Beliefs formation and the puzzle of forward guidance power*

Giovanni Di Bartolomeo, Elton Beqiraj, and Marco Di Pietro†

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Abstract

We study the extent to which the belief-formation process affects the dynamics of macroeconomic variables when the central bank uses forward guidance. Standard sticky-price models imply that far future forward guidance has huge and implausible effects on current outcomes, these effects grow in its horizon (forward guidance power puzzle). By a parsimonious macro-model that allows for the role of bounded rationality and heterogeneous agents, we obtain tempered responses for real and nominal variables.

JEL: E40, E50, E21.

Keywords: forward guidance power, heterogeneous agents, bounded rationality, monetary policy, announcements.

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†Corresponding author: Giovanni Di Bartolomeo, Department of Economics and Law, Sapienza University of Rome, Via del Castro Laurenziano, 9 00161, Roma, Italia, giovanni.dibartolomeo@uniroma1.it.

1 Introduction

In the aftermath of the Great Crisis, the effectiveness of monetary policy has been challenged by the zero lower bound (ZLB) constraint. As a result, many central banks have largely adopted the so–called "forward guidance." Essentially, forward guidance is the practice of communicating the future path of the policy rate (Svensson, 2015). In forward–looking economies, by announcing intentions about the future monetary stance, the central bank may be able to manipulate private sector expectations and affect current outcomes in spite of the ZLB. Therefore, a growing number of papers have analyzed forward guidance from several perspectives.[1]

We aim to study the extent to which the belief–formation process affects the dynamics of macroeconomic variables when the central bank uses forward guidance. In the spirit of, among others, Krusell and Smith (1996) and Reis (2006), we introduce a simple, small cost behavioral sophistication in an otherwise standard model. Agents' optimal decisions are modeled to be consistent with their forecasts, but their expectation operators may differ across them. We assume two types of individuals: rational and boundedly rational agents. The latter form their beliefs on the basis of a simple perceived linear law of motion on past observed values (as Brock and Hommes, 1997; Branch and McGough, 2009).[2]

Our assumption is consistent with the empirical macro–evidence. By using expectation surveys, Mankiw *et al.* (2004) find a substantial heterogeneity in beliefs and reject the rationality of US consumers' inflation

[1] See, among others, Eggertsson and Woodford (2003), Gurkanyak *et al.* (2005), Laséen and Svensson (2011), Campbell *et al.* (2012), Woodford (2012), Carlstrom *et al.*, (2012), Del Negro *et al.* (2012) and Chung *et al.* (2015).

[2] Alternatively, our framework can be equivalently interpreted as a model composed by homogeneous agents who all form their expectations by some degrees of bounded rationality (as, e.g., Bomfim and Diebold, 1997; Ball, 2000; Weder, 2004).

forecasts.[3] Similar results are obtained by Carroll (2003), Branch (2004), Andolfatto *et al.* (2007), Pfajfar and Santoro (2010), Andrade and Le Bihan (2013) and Coibion and Gorodnichenko (2015).[4] We use the latter's methodology to calibrate our model.

Our paper is in line with a wave of research that aims to rethink how to model the process by which people form their expectations. We explicitly consider non–homogeneous expectations in New Keynesian models as many recent authors.[5] Others have instead explored different forms of inattentiveness, i.e., infrequent information updating,[6] or simple least squares learning algorithms to form expectations (Evans and Honkapohja, 2001, for a survey). A virtue of our approach, shared with many works in this wave, is that we remain rooted in classical economics and its powerful tools as our agents are modelled as maximizing utility subject to constraints.

By our parsimonious behavioral macro-model, we focus on the puzzle of forward guidance power (FGP, henceforth). The puzzle consists of the fact that standard New Keynesian monetary models imply that far future forward guidance has huge and implausible effects on current outcomes. These effects grow in the horizon of the forward guidance (McKay *et al.*, 2016a; 2016b).

In our behavioral model, we obtain tempered responses for real and nominal variables to future forward guidance. The idea is that bounded rationality prevents a fraction of agents to smooth their (ex–post) consumption. On aggregate, this mitigates output responses to changes in future interest

[3] Early studies are Roberts (1997) and Campbell and Mankiw (1989).

[4] Hommes *et al.* (2005), Adam (2007), and Hommes (2011) find evidence for heterogeneity in beliefs by laboratory experiments.

