budget constraint is therefore given by:

$$G = N\sum_{c} t_c x_c + N\sum_{d} t_d x_d + Nt_{\ell} x_{\ell} + NT, \tag{3}$$

where t_{ℓ} denotes the tax rate on labour while t_c and t_d denote the commodity taxes on the clean and the dirty goods, respectively.

The preferences of a household with respect to both clean and dirty private goods, leisure x_n, the public good *G*, and the environmental quality *E* can be represented by a twice continuously differentiable, strictly quasi-concave utility function

$$U = u(x_1, \ldots, x_C, \ldots, x_{C+D}, x_v, G, E), \tag{4}$$

with $u_i > 0,\ i = 1, \ldots, C + D, v, G, E,$ denoting the marginal utility of good *i*. To simplify the analysis, we assume separability between private consumption and the environment *E*, and private consumption and the public good *G*, respectively, i.e. all marginal rates of substitution between private goods are independent of *E* and *G*. The time endowment is normalized to one, hence $x_v + x_{\ell} = 1$. The budget constraint of the household is given by

$$\sum_{c} (1+t_c)x_c + \sum_{d} (1+t_d)x_d = (1-t_{\ell})(1-x_v) - T. \tag{5}$$

We assume that when she maximizes utility the agent does not take account of the negative effect of her own consumption on the environmental quality. This simplifies the analysis and is justified by the idea that each individual and its effect on the environment is negligible.[2]

The benevolent government maximizes a utilitarian welfare function

$$W = Nu(x_1,\ldots,x_C,\ldots,x_{C+D},x_v,G,E)\,. \tag{6}$$

subject to its budget constraint (3). When maximizing, it takes account of the individual optimizing behaviour.

III. The second-best internalization tax

In the presence of externalities, Pareto efficiency requires the equality of *social* and *private* marginal welfare of consuming a dirty good. In a first-best world, characterized by the feasibility of lump-sum taxes, this can be achieved by imposing a tax on a polluting good which equals the marginal environmental damage, i.e. the induced change in the environmental quality multiplied by the marginal rate of substitution between the environment and a numéraire. Such a Pigovian tax fully *internalizes* the external costs at the margin as the individual has to take account of all marginal costs resulting from her decision.

If lump-sum taxes are not available, we are in a second-best environment, characterized by the use of distortionary taxes. In such a second-best world we can apply the concept of *internalizing externalities* by looking for a tax rate t_d^E on any given dirty good which would exactly internalize the external effect of this dirty good. Consider therefore the following thought experiment, which is a special – but for our purpose more intuitive – case of the Diamond (1975) approach to analyzing the welfare effect of marginal changes of

[2] Strictly speaking this is correct only for an atomless continuum of agents.

private income, which we will consider below. One of the *N* households obtains an additional marginal unit of exogenous income *Y*. In the household optimum the household is indifferent to how to spend the additional income. Hence without loss of generality we assume that the household increases the consumption of *d* only, i.e. by $1/(1+t_d^E)$. The government uses the additional tax revenues to increase the supply of the public good by $t_d^E/(1+t_d^E)$. The effect of a marginal increase in income for one household on social welfare is therefore:

$$\frac{dW}{dY} = \frac{u_d + Nu_E e_d + Nu_G t_d^E}{1+t_d^E}. \tag{7}$$

The first term of the right-hand side denotes the increase in private utility while the second term denotes the external effect imposed on all households by the additional consumption of the dirty good *d*. The last term is the increase in all household's utility due to the additional provision of the public good *G* which is financed by the internalization tax imposed on the dirty good *d*.

Full internalization requires that the *private marginal utility* of consuming the dirty good, which is $du/dY = u_d/(1+t_d^E)$, is equal to the *social marginal welfare* of consuming the dirty good:

$$\frac{u_d}{1+t_d^E} = \frac{dW}{dY}. \tag{8}$$

The external effect is exactly internalized if and only if the tax on the dirty good is equal to $-u_E e_d/u_G$. Therefore, we define

$$t_d^E \equiv -\frac{u_E}{u_G} e_d \tag{9a}$$

as the component of the tax on the dirty good *d* the government has to impose in order to exactly internalize the external effect. It will be called the *second-best internalization tax*, or, synonymously, the *Pigovian component* of t_d. An important property of the second-best internalization tax t_d^E, as defined in (9a), is that it depends only on the real variables u_E, u_G

and e_d and thus is itself a real variable. Therefore, although the tax rates themselves can be arbitrarily normalized (see n.6 below), the second-best internalization tax t_d^E is given independently of the normalization. It will not be affected by any change of this normalization.

