Three family policies to reconcile fertility and labor supply

Robert Fenge*
University of Rostock,
and CESifo

Lisa Stadler**
University of Munich

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Abstract
In this model we analyze three instruments of family policies. A direct child transfer is compared with a subsidy for external child care and a parental leave payment. The latter instruments change the quantity and quality of children by affecting the secondary earner's labor supply. We find that subsidizing external child care is the most effective policy instrument compared to child benefits and parental leave payments in increasing fertility and at the same time raising the labor supply. However, reconciling children and labor supply can only be achieved for secondary earners consuming above average external child care. For secondary earners having relatively high opportunity costs of staying at home with their children on the other hand, parental leave payments have the strongest impact on fertility but decrease their labor supply. The welfare analysis shows that for secondary earners whose ratio of own to average external child care is larger than their ratio of own to average opportunity costs of staying at home and who have an above average number of children a subsidy for external bought-in child care is advantageous against the other two policies.

JEL: H31, H53, J13, J22
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* University of Rostock, Department of Economics, Ulmenstraße 69, 18057 Rostock, Tel: ++49-381-4984339, E-mail: robert.fenge@uni-rostock.de
** University of Munich, Department of Economics, Ludwigstraße 28, 80539 Munich, Tel: ++49-89-21802246, E-mail: Lisa.Stadler@lrz.uni-muenchen.de

1 Introduction

There has been a steady and significant decline in birth rates in most OECD countries over the last 40 years. Despite this negative trend, a large heterogeneity within the countries' fertility rates can be observed. According to OECD statistics, total fertility rates in 2009 were as low as 1.4 children per woman in Italy, Spain, Germany, and Japan. Within the high-income countries of the world, no countries are solidly above the fertility rate of 2.1 children per women that is needed to replace the population at a constant level. Some other countries like France, Sweden, the United Kingdom or the United States managed to counteract this downside trend and to reincrease their birth rates. Therefore those countries could avert an as dramatic population decrease as for example in Germany. Due to the aging process associated with this decline in fertility, the developed countries are facing significant challenges.

Important factors linked to the decline of birth rates are higher incomes, and hence higher opportunity costs of children, as well as the rise in labor-force participation of women. According to Becker (1960 and 1981) and Becker and Lewis (1973), income increases may reduce fertility if the income elasticity for the quality of children exceeds the income elasticity for the quantity of children. Willis (1973) points out that increasing female wages will increase female labor-force participation and thus have a negative impact on the demand for children because of the higher opportunity costs.

It is often argued that the expansion of the welfare state and the social security system can also be blamed for a decline in fertility rates in the developed countries and especially in Europe. In the presence of a pay-as-you-go (PAYG) pension system there is a positive externality associated with having children, the so-called intergenerational transfer effect. The fertility distortion of the PAYG pension system arises because parents only obtain a small fraction of the pension contributions of their own children. Parts of the benefits of having children are socialized whilst the cost of raising children remains private. As a result, the number of children in a decentralized economy can be expected to be suboptimal. To counter this, most developed countries have in the last decades implemented political incentives to correct this externality and to improve the income position of families.

However, increasing fertility may not be a goal of public policies in itself. A higher number of children comes at a cost in terms of consumption and income of the parents. Even for children the quality of life may decrease if policies address only the quantity of children. Therefore the instruments of family policy have to be analyzed very carefully with respect to these effects and their implications for welfare. This paper presents a comparison of benefit programs on welfare, fertility and investments in quality of children within a model with endogenous fertility and labor supply of a secondary earner. We analyze the effects of changes in child benefits, in a child subsidy on bought-in child care as well as in parental leave payments.

Child benefits have been implemented in almost all OECD countries and there have been several empirical studies (e.g. Gauthier and Hatzius, 1997, Cigno et al, 2003, Laroque and Salanié, 2005) showing that they have a positive impact on the demand for children. Nevertheless countries such as Germany with very low fertility rates and relatively low female employment rates pay relatively high child benefits. Policy differences between high and low fertility countries as well as countries with high and low female employment rates can rather be found in the rates for parental leave payments, child care subsidies, and tax breaks towards families.

Both Sweden and France have achieved to keep their fertility rates relatively high and both countries have well developed subsidized care systems. This might lead to the conclusion that investing in child care is an important political instrument to help increasing fertility rates. In the empirical literature one finds mixed evidence about the success of child care subsidies in fostering fertility. While Hank et al (2004) find positive effects of full-time subsidized child care on fertility for Germany, Haan and Wrohlich (2009) only find significant effects for highly-educated women and women who give birth for the first time.

The third policy parameter we want to analyze, the rate of parental leave payments, has especially been implemented in Germany and Sweden. In the empirical literature one also finds mixed evidence on the effects of parental leave payments on fertility and the secondary earner's labor supply. Spiess and Wrohlich (2008) simulate fiscal costs and expected labor market outcomes of a parental leave benefit reform in Germany. They provide evidence that all income groups benefit and that in the second year, mothers increase their working hours and labor market participation significantly. Lalive and Zweimüller (2009) show that an extension of the Austrian parental leave period increases fertility but lengthens the time women spend at home. Other studies also show that leave expansions are associated with increased leave-taking (e.g. Pronzato, 2009, Han et al., 2009). Bergemann and Riphahn (2011) study the labor supply effects of a major change in the maternity leave benefit system in Germany on the intention of mothers to return to the labor market. They find that the change to a benefit system that replaced two-thirds of pre-birth earnings for at most one year succeeded in speeding up mothers' return to work.

In the following we provide a welfare analysis of the three policies. This enables us to calculate the distortions of the different policies and to compare them regarding to the utility of the parents. Our main results are that for secondary earners consuming above average external child care only a subsidy for bought-in child care has a positive impact on both fertility and labor supply. For secondary earners spending relative to their net wage more than the average time at home with their children on the other hand, parental leave payments have the strongest impact on fertility but decrease their labor supply.

In the next section we introduce the model. Section 3 presents the comparative static results. In section 4, we calculate the welfare effects of exchanging family policies and section 5 concludes.

2 The model

For simplicity, we divide the life cycle of each person into two phases of the same duration. During the first phase, a person entirely depends on parental support, while in the second, the adult person allocates his or her time to either working and thus contributing to family income or to raising children. For ease of exposition, we also assume that all men and women are neatly paired off into conventional families. Family i's decisions are assumed to be taken by the parents who derive utility from their own consumption, c_i, their number of children, n_i, and their children's quality of life, q_i, according to the additively separable utility function

$$U(c_i, n_i, q_i) = u(c_i) + u(n_i) + u(q_i) \tag{1}$$

for $i \in \{1, \dots, N\}$. We assume the utility function to be continuous, strictly concave, and strictly increasing in all arguments. The quality per child, q_i, can be understood as a good produced domestically by the parents who use as inputs time spent with the child and a child-specific consumption good, z_i, bought on the market. The price on the market for the child-specific consumption good is B. For simplicity, we assume that only the secondary earner of family i spends time with the children. Time spend with a child can be divided into the secondary earner's own time, h_i, and the time the child spends at external child care, g_i. The market price for child care, g_i, is denoted by π. The strictly concave domestic production function for quality is given by

$$q_i = q(h_i, g_i, z_i) \tag{2}$$

and increases monotonically in all arguments.

The secondary earner allocates her time to working which yields wage at the rate w_i and to leisure time. We assume that child rearing is the only domestic time requiring parental time so that she spends her leisure time completely with the children. Through the endogeneity of n_i, the secondary earner's labor supply is also endogenous. If she has n_i children her parental time equals $h_i n_i$. The rest of secondary earner i's total time is working time and given by $L_i = 1 - h_i n_i$, her gross income therefore equals $w_i L_i$. Secondary earners carrying a larger wage rate w_i thus have higher opportunity costs of raising children. The primary earner allocates all her time to working and her gross salary is Y.

The family's budget constraint is given by

$$(1-t)(Y + w_i L_i) + \alpha n_i + \gamma w_i (1-t) h_i n_i = c_i + B z_i n_i + (1-\beta)\pi g_i n_i \tag{3}$$

where α represents the child benefit, β the share of bought-in child care which is subsidized and γ the share of foregone net wage income of the secondary earner staying at home with the children which is granted as parental leave payment by the government.

The parents choose consumption, c_i, the number of children, n_i, the secondary earner's time spent with a child, h_i, the amount of bought-in child care, g_i, and the child-specific consumption, z_i, so as to maximize their utility, $u(c_i, n_i, q_i)$, by taking account of the child's quality production and their budget constraint. We abbreviate the first derivative of a function $y(x)$ by y_x.

The household decision problem is given by

$$\begin{aligned} &\max_{c_i, n_i, h_i, g_i, z_i} u\big(c_i, n_i, q_i(h_i, g_i, z_i)\big) \\ &s.t.\ (1-t)(Y + w_i L_i) + \alpha n_i + \gamma w_i (1-t) h_i n_i = c_i + B z_i n_i + (1-\beta)\pi g_i n_i \end{aligned} \tag{4}$$

The first-order conditions yield the following necessary and sufficient conditions of the concave maximization problem:

$$\frac{u_n}{u_c} = B z_i + (1-\beta)\pi g_i + w_i (1-t)(1-\gamma) h_i - \alpha \equiv P_{n,i} \tag{5}$$

$$\frac{u_q}{u_c} q_h = w_i(1-t)(1-\gamma)n_i \equiv P_{q_h,i} \quad (6)$$

$$\frac{u_q}{u_c} q_g = (1-\beta)\pi n_i \equiv P_{q_g,i} \quad (7)$$

$$\frac{u_q}{u_c} q_z = Bn_i \equiv P_{q_z,i} \quad (8)$$

All conditions have the well-known meaning that the marginal rate of substitution between the respective decision variables has to be equal to the marginal rate of transformation at the utility maximum. A variation in any of the policy parameters may affect the price of quantity as well as quality of children. Next to costs of parental time, the upbringing of children also incurs a cost per child, Bz_i, which covers child-specific consumption expenditure. The net cost of children $P_{n,i}$ in (5) is therefore composed of family i's consumption cost per child plus the net cost of external child care plus the opportunity cost of forgone net wage income of the secondary earner minus the child benefits. Children are considered consumption goods with positive net costs. The marginal net price of a child, $P_{n,i}$, decreases with a higher child benefit, α, as well as with a higher subsidy for child care, β, and higher parental leave payments, γ.