[5] These include Brock and Hommes (1997), Preston (2006), Branch and Evans (2006), Branch and McGough (2009), Massaro (2013), Gasteiger (2014), Di Bartolomeo *at al.* (2016).

[6] See, among others, Gabaix and Laibson, (2002), Mankiw and Reis (2002, 2003), Sims (2003), Moscarini (2004)

rates. Consequently, forward guidance has substantially less power on the current economic outcomes and, in general, to stimulate the economy. We show that our assumption implies that output behaves as though there is a discount factor on future consumption in the Euler equation, discounting mitigates the effects of forward guidance policies designed to produce real interest rate changes more and more the further in the future.

It is worth noting that our boundedly rational agents fully understand and believe the central bank announcements about future interest rates, but they are not able to correctly forecasts the effects of the forward guidance.

Related to our work, McKay *et al.* (2016a) propose an alternative solution to the puzzle of FGP. They assume that agents face uninsurable income risk and borrowing constraints,[7] a precautionary savings effect then tempers their responses to changes in future interest rates. As a consequence, an announcement of a policy plan implying a reduction of the real interest rate in the future is not fully anticipated in the consumption plans and, after the announcement, output rises gradually until the interest rate falls and then return on the steady state, after a short recession.

In McKay *et al.* (2016a), output responses to changes in future interest rates are mitigated. However, the responses of nominal variables are not. Announcement of future reduction in the real interest rate implies an immediate increase in the inflation rate, which grows with the horizon of the forward guidance. Consequently, inflation responses and announced current interest rates may be very high as long as the horizon of the forward guidance is far in the future.[8] By contrast, the novelty of our model is that it

[7] To some extents, the assumption of borrowing constraints is similar to ours since it could be interpreted as the result of bounded rationality behaviors (e.g., Amato and Laubach, 2003; Galí *et al.*, 2004, 2007).

[8] Apart from the period in which the cut is planned, the central bank should announce a path for the nominal interest rate that match inflation to keep the real interest rate at zero.

provides tempered responses for both real and nominal variables. In our knowledge, no other solutions avoiding a FGP puzzle in both nominal and real variables have been proposed.

The rest of the paper is organized as follows. Section 2 presents our parsimonious sticky price model consistent with heterogeneous agents. Section 3 illustrates our results: We first derive them analytically and then we provide a quantitative illustration. For the sake of comparison, the policy experiments proposed are the same used in McKay *et al.* (2016a). Section 4 concludes.

2 The HE–DSGE model

We consider a simple generalization of the small–scale New Keynesian DSGE model to account for bounded rationality. In particular, we use the HE (heterogeneous expectations)–DSGE model developed by Branch and McGough (2009).[9] Heterogeneous expectations are introduced in a New Keynesian model by an axiomatic approach, i.e., imposing on the possible expectation formation mechanisms the minimum constraints to obtain two aggregate IS and AS relations that only differ from the standard framework in the expectation aggregate operator.[10] The general mechanism behind is described by Jump and Levine (2017). The bounded rationality is founded on a fixed leaning cost.[11]

[9]This section aims to give an insight on the model. All details about its derivation and micro–foundations can be found in Branch and McGough (2009) or Di Bartolomeo *et al.* (2016). The same framework, and results, can be also obtained by assuming homogeneous agents who form their expectations by a near–rational mechanism theorized by, e.g., Bomfim and Diebold (1997), Ball (2000) or Weder (2004).

[10]An alternative approach is proposed by Preston (2006) and Massaro (2013).

[11]Opposed with the Euler learning approach of most of the literature, agents are assumed to be internally rational, i.e., they optimize given their beliefs of aggregate states and prices and face a fixed cost of being fully rational. Deak *et al.* (2016), Jump and Levine (2017) for details.

Our economy is populated by two kind of agents, who differ in the way they form their expectations. A fraction α have rational expectations (rational households), whereas the remaining $1 - \alpha$ form expectations according to a mechanism of bounded rationality (non–rational households). For the sake of brevity, all non–rational households use the same predictor and α is fixed.[12] The two kinds of households are indexed by $\mathcal{R}$ and $\mathcal{B}$. Apart from the heterogeneity in the expectation formation, the model is standard, i.e., it is characterized by monopolistic competition in the goods market and by the presence of nominal price rigidities.