Within a first-best framework where lump-sum taxes are available and consequently $t_c = t_\ell = 0,\ \forall\ c \in \{1,\ldots,C\}$ in the optimum, the budget constraint (3) becomes $G = N\sum_d t_d^E x_d + NT$. Optimality requires that the marginal welfare of public expenditures, i.e., Nu_G is equal to the marginal welfare of private expenditures, which for all clean goods $c \in \{1,\ldots,C\}$ equals the marginal utility u_c (recall that all producer prices are normalized at unity). Hence, the first-best Pigovian tax component for good *d* becomes

$$t_d^E = -\frac{Nu_E}{u_c} e_d\,. \tag{9b}$$

This is the well-known Pigovian tax, which equals the marginal environmental damage measured in terms of private income (cf. e.g. Baumol and Oates 1988, p. 42).

If lump-sum taxes are not feasible, the dirty good might also be taxed for reasons other than internalizing the external effect. Denoting $t_d^R \equiv t_d - t_d^E$ as the *additional* tax on the dirty good on top of the Pigovian component (a justification of such a tax is given in section IV below), and allowing the household to allocate a marginal unit of additional income optimally over all goods, we obtain the social marginal utility of private income as defined by Diamond [1975, his equation (6)]:

$$\begin{aligned}\frac{dW}{dY} &= Nu_G\left[\frac{u_Y}{Nu_G} + \frac{u_E e_d}{u_G}\frac{\partial x_d}{\partial Y} + t_c\frac{\partial x_c}{\partial Y} + (t_d^E + t_d^R)\frac{\partial x_d}{\partial Y} + t_\ell\frac{\partial x_\ell}{\partial Y}\right] \\ &= Nu_G\left[\frac{u_Y}{Nu_G} + t_c\frac{\partial x_c}{\partial Y} + t_d^R\frac{\partial x_d}{\partial Y} + t_\ell\frac{\partial x_\ell}{\partial Y}\right],\end{aligned} \tag{7'}$$

where $u_Y \equiv du/dY$. For $t_d^R = 0$ and $\partial x_c / \partial Y = \partial x_\ell / \partial Y = 0$ this reduces to (7). The *social marginal utility of private income* now depends (a) on the social evaluation of increased utility of the household made possible by higher income, and (b) on the social evaluation of the additional tax revenues - net of those arising from the internalization tax component - which are collected as a consequence of the higher private income. As the revenues arising from the internalization tax component exactly compensate for the marginal environmental damage, this also shows that the definition (9a) gives the tax on the dirty good which exactly internalizes the external effect.

In a world where distortionary taxes exist, a dollar in the public purse is worth more (or less) than a dollar in the private purse. As the tax revenues from internalizing the external effect accrue at the governmental level, these tax revenues should be measured by the marginal welfare of public expenditures, i.e. the marginal welfare of the public good *G*. Consequently, the second-best internalization tax as defined in equation (9a) is measured in terms of public expenditures.[3]

To summarize this section: Measuring the social costs of pollution in terms of public expenditures allows for a more general definition of an optimal environmental tax which encompasses the first-best Pigovian tax as a special case. As already indicated above, in a second-best world this environmental tax is not necessarily the only tax on polluting goods. Therefore, we have to analyse the optimal tax structure in a broader context where we will make use of the definition developed here.

[3] Notice that the equivalence is only true, if *G* is determined endogenously by the optimizing government. If *G* is exogenously fixed, the welfare change from additional public good provision and rebating additional tax revenues via reducing other taxes may differ.