The marginal net price of parental time spent with the children, $P_{q_{h,i}}$, in (6) consists of the net wage loss while the marginal price for external child care, $P_{q_g,i}$, in (7) equals the net cost for the utilization of this service. Obviously, child benefits have no effect on the price of quality while the subsidy for child care decreases the price for bought-in child care and the parental leave payment reduces the net price of parental time spent with the child.

In the following, the net wage is abbreviated by $\hat{w}_i = (1-t)w_i$ and the first derivative of utility with respect to the quality inputs $x_i = h_i, g_i, z_i$, $U_q q_x > 0$, by U_{q_x} and the second derivative, $U_q q_{xx} + U_{qq} q_x q_x < 0$, by $U_{q_x q_x}$.

3 Comparative Statics: The effects of changes in the benefit system

First, we analyze the direct effects of the policy instruments on fertility, secondary earner's labor supply, the demand for external child care and parental and child-specific consumption. Second, we compare the policies by investigating the differential effects of exchanging mutually the instruments in a budget-neutral reform. By implicit differentiation of the first-order conditions (5)-(8) we derive the results and present in the following the impact on fertility, labor supply and demand for external child care. The derivation and the other effects on consumption can be found in Appendix A.

We start by analyzing the effects of a variation in the child benefit rate on quantity and quality of children. The effect of an increase of the child benefit rate on the quantity of children

$$\frac{\partial n_i}{\partial \alpha} = -s_{nn} - n_i i_n = \frac{1}{D_i} u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} \left(u_c - n_i u_{cc} P_{n,i}\right) > 0 \tag{10}$$

is positive as the determinant of the bordered Hessian matrix is negative ($D_i < 0$) and $\left(u_c - n_i u_{cc} P_{n,i}\right) > 0$. As expected, additional child benefits encourage fertility as they reduce the cost of having children. The income effect, $-n i_n$, with respect to α is positive and the substitution effect, $-s_{nn}$, is positive since an increase in α decreases the marginal net price of a child in (5). The size of the effect is driven by family i's number of children, n_i, and their marginal price for a child, $P_{n,i}$. The impact of an increase of α is therefore larger for high income families. On the contrary, secondary earners with a smaller wage rate invest more in parental consumption when α is increased (see Appendix A).

An increase of the child benefit rate has also a positive effect on both parental time and time the child spends in external child care as $\left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right) > 0$ (see Appendix A):

$$q_h \frac{\partial h_i}{\partial \alpha} = s_{nq_h} + n_i i_{q_h} = -\frac{1}{D_i} u_{q_g q_g} u_{q_z q_z} P_{q_h,i} \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right) > 0 \tag{11}$$

$$q_g \frac{\partial g_i}{\partial \alpha} = -s_{nq_g} - n_i i_{q_g} = -\frac{1}{D_i} u_{q_h q_h} u_{q_z q_z} P_{q_g,i} \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right) > 0 \tag{12}$$

Regarding the effects of an increase of the child benefit rate on the inputs of children's quality, parental time and child care, the substitution effects of the marginal cost of quantity on the demand for quality are negative. As child benefits have no direct effect on the quality of children, the ratio of quality and quantity will fall since the relative price of quality to quantity rises with α. This can be illustrated by a comparison of the net price of children with respect to the number of children in (5) and to the quality of children in (6) and (7). The price for quality of children is not affected by changes in the child benefit rate. Therefore, the change of the relative price in favor of the quantity of children reduces the parental time and the child care and, hence, the quality of children. However, the positive income effect dominates this substitution effect.

The size of the effect of an increase in α on parental time in (11) is driven by the marginal net price of parental time, $P_{q_h,i}$, and is thus larger for high income families. The size of the effect on the time the children spend in external child care in (12) depends on the marginal price for external child care, $P_{q_g,i}$, which is independent of the wage income. Hence, child benefits have a stronger impact on high income earners to stay home with the children than on low income earners whereas the effect on the demand for external child care is the same. Thus, the labor supply of the high income secondary earner decreases by more.

Proposition 1: *Increasing the child benefit rate encourages fertility and the demand for external child care while it discourages the secondary earner's labor supply. The effects are larger for high income secondary earners. As child benefits only have a direct effect on the quantity of children, an increase in the child benefit rate leads to a decrease in the ratio of quality and quantity of children.*

The effects of an increase of the subsidy for bought-in child care affect both the quantity and quality of children and rise in the prize of external child care, π. We now have to differentiate between two groups of families: families whose demand for external child care time is elastic, that is $\left(1+\frac{g_i u_{q_g q_g}}{u_{q_g}}\right) \equiv \left(1+\frac{1}{\varepsilon_{g,i}}\right) > 0$, and families whose demand for external child care is inelastic, that is $\left(1+\frac{g_i u_{q_g q_g}}{u_{q_g}}\right) \equiv \left(1+\frac{1}{\varepsilon_{g,i}}\right) < 0$. The effects of the subsidy for external child care thus depend inversely on family i's price elasticity of the demand for external child care time, $\varepsilon_{g,i} = \frac{dg_i}{dP_{q_g,i}} \frac{P_{q_g,i}}{g_i} < 0$. Therefore, family i's demand for external child care is elastic if $\varepsilon_{g,i} < -1$ and inelastic if $\varepsilon_{g,i} > -1$ holds.[1]

$$\frac{\partial n_i}{\partial \beta} = -\pi\left(g_i s_{nn} + n_i s_{q_g n} + g_i n_i i_n\right) = \frac{\pi}{D_i} u_{q_g} u_{q_h q_h} u_{q_z q_z}\left(u_c - n_i u_{cc} P_{n,i}\right)\left(1+\frac{1}{\varepsilon_{g,i}}\right) \gtreqless 0 \qquad (13)$$

The size of the effect on fertility in (13) depends on the secondary earner's income. A higher β decreases the net price of a child, $P_{n,i}$, which induces as before a positive income effect, $-\pi g_i n_i i_n$, and a positive substitution effect on the demand for the number of children,

[1] Compare to Atikson and Stiglitz (1972) for a detailed derivation of the elasticity of marginal utility of consumption

$-\pi g_i s_{nn}$. Both effects exceed the negative substitution effect of the marginal cost of bought-in child care on the demand for quantity, $-\pi n_i s_{q_g n}$, if $\varepsilon_{g,i} > -1$ holds. For families with $\varepsilon_{g,i} < -1$, the negative substitution effect dominates the other two effects and the total effect on fertility is thus negative. This negative substitution effect arises because a higher β decreases the net price of a child less than the net price of external child care. Thus the relative price between the quantity of children and child care increases so that the family decides for fewer children relative to the demand for child care.

Considering the effect of an increase of β on the quality of children, we find an ambiguous impact on parental time and a positive impact on the time children spend in external child care.

$$q_h \frac{\partial h_i}{\partial \beta} = \pi \left(g_i s_{nq_h} + n_i s_{q_g q_h} + g_i n_i i_{q_h} \right) = -\frac{\pi}{D_i} u_{q_g} u_{q_z q_z} P_{q_h,i} \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i} \right) \left(1 + \frac{1}{\varepsilon_{g,i}} \right) \gtreqless 0 \quad (14)$$

$$q_g \frac{\partial g_i}{\partial \beta} = -\pi \left(g_i s_{nq_g} + n_i s_{q_g q_g} + g_i n_i i_{q_g} \right) = \frac{\pi}{D_i} \left[\left(u_c u_{q_z q_z} P_{q_h,i}^2 + u_c u_{q_h q_h} P_{q_z,i}^2 - g_i u_{q_h q_h} u_{q_z q_z} P_{q_g,i} \right) \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i} \right) + n_i u_c u_{q_h q_h} u_{q_z q_z} \left(u_{cc} P_{n,i}^2 + u_{nn} \right) \right] > 0 \quad (15)$$

The effect of an increase of the subsidy on external child care on parental time in equation (14) is positive if $\varepsilon_{g,i} > -1$ and negative if $\varepsilon_{g,i} < -1$ holds.

Both of the substitution effects in (14) are negative for the following reason: A higher β decreases the price of a child but not the opportunity cost of parental leave to spend time with children. Hence, the quantity of children becomes relatively less costly than increasing the quality by staying at home. Furthermore, as the net cost of external child care decreases it becomes relatively more attractive to buy more child care on the market than to provide own parental time for the children. However, the positive income effect, $\pi g_i n_i i_{q_h}$, exceeds the negative substitution effects for families with $\varepsilon_{g,i} > -1$. As before, the size of the total effect increases with the secondary earner's income.

Regarding the demand for external child care in (15), both the income effect, $-\pi g_i n_i i_{q_g}$, and the own substitution effect of the marginal costs of bought-in child care on the time the child spends in external child care, $-\pi n_i s_{q_g q_g}$, are positive and they exceed the negative substitution effect of the marginal cost of quantity on the time the child spends in child care,

$-\pi g_i s_{nq_g}$. The more the government subsidizes child care, the more family i takes advantage of external child care and the share of external child care in total time spent with the children increases disproportionately.

Proposition 2: *The subsidy for external child care affects both the marginal price for quantity and for external child care, and has thus a stronger impact on high income secondary earners. Child care subsidies have a positive (negative) effect on fertility and parental time for families whose demand for external child care is inelastic (elastic) and increase the demand for external child care for all families.*

The effects of an increase of the parental leave payments affect also both the quantity and quality of children and increase in the secondary earner's net wage rate, $\widehat{w}_i$. We now have to differentiate between two groups of families: families whose demand for parental child care time is elastic, that is $\left(1+\frac{h_i u_{q_h q_h}}{u_{q_h}}\right) \equiv \left(1+\frac{1}{\varepsilon_h}\right) > 0$, and families whose demand for parental child care time is inelastic, that is $\left(1+\frac{h_i u_{q_h q_h}}{u_{q_h}}\right) \equiv \left(1+\frac{1}{\varepsilon_h}\right) < 0$. The effects of the subsidy for external child care thus depend inversely on the family i's price elasticity of parental child care time, $\varepsilon_{h,i} = \frac{dh_i}{dP_{q_h,i}} \frac{P_{q_h,i}}{h_i} < 0$. Therefore, family i's demand for external child care is elastic if $\varepsilon_{h,i} < -1$ and inelastic if $\varepsilon_{h,i} > -1$ holds.

