An Experimental Investigation of Colonel Blotto Games

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Abstract

This article examines behavior in the Colonel Blotto game with asymmetric resources. In this constant-sum game, two players simultaneously allocate their resources across *n*-battlefields, with the objective of maximizing the expected number of battlefields won. The experimental results support all major theoretical predictions. In the auction treatment, where winning a battlefield is deterministic, disadvantaged players often use a "guerilla warfare" strategy which stochastically allocates zero resources to a subset of battlefields. Advantaged players often employ a "stochastic complete coverage" strategy, allocating random, but positive, resource levels across the battlefields. In the lottery treatment, where winning a battlefield is probabilistic, both players divide their resources equally across all battlefields. Due to the constant-sum nature of the game, we examine behavior under both strangers and partners matching protocols. In the auction treatment, under the strangers protocol, players have significant serial correlation in allocations to a given battlefield across time. Under the partners protocol this correlation is significantly reduced, and disappears for the disadvantaged player.

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1. Introduction

This paper represents a first attempt to experimentally investigate the classic Colonel Blotto game with asymmetric resources. In this constant-sum game, two players simultaneously allocate their endowments of resources across *n*-battlefields, with the objective of maximizing the expected number of battlefields won. The probability of winning a battlefield depends on the resources allocated by both players to that field. The function that maps the two players' resource allocations into their respective probabilities of winning is called the contest success function (CSF). We examine two types of contest success functions (CSFs): the "auction" CSF, in which the player allocating more resources to a battlefield wins that battlefield with certainty, and the "lottery" CSF, in which the probability of winning a battlefield equals the ratio of a player's resource allocation to the sum of the players' resource allocations in that battlefield.

The experimental results support all major theoretical predictions. In the auction treatment, where the winner of each battlefield is determined according to the auction CSF, disadvantaged players often use a "guerilla warfare" strategy which stochastically allocates zero resources to a subset of battlefields. Advantaged players often employ a "stochastic complete coverage" strategy, allocating random, but positive, resource levels to each battlefield. Under the lottery treatment, where the winner of each battlefield is determined according to the lottery CSF, there is support for the equilibrium prediction of a constant allocation across battlefields for both players. Deviations from equilibrium behavior by employing either greater dispersion of resources across battlefields in the lottery treatment or less dispersion across battlefields (or within a battlefield, across time) in the auction treatment are associated with lower payoffs. Due to the constant-sum nature of the game, we examine behavior under both strangers and partners matching protocols. In the auction treatment, under the strangers protocol, players have significant serial correlation in allocations to a given battlefield across time. Under the partners protocol this correlation is significantly reduced, and disappears for the disadvantaged player.

The Colonel Blotto game is the prototype of models of multidimensional strategic resource allocation. Originally formulated by Borel (1921), it is among the first strategic situations to be subject to formal mathematical analysis. Over the years, variants of the game have been examined by prominent scholars across a wide range of disciplines (Tukey, 1949; Blackett, 1954, 1958; Bellman, 1969; Shubik and Weber, 1981; Snyder, 1989; Powell, 2007; and Hart, 2008). Interest in the game is derived from its wide potential for application, including to problems in military and systems defense (Blackett, 1954, 1958; Shubik and Weber, 1981; Clark and Konrad 2007; Powell 2007, Hausken, 2008; and Kovenock and Roberson, 2008), advertising (Friedman, 1958), research and development portfolio selection (Clark and Konrad, 2008), political campaign resource allocation (Snyder, 1989; Klumpp and Polborn, 2006; and Strömberg, 2008) and redistributive politics (Laslier, 2002; Laslier and Picard, 2002; and Roberson, 2008).[1]

Borel's original version of the Colonel Blotto game employed an auction CSF and was solved for the special case of three battlefields and symmetric resources by Borel and Ville (1938). Gross and Wagner (1950), extended the Borel and Ville analysis of the case of symmetric resources to allow for any finite number of battlefields. Friedman (1958) provided a partial characterization of the solution to Borel's problem for n-battlefields and asymmetric resources. More recently, Roberson (2006) has applied the theory of copulas to prove the uniqueness of equilibrium payoffs under the auction CSF for n-battlefields and arbitrary asymmetric resources and prove that uniform univariate marginal distributions are necessary for

[1] Conceptually related, but somewhat different technically, are the models of redistributive politics with a continuum of battlefields following Myerson (1993). Contributions in this line of research include Lizzeri (1999), Lizzeri and Persico (2001), Sahuguet and Persico (2006), Crutzen and Sahuguet (2009), and Kovenock and Roberson (2009).

equilibrium over a wide range of endowments of resources.[2] To our knowledge, Friedman (1958) was also the first to examine the Blotto game under the lottery CSF and solved the game for n-battlefields and asymmetric resources. A recent extension is Robson (2005), who extends the analysis from the lottery CSF to more general CSF's of the ratio form in which the probability that player i wins the contest as a function of the two levels of expenditure x_i and x_j is $x_i^r/(x_1^r + x_2^r)$, where $0 < r \leq 1$.[3]

Which of the contest success functions better describes a given strategic multi-dimensional resource allocation problem depends on the nature of the conflict within each contested battlefield. An auction CSF might well approximate environments in which exogenous noise plays little role in influencing the outcome of the battle. The lottery CSF is a popular method of modeling environments in which victory in a given battlefield is determined not just by the respective resource allocations, but also a substantial random component.

The rest of the paper is organized as follows. In section 2, we describe our experimental design and theoretical predictions. Section 3 presents the results of our experiment and compares these results to the corresponding theoretical predictions. Section 4 concludes.