IV. The optimal tax structure in a second-best framework

As a first step we focus on the three good case with one clean and one polluting consumption good and leisure. A generalisation of the results will be presented in Section VII. Assume that lump-sum taxes are not feasible. To derive the optimal tax structure, we make use of the indirect utility function $w(t_c,t_d,t_\ell,G,E)$, which already takes into account the utility maximizing behaviour of the household. With lump-sum taxes being infeasible, the government can only raise tax revenues by introducing taxes on the two private commodities or on labour. Hence, the government maximizes

$$W = Nw(t_c,t_d,t_\ell,G,E) = Nu\left[x_c(t_c,t_d,t_\ell), x_d(t_c,t_d,t_\ell), x_v(t_c,t_d,t_\ell), G, E\right], \qquad (10)$$

with respect to t_c,t_d,t_ℓ,G and subject to equations (2) and (3).[4] Since (2) is implied by Walras' Law and since maximizing W/N is equivalent to maximizing *W*, we define the Lagrangian as

$$\mathrm{L}(t_c,t_d,t_\ell,G,m) = w(t_c,t_d,t_\ell,G,E) - m\left(\frac{G}{N} - t_c x_c - t_d x_d - t_\ell x_\ell\right). \qquad (11)$$

Using Roy's identity the first-order conditions are given by:

$$\frac{\partial \mathrm{L}}{\partial t_c} = -\lambda x_c + u_E e_d N \frac{\partial x_d}{\partial t_c} + \mu\left(x_c + t_c \frac{\partial x_c}{\partial t_c} + t_d \frac{\partial x_d}{\partial t_c} + t_\ell \frac{\partial x_\ell}{\partial t_c}\right) = 0, \qquad (12a)$$

$$\frac{\partial \mathrm{L}}{\partial t_d} = -\lambda x_d + u_E e_d N \frac{\partial x_d}{\partial t_d} + \mu\left(x_d + t_c \frac{\partial x_c}{\partial t_d} + t_d \frac{\partial x_d}{\partial t_d} + t_\ell \frac{\partial x_\ell}{\partial t_d}\right) = 0, \qquad (12b)$$

$$\frac{\partial \mathrm{L}}{\partial t_\ell} = -\lambda x_\ell + u_E e_d N \frac{\partial x_d}{\partial t_\ell} + \mu\left(x_\ell + t_c \frac{\partial x_c}{\partial t_\ell} + t_d \frac{\partial x_d}{\partial t_\ell} + t_\ell \frac{\partial x_\ell}{\partial t_\ell}\right) = 0, \qquad (12c)$$

$$\frac{\partial \mathrm{L}}{\partial G} = u_G - \mu \frac{1}{N} = 0. \qquad (12d)$$

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[4] Note that because of the separability between private consumption and the environment *E*, and private consumption and the public good *G*, respectively, the demand functions do not depend on *E* and *G*.

The shadow price λ denotes the *private marginal utility of private income*. The shadow price μ equals the *marginal utility of public expenditures* [cf. equation (12d)].

Since x_c, x_d and x_ℓ are each homogenous of degree zero in consumer prices, the equations (12a) – (12c) are linearly dependent.[5] Hence, we can arbitrarily normalize the system of tax rates[6] or, equivalently, the consumer prices. In particular, we can set any of the three tax rates equal to zero. As we will make use of different normalizations, *all* first order conditions have been presented above.

Defining the *Ramsey component* t_d^R implicitly by

$$t_d = t_d^R + t_d^E, \tag{13}$$

we can rewrite the first order conditions:

$$-\lambda x_c + \mu(x_c + t_c \frac{\partial x_c}{\partial t_c} + t_d^R \frac{\partial x_d}{\partial t_c} + t_\ell \frac{\partial x_\ell}{\partial t_c}) = 0, \tag{14a}$$

$$-\lambda x_d + \mu(x_d + t_c \frac{\partial x_c}{\partial t_d} + t_d^R \frac{\partial x_d}{\partial t_d} + t_\ell \frac{\partial x_\ell}{\partial t_d}) = 0, \tag{14b}$$

$$-\lambda x_\ell + \mu(x_\ell + t_c \frac{\partial x_c}{\partial t_\ell} + t_d^R \frac{\partial x_d}{\partial t_\ell} + t_\ell \frac{\partial x_\ell}{\partial t_\ell}) = 0, \tag{14c}$$