The impact of an increase of the parental leave payment on fertility is thus ambiguous

$$\frac{\partial n_i}{\partial \gamma} = -\widehat{w}_i\left(h_i s_{nn} - n_i s_{q_h n} + h_i n_i i_n\right) = \frac{\widehat{w}_i}{D_i} u_{q_h} u_{q_g q_g} u_{q_z q_z}\left(u_c - n_i u_{cc} P_{n,i}\right)\left(1+\frac{1}{\varepsilon_h}\right) \gtreqless 0 \qquad (16)$$

The size of this effect in (16) depends next to the secondary earner's net wage rate, $\widehat{w}_i$, on the number of children of family i, n_i, and the marginal price for quantity, $P_{n,i}$. If $\varepsilon_{h,i} > -1$ holds, the positive income effect, $-\widehat{w}_i h_i n_i i_n$, and the positive substitution effect of the marginal price of quantity on the demand for children, $-\widehat{w}_i h_i s_{nn}$, exceed the negative substitution effect of the marginal costs of parental child care time on the demand for children, $-\widehat{w}_i n_i s_{q_h n}$, and the total effect on fertility is thus positive. The last substitution effect is negative because higher paternal leave payments reduce the opportunity costs of staying home with the children. This is an implicit tax on continued work which decreases labor supply. At the same time it improves the quality of the children. Since the net price of the number of children decreases by less than the net price of parental time, the quality of children increases relatively to the quantity.

The overall effect of an increase of γ on parental time is positive while the effect on external child care time is ambiguous

$$\begin{aligned} q_h \frac{\partial h_i}{\partial \gamma} &= \widehat{w}_i\left(h_i s_{nq_h} - n_i s_{q_h q_h} + h_i n_i i_{q_h}\right) \\ &= \frac{\widehat{w}_i}{D_i}\left[\left(u_c u_{q_g q_g} P_{q_z,i}^2 + u_{q_c} u_{q_z q_z} P_{q_g,i}^2 - h_i u_{q_g q_g} u_{q_z q_z} P_{q_h,i}\right)\left(n_i u_{cc} u_{nn} \right.\right. \\ &\quad \left.\left. + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right) + n_i u_c u_{q_g q_g} u_{q_z q_z}\left(u_{cc} P_{n,i}^2 + u_{nn}\right)\right] > 0 \end{aligned} \tag{17}$$

$$\begin{aligned} q_g \frac{\partial g_i}{\partial \gamma} &= -\widehat{w}_i\left(h_i s_{nq_g} - n_i s_{q_h q_g} + h_i n_i i_{q_g}\right) \\ &= -\frac{\widehat{w}_i}{D_i} u_{q_h} u_{q_z q_z} P_{q_g,i}\left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left(1 + \frac{1}{\varepsilon_h}\right) \gtreqless 0 \end{aligned} \tag{18}$$

Concerning parental time in (17), the positive income effect, $\widehat{w}_i h_i n_i i_{q_h}$, and the positive own substitution effect of the marginal costs of parental time on the demand for parental time, $-\widehat{w}_i n_i s_{q_h q_h}$, exceed the negative substitution effect of the marginal price for quantity on the demand for parental time, $\widehat{w}_i h_i s_{nq_h}$. Regarding the time the children spend at external child care on the other hand in (18), the two substitution effects are negative and only fall short of the positive income effect, $-\widehat{w}_i h_i n_i i_{q_g}$, if $\varepsilon_{h,i} > -1$ holds. For family i with $\varepsilon_{h,i} < -1$, the total effect on g_i is thus negative.

The more the government subsidizes parental child care, the more parents stay at home with their children and the share of parental child care in total time spent with the children increases disproportionately.

Proposition 3*: Increasing γ decreases both the marginal price for quantity and for parental time and the impact is thus stronger for high income secondary earners. Parental leave payments have a positive (negative) effect on fertility and the demand for external child care for families whose demand for parental child care time is inelastic (elastic) and increase parental time for all families.*

Now we compare mutually the effectiveness of the three policy instruments in raising the number of children and increasing the secondary earner's labor supply. We consider a budget neutral substitution of two instruments in order to determine the size of the effect. The government's budget is given by:

$$t(Y + \bar{w}L) = \alpha\bar{n} + \beta\pi\bar{g}\bar{n} + \gamma\bar{\hat{w}}\bar{h}\bar{n} \tag{19}$$

where $\bar{n}$, $\bar{g}$, $\bar{h}$, $\bar{w}$, and $\bar{\hat{w}}$ represent the average number of children, use of external child care, parental child care, wage and net wage respectively.

Looking at first at an exchange of child benefits and subsidies for bought-in child care, the budget keeps constant if $d\alpha = -\pi\bar{g}d\beta$. In this case we have to differentiate between two groups of families: families who initially consume above average and families who initially consume below average external child care. Families who consume below average external child care, that is $g_i \le \bar{g}$, are net contributors to the subsidy while families who consume above average bought-in child care, that is $g_i > \bar{g}$, are net recipients. As both the child benefit and the subsidy for external child care depend on the number of children, there is no redistribution with respect to the number of children.

Taking account of equations (10) and (13), an increase of the subsidy β accompanied by a reduction of the child benefit α so as to keep the budget constant has a negative effect on the number of children for all net contributors and is ambiguous for net recipients depending on the price elasticity of the demand for external child care time

$$\begin{aligned}\frac{dn_i}{d\beta}\Big|_{d\alpha=-\pi\bar{g}d\beta} &= \frac{\partial n_i}{\partial\beta}d\beta + \frac{\partial n_i}{\partial\alpha}d\alpha = -\pi\left[(s_{nn} + n_i i_n)(g_i - \bar{g}) + n_i s_{q_g n}\right] \\ &= \frac{\pi}{D_i}u_{q_g}u_{q_h q_h}u_{q_z q_z}\left(u_c - n_i u_{cc}P_{n,i}\right)\left[1 + \frac{(g_i - \bar{g})u_{q_g q_g}}{u_{q_g}}\right]\end{aligned} \tag{20}$$

The subsidy for child care thus has a weaker effect on fertility than child benefits for all families with $g_i \le \bar{g}$. In case of a subsidy for external child care the money is bound to this service whereas the child benefits are paid unconditional. Therefore, an increase in β has a smaller impact on fertility for families with $g_i \le \bar{g}$ than an increase of α.

For the secondary earner's parental time, h_i, the effect of a budget neutral increase of β in (14) falls also short off the effect of α in (11) for net contributors

$$\frac{dh_i}{d\beta}|_{d\alpha=-\pi\bar{g}d\beta} = \frac{\partial h_i}{\partial \beta}d\beta + \frac{\partial h_i}{\partial \alpha}d\alpha = \pi\left[\left(s_{nq_h} + n_i i_{q_h}\right)(g_i - \bar{g}) + n_i s_{q_g q_h}\right]$$
$$= -\frac{\pi}{D_i}u_{q_g}u_{q_z q_z}P_{q_h,i}\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[1 + \frac{(g_i - \bar{g})u_{q_g q_g}}{u_{q_g}}\right] \quad (21)$$

Increasing the subsidy for family i consuming $g_i \leq \bar{g}$, thus leads to a decrease in the secondary earner's parental time which is equivalent to an increase in her employment rate.
The effect of the budget neutral increase of β for net recipients however is ambiguous and depends on family i's price elasticity of the demand for external child care.
Hence, supporting the demand for child care is only a more promising way of fostering fertility than child benefits for net recipients if their demand for external child care is elastic. At the same time it decreases the labor supply of the secondary earner. For all families, the effect increases with $P_{n,i}$ and it therefore depends on the secondary earner's income.
A comparison of the effect of the child benefit in (12) to the effect of the subsidy on bought-in child care in (15) shows that a budget neutral increase of β clearly exceeds the effect of α for net consumers and that the budget neutral exchange is also likely to be beneficial for net contributors.

$$\frac{dg_i}{d\beta}|_{d\alpha=-\pi\bar{g}d\beta} = \frac{\partial g_i}{\partial \beta}d\beta + \frac{\partial g_i}{\partial \alpha}d\alpha = -\pi\left[\left(s_{nq_g} + n_i i_{q_g}\right)(g_i - \bar{g}) + n_i s_{q_g q_g}\right]$$
$$= -\frac{\pi}{D_i}\left\{\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[u_{q_h q_h}u_{q_z q_z}P_{q_g,i}(g_i - \bar{g}) \right.\right. \quad (22)$$
$$\left.\left. - u_c u_{q_z q_z}P_{q_h,i}^2 - u_c u_{q_h q_h}P_{q_z,i}^2\right] - n_i u_c u_{q_h q_h}u_{q_z q_z}\left(u_{cc}P_{n,i}^2 + u_{nn}\right)\right\}$$

Combining our results from (21) and (22), we can conclude that for most families the budget-neutral policy exchange leads to a decrease of parental child care and an increase in bought-in child care. Depending on the price elasticity, net recipients might also increase their parental time. For this group of families, the policy exchange may lead to a decrease in the secondary earner's labor supply.

The same policy exchange also leads to both less parental and child-specific consumption for net contributors while the effect is ambiguous for the others (See Appendix B). This negative effect on consumption can be explained by the fact that in the case of the subsidy the money is bound to external child care and thus does not benefit consumption in the way the child benefit does.

Proposition 4: *A budget-neutral increase of a subsidy for external child care accompanied by a decrease of child benefits has a negative effect on fertility and a positive effect on the secondary earner's labor supply for family i consuming $g_i \leq \bar{g}$. The same policy exchange leads to an increase in the demand for external child care for families with $g_i > \bar{g}$.*

Comparing child benefits and the rate of parental leave payments, a budget neutral substitution requires $d\alpha = -\bar{\hat{w}}\bar{h}d\gamma$. In this case we have to again differentiate between two groups of families: Net contributors, that is $\hat{w}_i h_i < \bar{\hat{w}}\bar{h}$, and net recipients, that is $\hat{w}_i h_i > \bar{\hat{w}}\bar{h}$, of the parental leave payments. As both the child benefit and the parental leave payments depend on the number of children, there is no redistribution with respect to the number of children.