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[2] See also Kvasov (2007) and Roberson and Kvasov (2008), who examine "non-constant sum" Blotto games in which budgets are not use-it-or-lose-it, Golman and Page (2009) who examine Blotto games with payoffs nonlinear in the number of battlefields won and externalities across battlefields, and Hart (2008) who examines a Blotto game with discrete strategy spaces.

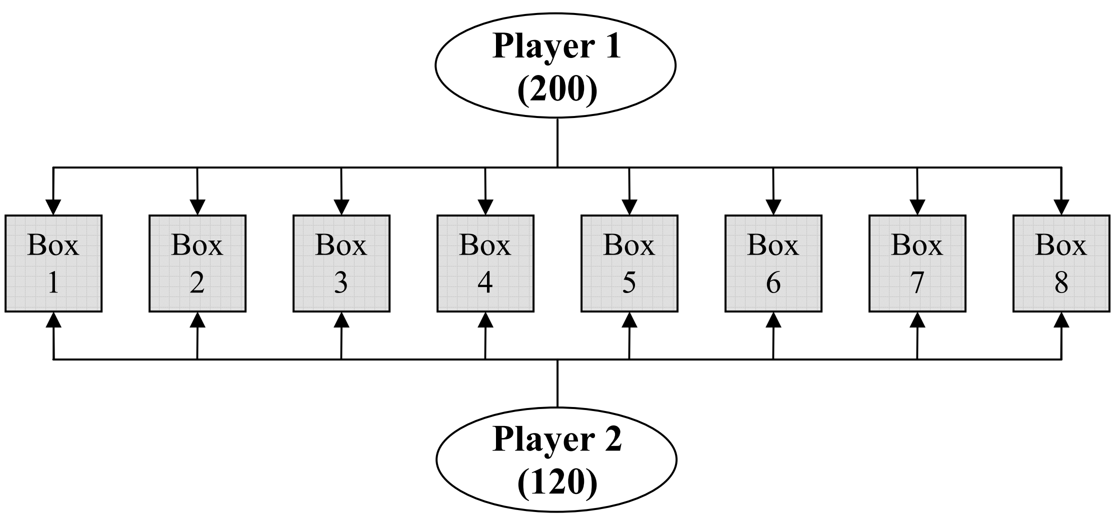
[3] Snyder (1989) examined a related game in which the CSF for each battlefield was of the type employed by Rosen (1986) and contained the lottery CSF as a special case. Snyder assumed no budget constraints, but instead a positive marginal cost of each unit resource employed. He also examined two different objectives, one involving a payoff linear in the number of battlefields won and the other a payoff that was discontinuous when a majority of battlefields was won.

2. Experimental Environment

2.1. Experimental Design and Theoretical Predictions

This paper examines experimentally whether behavior conforms to the Nash equilibrium predictions of the Colonel Blotto game with asymmetric budgets. Our experimental design is based on the constant-sum Colonel Blotto game, in which two players simultaneously allocate their resources across *n*-battlefields, with the objective of maximizing the expected number of battlefields won.[4] We study two treatments: the *auction* treatment, in which the player with the higher resource allocation to a battlefield wins that battlefield with certainty, and the *lottery* treatment, in which the probability of winning a battlefield equals the ratio of a player's resource allocation to the sum of the players' resource allocations in that battlefield. The auction treatment is based on Roberson (2006) and the lottery treatment is based on Friedman (1958). The structure of the game is shown in Figure 2.1. We use 8 battlefields (boxes) and two players with asymmetric resources. The resource endowment for player 1 is 200 tokens and for player 2 it is 120 tokens.

Figure 2.1 – The Structure of the Game



[4] Since the games examined are constant sum, Nash equilibrium strategies are also optimal strategies.

The outline of the experimental design along with the theoretical predictions is shown in Table 2.1. Under the auction CSF (auction treatment), as demonstrated by Roberson (2006), if the budgets are not too asymmetric there exists no pure strategy Nash equilibrium in this game. The qualitative nature of the mixed strategy equilibria that arise depends critically on the ratio of the two players' budgets. For a wide range of budgets, including those examined in this paper, the equilibrium marginal distributions of each player's resource allocation within each battlefield are uniquely determined. The disadvantaged player 2 allocates zero resources to a given battlefield with positive probability (0.4) and then employs a uniform marginal distribution between zero (0 tokens) and a common upper bound (50 tokens). Hence, the disadvantaged player 2 uses a "guerilla warfare" strategy which stochastically allocates zero resources to a subset of battlefields. The advantaged player 1's equilibrium strategy must generate marginal distributions that are uniform over the complete support, which coincides with that of the disadvantaged player. Hence, equilibrium strategies for the advantaged player exhibit "stochastic complete coverage," allocating random, but positive, resource levels across the battlefields. The unique equilibrium expected payoff for the advantaged player is 0.7 and for the disadvantaged player it is 0.3.