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[5] Let $q_c = 1 + t_c$, $q_d = 1 + t_d$, and $q_\ell = 1 - t_\ell$ denote the consumer prices. Homogeneity implies

$$0 = q_c \frac{\partial x_i}{\partial q_c} + q_d \frac{\partial x_i}{\partial q_d} + q_\ell \frac{\partial x_i}{\partial q_\ell} = q_c \frac{\partial x_i}{\partial t_c} + q_d \frac{\partial x_i}{\partial t_d} - q_\ell \frac{\partial x_i}{\partial t_\ell}, \quad i = c, d, \ell .$$

Note that the household's budget constraint (6) with $T = 0$ implies: $\frac{q_c}{q_\ell} x_c + \frac{q_d}{q_\ell} x_d = x_\ell$. Multiplying (12a) by q_c / q_ℓ, (12b) by q_d / q_ℓ, making use of the homogeneity property and adding up gives (12c).

[6] Given any tax vector $t = (t_c, t_d, t_\ell)$, the tax vector $t' = (t_c', t_d', t_\ell')$, where

$$t_c' = \frac{1 + t_c}{\gamma} - 1,\; t_d' = \frac{1 + t_d}{\gamma} - 1,\; t_\ell' = 1 - \frac{1 - t_\ell}{\gamma}$$

and $\gamma > 0$, gives identical budget equations for all households and the government for any parameter $\gamma > 0$. Consequently, t' leads to the same allocation as t. With $\gamma = 1 + t_c$, we get $t_c' = 0$, with $\gamma = 1 - t_\ell$, we get $t_\ell' = 0$. However, it follows from (9a) that the second-best internalization tax t_d^E is the *same* for all normalizations since neither u_E nor u_G nor e_d are affected by normalization. Only the difference between the total tax on the dirty good and the second-best internalization tax, i.e. $t_d - t_d^E$, which will be called the *Ramsey component* below, depends on the chosen normalization.

$$Nu_G = \mu, \tag{14d}$$

$$t_d^E = -\frac{u_E}{u_G} e_d, \tag{14e}$$

where (14e) restates (9a). The first-order conditions (14a) – (14d) are very similar to the first-order conditions we would obtain for an optimization problem without considering externalities. The presence of externalities only implies one modification of these first-order conditions for the standard case. Instead of the total tax on the dirty good we have to consider the Ramsey component of the total tax. The Ramsey component is determined by *non-environmental* welfare considerations only, i.e. by minimizing the excess burden of the tax system; *environmental* welfare considerations, on the other hand, are fully captured by the Pigovian component given by (14e).

The structure of the first-order conditions shows that we can separate the Pigovian component and the Ramsey component of the second-best optimal tax t_d. Given the second-best allocation, the equation system (14a) – (14c) yields the optimal tax rates (subject to normalization) for goods which do not generate an external effect plus the optimal Ramsey component for the tax on the dirty good. Substituting equation (14e) into equation (13) then yields the optimal tax on the dirty good. Notice, however, that the household's decision concerning x_c, x_d and x_ℓ depend on $t_d = t_d^R + t_d^E$ and that therefore all equations (14a) – (14e) have to be solved simultaneously. The separation of the two tax components is hence a conceptual and not a computational separation.

The fact that the marginal environmental damage of the polluting good enters the tax formula for the polluting good as an additional term only has already been pointed out by Sandmo (1975, p. 92). He calls this the additivity property of a second-best tax system in the presence of externalities. However, Sandmo's result was derived by solving the equation system (12a) - (12d) explicitly for the optimal tax structure while our approach suggests to separate the internalization tax initially and then solve the equation system (14a) - (14d) for the optimal Ramsey tax components. The advantage this approach has will be exemplified in the next section by showing how standard results of the optimal taxation literature may easily be carried over to the case where externalities are present.

V. Optimal Taxation Rules in the Presence of Externalities: A Reconsideration

The separation of the internalization problem from the optimal taxation problem makes it easy to carry over the results from the optimal taxation literature into a more general framework where externalities occur in the economy. This advantage is a general property of our approach and will be exemplified for two well-known optimal taxation rules. In this section we show how the Ramsey rule and the Corlett-Hague rule, derived for the case without externalities, have to be modified in order to apply to the case where externalities are present.[7] Again we confine the analysis to the three good case.