Taking account of (10) and (16), the effect of increasing the parental leave payments on the number of children is negative for secondary earners whose income weighted parental time is smaller than the average, that is $\hat{w}_i h_i \leq \bar{\hat{w}}\bar{h}$

$$\begin{aligned} \frac{dn_i}{d\gamma}\Big|_{d\alpha=-\bar{\hat{w}}\bar{h}d\gamma} &= \frac{\partial n_i}{\partial \gamma}d\gamma + \frac{\partial n_i}{\partial \alpha}d\alpha = \left(\bar{\hat{w}}\bar{h} - \hat{w}_i h_i\right)(s_{nn} + n_i i_n) + \hat{w}_i n_i s_{q_h n} \\ &= \frac{\hat{w}_i}{D_i} u_{q_h} u_{q_g q_g} u_{q_z q_z}\left(u_c - n_i u_{cc} P_{n,i}\right)\left[1 + \frac{\left(\hat{w}_i h_i - \bar{\hat{w}}\bar{h}\right)u_{q_h q_h}}{\hat{w}_i u_{q_h}}\right] \end{aligned} \tag{23}$$

The parental leave payment therefore has in case of this group of secondary earners a weaker effect on fertility than child benefits. Hence, for families with low opportunity costs of staying home and taking care of their children, a child benefit is a more effective instrument to set incentives for children than a parental leave payment.

Looking at the effect of the policy exchange on the time children spend at external child care, we find that the budget-neutral exchange of α and γ also leads to a decrease of the demand for external child care of family i if $\hat{w}_i h_i \leq \bar{\hat{w}}\bar{h}$

$$\begin{aligned} \frac{dg_i}{d\gamma}\Big|_{d\alpha=-\bar{\hat{w}}\bar{h}d\gamma} &= \frac{\partial g_i}{\partial \gamma}d\gamma + \frac{\partial g_i}{\partial \alpha}d\alpha = \left(\bar{\hat{w}}\bar{h} - \hat{w}_i h_i\right)\left(s_{n q_g} + n_i i_{q_g}\right) + \hat{w}_i n_i s_{q_h q_g} \\ &= -\frac{\hat{w}_i}{D_i} u_{q_h} u_{q_z q_z} P_{q_g,i}\left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[1 + \frac{\left(\hat{w}_i h_i - \bar{\hat{w}}\bar{h}\right)u_{q_h q_h}}{\hat{w}_i u_{q_h}}\right] \end{aligned} \tag{24}$$

For secondary earners with $\hat{w}_i h_i > \bar{\hat{w}}\bar{h}$ the effects in (23) and (24) are both ambiguous. However, for family i having initial opportunity costs $\hat{w}_i h_i \gg \bar{\hat{w}}\bar{h}$ such that $\left[\left(\hat{w}_i h_i - \bar{\hat{w}}\bar{h}\right)u_{q_h q_h} + \hat{w}_i u_{q_h}\right] < 0$, the budget-neutral increase of parental leave payments leads to an increase in both fertility and the demand for external child care. Net recipients of parental leave payments therefore may benefit more from an increase in γ than from an increase in α.

Regarding the effect of a budget-neutral exchange on parental time, h_i, the impact of an increase in γ by $d\alpha = -\bar{\bar{w}}\bar{h}d\gamma$ in (17) exceeds the effect of α in (11) for secondary earners with large initial opportunity costs, $\hat{w}_i h_i \geq \bar{\bar{w}}\bar{h}$

$$\begin{aligned}\frac{dh_i}{d\gamma}\Big|_{d\alpha=-\bar{\bar{w}}\bar{h}d\gamma} &= \frac{\partial h_i}{\partial\gamma}d\gamma + \frac{\partial h_i}{\partial\alpha}d\alpha = -\left(\bar{\bar{w}}\bar{h} - \hat{w}_i h_i\right)\left(s_{nq_h} + n_i i_{q_h}\right) - \hat{w}_i n_i s_{q_h q_h} \\ &= -\frac{1}{D_i}\left\{\left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[\left(\hat{w}_i h_i - \bar{\bar{w}}\bar{h}\right)u_{q_g q_g} u_{q_z q_z} P_{q_h,i}\right.\right. \\ &\left.\left. - \hat{w}_i u_c\left(u_{q_g q_g} P_{q_z,i}^2 + u_{q_z q_z} P_{q_g,i}^2\right)\right] - \hat{w}_i n_i u_c u_{q_g q_g} u_{q_z q_z}\left(u_{cc} P_{n,i}^2 + u_{nn}\right)\right\}\end{aligned} \quad (25)$$

An increase of the parental leave rate on the expense of the child benefits has thus a positive impact on parental time for this group of secondary earners. The effect on families with $\hat{w}_i h_i < \bar{\bar{w}}\bar{h}$ is ambiguous but also likely to be positive.

Combining our results from (24) and (25), we can conclude that the policy exchange has a negative effect on the demand of external child care for families with $\hat{w}_i h_i \leq \bar{\bar{w}}\bar{h}$ and a positive effect on parental child care for secondary earners with $\hat{w}_i h_i \geq \bar{\bar{w}}\bar{h}$. Only families who initially have very large opportunity costs of staying at home, may use the additional parental leave payments to consume more external child care.

The policy exchange of child benefits and parental leave payments by $d\alpha = -\bar{\bar{w}}hd\gamma$ leads also to less parental and child-specific consumption for families with $\bar{\bar{w}}\bar{h} > \hat{w}_i h_i$. (See Appendix B). This reduction in parental and child-specific consumption can be explained by the increased parental child care time and thus reduced family income.

<u>Proposition 5</u>*: A budget-neutral increase of* γ *accompanied by a decrease of* α *leads to a decrease (increase) in fertility and a lower (higher) demand for external child care for secondary earners with* $\hat{w}_i h_i \leq \bar{\bar{w}}\bar{h}$ *(*$\hat{w}_i h_i \gg \bar{\bar{w}}\bar{h}$*). The same policy exchange leads to an increase in parental child care for families with* $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$*.*

When comparing the effects of the budget-neutral increase of the subsidy for external child care and the parental leave payment such that $d\gamma = -\frac{\pi\bar{g}}{\bar{\bar{w}}\bar{h}}d\beta$

$$
\begin{aligned}
\frac{dn_i}{d\beta}\Big|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\widehat{w}}\bar{h}}d\beta} &= \frac{\partial n_i}{\partial\beta}d\beta + \frac{\partial n_i}{\partial \gamma}d\gamma \\
&= -\pi\left(g_i - \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)(s_{nn} + n_i i_n) - \pi n_i\left(s_{q_g n} + \frac{\bar{g}}{\bar{h}}\frac{\widehat{w}_i}{\bar{\widehat{w}}}s_{q_h n}\right) \\
&= -\frac{\pi}{D_i}u_{q_z q_z}(n_i u_{cc} P_{n,i} - u_c)\left[\left(g_i - \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)u_{q_h q_h}u_{q_g q_g}\right. \\
&\left. + u_c\left(u_{q_h q_h}P_{q_g,i} - \frac{\bar{g}}{\bar{h}}\frac{\widehat{w}_i}{\bar{\widehat{w}}}u_{q_g q_g}P_{q_h,i}\right)\right]
\end{aligned} \tag{26}
$$

we find that a budget-neutral increase in β leads to an increase in fertility for family i consuming $\left(g_i > \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)$ if $\left(u_{q_h q_h}P_{q_g,i} < \frac{\bar{g}}{\bar{h}}\frac{\widehat{w}_i}{\bar{\widehat{w}}}u_{q_g q_g}P_{q_h,i}\right)$ holds. For families whose relative demand for external child care is larger than the relative opportunity costs of staying at home, that is $\left(\frac{g_i}{\bar{g}} > \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\right)$, and whose elasticity of marginal utility w.r.t. h is smaller than w.r.t. g weighted by the relative net income at the average, that is $\left(\frac{u_{q_h q_h}}{u_{q_h}}\bar{h} < \frac{\widehat{w}_i}{\bar{\widehat{w}}}\frac{u_{q_g q_g}}{u_{q_g}}\bar{g}\right)$, the subsidy for external child care is a more effective instrument to set incentives for children than a parental leave payment.

The budget-neutral increase of β accompanied by a decerase in γ has also a positive effect on the time family i's children spend in external child care if $\left(g_i > \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)$ and likely also for the other group.

$$
\begin{aligned}
\frac{dg_i}{d\beta}\Big|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\widehat{w}}\bar{h}}d\beta} &= \frac{\partial q_g}{\partial\beta}d\beta + \frac{\partial q_g}{\partial \gamma}d\gamma \\
&= -\pi\left(g_i - \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)\left(s_{n q_g} + n_i i_{q_g}\right) - \pi n_i\left(s_{q_g q_g} + \frac{\bar{g}}{\bar{h}}\frac{\widehat{w}_i}{\bar{\widehat{w}}}s_{q_h q_g}\right) \\
&= -\frac{\pi}{D_i}\left\{\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[\left(g_i - \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)u_{q_h q_h}u_{q_z q_z}P_{q_g,i}\right.\right. \\
&\left. - u_c\left(u_{q_z q_z}P^2_{q_h,i} + u_{q_h q_h}P^2_{q_z,i} + \frac{\bar{g}}{\bar{h}}\frac{\widehat{w}_i}{\bar{\widehat{w}}}u_{q_z q_z}P^2_{q_g,i}\right)\right] \\
&\left. - n_i u_c u_{q_h q_h}u_{q_z q_z}\left(u_{cc}P^2_{n,i} + u_{nn}\right)\right\}
\end{aligned} \tag{27}
$$

Regarding the parental time, h_i, the budget-neutral increase of β leads to a decrease if $\left(g_i < \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\bar{g}\right)$ and likely also for the other group.

$$\frac{dh_i}{d\beta}\Big|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\hat{w}}\bar{h}}d\beta} = \frac{\partial h_i}{\partial \beta}d\beta + \frac{\partial h_i}{\partial \gamma}d\gamma$$
$$= \pi\left(g_i - \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\bar{g}\right)\left(s_{nq_h} + n_i i_{q_h}\right) + \pi n_i\left(s_{q_g q_h} + \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\hat{w}}}s_{q_h q_h}\right)$$
$$= -\frac{\pi}{D_i}\left\{\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[\left(g_i - \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\bar{g}\right)u_{q_g q_g}u_{q_z q_z}P_{q_h,i}\right.\right. \quad (28)$$
$$\left. + u_c u_{q_z q_z}P_{q_h,i}^2 + \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\hat{w}}}u_c\left(u_{q_g q_g}P_{q_z,i}^2 + u_{q_z q_z}P_{q_g,i}^2\right)\right]$$
$$\left. + \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\hat{w}}}n_i u_c u_{q_g q_g}u_{q_z q_z}\left(u_{cc}P_{n,i}^2 + u_{nn}\right)\right\}$$

Therefore, the budget-neutral increase of the subsidy for external child care has a negative effect on parental child care for all secondary earners whose relative demand for external child care is smaller than the relative opportunity costs of staying at home, that is $\left(\frac{g_i}{\bar{g}} < \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\right)$. Hence, this group's labor supply will increase due to the policy exchange. The effect is also likely to be negative for family i consuming $\left(\frac{g_i}{\bar{g}} > \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\right)$.