The HE–DSGE model can be represented as follows:

$$y_t = \mathcal{E}_t y_{t+1} - \sigma r_t \quad (1)$$

$$\pi_t = \beta \mathcal{E}_t \pi_{t+1} + \kappa y_t \quad (2)$$

$$r_t = i_t - \mathcal{E}_t \pi_{t+1} \quad (3)$$

where y_t is the output gap; π_t is the inflation; i_t and r_t indicate the nominal and (average) real expected interest rate, respectively; the operator $\mathcal{E}_t$ indicates the average expectation; β, σ, and κ are positive parameters.

The model (1)–(3) differs from the standard model in one respect, the operator $\mathcal{E}_t$ averages the expectations of the different agents. Equation (1) represents the dynamic IS; (2) describes the New Keynesian Phillips curve; (3) defines the expected real interest rate. It is worth noting that, as agents are heterogeneous and may have different beliefs, expected real interest rate are different among individuals, thus r_t is an average expected real interest

[12] Our framework can be interpreted as a study on the effects of forward guidance in a HE equilibrium resulting from the convergence of different learning processes based on different specifications of the forecasting model. Berardi (2007), e.g., shows how a HE equilibrium can emerge as a learnable equilibrium when agents underparametrize their model with respect to the common factor representation.

rate, which is the relevant rate to determine the output gap and for the monetary policy design.

Average expectation at t for any variable x at $t+1$ is defined by the weighted average of expectations of rational ($\mathcal{R}$) and boundedly rational agents ($\mathcal{B}$), i.e.,

$$\mathcal{E}_t x_{t+1} = \alpha \mathcal{E}_t^{\mathcal{R}} x_{t+1} + (1-\alpha)\, \mathcal{E}_t^{\mathcal{B}} x_{t+1} \tag{4}$$

Consistently, rational agents' forecasts on economic variables are $\mathcal{E}_t^{\mathcal{R}} x_{t+1} = E_t x_{t+1}$. In contrast, non–rational individuals form their beliefs on the basis of a simple perceived linear law of motion, i.e., $x_t = \theta x_{t-1}$, where $\theta < 1$ is defined as the adaption operator. It follows that $\mathcal{E}_t^{\mathcal{B}} x_t = \theta x_{t-1}$ and, by the law of iterated expectations, $\mathcal{E}_t^{\mathcal{B}} x_{t+1} = \theta^2 x_{t-1}$. By substituting (4) into (1)–(3), we obtain the rational expectation form associated to our HE–DSGE model, which can be used to study the aggregate properties of our heterogeneous–agent model (see Branch and McGough, 2009).

3 Expectation formation and the effects of forward guidance

3.1 Analytical results: The *discounted* Euler equation

We use the HE–DSGE model to study the effects of forward guidance, i.e., the communication of the future path of monetary policy instruments (nominal interest rate). For the sake of comparison with the standard case, we assume that the central bank is fully rational and thus it is able to announce the nominal interest rate path consistent with its target (i.e., the path of the average real interest rate).

The IS curve (1) of our HE–DSGE model can be solved forward, yielding

$$y(t) = -\sigma \sum_{i=0}^{\infty} \underbrace{\frac{2}{1+K(\alpha,\theta)} \left[\frac{1}{2}\frac{1-K(\alpha,\theta)}{(1-\alpha)\theta^2}\right]^i}_{\Lambda(t+i)} r(t+i) \tag{5}$$

where $K(\alpha,\theta) = \sqrt{1-4\alpha(1-\alpha)\theta^2} < 1$.[13] Equation (5) behaves as a discounted Euler equation. Agents' beliefs formation processes affect aggregate expectations and, through these, imply an effect equivalent to discounting on output. Piergallini (2006), Nisticò (2012), and McKay *et al.* (2016a, 2016b) provide alternative micro-foundations for discounting in the consumption Euler equation. However, in our model the discounting effect depends on the expectations formation process and, differently from these paper, it also affects the nominal variables dynamics. This is relevant for the puzzle of FGP, as we will later show (see Figure 5).