The Ramsey rule

The Ramsey rule states that, if commodity taxes are set optimally, a small equiproportional increase in all tax rates will cause all compensated commodity demands to fall by the same proportion. We show that with external effects being present we only have to substitute "Ramsey components" for "tax rates".

Recall that many tax rate normalizations lead to the same allocation (see n. 6 above). Thus we are free to normalize the tax rate on labour to zero, $t_\ell = 0$. Defining $t_c^R \equiv t_c$ and using the Slutzky decomposition

$$\frac{\partial x_i}{\partial(1+t_j)} = s_{ij} - x_j \frac{\partial x_i}{\partial Y}, \tag{15}$$

[7] For the derivation of a third well-known optimal taxation rule, the inverse elasticity rule in the presence of externalities [see Sandmo (1975)], our approach can be similarly applied.

with s_{ij} denoting the compensated (cross-)price effect. From this and (14a), or (14b) respectively, we obtain after some re-arrangement

$$\frac{\sum_{i=c,d} t_i^R s_{ic}}{x_c} = \left(\frac{l - m}{m} + \sum_{i=c,d} t_i^R \frac{\P x_i}{\P Y} \right) \equiv -(1-b)\,, \tag{16a}$$

$$\frac{\sum_{i=c,d} t_i^R s_{id}}{x_d} = \left(\frac{l - m}{m} + \sum_{i=c,d} t_i^R \frac{\P x_i}{\P Y} \right) \equiv -(1-b)\,, \tag{16b}$$

where $b \equiv \left(\frac{l}{m} + \sum_{i=c,d} t_i^R \frac{\P x_i}{\P Y} \right)$ is to be interpreted as the social marginal utility of private income (cf. Diamond 1975, p.338).[8] However, in the presence of externalities, the definition of b is *conditional on the full internalization of the external effect* as suggested in Section III. Though the equations (16a) and (16b) are similar to the Ramsey rule (cf. Atkinson and Stiglitz 1980, p. 372), the interpretation has to be slightly modified, as in the presence of externalities we have to consider the Ramsey components instead of the total tax rate.

> **Ramsey rule in the presence of externalities:** If commodity taxes are set optimally, a small equiproportional increase in all *Ramsey components*, i.e. in all *non-environmental tax components*, will cause all compensated commodity demands to fall by the same proportion.

A formal derivation of this result is given in the appendix.

[8] Note that b as well as λ depend on the chosen normalization since the normalization influences consumer prices and thus the real value of nominal income.

Corlett-Hague rule

The Corlett-Hague rule states that relative to the consumer price the tax rate should be higher for goods which are complementary to leisure than for goods which are substitutes for leisure. Applying our approach, this rule can easily be generalized to allow for external effects.

Multiplying (16a) by x_c and (16b) by x_d respectively, and using Cramer's rule, we can solve for the optimal Ramsey components:

$$t_c = t_c^R = \frac{(1-b)(x_d s_{dc} - x_c s_{dd})}{s_{dd} s_{cc} - s_{dc} s_{cd}}, \tag{17a}$$

$$t_d^R = \frac{(1-b)(x_c s_{cd} - x_d s_{cc})}{s_{dd} s_{cc} - s_{dc} s_{cd}}. \tag{17b}$$

Dividing (17a) and (17b) by the consumer prices q_j (cf. n. 5) and defining the elasticity of compensated demand as $\varepsilon_{ij} = q_j s_{ij} / x_i$; $i, j = c, d, v$, we can derive the following expressions:

$$\frac{t_c^R}{q_c} = \frac{(1-b)}{(s_{dd} s_{cc} - s_{dc} s_{cd})} \frac{x_c x_d}{q_c q_d} \left(e_{cd} - e_{dd} \right), \tag{18a}$$

$$\frac{t_d^R}{q_d} = \frac{(1-b)}{(s_{dd} s_{cc} - s_{dc} s_{cd})} \frac{x_c x_d}{q_c q_d} \left(e_{dc} - e_{cc} \right), \tag{18b}$$