Combined with the result for the effect of the budget-neutral policy exchange on g_i in (27), we find that the ratio of external to parental child care will increase for all families. Increasing the subsidy for external child care has thus a positive effect on secondary earner i's labor supply.

Proposition 6*: A budget-neutral increase of β accompanied by a decrease of γ has a positive effect on the secondary earner's labor supply of households with $\frac{g_i}{\bar{g}} < \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}$.The same budget-neutral exchange leads to an increase in both fertility and the demand for external child care for all families whose relative demand for external child care is larger than the relative opportunity costs of staying at home, that is $\frac{g_i}{\bar{g}} > \frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}$, if $\left(\frac{u_{q_h q_h}}{u_{q_h}}\bar{h} < \frac{\hat{w}_i}{\bar{\hat{w}}}\frac{u_{q_g q_g}}{u_{q_g}}\bar{g}\right)$ holds.*

4 Welfare Analysis

In the welfare analysis we analyze the redistribution effects of budget-neutral policy exchanges on different income groups. We assume that the benevolent government maximizes the household's indirect utility function subject $V(\alpha, \beta, \gamma)$ to the government's budget constraint in (19). The maximization problem can be written as:

$$\max_{\alpha,\beta,\gamma} V(\alpha,\beta,\gamma) = u\left(c_i(\alpha,\beta,\gamma), n_i(\alpha,\beta,\gamma), q_i\big(h_i(\alpha,\beta,\gamma), g_i(\alpha,\beta,\gamma), z_i(\alpha,\beta,\gamma)\big)\right) \\ + \mu\left[t\left(Y + \bar{w}\left(1 - \bar{h}(\alpha,\beta,\gamma)\bar{n}(\alpha,\beta,\gamma)\right)\right) - \alpha\bar{n}(\alpha,\beta,\gamma) \\ - \beta\pi\bar{g}(\alpha,\beta,\gamma)\bar{n} - \gamma\bar{\bar{w}}\bar{h}(\alpha,\beta,\gamma)\bar{n}(\alpha,\beta,\gamma)\right] \quad (29)$$

The total derivative of $V(\alpha,\beta,\gamma)$ is then given by:

$$dV = \left\{\frac{\partial u}{\partial c_i}\frac{\partial c_i}{\partial \alpha} + \frac{\partial u}{\partial n_i}\frac{\partial n_i}{\partial \alpha} + \frac{\partial u}{\partial q_h}\frac{\partial h_i}{\partial \alpha} + \frac{\partial u}{\partial q_g}\frac{\partial g_i}{\partial \alpha} + \frac{\partial u}{\partial q_z}\frac{\partial z_i}{\partial \alpha} \right. \\ \left. - \mu\left[\left(t\bar{w} + \gamma\bar{\bar{w}}\right)\left(\frac{\partial \bar{h}}{\partial \alpha}\bar{n} + \bar{h}\frac{\partial \bar{n}}{\partial \alpha}\right) + \bar{n} + \alpha\frac{\partial \bar{n}}{\partial \alpha} + \beta\pi\left(\frac{\partial \bar{g}}{\partial \alpha}\bar{n} + \bar{g}\frac{\partial \bar{n}}{\partial \alpha}\right)\right]\right\} d\alpha \\ + \left\{\frac{\partial u}{\partial c_i}\frac{\partial c_i}{\partial \beta} + \frac{\partial u}{\partial n_i}\frac{\partial n_i}{\partial \beta} + \frac{\partial u}{\partial q_h}\frac{\partial h_i}{\partial \beta} + \frac{\partial u}{\partial q_g}\frac{\partial g_i}{\partial \beta} + \frac{\partial u}{\partial q_z}\frac{\partial z_i}{\partial \beta} \right. \\ \left. - \mu\left[\left(t\bar{w} + \gamma\bar{\bar{w}}\right)\left(\frac{\partial \bar{h}}{\partial \beta}\bar{n} + \bar{h}\frac{\partial \bar{n}}{\partial \beta}\right) + \alpha\frac{\partial \bar{n}}{\partial \beta} + \pi\bar{g}\bar{n} + \beta\pi\left(\frac{\partial \bar{g}}{\partial \beta}\bar{n} + \bar{g}\frac{\partial \bar{n}}{\partial \beta}\right)\right]\right\} d\beta \\ + \left\{\frac{\partial u}{\partial c_i}\frac{\partial c_i}{\partial \gamma} + \frac{\partial u}{\partial n_i}\frac{\partial n_i}{\partial \gamma} + \frac{\partial u}{\partial q_h}\frac{\partial h_i}{\partial \gamma} + \frac{\partial u}{\partial q_g}\frac{\partial g_i}{\partial \gamma} + \frac{\partial u}{\partial q_z}\frac{\partial z_i}{\partial \gamma} \right. \\ \left. - \mu\left[\left(t\bar{w} + \gamma\bar{\bar{w}}\right)\left(\frac{\partial \bar{h}}{\partial \gamma}\bar{n} + \bar{h}\frac{\partial \bar{n}}{\partial \gamma}\right) + \alpha\frac{\partial \bar{n}}{\partial \gamma} + \beta\pi\left(\frac{\partial \bar{g}}{\partial \gamma}\bar{n} + \bar{g}\frac{\partial \bar{n}}{\partial \gamma}\right) + \bar{\bar{w}}\bar{h}\bar{n}\right]\right\} d\gamma \quad (30)$$

First, we keep the parental leave rate, γ, constant and consider a budget neutral substitution of child benefits, α, and subsidies for external child care, β. Taking account of the results of the comparative statics, an increase in β accompanied by a reduction in α keeps the government's budget constant if $d\alpha = -\pi\bar{g}d\beta$:

$$\frac{dV}{d\beta}\Big|_{d\alpha=-\pi\bar{g}d\beta} = \lambda\pi n_i(g_i - \bar{g}) \\ + \lambda\pi\bar{n}\frac{\mu}{\bar{D}}\left\{\left[\left(t\bar{w} + \gamma\bar{\bar{w}}\right)u_{q_zq_z}\overline{P_{q_h}P_{q_g}} - \beta\pi\left(u_{q_zq_z}\overline{P_{q_h}^2} + u_{q_hq_h}\overline{P_{q_z}^2}\right)\right]\left(\bar{n}u_{cc}u_{nn} \right.\right. \\ \left. + 2u_nu_{cc} - \frac{u_c^2}{\bar{n}}\right) + u_{q_hq_h}u_{q_zq_z}\overline{P_{q_g}}\left[\alpha + \beta\pi\bar{g} + \bar{h}\left(\gamma\bar{\bar{w}} + t\bar{w}\right)\right]\left(u_{cc}\overline{P_n} - \frac{u_c}{\bar{n}}\right) \\ \left. - \beta\pi\bar{n}u_{q_hq_h}u_{q_zq_z}\left(u_{cc}\overline{P_n^2} + u_{nn}\right)\right\} \quad (31)$$

where the determinant of the bordered Hessian matrix for the average consumer is negative ($\bar{D} < 0$). This policy exchange leads to an increase of welfare for family i having more than the average number of children, that is $n_i > \bar{n}$, and consuming more than the average amount of external child care, that is $g_i > \bar{g}$, if the second term on the right hand sight is positive or relatively small. As before, we observe redistribution with respect to the consumption of external child care but in case of the parents' welfare additionally with respect to the number of children due to the redistribution of income for family policies. The size of the effect in

(31) therefore depends on the number of children of family i and the price for external child care which influences the significance of a subsidy for bought-in child care.

When looking at the case of $\beta = 0$, we find that the policy exchange is positive for all families consuming at least the average time of external child care, that is $g_i \geq \bar{g}$, as the second term is positive:

$$\frac{dV}{d\beta}|_{d\alpha=-\pi\bar{g}d\beta,\beta=0} \\ = \lambda\pi n_i(g_i - \bar{g}) \\ + \lambda\pi\bar{n}u_{q_z q_z}\overline{P_{q_g}}\frac{\mu}{\bar{\bar{D}}}\left\{\overline{P_{q_h}}(t\bar{w} + \gamma\bar{\bar{w}})\left(\bar{n}u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{\bar{n}}\right) \\ + u_{q_h q_h}[\alpha + \bar{h}(\gamma\bar{\bar{w}} + t\bar{w})]\left(u_{cc}\overline{P_n} - \frac{u_c}{\bar{n}}\right)\right\} \quad (32)$$

The budget-neutral policy exchange of child benefits to a subsidy for external child care is therefore positive for all families consuming $g_i > \bar{g}$ starting at the introduction up to a certain initial provision level of the subsidy and depending on their number of children. The larger the initial provision level of the subsidy for external child care the smaller becomes the group of families who benefit from the policies exchange.

<u>Proposition 7</u>: *A budget-neutral increase of a subsidy on external child care accompanied by a decrease in child benefits leads to an increase in parental welfare for families consuming $g_i > \bar{g}$ if the subsidy is being introduced and up to a certain initial provision level. The size of the welfare effect depends on the family's own relative to the average number of children.*

Keeping β constant, a budget neutral substitution of child benefits and the rate of parental leave payments requires $d\alpha = -\bar{\bar{w}}\bar{h}d\gamma$. Taking account of our results in the comparative statics, an increase of γ accompanied by a reduction of α has the following effect on the parents' welfare:

$$\frac{dV}{d\gamma}|_{d\alpha=-\bar{\bar{w}}\bar{h}d\gamma} = \lambda(\hat{w}_i h_i - \bar{\bar{w}}\bar{h})n_i + \lambda\bar{\bar{w}}\bar{n}\frac{\mu}{\bar{\bar{D}}}\Big\{\Big[\beta\pi u_{q_zq_z}\overline{P_{q_h}P_{q_g}} - (t\bar{w}+\gamma\bar{\bar{w}})\Big(u_{q_gq_g}\overline{P_{q_z}^2} + u_{q_zq_z}\overline{P_{q_g}^2}\Big)\Big]\Big(\bar{n}u_{cc}u_{nn} + 2u_nu_{cc} - \frac{u_c^2}{\bar{n}}\Big) + \big[\alpha+\beta\pi\bar{g}+\bar{h}(t\bar{w}+\gamma\bar{\bar{w}})\big]u_{q_gq_g}u_{q_zq_z}\overline{P_{q_h}}\Big(u_{cc}\overline{P_n} - \frac{u_c}{\bar{n}}\Big) - (t\bar{w}+\gamma\bar{\bar{w}})\bar{n}u_{q_gq_g}u_{q_zq_z}\big(u_{cc}\overline{P_n^2} + u_{nn}\big)\Big\} \tag{33}$$

The welfare effect in (33) is positive for secondary earners whose opportunity costs for parental time are above average, that is $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$, and have an above average quantity of children, $n_i > \bar{n}$, if the last term is positive or relatively small. As before, we observe redistribution with respect to the income weighted parental child care time but in case of the parents' welfare additionally with respect to the number of children due to the redistribution of income for family policies. The size of this effect therefore depends on the secondary earner's net wage rate, her parental child care time, and the family's number of children.