Table 2.1 – Experimental Design and Theoretical Predictions

Treatment	Number of Boxes	Player	Budget	Equilibrium Marginal Distribution of Tokens	Expected Payoff per Box
Lottery	8	1	200	$x_1 = 25$	0.625
		2	120	$x_2 = 15$	0.375
Auction	8	1	200	$F_1(x) = \frac{x}{50}$ where $x \in [0, 50]$	0.7
		2	120	$F_2(x) = \frac{2}{5} + \frac{3}{250}x$ where $x \in [0, 50]$	0.3

The Colonel Blotto game with the lottery CSF (lottery treatment) applied in each battlefield yields markedly different equilibrium predictions. For all positive budget pairs of the two players, the unique equilibrium requires that players employ pure strategies that divide their budgets equally across the *n*-battlefields (Friedman, 1958). Given the specific parametric restrictions used in our experiment, equilibrium requires that the advantaged player 1 allocates 25 tokens, whereas the disadvantaged player 2 allocates 15 tokens to each box. It is straightforward to calculate the expected payoff in the lottery treatment. The expected payoff per box is equal to the probability of winning that box. Hence, player 1's expected payoff is 25/(25+15) = 0.625 and player 2's expected payoff is 0.375.

2.2. Experimental Procedures

The experiment was conducted at the Vernon Smith Experimental Economics Laboratory during March and May of 2008. The computerized experimental sessions were run using z-Tree (Fischbacher, 2007). A total of 128 subjects participated in eight sessions. All subjects were Purdue University undergraduate students who participated in only one session of this study. Some students had participated in other economics experiments that were unrelated to this research.

Table 2.2 – Experimental Design

Session Number	Design	Matching Protocol	Participants per Session	Periods per Treatment
1-2	Lottery → Auction	Strangers	16	15
3-4	Auction → Lottery	Strangers	16	15
5-6	Lottery → Auction	Partners	16	15
7-8	Auction → Lottery	Partners	16	15

Table 2.2 summarizes the design of the experiment. We employ two treatment variables: CSF (lottery versus auction) and matching protocol (strangers versus partners). Each experimental session had 16 subjects and it proceeded in three parts. In the first part subjects made 15 choices in simple lotteries, similar to Holt and Laury (2002).[5] This method was used to elicit subjects' risk preferences. The second and the third parts corresponded to the lottery and auction treatments. Each subject played for 15 periods in the lottery treatment and 15 periods in the auction treatment. The sequence was varied so that half the sessions had the auction treatment first, and half had the lottery treatment first.

At the beginning of each treatment subjects were given instructions, available in the Appendix, and the experimenter read the instructions aloud. Before the first period of the experiment subjects were randomly and anonymously assigned as player 1 or player 2. All subjects remained in the same role assignment throughout the entire experiment. In sessions 1-4, where we employed the strangers matching protocol, subjects of opposite assignments were randomly re-paired each period to form a two player group. In sessions 5-8, where we employed the partners matching protocol, subjects were paired with the same participant of opposite assignment for the entire experiment. In each period, player 1 received 200 tokens and player 2 received 120 tokens. Both players where asked to choose how to allocate their tokens across 8 boxes. Player 1 could allocate any number of tokens between 0 and 200 (with a mesh of 0.5 tokens) to each box. In each period the total number of tokens had to sum to 200 or the computer did not accept the allocation of player 1. A corresponding rule was applied to player 2 up to his budget of 120 tokens. After all subjects in the session submitted their allocations for a given

[5] They were asked to state whether they preferred safe option A or risky option B. Option A yielded $1 payoff with certainty, while option B yielded a payoff of either $3 or $0. The probability of receiving $3 or $0 varied across all 15 lotteries. The first lottery offered a 5% chance of winning $3 and a 95% chance of wining $0, while the last lottery offered a 70% chance of winning $3 and a 30% chance of winning $0.

period, the computer informed each player which boxes they had won. The winner of each box received 1 franc (experimental currency). In the lottery treatment, the winner of each box was chosen according to the lottery CSF. A simple lottery was used to explain how the computer chose the winner. In the auction treatment, the player who allocated more tokens to a particular box was chosen as the winner of that box. In the case that both players allocated the same amount to a given box, the computer always chose player 1 as a winner of that box. In each period, after all subjects in the session submitted their allocations, the computer displayed on the outcome screen each player's allocation, the allocation of tokens by the player's opponent, the player's period earnings in francs, and the player's cumulative earnings.

At the end of the experiment, 1 out of the 15 decisions subjects made in part one was randomly selected for payment. Subjects were also paid for each of the 15 periods in the lottery treatment and each of the 15 periods in the auction treatment. The earnings were converted into US dollars at the end of the experiment. For player 1, the conversion rate was 8 francs to $1, and for player 2, the conversion rate was 4 francs to $1. The conversion rates were private information. On average, subjects earned $25 each which was paid in cash. The experimental sessions lasted for about 100 minutes.

3. Results

3.1. General Results

Table 3.1 summarizes the average allocation of tokens to each box and the average payoff of each player. The first support for the "Colonel Blotto" theory comes from the fact that the actual payoffs in Table 3.1 are very close to the predicted payoffs in Table 2.1. Specifically, the theory predicts that player 1's expected payoff per box is 0.63 in the lottery treatment and

0.70 in the auction treatment. The actual payoffs are 0.64 and 0.71. The actual payoffs of player 2 are also consistent with the payoffs predicted by the theory.[6] This result is striking because the vast majority of experimental studies on contests document that payoffs do not conform to theoretical predictions (Davis and Reilly, 1998; Gneezy and Smorodinsky, 2006; Sheremeta, 2008, 2009). In contrast to the previous studies, however, our experiments investigate a constant-sum game, where players cannot over-dissipate. In the absence of the possibility of over-dissipation, we find that the actual payoffs are consistent with the theoretical payoffs.