In the HE–DSGE model, the impact of interest rates T periods in the future on current output (future forward guidance) are mitigated by a sort of discount. Formally, the current impact of future forward guidance at T on the output gap is described by the following expression:

$$y(0) = -\frac{2\sigma}{1+K(\alpha,\theta)} \left[\frac{1}{2}\frac{1-K(\alpha,\theta)}{(1-\alpha)\theta^2}\right]^T r(T) \tag{6}$$

The impact of forward guidance on current output falls in the horizon T. Output behaves as though there is a discount factor ($\Lambda(t+T)$) on future consumption in the Euler equation that tempers the effects of real interest rate changes more and more the further in the future.

Our framework generalizes the homogenous standard model. Clearly, the HE–DSGE model collapses to the standard New Keynesian framework when

[13] Note that the highest value for $\alpha(1-\alpha)$ is $1/4$ (achieved when $\alpha = 0.5$). Therefore, $K(\alpha,\theta)$ is smaller than one as long as $\theta < 1$.

$\alpha = 1$. In such a case, $\lim_{\alpha=1} \Lambda(t+i) = 1 \; \forall \; i$ and an announcement $r(T) = -\bar{r}$ at time $t < T$ always impacts the current output by $\sigma\bar{r}$ independently of the length of T. Output instantaneously jumps to $\sigma\bar{r}$ at time t and drops back to the steady state at $T+1$. The response of current output and consumption is just a function of an undiscounted sum of log changes in future real interest rates.[14]

3.2 Future forward guidance

In order to quantify the effects of our analytical results on the puzzle of FGP, we use numerical simulations. We provide some simulations of future–forward guidance that refers to announcements as in McKay *et al.* (2016a; 2016b).

In our simulations, we calibrate the model to the US economy. The time unit is one quarter. The calibration of the structural parameters is chosen in line with other studies (see, e.g., Rotemberg and Woodford, 1997; Smets and Wouters, 2007). We assume that the subjective discount rate β is 0.99 such that $(\beta^{-1} - 1)$ equals the long–run average real interest rate. The price elasticity of demand ε is calibrated to 7.84, which implies a markup of 15%, the frequency of price adjustment (ξ_p) is set at 0.66, i.e., prices are sticky, on average, for three quarters. Finally, the inverse of Frisch elasticity, η, is calibrated to 0.47. The relative risk aversion coefficient, σ, is assumed to be equal to 1, involving a logarithmic utility function in consumption. The slope of the Phillips curve κ is a convolution of the latter parameters and equal to 0.056.

The parameters governing the expectation-formation process, α and θ, are calibrated to fit the relationship between ex-post mean forecast errors

[14] See McKay *et al.* (2016a; 2016b).

and ex-ante mean forecast revisions, following Coibion and Gorodnichenko (2015).[15] Empirical estimation of the expectation-formation process suggest a model consistent share of fully rational agents α equal to 0.77 and an adaption operator θ equal to 0.95, i.e., data are consistent with adaptive expectations.[16] As a result, we consider two scenarios for belief-formation parameters (α and θ).

1. In the first scenario, we set $\alpha = 1$, all the agents are rational and our model encompasses the standard small–scale New Keynesian specification. We refer to this scenario as the rational expectations (RE) case.

2. In the second scenario, we assume, $\alpha = 0.77$, implying that 23% of households form their expectations using a mechanism of bounded rationality, and $\theta = 0.95$, which entail that expectations are adaptive. We refer to this scenario as the heterogeneous beliefs case (BR, bounded rationality). Our findings are, however, qualitatively robust for different calibrations of α and θ.[17]

The calibration is summarized in Table 1.

[15] Details are provided in Appendix.

[16] Large values of θ tend to be associated to indeterminacy and instability (see, Branch and McGough, 2009).

[17] Results are available upon request.

Table 1 – Calibration

		Common	Scenario RE	Scenario BR
β	discount factor	0.99		
σ	relative risk aversion coefficient	1.00		
ε	price elasticity of demand	7.84		
η	inverse of the Frisch elasticity	0.47		
ξ_p	Calvo parameter	0.66		
κ	Phillips curve slope	0.056		
α	degree of bounded rationality		1.00	0.77
θ	adaption parameter		1.00	0.95

The central bank announces a certain path for the nominal interest rate (forward guidance). As in the experiment proposed in McKay *et al.* (2016), we assume that the announced path for the policy rate is designed such as the real interest rate will drop of 1% after 5 years (20 quarters), as described in Figure 1. In other words, the announcement is designed such as the real interest rate (r_t) will be lower by 1% for a single quarter 5 years in the future, but it will be maintained at the natural real rate of interest in all other quarters.