Applying the property of the Slutzky term for the compensated elasticities, $\varepsilon_{ic} + \varepsilon_{id} + \varepsilon_{iv} = 0,\ i = c, d$ (cf. e.g. Deaton and Muellbauer 1980, p. 62), we obtain $\varepsilon_{cd} - \varepsilon_{dd} = -\varepsilon_{cc} - \varepsilon_{dd} - \varepsilon_{cv}$ and $\varepsilon_{dc} - \varepsilon_{cc} = -\varepsilon_{cc} - \varepsilon_{dd} - \varepsilon_{dv}$. Together with equations (18a) and (18b) and $(1-b)/(s_{dd} s_{cc} - s_{dc} s_{cd}) > 0$ this implies

$$\varepsilon_{cv} > \varepsilon_{dv} \quad \Leftrightarrow \quad \frac{t_d^R}{q_d} > \frac{t_c^R}{q_c}. \tag{19}$$

The left-hand side of condition (19) considers the case where the dirty good is a relatively stronger complement (weaker substitute) to leisure than the clean good. This implies that the optimal Ramsey component of the tax on the dirty good relative to its consumer price should be higher than the optimal Ramsey component of the tax on the clean good relative to *its* consumer price. This is a result which is close to the Corlett-Hague (1953) rule.

Corlett-Hague rule in the presence of externalities: The Ramsey component, i.e. the non-environmental tax component, relative to the consumer price should be higher for goods which are complementary to leisure than for goods which are substitutes for leisure.

VI. The result from Bovenberg and de Mooij (1994) reconsidered

Bovenberg and de Mooij (1994) have recently derived a surprising result. For the case where only labour taxes and green taxes are considered - which is equivalent to the normalization $t_c = 0$ - they show "that, in the presence of pre-existing distortionary taxes, the optimal pollution tax typically lies below the Pigovian tax, which fully internalizes the marginal social damage from pollution". Furthermore, from this they infer that "environmental taxes typically exacerbate, rather than alleviate, pre-existing tax distortions" and that "marginal costs of environmental policy rise with the marginal costs of public funds" (Bovenberg and de Mooij, 1994, p.1085).

We can apply our approach to derive and illuminate this interesting result. Bovenberg and de Mooij (1994) assume, in addition to the assumptions made in Section II above, that the utility function is separable between leisure and consumption goods and homothetic in consumption goods. These assumptions imply $t_d^R = 0$, i.e. the Ramsey component of the tax on the dirty good is zero.[9] Therefore, the optimal tax is given by

[9] This result is closely related to the literature on uniform commodity taxation. For the case without externalities, Sandmo (1974) shows that uniform commodity taxation is optimal for a utility function which satisfies the assumptions made above. Applying our approach leads to the result that the *Ramsey components* of all taxes have to be identical when externalities are present (for the clean good the Ramsey component coincides with the total tax rate). Since the Ramsey components of the commodity taxes are all identical, the normlization $t_c = 0$ of Bovenberg and de Mooij implies $t_d^R = 0$.

$$t_d^* = t_d^E . \tag{20}$$

In the presence of distortionary taxes we can expect that $Nu_G > u_c$ (for a discussion of this issue, see Atkinson and Stern 1974). Hence, a comparison of equations (9b) and (14e) shows that the second-best optimal pollution tax has to be below the (hypothetical) first-best Pigovian tax rate associated with the second-best allocation.

The comparison of the total tax rate on the dirty good with the first-best Pigovian tax rate, however, is somewhat questionable because the total tax rate depends on the normalization whereas the Pigovian tax rate does not. Given any system of tax rates (t_c, t_d, t_ℓ) and any real number $\tau \in (-1, \infty)$, we can re-normalize the tax rates in such a way that we get an equivalent system of taxe rates (t_c', t_d', t_ℓ') with $t_d' = \tau$ (see n.6; for $\gamma = (1+t_d)/(1+\tau)$ we get $t_d' = \tau$). As any re-normalization of prices, this re-normalization of tax rates will have no real effects. Consequently, the difference between the total tax rate t_d and the first-best Pigovian tax rate has no unambiguous meaning unless we can argue that this comparison should be performed at a particular normalization (cf. also Schoeb 1994, Fullerton 1996). On the other hand, the second-best internalization tax, i.e. the Pigovian component t_d^E, is, as shown above, a real variable and thus independent of the normalization. Therefore, the difference between this Pigovian component and the first-best Pigovian tax rate is a *real* variable and consequently an unambiguous measure of whether the internalizing tax rate on the dirty good is higher or lower in the second-best environment relative to the first-best environment.