Table 3.1 – Average Allocations and Payoffs

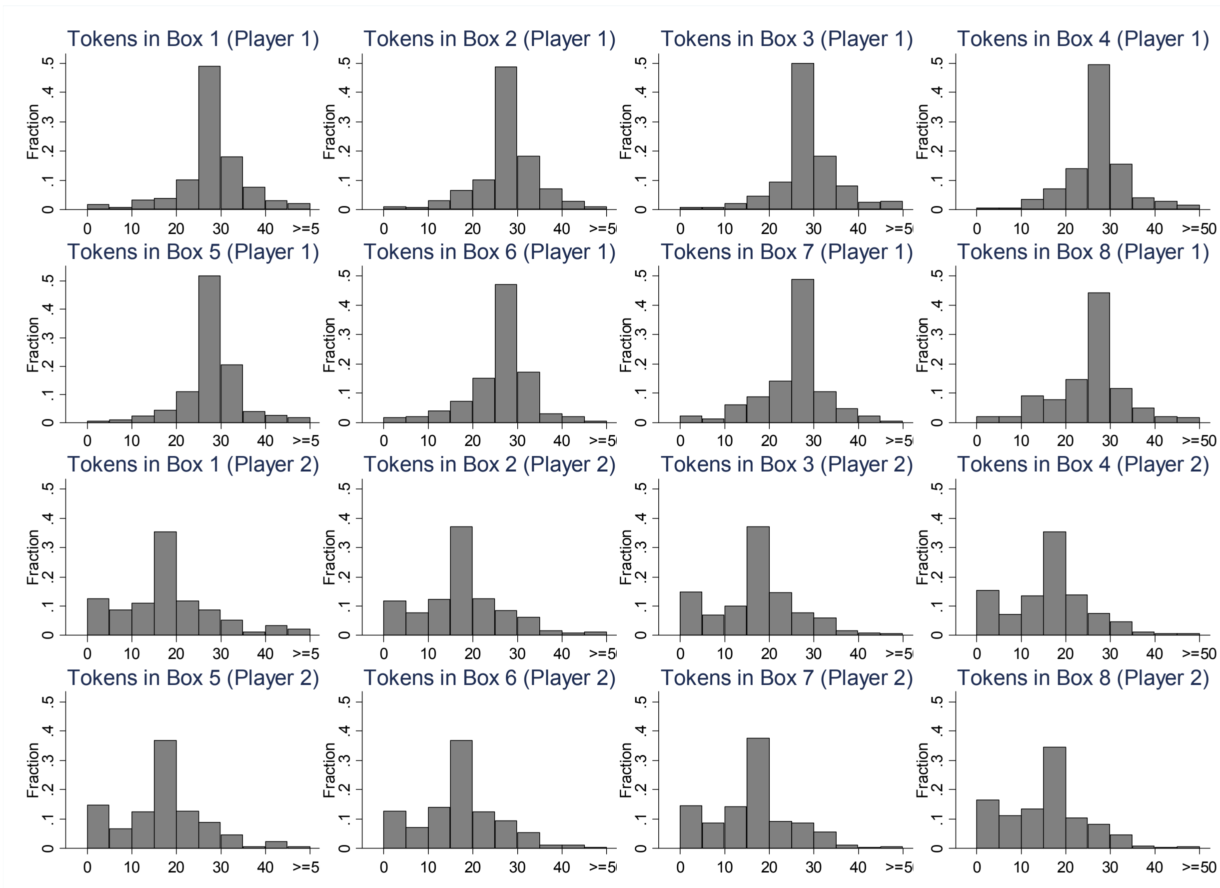
Field	Lottery		Auction	
	Player 1	Player 2	Player 1	Player 2
1	25.9 (7.6)	16.3 (10.9)	26.2 (11.9)	16.2 (15.8)
2	25.5 (7.0)	15.7 (9.4)	26.0 (11.3)	14.6 (15.3)
3	26.6 (7.6)	15.1 (9.2)	27.0 (10.9)	16.2 (15.4)
4	25.1 (7.2)	14.6 (9.1)	25.2 (11.2)	14.8 (14.7)
5	25.9 (6.8)	15.0 (9.5)	26.0 (10.9)	15.2 (15.0)
6	23.9 (7.2)	15.1 (8.7)	24.2 (10.8)	15.6 (15.1)
7	23.5 (7.6)	14.5 (8.9)	23.0 (11.5)	15.4 (14.8)
8	23.5 (9.0)	13.7 (8.9)	22.3 (12.5)	12.0 (14.0)
Payoff	0.64 (0.17)	0.36 (0.17)	0.71 (0.13)	0.29 (0.13)

Table 3.1 also shows that each player's average allocation of tokens does not vary much across boxes. Nevertheless, there seems to be a slight allocation bias towards boxes 1-4. One explanation for this bias comes from the theory of "focal points" introduced by Schelling (1960). In our experiment all 8 boxes were symmetric from a strategic standpoint but they were located in a row from left-to-right. Thus, for people whose native language reads from left-to-right, allocating more tokens to boxes 1-4 (on the left) might appear natural. At the end of the

[6] Separately for each treatment, we estimated a random effects model, with individual subject effects, where the dependent variable is payoff and the independent variables are a constant and session dummy-variables. A standard Wald test, conducted on estimates of a model, cannot reject the hypothesis that the constant coefficients are equal to the predicted theoretical values as in Table 2.1 (*p-value* > 0.01).

experiment we conducted a questionnaire in which we asked all subjects to state whether in their native language they write from right-to-left or from left-to-right. Around 90% of all subjects answered that they write from left-to-right.