The central bank is always able to implement correctly the path described in the above figure by an appropriate path for the nominal interest rate. In fact, our minimal deviation from the standard New Keynesian framework consists in the assumption that a fraction of agents may not be able to forecast the future, but the central bank is always able to do it.

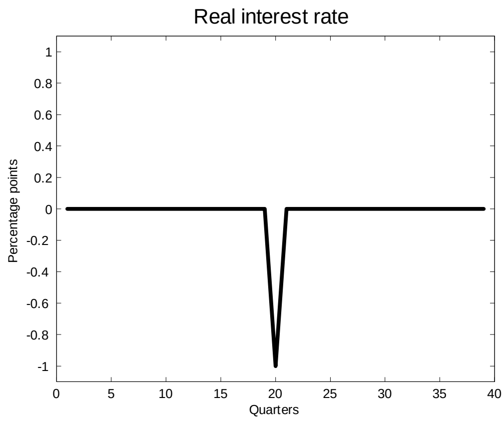

Figure 1 – The real interest rate implemented by the central bank though forward guidance announcements (deviations from the natural rate).

The forward guidance (i.e., the published policy rate path) and its effects on output and inflation are described in Figure 2-4. Specifically, Figure 2 and Figure 3 report dynamics of output and inflation. Figure 4 illustrates the published policy rate path consistent with that of the real interest rate in Figure 1. All figures compare the standard New Keynesian scenario (RE case) to our alternative one (BR case). In the former all the agents are assumed to form their expectations in a rational manner (RE case), whereas in the latter beliefs are heterogeneous.

Figure 2 shows the impulse response function (IRF) of output to forward guidance. In the New Keynesian scenario, the IRF is a step function. Although the real interest rate will drop only for a single quarter 5 years after the central bank's announcement, output jumps up immediately by 1% and return to the steady state after 5 years.

As explained by McKay *et al.* (2015), "the forward guidance does not change the relative price of consumption for any two dates before the date of the interest rate change. All these dates must therefore have the same level of consumption." As long as monetary policies have no effect on real outcomes in the long run, the end–point of consumption is instead pinned down at the old steady state.

The picture changes substantially when bounded rationality is considered. Now, output gradually rises, as the real interest rate fall gets closer. The outcome is consistent with the fact that (some) agents understand the effects of the announcements about policy rates as far as the time of the planned cut in the real interest rate occurs (i.e., period 20).

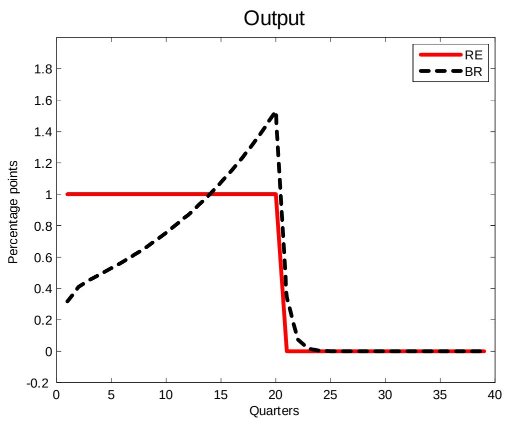

Figure 2 – Output associated with forward guidance announcements (deviations from the steady state).

The inflation dynamics associated to the forward guidance in the two scenarios is illustrated in Figure 3. The difference in the two paths is evident. In the RE scenario inflation jumps immediately and after the interest rate cut goes back to the equilibrium. In the BR scenario, the response of inflation is hump shaped with a peak about in the period when the real interest rate falls.

The different paths depend on expectations. Sticky prices in both cases imply that firms in setting their prices should account for changes in future marginal costs (measured by inflation expectations). Firms anticipate the increase when the real interest rate falls, but they will also anticipate the anticipations. In other words, at quarter 19, the increase in prices is unnecessary if prices were flexible, but firms understand that they may be not able to raise prices at quarter 20 (when marginal cost increases) and partially anticipate the price increase. Nevertheless, at quarter 18, in a similar manner, they anticipate the increases in quarter 19 and 20, and so on.

As a result, under rational expectations, the effects of far future forward guidance on current inflation are magnified. Planned cuts in the interest rate very far in the future have unreasonable effects on current inflation (and need of unbelievable announcements of high current nominal interest rate to be sustainable, see below). It is worth noting that the same occurs in McKay *et al.* (2016a) when incomplete markets are considered.