When looking at the case of $\gamma = 0$, we find that the size of the welfare effect depends on the size of the average tax payments $t\bar{w}$ of secondary earners

$$\frac{dV}{d\gamma}|_{d\alpha=-\bar{\bar{w}}\bar{h}d\gamma,\gamma=0} = \lambda(\hat{w}_i h_i - \bar{\bar{w}}\bar{h})n_i + \lambda\bar{\bar{w}}\bar{n}\frac{\mu}{\bar{\bar{D}}}\Big\{\Big[\beta\pi u_{q_zq_z}\overline{P_{q_h}P_{q_g}} - t\bar{w}\Big(u_{q_gq_g}\overline{P_{q_z}^2} + u_{q_zq_z}\overline{P_{q_g}^2}\Big)\Big]\Big(\bar{n}u_{cc}u_{nn} + 2u_nu_{cc} - \frac{u_c^2}{\bar{n}}\Big) + (\alpha+\beta\pi\bar{g}+t\bar{w}\bar{h})u_{q_gq_g}u_{q_zq_z}\overline{P_{q_h}}\Big(u_{cc}\overline{P_n} - \frac{u_c}{\bar{n}}\Big) - t\bar{w}\bar{n}u_{q_gq_g}u_{q_zq_z}\big(u_{cc}\overline{P_n^2} + u_{nn}\big)\Big\} \tag{34}$$

If the average secondary earners' tax payments $t\bar{w}$ are relatively small, the welfare effect is likely to be positive for secondary earners whose opportunity costs for parental time are above average, that is $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$.

For $\gamma = 1$ we find that the size of the welfare effect depends only on the average wage rate of the secondary earners:

$$\frac{dV}{d\gamma}\Big|_{d\alpha=-\bar{\bar{w}}\bar{h}d\gamma,\gamma=1}$$

$$= \lambda\left(\hat{w}_i h_i - \bar{\bar{w}}\bar{h}\right)n_i + \lambda\bar{\bar{w}}\bar{n}\frac{\mu}{\bar{D}}\left\{\left[\beta\pi u_{q_z q_z}\overline{P_{q_h}P_{q_g}} - \bar{w}\left(u_{q_g q_g}\overline{P_{q_z}^2} + u_{q_z q_z}\overline{P_{q_g}^2}\right)\right]\left(\bar{n}u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{\bar{n}}\right) + \left(\alpha + \beta\pi\bar{g} + \bar{w}\bar{h}\right)u_{q_g q_g}u_{q_z q_z}\overline{P_{q_h}}\left(u_{cc}\overline{P_n} - \frac{u_c}{\bar{n}}\right) - \bar{w}\bar{n}u_{q_g q_g}u_{q_z q_z}\left(u_{cc}\overline{P_n^2} + u_{nn}\right)\right\} \quad (35)$$

The welfare effect is now positive for secondary earners whose opportunity costs for parental time are above average, that is $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$, if the average wage rate is relatively small. This shows us that with increasing provision level of parental leave payments, the size of the tax rate becomes less important and the welfare effect less likely to be positive. As described before, families with $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$ are net recipients of the parental leave payments. Nevertheless families with $\hat{w}_i > \bar{\bar{w}}$ finance the policy instruments and therefore only benefit from a budget-neutral increase in parental leave payments if the average secondary earners' income is low and there is thus little redistribution with respect to parental leave payments.

Proposition 8: *A budget-neutral increase of parental leave payments accompanied by a decrease in child benefits leads to an increase in parental welfare for families with* $\hat{w}_i h_i > \bar{\bar{w}}\bar{h}$ *from the introduction up to a certain initial provision level if the average tax payments of secondary earners are relatively small. The size of the welfare effect depends on the family's own relative to the average number of children.*

Keeping α constant, an increase of a subsidy for child care accompanied by a decrease of the rate of parental leave payments is budget neutral if $d\gamma = -\frac{\pi\bar{g}}{\bar{\bar{w}}\bar{h}}d\beta$. This equals a comparison of the two aforementioned welfare effects of the budget-neutral exchanges of α and β as well as α and γ. Substituting γ for β has the following effect on the parents' welfare:

$$
\begin{aligned}
\frac{dV}{d\beta}\Big|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\hat{w}}\bar{h}}d\beta} &= \lambda\pi n_i\left(g_i-\frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\bar{g}\right)\\
&+\lambda\pi\bar{n}\frac{\mu}{\bar{D}}\Bigg\{\Bigg[\beta\pi\left(u_{q_zq_z}\overline{P_{q_h}^2}+u_{q_hq_h}\overline{P_{q_z}^2}+\frac{\bar{g}}{\bar{h}}u_{q_zq_z}\overline{P_{q_h}P_{q_g}}\right)\\
&-\left(t\bar{w}+\gamma\bar{\hat{w}}\right)\left(\frac{\bar{g}}{\bar{h}}u_{q_zq_z}\overline{P_{q_g}^2}+\frac{\bar{g}}{\bar{h}}u_{q_gq_g}\overline{P_{q_z}}+u_{q_zq_z}\overline{P_{q_h}P_{q_g}}\right)\Bigg]\Bigg(\bar{n}u_{cc}u_{nn}\\
&+2u_nu_{cc}-\frac{u_c^2}{\bar{n}}\Bigg)\\
&-u_{q_zq_z}\left(u_{q_hq_h}\overline{P_{q_g}}-\frac{\bar{g}}{\bar{h}}u_{q_gq_g}\overline{P_{q_h}}\right)\left[\alpha+\beta\pi\bar{g}+\bar{h}\left(t\bar{w}+\gamma\bar{\hat{w}}\right)\right]\left(u_{cc}\overline{P_n}-\frac{u_c}{\bar{n}}\right)\\
&+\bar{n}u_{q_zq_z}\left[\beta\pi u_{q_hq_h}-\frac{\bar{g}}{\bar{h}}\left(t\bar{w}+\gamma\bar{\hat{w}}\right)u_{q_gq_g}\right]\left(u_{cc}\overline{P_n^2}+u_{nn}\right)\Bigg\}
\end{aligned}
\tag{36}
$$

The size of the welfare effect of the budget neutral increase of the subsidy for child care in (36) depends to a large extent on family i's ratio of consumption of g_i to h_i as well as on the average ratio of consumption of $\bar{g}$ to $\bar{h}$. The budget-neutral increase of the subsidy for external child care has a positive effect on the parents' welfare for all secondary earners whose ratio of own to average external child care is larger than their ratio of own to average opportunity costs of staying at home, that is $\left(\frac{g_i}{\bar{g}}>\frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\right)$, and who have more than the average number of children if the second term on the right hand side is positive or relatively small.

When looking at the case of $\beta=0$

$$
\begin{aligned}
&\frac{dV}{d\beta}\Big|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\hat{w}}\bar{h}}d\beta,\beta=0}\\
&= \lambda\pi n_i\left(g_i-\frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\bar{g}\right)\\
&-\lambda\pi\bar{n}\frac{\mu}{\bar{D}}\Bigg\{\Bigg[\left(t\bar{w}+\gamma\bar{\hat{w}}\right)\left(\frac{\bar{g}}{\bar{h}}u_{q_zq_z}\overline{P_{q_g}^2}+\frac{\bar{g}}{\bar{h}}u_{q_gq_g}\overline{P_{q_z}}+u_{q_zq_z}\overline{P_{q_h}P_{q_g}}\right)\Bigg]\Bigg(\bar{n}u_{cc}u_{nn}\\
&+2u_nu_{cc}-\frac{u_c^2}{\bar{n}}\Bigg)\\
&+u_{q_zq_z}\left(u_{q_hq_h}\overline{P_{q_g}}-\frac{\bar{g}}{\bar{h}}u_{q_gq_g}\overline{P_{q_h}}\right)\left[\alpha+\bar{h}\left(t\bar{w}+\gamma\bar{\hat{w}}\right)\right]\left(u_{cc}\overline{P_n}-\frac{u_c}{\bar{n}}\right)\\
&+\bar{n}\frac{\bar{g}}{\bar{h}}\left(t\bar{w}+\gamma\bar{\hat{w}}\right)u_{q_gq_g}u_{q_zq_z}\left(u_{cc}\overline{P_n^2}+u_{nn}\right)\Bigg\}
\end{aligned}
\tag{37}
$$

we find that the welfare effect is negative for all secondary earners whose ratio of own to average external child care is smaller than their ratio of own to average opportunity costs of staying at home, that is $\left(\frac{g_i}{\bar{g}}\leq\frac{\hat{w}_i h_i}{\bar{\hat{w}}\bar{h}}\right)$, if $u_{q_hq_h}\overline{P_{q_g}}>\frac{\bar{g}}{\bar{h}}u_{q_gq_g}\overline{P_{q_h}}$. This group of secondary earners therefore does not benefit from the budget-neutral increase of the subsidy for external child care as long as the provision level is low. For an increasing provision level, the welfare effect is ambiguous for families with $\left(\frac{g_i}{\bar{g}} \leq \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\right)$ and likely to be positive for families with $\left(\frac{g_i}{\bar{g}} > \frac{\widehat{w}_i h_i}{\bar{\widehat{w}}\bar{h}}\right)$.