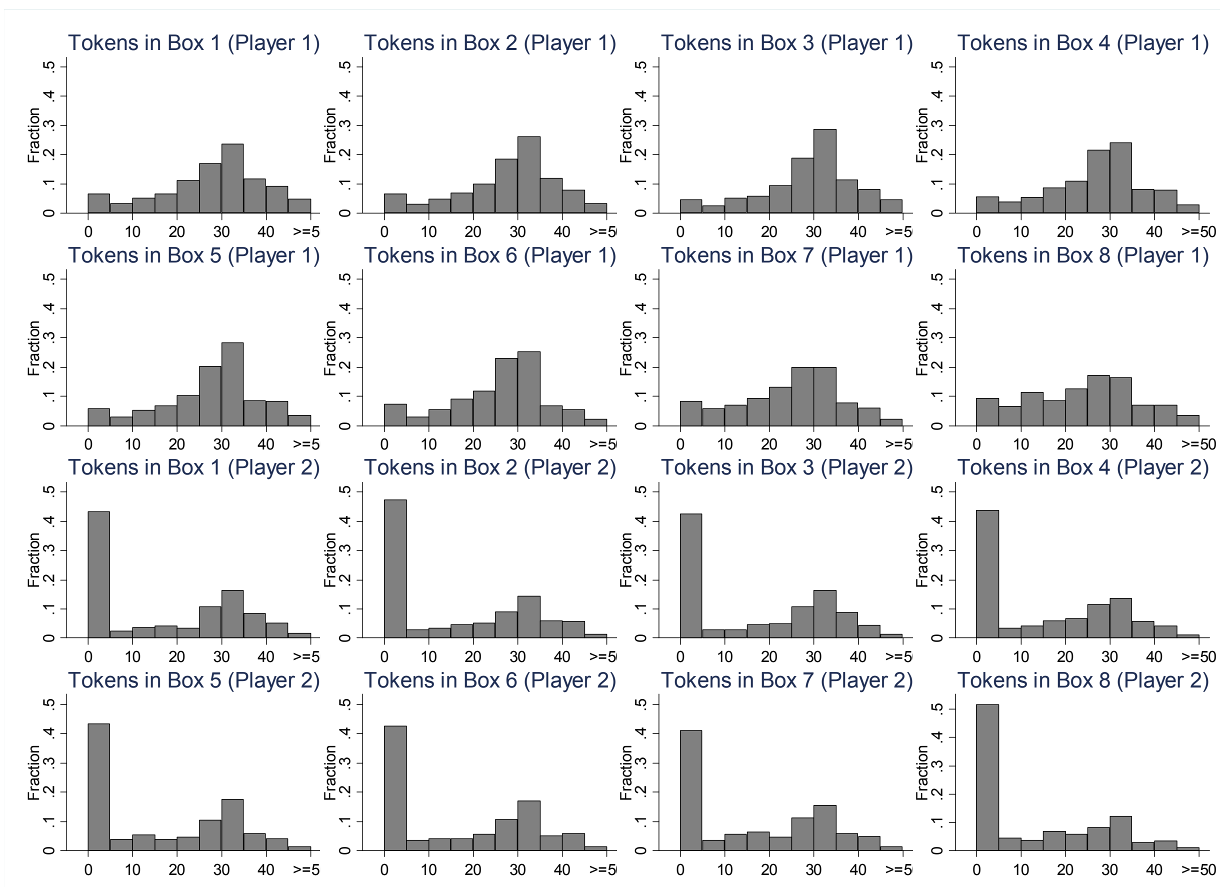
Figure 3.1 – Distribution of Tokens in the Lottery Treatment



In the lottery treatment, equilibrium requires a constant allocation across boxes for both players (Table 2.1). To see whether this prediction is supported, Figure 3.1 displays the distribution of tokens within each box in the lottery treatment. There is support for the equilibrium prediction as the majority of player 1's allocations are centered at 25 while the majority of player 2's allocations are centered at 15. Contrary to the equilibrium predictions, there is substantial variance in the allocation of tokens. This variance is consistent with previous experimental findings in the literature on contests with lottery CSFs (Davis and Reilly, 1998;

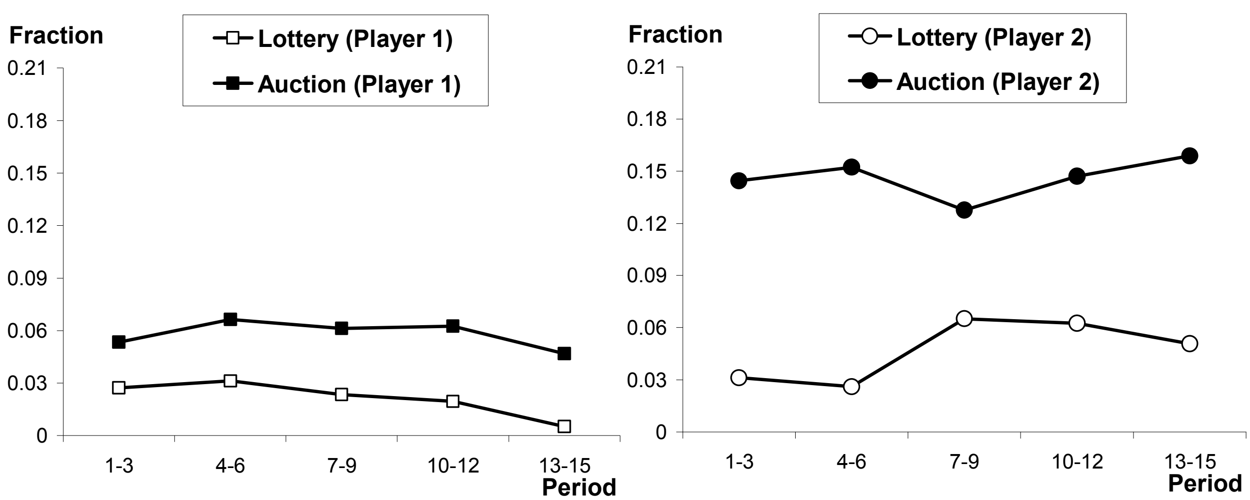
Potters et al., 1998; Sheremeta, 2009). Bounded rationality is one prominent explanation for this phenomenon; players may commit errors and potentially learn over time. This process is complicated by the probabilistic nature of the lottery CSF. Each period, the computer makes a random draw to determine the winner of each box. The random draw in period *t*-1 may affect a player's behavior in period *t*, which can explain why there is substantial variance in the allocations in all periods of the experiment. The mechanisms through which period *t*-1 random outcomes influence decisions in period *t* are described further in section 3.2.