Bounded rationality changes the picture. The intuition is simple. Firms still set prices according to their expectations, but these are smoothed by the bounded–rationality mechanism and thus their effects on current outcomes are not magnified. As we will later discuss, the different paths have crucial consequences on the analysis of the effects of forward guidance horizon on current real and nominal outcomes (i.e., the FGP puzzle).

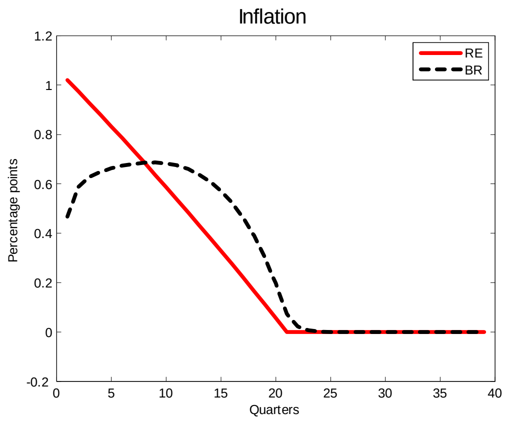

Figure 3 – Inflation associated with forward guidance announcements (deviations from the steady state).

Figure 4 shows the forward guidance, i.e., the published path for the nominal interest rates consistent with the desired path for the real rate described in Figure 1. The announced path for the interest rate matches the inflation dynamics, anticipated by the central bank, to implement the real desired interest rate policy.

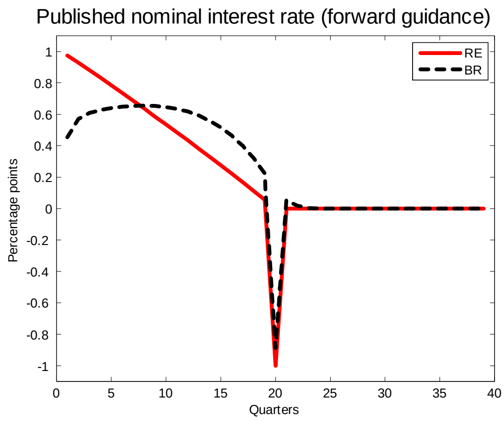

Figure 4 – Published interest rate path (forward guidance) consistent with a 1% fall in the real interest rate after 5 year and the natural rate otherwise (in deviations from the steady state).

The same logic described above for forward guidance 20 quarters in the future, applies for forward guidance at any horizon. Figure 5 plots the responses of current output and inflation to forward guidance announcements consistent with equally large cuts in the real interest rate at different horizons.

In the RE case, the figure shows that the impact of forward guidance on the current output is independent of the horizon of forward guidance. Output and consumption rise by 1% immediately independently of the time of the 1% planned cut in the real interest rate. By contrast, the current response of inflation to forward guidance rises in its horizon. The response of inflation to forward guidance about interest rates 5 years in the future is roughly 18 times larger than the response of inflation to an equally sized change in the current real interest rate.

The intuition is as follows. For any horizon, output and consumption rise by 1% immediately and fall back to steady state after the real interest rate drops, as shown in Figure 2. Formally, the response of current output and consumption is a function of an undiscounted sum of log changes in future real interest rates as it is determined by a step–function. Due to the sticky prices, inflation is anticipated, and anticipations are anticipated as well. Thus, the current impact of forward guidance on current inflation is magnified by the forward guidance horizon. Of course, the path for inflation implies an opposite path in the announced current nominal interest rate (which is needed to keep the real interest rate equal to the natural rate).

The picture changes when heterogeneous beliefs are introduced. Assuming a small fraction of consumers and price setters who are boundedly rational, the impact of forward guidance on current output and inflation is mild and falls in its horizon. Specifically, in the BR scenario, as a fraction of agents do not perfectly smooth consumption, output behaves as though there is a discount factor on future consumption in the Euler equation that tempers the effects of real interest rate changes more and more the further in the future. Regarding inflation, as said commenting the canonical case, inflation *is anticipated* and anticipations *are anticipated* as well due to price stickiness, but now expectations of the fraction of boundedly rational price–setters are adaptive; as a consequence, they smooth the aggregate inflation dynamics.