Proposition 9: *A budget-neutral increase of a subsidy on external child care accompanied by a decrease in parental leave payments leads to a decrease in parental welfare for families whose ratio of own to average external child care is smaller than their ratio of own to average opportunity costs of staying at home if* $u_{q_h q_h}\overline{P_{q_g}} > \frac{\bar{g}}{\bar{h}} u_{q_g q_g}\overline{P_{q_h}}$ *and the provision level of the subsidy is low. The size of the welfare effect depends on the family's own relative to the average number of children.*

5 Conclusion

Summarizing our comparative static results and our results of the welfare analysis leads to the conclusion that the effects on fertility and parental welfare depend on the secondary earner's income and her demand for external and parental child care. We find that secondary earners consuming above average external child care benefit the most from a subsidy for bought-in child care with respect to both fertility and labor supply. For secondary earners spending relative to their net wage more than the average time at home with their children on the other hand, parental leave payments have the strongest impact on fertility but decrease their labor supply.

The analysis has shown that a budget-neutral increase of parental leave payments accompanied by a decrease of child benefits has a negative impact on the secondary earner's labor supply for all families. Regarding fertility, the effect depends on the size of the opportunity costs of parental child care relative to the average opportunity costs. Only for secondary earners who spend above average time at home relative to her net wage rate the policy exchange has a positive effect on fertility. Regarding the parents' welfare, a budget-neutral increase of parental leave payments together with a decrease in child benefits leads to an increase for families having above average opportunity costs for staying at home if the average tax payments of secondary earners are relatively small. With growing provision level of parental leave payments, the size of the tax rate becomes less important and the size of the welfare effect is driven by the average secondary earner’s wage rate. The size of the welfare effect also depends on the family’s own relative to the average number of children.

A budget-neutral increase of a subsidy on external child care accompanied by a decrease in child benefits on the other hand can have both a positive effect on the secondary earner’s labor supply and on fertility. We find that due to this policy exchange all secondary earners will take more advantage of external child care and thus increase their labor supply. The effect on fertility depends on the family’s demand for external child care: only for families consuming more than the average amount of bought-in child care the effect on fertility is positive. Concerning the parents’ welfare, a budget-neutral increase of a subsidy on external child care accompanied by a decrease in child benefits leads to an increase for families consuming above average external child care if the subsidy is being introduced and up to a certain provision level. A budget-neutral increase of a subsidy on external child care accompanied by a decrease in parental leave payments on the other hand has an ambiguous effect on parental welfare. For families whose ratio of own to average external child care is smaller than their ratio of own to average opportunity costs of staying at home the effect is likely to be negative if the provision level of the subsidy is low. The size of the welfare effects depend in both cases on the family’s own relative to the average number of children.

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Appendix

A: Derivation of the comparative statics results

Differentiation of the first-order conditions of individual utility maximization (6) - (8) yields:

$$\begin{pmatrix} u_{cc} & 0 & 0 & 0 & 0 & -1 \\ 0 & u_{nn} & -\lambda\widehat{w}_i(1-\gamma) & -\lambda(1-\beta)\pi & -\lambda B & -P_{n,i} \\ 0 & -\lambda\widehat{w}_i(1-\gamma) & u_{q_h q_h} & 0 & 0 & -P_{q_h,i} \\ 0 & -\lambda(1-\beta)\pi & 0 & u_{q_g q_g} & 0 & -P_{q_g,i} \\ 0 & -\lambda B & 0 & 0 & u_{q_z q_z} & -P_{q_z,i} \\ -1 & -P_{n,i} & -P_{q_h,i} & -P_{q_g,i} & -P_{q_z,i} & 0 \end{pmatrix} \begin{pmatrix} dc \\ dn \\ dh \\ dg \\ dz \\ d\lambda \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ -\lambda & -\lambda\pi g_i & -\lambda\widehat{w}_i h_i \\ 0 & 0 & -\lambda\widehat{w}_i n_i \\ 0 & -\lambda\pi n_i & 0 \\ 0 & 0 & 0 \\ -n_i & -\pi g_i n_i & -\widehat{w}_i h_i n_i \end{pmatrix} \begin{pmatrix} d\alpha \\ d\beta \\ d\gamma \end{pmatrix}$$

where the determinant of the matrix on the left-hand side is denoted by D.

The Cramer rule yields the following derivatives:

$$dc_i = \left(\lambda \frac{D_{21}}{D} + n_i \frac{D_{61}}{D}\right) d\alpha + \left(\lambda\pi g_i \frac{D_{21}}{D} + \lambda\pi n_i \frac{D_{41}}{D} + \pi g_i n_i \frac{D_{61}}{D}\right) d\beta + \left(\lambda\widehat{w}_i h_i \frac{D_{21}}{D} - \lambda\widehat{w}_i n_i \frac{D_{31}}{D} + \widehat{w}_i h_i n_i \frac{D_{61}}{D}\right) d\gamma$$

$$dn_i = \left(-\lambda \frac{D_{22}}{D} - n_i \frac{D_{62}}{D}\right) d\alpha + \left(-\lambda\pi g_i \frac{D_{22}}{D} - \lambda\pi n_i \frac{D_{42}}{D} - \pi g_i n_i \frac{D_{62}}{D}\right) d\beta + \left(-\lambda \widehat{w}_i h_i \frac{D_{22}}{D} + \lambda \widehat{w}_i n_i \frac{D_{32}}{D} - \widehat{w}_i h_i n_i \frac{D_{62}}{D}\right) d\gamma$$

$$dh_i = \left(\lambda \frac{D_{23}}{D} + n_i \frac{D_{63}}{D}\right) d\alpha + \left(\lambda\pi g_i \frac{D_{23}}{D} + \lambda\pi n_i \frac{D_{43}}{D} + \pi g_i n_i \frac{D_{63}}{D}\right) d\beta + \left(\lambda \widehat{w}_i h_i \frac{D_{23}}{D} - \lambda \widehat{w}_i n_i \frac{D_{33}}{D} + \widehat{w}_i h_i n_i \frac{D_{63}}{D}\right) d\gamma$$

$$dg_i = \left(-\lambda \frac{D_{24}}{D} - n_i \frac{D_{64}}{D}\right) d\alpha + \left(-\lambda\pi g_i \frac{D_{24}}{D} - \lambda\pi n_i \frac{D_{44}}{D} - \pi g_i n_i \frac{D_{64}}{D}\right) d\beta + \left(-\lambda \widehat{w}_i h_i \frac{D_{24}}{D} + \lambda \widehat{w}_i n_i \frac{D_{34}}{D} - \widehat{w}_i h_i n_i \frac{D_{64}}{D}\right) d\gamma$$

$$dz_i = \left(\lambda \frac{D_{25}}{D} + n_i \frac{D_{65}}{D}\right) d\alpha + \left(\lambda\pi g_i \frac{D_{25}}{D} + \lambda\pi n_i \frac{D_{45}}{D} + \pi g_i n_i \frac{D_{54}}{D}\right) d\beta + \left(\lambda \widehat{w}_i h_i \frac{D_{25}}{D} - \lambda \widehat{w}_i n_i \frac{D_{35}}{D} + \widehat{w}_i h_i n_i \frac{D_{565}}{D}\right) d\gamma$$

Using

$$\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) > 0$$

$$\left(u_{nn} + \frac{u_n}{n_i}\right) < 0$$

$$\left(h_i u_{q_h q_h} + u_{q_h}\right) \lesseqgtr 0$$

$$\left(g_i u_{q_g q_g} + u_{q_g}\right) \lesseqgtr 0$$

$$D_i = -\left(u_{q_g q_g} u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} u_{q_z q_z} P_{q_g,i}^2 + u_{q_h q_h} u_{q_g q_g} P_{q_z,i}^2\right)\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) - u_{q_h q_h} u_{q_g q_g} u_{q_z q_z}\left(u_{cc}P_{n,i}^2 + u_{nn}\right) < 0$$

$$D_{21} = -u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} P_{n,i} - \frac{u_c}{n_i}\left(u_{q_g q_g} u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} u_{q_z q_z} P_{q_g,i}^2 + u_{q_h q_h} u_{q_g q_g} P_{q_z,i}^2\right) < 0$$

$$D_{22} = -u_{cc}\left(u_{q_g q_g} u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} u_{q_z q_z} P_{q_g,i}^2 + u_{q_h q_h} u_{q_g q_g} P_{q_z,i}^2\right) - u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} > 0$$

$$D_{23} = D_{32} = -u_{q_g q_g} u_{q_z q_z} P_{q_h,i}\left(u_{cc}P_{n,i} - \frac{u_c}{n_i}\right) > 0$$

$$D_{24} = D_{42} = u_{q_h q_h} u_{q_z q_z} P_{q_g,i}\left(u_{cc}P_{n,i} - \frac{u_c}{n_i}\right) < 0$$

$$D_{25} = -u_{q_h q_h} u_{q_g q_g} P_{q_z,i}\left(u_{cc}P_{n,i} - \frac{u_c}{n_i}\right) > 0$$

$$D_{31} = u_{q_g q_g} u_{q_z q_z} P_{q_h,i}\left(u_{nn} + \frac{u_n}{n_i}\right) < 0$$

$$D_{33} = -\left(u_{q_g q_g} P_{q_z,i}^2 + u_{q_z q_z} P_{q_g,i}^2\right)\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) - u_{q_g q_g} u_{q_z q_z}\left(u_{cc}P_{n,i}^2 + u_{nn}\right) > 0$$

$$D_{34} = D_{43} = -u_{q_z q_z} P_{q_h,i} P_{q_g,i}\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) > 0$$

$$D_{35} = u_{q_g q_g} P_{q_h,i} P_{q_z,i}\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) < 0$$

$$D_{41} = -u_{q_h q_h} u_{q_z q_z} P_{q_g,i}\left(u_{nn} + \frac{u_n}{n_i}\right) > 0$$

$$D_{44} = -\left(u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} P_{q_z}^2\right)\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) - u_{q_h q_h} u_{q_z q_z}\left(u_{nn} + u_{cc}{P_{n,i}}^2\right) > 0$$

$$D_{45} = -u_{q_h q_h} P_{q_g,i} P_{q_z,i}\left(u_{cc}u_{nn} + \frac{2u_{cc}u_n}{n_i} - \frac{u_c^2}{n_i^2}\right) > 0$$

$$D_{61} = -u_{nn} u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} + \frac{u_c^2}{n_i^2}\left(u_{q_g q_g} u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} u_{q_z q_z} P_{q_g,i}^2 + u_{q_h q_h} u_{q_g q_g} P_{q_z,i}^2\right) < 0$$

$$D_{62} = u_{cc}\left[u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} P_{n,i} + \frac{u_c}{n_i}\left(u_{q_g q_g} u_{q_z q_z} P_{q_h,i}^2 + u_{q_h q_h} u_{q_z q_z} P_{q_g,i}^2 + u_{q_h q_h} u_{q_g q_g} P_{q_z,i}^2\right)\right] > 0$$

$$D_{63} = -u_{cc} u_{q_g q_g} u_{q_z q_z} P_{q_h,i}\left(u_{nn} + \frac{u_n}{n_i}\right) < 0$$

$$D_{64} = u_{cc} u_{q_h q_h} u_{q_z q_z} P_{q_g,i}\left(u_{nn} + \frac{u_n}{n_i}\right) > 0$$

$$D_{65} = -u_{cc} u_{q_h q_h} u_{q_g q_g} P_{q_z,i}\left(u_{nn} + \frac{u_n}{n_i}\right) < 0$$

and abbreviations for the substitution and income effects as follows:

$s_{nc} \equiv \lambda \frac{D_{21}}{D},\ s_{nn} \equiv \lambda \frac{D_{22}}{D},\ s_{nq_h} \equiv \lambda \frac{D_{23}}{D},\ s_{nq_g} \equiv \lambda \frac{D_{24}}{D},\ s_{nq_z} \equiv \lambda \frac{D_{25}}{D},$

$s_{q_h c} \equiv \lambda \frac{D_{31}}{D},\ s_{q_h n} \equiv \lambda \frac{D_{32}}{D},\ s_{q_h q_h} \equiv \lambda \frac{D_{33}}{D},\ s_{q_h q_g} \equiv \lambda \frac{D_{34}}{D},\ s_{q_h q_z} \equiv \lambda \frac{D_{35}}{D},$

$s_{q_g c} \equiv \lambda \frac{D_{41}}{D},\ s_{q_g n} \equiv \lambda \frac{D_{42}}{D},\ s_{q_g q_h} \equiv \lambda \frac{D_{43}}{D},\ s_{q_g q_g} \equiv \lambda \frac{D_{44}}{D},\ s_{q_g q_z} \equiv \lambda \frac{D_{45}}{D},$

$i_c \equiv \frac{D_{61}}{D},\ i_n \equiv \frac{D_{62}}{D},\ i_{q_h} \equiv \frac{D_{63}}{D},\ i_{q_g} \equiv \frac{D_{64}}{D},\ i_{q_z} \equiv \frac{D_{65}}{D}.$

the comparative static results in (10) – (18) follow. The effects for parental and child-specific consumption are the following

$$\frac{\partial c}{\partial \alpha} = s_{nc} + n_i i_c = -\frac{1}{D} u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} (n_i u_{nn} + u_n) > 0$$

$$\frac{\partial q_z}{\partial \alpha} = s_{nq_z} + n_i i_{q_z} = -\frac{1}{D} u_{q_h q_h} u_{q_g q_g} P_{q_z,i} \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i} \right) > 0$$

$$\frac{\partial c}{\partial \beta} = \pi \left(g_i s_{nc} + n_i s_{q_g c} + g_i n_i i_c \right) = -\frac{\pi}{D} u_{q_h q_h} u_{q_z q_z} \left(g_i u_{q_g q_g} + u_{q_g} \right) (n_i u_{nn} + u_n) \gtreqless 0$$

$$\frac{\partial q_z}{\partial \beta} = \pi \left(g_i s_{nq_z} + n_i s_{q_g q_z} + g_i n_i i_{q_z} \right)$$
$$= -\frac{\pi}{D} u_{q_h q_h} P_{q_z,i} \left(g_i u_{q_g q_g} + u_{q_g} \right) \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i} \right) \gtreqless 0$$

$$\frac{\partial c}{\partial \gamma} = \widehat{w}_i \left(h_i s_{nc} - n_i s_{q_h c} + h_i n_i i_c \right) = -\frac{\widehat{w}_i}{D} u_{q_g q_g} u_{q_z q_z} \left(h_i u_{q_h q_h} + u_{q_h} \right) (n_i u_{nn} + u_n) \gtreqless 0$$

$$\frac{\partial q_z}{\partial \gamma} = \widehat{w}_i \left(h_i s_{nq_z} - n_i s_{q_h q_z} + h_i n_i i_{q_z} \right)$$
$$= -\frac{\widehat{w}_i}{D} u_{q_g q_g} P_{q_z,i} \left(h_i u_{q_h q_h} + u_{q_h} \right) \left(n_i u_{cc} u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i} \right) \gtreqless 0$$

B: Derivation of welfare effects

Using

$$\bar{D} = -\left(u_{q_g q_g} u_{q_z q_z} \overline{P_{q_h}^2} + u_{q_h q_h} u_{q_z q_z} \overline{P_{q_g}^2} + u_{q_h q_h} u_{q_g q_g} \overline{P_{q_z}^2} \right) \left(u_{cc} u_{nn} + \frac{2u_{cc} u_n}{\bar{n}} - \frac{u_c^2}{\overline{\overline{n^2}}} \right)$$
$$- u_{q_h q_h} u_{q_g q_g} u_{q_z q_z} \left(u_{cc} \overline{P_n^2} + u_{nn} \right) < 0$$

and the comparative static results in (10) – (18) as well as the comparative static results for parental and child-specific consumption and the government's budget constraint in (19) the effects in (20) to (27) as well as (30) to (32) follow:

- <u>For $d\gamma = 0$</u>: Budget-neutral increase of β by $d\alpha = -\pi \bar{g} d\beta$

$$dc_i|_{d\alpha = -\pi \bar{g} d\beta} = \frac{\partial c}{\partial \beta} d\beta + \frac{\partial c}{\partial \alpha} d\alpha = \pi \left[(s_{nc} + n_i i_c)(g_i - \bar{g}) + n_i s_{q_g c} \right]$$
$$= -\frac{\pi}{D} (n_i u_{nn} + u_n) u_{q_h q_h} u_{q_z q_z} \left[(g_i - \bar{g}) u_{q_g q_g} + u_{q_g} \right]$$

$$dz_i|_{d\alpha=-\pi\bar{g}d\beta} = \frac{\partial q_z}{\partial\beta}d\beta + \frac{\partial q_z}{\partial\alpha}d\alpha = \pi\left[\left(s_{nq_z} + n_i i_{q_z}\right)(g_i - \bar{g}) + n_i s_{q_g q_z}\right]$$

$$= -\frac{\pi}{D}\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)u_{q_h q_h}P_{q_z,i}\left[(g_i - \bar{g})u_{q_g q_g} + u_{q_g}\right]$$

- <u>For $d\beta = 0$</u>: Budget-neutral increase of γ by $d\alpha = -\bar{\bar{w}}\bar{h}d\gamma$

$$dc_i|_{d\alpha=-\bar{\bar{w}}\bar{h}d\gamma} = \frac{\partial c}{\partial\gamma}d\gamma + \frac{\partial c}{\partial\alpha}d\alpha = -\left(\bar{\bar{w}}\bar{h} - \hat{w}_i h_i\right)(s_{nc} + n_i i_c) - \hat{w}_i n_i s_{q_h c}$$

$$= \frac{1}{D}u_{q_g q_g}u_{q_z q_z}(n_i u_{nn} + u_n)\left[\left(\bar{\bar{w}}\bar{h} - \hat{w}_i h_i\right)u_{q_h q_h} - \hat{w}_i u_{q_h}\right]$$

$$dz_i|_{d\alpha=-\bar{\bar{w}}hd\gamma} = \frac{\partial q_z}{\partial\gamma}d\gamma + \frac{\partial q_z}{\partial\alpha}d\alpha = -\left(\bar{\bar{w}}\bar{h} - \hat{w}_i h_i\right)\left(s_{nq_z} + n_i i_{q_z}\right) - \hat{w}_i n_i s_{q_h q_z}$$

$$= \frac{1}{D}u_{q_g q_g}P_{q_z,i}\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)\left[\left(\bar{\bar{w}}\bar{h} - \hat{w}_i h_i\right)u_{q_h q_h} - \hat{w}_i u_{q_h}\right]$$

- <u>For $d\alpha = 0$</u>: Budget-neutral increase of β by $d\gamma = -\frac{\pi\bar{g}}{\bar{\bar{w}}\bar{h}}d\beta$

$$dc_i|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\bar{w}}\bar{h}}d\beta} = \frac{\partial c}{\partial\beta}d\beta + \frac{\partial c}{\partial\gamma}d\gamma = \pi\left(g_i - \frac{\hat{w}_i}{\bar{\bar{w}}}\frac{h_i}{\bar{h}}\bar{g}\right)(s_{nc} + n_i i_c) + \pi n_i\left(s_{q_g c} + \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\bar{w}}}s_{q_h c}\right)$$

$$= -\frac{\pi}{D}(n_i u_{nn} + u_n)u_{q_z q_z}\left[\left(g_i - \frac{\hat{w}_i}{\bar{\bar{w}}}\frac{h_i}{\bar{h}}\bar{g}\right)u_{q_h q_h}u_{q_g q_g}\right.$$

$$\left. + u_c\left(u_{q_h q_h}P_{q_g,i} - \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\bar{w}}}u_{q_g q_g}P_{q_h,i}\right)\right]$$

$$dz_i|_{d\gamma=-\frac{\pi\bar{g}}{\bar{\bar{w}}\bar{h}}d\beta} = \frac{\partial q_z}{\partial\beta}d\beta + \frac{\partial q_z}{\partial\gamma}d\gamma = \pi\left(g_i - \frac{\hat{w}_i}{\bar{\bar{w}}}\frac{h_i}{\bar{h}}\bar{g}\right)\left(s_{nq_z} + n_i i_{q_z}\right) + \pi n_i\left(s_{q_g q_z} + \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\bar{w}}}s_{q_h q_z}\right)$$

$$= -\frac{\pi}{D}\left(n_i u_{cc}u_{nn} + 2u_n u_{cc} - \frac{u_c^2}{n_i}\right)P_{q_z,i}\left[\left(g_i - \frac{\hat{w}_i}{\bar{\bar{w}}}\frac{h_i}{\bar{h}}\bar{g}\right)u_{q_h q_h}u_{q_g q_g}\right.$$

$$\left. + u_c\left(u_{q_h q_h}P_{q_g,i} - \frac{\bar{g}}{\bar{h}}\frac{\hat{w}_i}{\bar{\bar{w}}}u_{q_g q_g}P_{q_h,i}\right)\right]$$