Figure 3.2 – Distribution of Tokens in the Auction Treatment



In the auction treatment, equilibrium requires that player 1 employ a joint distribution which generates for each box a uniform marginal distribution over the interval [0, 50]. On the other hand, player 2's joint distribution generates a marginal distribution in each box that allocates 0 tokens with probability 0.4, and randomizes uniformly over the interval [0, 50] with the remaining probability. From Figure 3.2 we can see that behavior conforms substantially to the predictions of equilibrium. The interval over which players randomize is between 0 and 50, with only 0.5% of observed allocations above 50 tokens. Consistent with the theory, the advantaged player 1 employs a "stochastic complete coverage" strategy, allocating a random, but positive, number of tokens across the boxes. The disadvantaged player 2 uses a "guerilla warfare" strategy which stochastically allocates zero tokens to a subset of the boxes. As in studies of non-constant sum contests (Barut et al, 2002; Gneezy and Smorodinsky, 2006), behavior is more dichotomous than predicted by the equilibrium, with players choosing either very low allocations or moderately high allocations to a given box more often than predicted.

Figure 3.3 – Fraction of Players who use Decimals



The theoretical predictions as well as the observed behavior in the lottery treatment are very different from the auction treatment. The difference also comes from the observation that players use decimals in the allocation of tokens in the auction treatment more often than in the lottery treatment. Figure 3.3 shows the fraction of players who use decimal points in their allocations. In the auction treatment both players 1 and 2 use decimal points more frequently than in the lottery treatment. This finding is due to several factors. First, equilibrium under the auction CSF requires nondegenerate marginal distributions with support [0, 50]. Second, the tie-breaking rule under the auction CSF favors player 1, so one might expect player 2 to attempt to avoid ties. Finally, for the parameters we examine equilibrium in the lottery treatment requires pure strategies that are whole numbers.

3.2. Strangers versus Partners: Serial Correlation and "Hot Box"

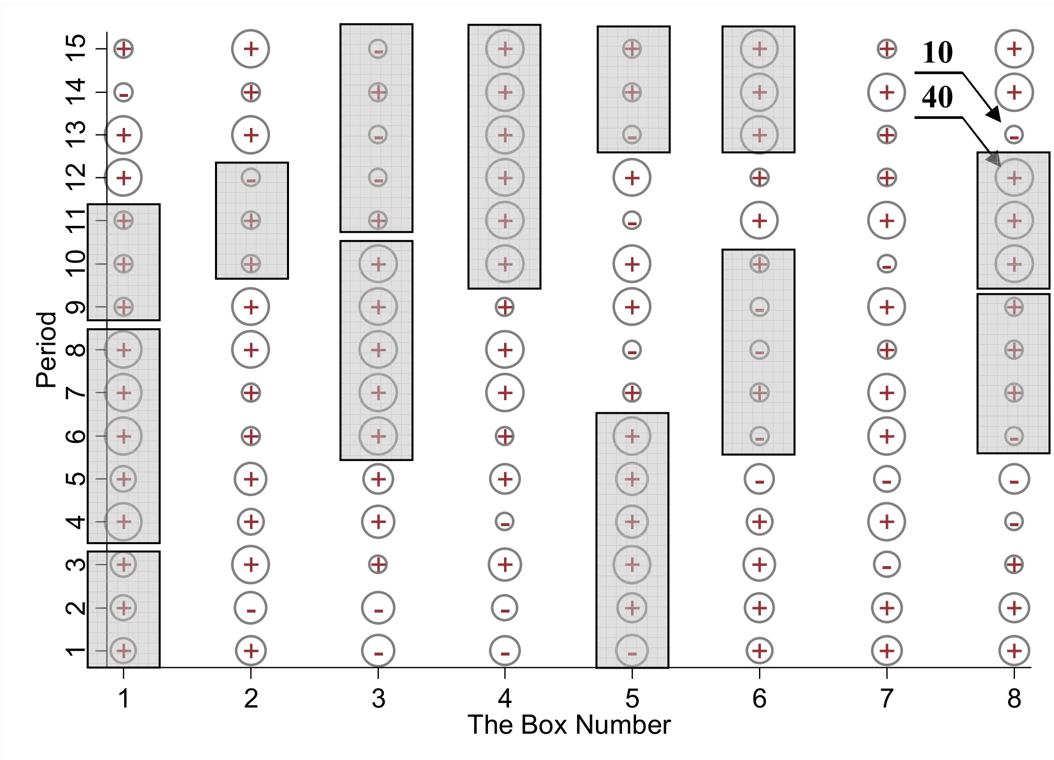
In non-constant sum games repetition with the same set of players (a partners protocol) may change the nature of equilibrium since subjects have incentive to collude (Kreps et al., 1982). A common way to deal with this is to randomly re-group players (a strangers protocol) after each iteration of the game.[7] In contrast to the standard auction literature (Klemperer, 2002), collusion is not an issue in our experiment since the Colonel Blotto game presented in this article is a constant sum game. Every gain for one player is a loss for the other. However, after we ran the first set of experiments using the conventional strangers protocol, we realized that several very interesting behavioral patterns were caused by this matching protocol. Specifically, we found that players have significant serial correlation in allocations to a given box across periods.