Comparing our results to McKay *et al.* (2016a, 2016b), they obtain a similar path for output and consumption by assuming incomplete markets and/or a discount factor on future consumption in the Euler equation. However, McKay *et al.* (2016a) is still characterized by a FGP puzzle in the nominal variables: Announcement of future reduction in the real interest rate implies an immediate increases in the inflation and nominal interest rates, which grow with the horizon of the forward guidance.[18] This does not occur in our setup.

The rationale of the difference between us and McKay *et al.* (2016a) is that incomplete markets do not introduce a discounting mechanism for price–setters. As a result, in McKay *et al.* (2016a), the impact of forward guidance on current inflation is magnified by the horizon and a far horizon for the forward guidance has a huge and implausible impact on current inflation. An equal (but in the opposite direction) behavior for announced current nominal interest rate is also implied to keep the current real interest rate equal to the natural one. In our setup, a small fraction of price setters who are not able to fully anticipate the effects of monetary policy instead implies that a FGP puzzle is not observed for nominal variables too.

[18] Qualitatively, in the solution proposed by McKay *et al.* (2016a), the response of current inflation to the horizon of forward guidance is the same as that illustrated in Figure 5 for the RE scenario. Quantitatively, the effects are sensibly smaller..

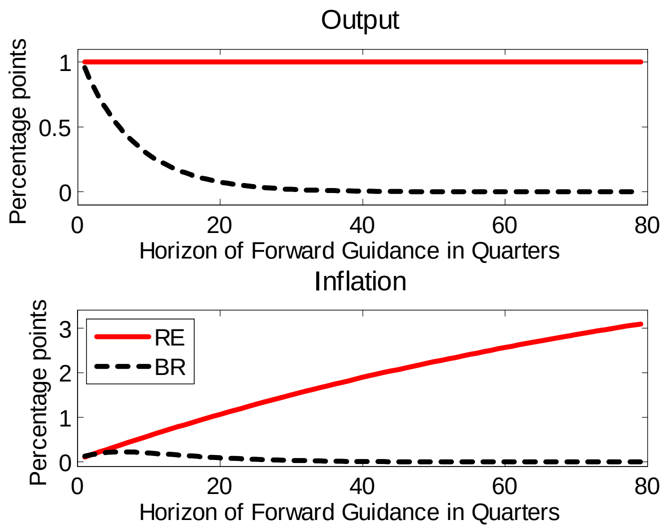

Figure 5 – Response of current output and inflation to forward guidance about interest rates at different horizons (deviations from the steady state).

4 Conclusions

In standard sticky price models, policy anticipations of forward–looking agents imply that far future forward guidance has huge and implausible effects on current real and nominal outcomes. For instance, under a reasonable calibration, a planned cut in the real interest rate in 10 years in the future requires an announced increase in the current nominal interest rate of 2%. This occurs because rational expectations magnify the anticipation effects associated to the announcement when prices are sticky. Due to the strong anticipation effects, after the announcement, current inflation in fact increases by 2% – even if the cut of the interest rate is planned only 10 years in the future. These effects grow with the horizon of the forward guidance.

Moreover, the impact of the announcement on current output is immediate and independent of the horizon of the forward guidance (McKay *et al.*, 2016a; 2016b).

Our paper proposed a simple solution to the puzzle of forward guidance power described above. We used a parsimonious sticky–price model consistent with heterogeneous agents and assumed that a fraction of them use a bounded rationality mechanism to form their expectations. The intuition is that the puzzle is driven by the huge anticipation effects implied by rational expectations. Therefore, heterogeneous beliefs might provide a solution for the puzzle as long as they can prevent some agents (consumers and price–setters) to react to the forward guidance announcements by a fully forward lookiness behavior.

Our results, somehow, complement those presented by McKay *et al.* (2016a, 2016b). Differently from alternative solutions proposed, in fact, we obtained realistic tempered responses for both real and nominal variables. Incomplete markets or simple Euler discounting only prevent consumers' behavior. Thus they does not fully solve the puzzle as nominal variables still exhibit implausible behaviors. Bounded rationality instead affects both consumers and price–setters, and therefore, it implies a tempered responses in both real and nominal variables to forward guidance announcements.