From these considerations we conclude that for the comparison of the *total* second-best tax rate on the dirty good with the first-best Pigovian tax rate there is a unique "correct" normalization of the tax rates. This is the normalization which makes the total tax rate equal to the Pigovian component because then - and only then - the comparison relates to two *real* variables. Bovenberg and de Mooij (1994) have, in fact, chosen the *only* normalization for which their comparison (between a nominal and a real variable) is equivalent to a comparison between real variables.

Our approach illuminates the role of the particular normalization in Bovenberg and de Mooij (1994) and thereby helps to achieve a deeper understanding of their surprising result. Having derived (20), i.e. the equality of the total tax rate on the dirty good and the Pigovian component, and assuming $Nu_G > u_c$ (for the reasons spelled out above), we can restate their result as follows: The *tax component necessary to internalize external effects* is smaller in a world with distortionary taxes than it is in a world without - whatever the Ramsey-optimal taxes on the dirty good might be.

To give an intuition for this result, notice that tax revenues from internalizing external costs are measured in terms of public expenditures. The second-best internalization tax measures the marginal external costs in terms of public expenditures, rather than private income. In a world with distortionary taxes a dollar in the public purse is worth more than a dollar in the private purse. "This implies that each unit of pollution does not have to yield as much public revenue to offset the environmental damage if this revenue becomes more valuable" (cf. Bovenberg and van der Ploeg 1994, p. 361, who refer to an "externality-correction term" of the second-best optimal pollution tax).

The analysis of this section has illustrated once more that our definition of a second-best internalization tax is a very useful concept. Not only can it be employed to separate the internalization problem from the optimal taxation problem, as shown in Section IV; in addition, it has the advantage of being a real concept which, therefore, facilitates meaningful comparisons with other real variables and leads to straightforward interpretations.

VII. The case of multiple externalities

Consider now the general case where we have *C* clean goods and *D* dirty goods. With lump-sum taxes not feasible, the government can raise taxes only on private commodities and on labour. Hence the government maximizes

$$W = Nw(t_1,\ldots t_C, t_{C+1},\ldots,t_{C+D}, t_\ell, G, E) \tag{10'}$$

subject to equation (3), as in Section IV. Assuming again separability between private consumption and the environment *E*, and private consumption and the public good *G*, respectively, the first-order conditions for the general case become:

$$-\lambda x_c + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_c} + \mu\Big(x_c + \sum_c t_c \frac{\partial x_c}{\partial t_c} + \sum_d t_d \frac{\partial x_d}{\partial t_c} + t_\ell \frac{\partial x_\ell}{\partial t_c}\Big) = 0, \quad \forall c \in \{1,\ldots C\} \tag{12a'}$$

$$-\lambda x_d + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_d} + \mu\Big(x_d + \sum_c t_c \frac{\partial x_c}{\partial t_d} + \sum_d t_d \frac{\partial x_d}{\partial t_d} + t_\ell \frac{\partial x_\ell}{\partial t_d}\Big) = 0, \quad \forall d \in \{C+1,\ldots C+D\} \tag{12b'}$$

$$-\lambda x_\ell + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_\ell} + \mu\Big(x_\ell + \sum_c t_c \frac{\partial x_c}{\partial t_\ell} + \sum_d t_d \frac{\partial x_d}{\partial t_\ell} + t_\ell \frac{\partial x_\ell}{\partial t_\ell}\Big) = 0\,, \tag{12c'}$$

$$u_G - \mu \frac{1}{N} = 0. \tag{12d'}$$

Defining the Pigovian tax components for all dirty goods according to definition (9a) and using (12d'), we obtain:

$$\mu \sum_d t_d^E \frac{\partial x_d}{\partial t_j} = -\sum_d u_E e_d N \frac{\partial x_d}{\partial t_j}, \qquad j = 1,\ldots,C,\ldots,C+D,\ell. \tag{22}$$