Serial correlation of allocations to a given box is clearly illustrated in Figure 3.4, where we display the allocations to each box of one of the subjects taking the role of player 1 over the 15 periods in the auction treatment. This player 1 received the highest payoff among all players under the strangers matching protocol. The size of a bubble in the figure indicates the size of the allocation. For example, the biggest bubbles in the figure correspond to the allocation of 40

[7] There is no general agreement on how matching protocol influences individual behavior. In public good games, some studies find more cooperation among strangers, some find more by partners, and some fail to find any difference at all (Andreoni and Croson, 2008). In auctions, there is some evidence that subjects cooperate more under the partners matching protocol (Lugovskyy et al., 2008).

tokens, while the smallest bubbles correspond to the allocation of 10 tokens. The "+" or "–" correspond to winning or losing. Note that this player 1 has a tendency to allocate the same amount of tokens to a given box across periods (we have highlighted these boxes). It is worth examining whether this behavior results from randomization over periods or from some type of individual bias.

Figure 3.4 – Allocation by Player 1 in the Auction Treatment (Strangers)



To control for individual and period effects, we provide a multivariate analysis. To capture heterogeneity across individuals we use random effects models with individual subject effects. We also include dummy variables to capture session effects. The regressions are of the following form:

$$allocation_{itn} = \beta_0 + \beta_1 own\text{-}lag_{i(t\text{-}1)n} + \beta_2 own\text{-}lag_{i(t\text{-}1)n} \times win\text{-}lag_{i(t\text{-}1)n} + \beta_3 opponent\text{-}lag_{i(t\text{-}1)n} +$$
$$+ \beta_4 box1234 + \beta_5 box1234 \times (1/t) + \beta_6 box5678 \times (1/t) + \sum_h \beta_{7h} S_h + u_i + \varepsilon_{it} ,$$

where *allocation* is player *i*'s allocation of tokens to the *n*-th box in a period *t*, *own-lag* denotes player *i*'s allocation to the same box in the previous period, *win-lag* denotes whether player *i* won that box in the previous period, and *opponent-lag* denotes the opponent's allocation to that box in the previous period. The variable *box*1234 is a dummy variable which is designed to capture allocation bias towards boxes 1-4. All regressions also include dummy-variables to capture session effects. The results of the estimation are presented in Table 3.2. In specifications (1) and (2) we use the data from the lottery treatment while in specifications (3) and (4) we use the data from the auction treatment.

Table 3.2 – Determinants of Allocation to a Specific Box (Strangers)

Treatments	Lottery	Lottery	Auction	Auction
Dependent variable, allocation of tokens	(1) Player 1	(2) Player 2	(3) Player 1	(4) Player 2
own-lag	0.36**	0.24**	0.31**	0.15**
[own tokens in period *t* -1]	(0.03)	(0.03)	(0.03)	(0.03)
own-lag x *win-lag*	0.01	0.02	0.05*	0.07**
[own tokens if subject won the field in period *t* -1]	(0.01)	(0.02)	(0.02)	(0.03)
opponent-lag	0.06**	0.02	0.10**	-0.05
[opponent's tokens in period *t* -1]	(0.01)	(0.02)	(0.02)	(0.03)
box 1234	0.96*	0.17	1.26*	-1.02
[1 if field is 1, 2, 3, or 4]	(0.41)	(0.53)	(0.58)	(0.83)
1/*t* x *box* 1234	-0.26	1.02	-0.19	3.64
[inverse of a period trend for *box* 1234]	(1.48)	(1.98)	(1.74)	(2.81)
1/*t* x *box* 5678	0.19	-1.08	0.27	-3.35
[inverse of a period trend for *box* 5678]	(1.76)	(1.85)	(1.88)	(2.56)
Constant	14.50**	10.51**	14.13**	13.81**
	(0.86)	(0.69)	(0.83)	(0.96)
Observations	3584	3584	3584	3584
Number of subject	32	32	32	32

Robust standard errors in parentheses. * significant at 5%, ** significant at 1%
In each regression we include dummies to control for for session effects
All models include a random effects error structure, with individual subject effects

In all specifications the *own-lag* coefficient is positive and significant, indicating the presence of serial correlation. Several experimental studies have shown that winning the contest in period *t*-1 affects a player's behavior in period *t* (Sheremeta, 2008, 2009). To capture this dynamic aspect of the game, we also used an interaction between *own-lag* and *win-lag*. The *win-*

lag variable takes on the value of 1 if the subject won the box in period *t*-1 and 0 otherwise. In specifications (3) and (4) the interaction between *own-lag* and *win-lag* is positive and significant. We call this the "hot box" effect.[8] This is a robust finding since in both specifications we control for the number of tokens allocated to each box by the opponent in period *t*-1. The *opponent-lag* variable is positive and significant only for player 1, indicating that player 1 allocates more tokens to the boxes where his opponent allocated more tokens in period *t*-1.