Our way to introduce bounded rationality is admittedly *ad hoc*. However, a virtue of the approach used is that it remains firmly rooted in *classical* economics, where agents are modelled as maximizing utility subject to constraints (which, in our case, also include cognitive limitations). By using the axiomatic approach introduced by Branch and McGough (2009), we have focused on the simplest form of bounded rationality consistent with the micro–foundations of a New Keynesian model and the data.[19] So far, our results would be quite general and should apply to more sophisticated bounded rationality mechanisms, which could also smooth the anticipation effects of forward guidance.[20] We let the issue to future researches.

Appendix

The aggregate time t forecast, F_t, of a variable x at time $t+j$ can be written as $F_t x_{t+j} = \alpha E_t x_{t+j} + (1-\alpha)\,\theta^{j+1} x_{t-1}$, which is the weighted average of rational agents j–step ahead forecast on x, i.e., $\mathcal{E}_t^{\mathcal{R}} x_{t+j} = E_t x_{t+j} = x_{t+j} + \varepsilon_{t+j}$ (where ε_{t+j} is an i.i.d. term), and of non–rational individuals beliefs based on their perceived linear law of motion, $\mathcal{E}_t^{\mathcal{B}} x_{t+j} = \theta^{j+1} x_{t-1}$. Then, by simple manipulations, the forecast equation can be rewritten in terms of forecast error, $FE_t x_{t+j} = x_{t+j} - F_t x_{t+j}$, as

$$FE_t x_{t+j} = \frac{(1-\alpha)}{\alpha} F_t x_{t+j} - \frac{(1-\alpha)}{\alpha} \theta^{j+1} x_{t-1} + \varepsilon_{t+j} \quad (7)$$

As Coibion and Gorodnichenko (2015), from the theoretical formulation (7) and using data on inflation forecasts from the US Survey of Professional Forecasters $(1971-2014)$,[21] we test the model-consistent-bounded-rational hypothesis based on the following empirical specification:

$$FE_t x_{t+3} = c + \beta F_t \pi_{t+3} + \gamma \pi_{t-4} + error_t. \quad (8)$$

The results based on the above empirical estimation are reported in Table

[19] We take the model o the data following Coibion and Gorodnichenko's (2015) methodology.

[20] Our results can be obtained, e.g., by assuming homogeneous agents who have near rational expectations of the kind introduced by Roberts (1995, 1997), Bomfim and Diebold (1997), Ball (2000), Weder (2004). Similar results would also emerge by assuming long horizon forecasts and bounded rationality as in Preston (2006) and Massaro (2013).

[21] See Coibion and Gorodnichenko (2015) for details on data and methodology.

A1. They suggest a model consistent share of boundedly rational agents equal to $1-\alpha = 1-(1+\beta)^{-1} = 0.23$ and an adaption operator equal to $\theta = \left(-\gamma\frac{1-\alpha}{\alpha}\right)^{\frac{1}{j+1}} = 0.95$ (see column (1), where $\beta = 0.3$ and $\gamma = -0.223$). The signs of α and θ are in line with the theoretical model predictions.

Based on the empirical results, we reject the full-information rational expectation hypothesis in favor of the presence of aggregate information rigidities (modelled as heterogenous expectations) at the 5% percent level of statistical significance. The empirical results are qualitatively and quantitatively are robust to the cases of augmented empirical estimation to allow for additional controls such as interest rates, oil prices and unemployment rate (see columns (2)-(4)).

Table A1 – Tests of the inflation expectations process[22]

	(1)	(2)	(3)	(4)
Constant	-0.337	-0.224	-0.307	0.908
	(0.220)	(0.205)	(0.198)	(0.650)
$F_t\pi_{t+3,t}$	0.300**	0.465***	0.270*	0.288**
	(0.152)	(0.164)	(0.144)	(0.137)
π_{t-4}	-0.223**	-0.209**	-0.208**	-0.187**
	(0.100)	(0.096)	(0.098)	(0.087)
z_{t-1}	–	-0.149***	1.405*	-0.207*
		(0.046)	(0.766)	(0.113)
Obs.	168	168	168	168
R^2	0.085	0.152	0.114	0.157

[22] Columns report the augmented empirical estimation to allow for additional control variables (z_{t-1}). In column (1), there are not additional controls. In column (2), we

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consider the average quarterly 3-month Tbill rate; in column (3) uses the quarterly change in log of oil price; column (4) considers the average unemployment rate.

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