Analogously to (13), the Ramsey components are implicitly defined by

$$t_d = t_d^R + t_d^E, \qquad \forall d \in \{C+1,\ldots C+D\}. \tag{13'}$$

Substituting the equations (13') and (22) into (12a') - (12c'), we can rewrite the first order conditions as:

$$-\lambda x_c + \mu\Big(x_c + \sum_c \tau_c \frac{\partial x_c}{\partial \tau_c} + \sum_d \tau_d^R \frac{\partial x_d}{\partial \tau_c} + \tau_\ell \frac{\partial x_\ell}{\partial \tau_c}\Big) = 0, \qquad \forall c \in \{1, \ldots C\} \tag{14a'}$$

$$-\lambda x_d + \mu\Big(x_d + \sum_c \tau_c \frac{\partial x_c}{\partial \tau_d} + \sum_d \tau_d^R \frac{\partial x_d}{\partial \tau_d} + \tau_\ell \frac{\partial x_\ell}{\partial \tau_d}\Big) = 0, \qquad \forall d \in \{C+1, \ldots C+D\} \tag{14b'}$$

$$-\lambda x_\ell + \mu\Big(x_\ell + \sum_c \tau_c \frac{\partial x_c}{\partial \tau_\ell} + \sum_d \tau_d^R \frac{\partial x_d}{\partial \tau_\ell} + \tau_\ell \frac{\partial x_\ell}{\partial \tau_\ell}\Big) = 0\,. \tag{14c'}$$

The separability of the Pigovian tax component from the non-environmental tax component is therefore also applicable in the case of multiple externalities. Hence, all optimal taxation rules derived for the case of many goods in a world without externalities can be modified for the case of multiple externalities in the same way as has been demonstrated for the case with three goods and one externality in Section IV.

VIII. Conclusions

We have examined the analytical structure of the optimal taxation of polluting and non-polluting goods in a second-best world where lump-sum taxes are not feasible. In this area there is some controversy and confusion, which is due to the use of concepts which are not independent of the arbitrary normalization of tax rates.

By clarifying what is meant by internalization of external effects in a world with distortionary taxes we offer a definition of a second-best internalization tax (component) which is uniquely determined by the real allocation and consequently independent of the normalization of tax rates. The application of this definition enables us to separate the analysis of second-best optimal environmental taxes from the analysis of the tax structure which minimizes the excess burden. This separation illuminates the basic structure of the optimal taxation in the presence of external effects and distortionary taxes and is, therefore, helpful for a better understanding of this area.

By simplifying the analysis in this way, we can show that the standard results of the optimal taxation literature easily carry over to economies with externalities. It also allows for a generalization of the results to the case of multiple externalities. Furthermore, our approach can help to clarify the controversy about the relation between optimal environmental taxation (i) in a first-best world, where lump-sum taxes are feasible, on the one hand and (ii) in a second-best world, where taxes are distortionary, on the other.

Appendix

Consider the compensated demand function for e.g. the clean good $x_c^{com}(t_c, t_d, u)$. The effect of a change in the tax rates on compensated demand can be written as

$$dx_c^{com} = s_{cc} dt_c + s_{cd} dt_d \, . \tag{A1}$$

Now consider a proportional change of all Ramsey components, i.e. $dt_c = \alpha t_c = \alpha t_c^R$ and $dt_d = \alpha t_d^R$. Making use of equation (16a) and of the symmetry property of the compensated cross-price effects, i.e. $s_{cd} = s_{dc}$, we can reformulate (A1):

$$\frac{dx_c^{com}}{x_c} = \alpha \frac{s_{cc} t_c^R + s_{dc} t_d^R}{x_c} = -\alpha(1-b) \, . \tag{A2}$$

As this result applies analogously for all taxed goods, the interpretation for the Ramsey rule in the presence of externalities follows.

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