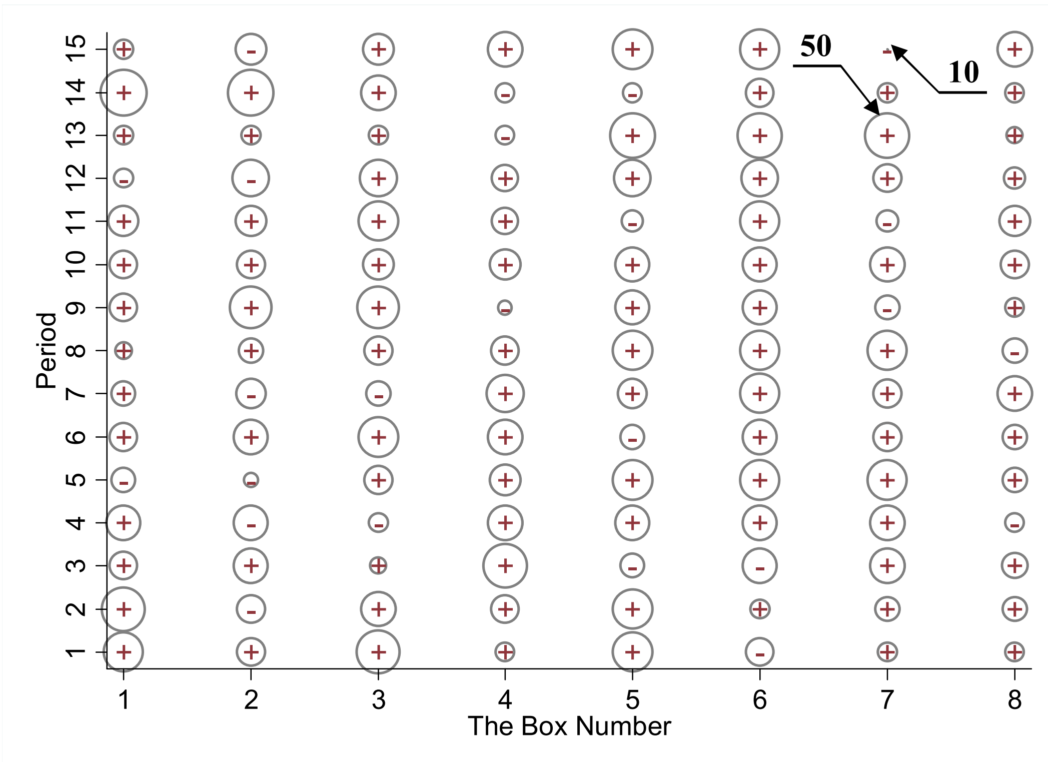
Another finding from Table 3.2 is the allocation bias effect. In specifications (1) and (3), the *box*1234 variable has a positive and significant effect on allocation. This means that player 1 allocates more tokens to boxes 1-4 than to boxes 5-8. As noted earlier, this finding can be explained by the fact that 90% of our subjects write from left-to-right. We re-estimated all specifications in Table 3.2 by controlling for language differences. However, we did not find any significant differences. Note that the inverse of a period trend interacted with box location is not significant in any of the specifications. This suggests that player 1's bias towards boxes 1-4 does not disappear with experience.

The presence of serial correlation, a "hot box" effect, and box location bias under the strangers matching protocol motivated us to run sessions with the partners matching protocol. Next we examine the behavior of the subjects under the partners protocol. Figure 3.5 displays the allocations to each box of one of the subjects taking the role of player 1 over 15 periods in the auction treatment. This player 1 received the highest payoff among all players under the partners matching protocol. The biggest bubbles in the figure correspond to the allocation of 50 tokens, while no bubble corresponds to the allocation of 0 tokens. Note that the striking difference from Figure 3.4 is that it is hard to detect serial correlation in Figure 3.5. This player 1 frequently

[8] This finding resembles a phenomenon known in the gambling literature as a "hot hand" – a belief in a positive autocorrelation of a non-autocorrelated random sequence. For a review see Chau and Phillips (1995) and Croson and Sundali (2005).

changes the allocation of tokens to a particular box across periods. It is important to emphasize, however, that the two figures present only the most successful players in the auction treatment under the strangers and partners protocols. For a robust comparison we need to control for individual differences and use the entire subject population.

Figure 3.5 – Allocation by Player 1 in the Auction Treatment (Partners)



As before, to capture individual differences we provide a multivariate analysis by employing random effects models with individual subject effects. The results of the estimation in Table 3.3 support our initial observation. The serial correlation effect is much lower under the partners protocol, which is illustrated by a much lower *own-lag* coefficient. Note that the *own-lag* coefficient is not significant for player 2 in the auction treatment. Moreover, the "hot box" effect disappears. On the other hand, the *box*1234 coefficient is still high and significant indicating that the partners protocol does not reduce the box location bias in these Colonel Blotto games.

Table 3.3 – Determinants of Allocation to a Specific Box (Partners)

Treatments	Lottery	Lottery	Auction	Auction
Dependent variable, number of tokens	(1) Player 1	(2) Player 2	(3) Player 1	(4) Player 2
own-lag	0.07**	0.17**	0.11**	0.01
[own tokens in period *t*-1]	(0.02)	(0.03)	(0.04)	(0.03)
own-lag x *win-lag*	-0.01	0	-0.02	-0.04
[own tokens if subject won the field in period *t*-1]	(0.01)	(0.02)	(0.03)	(0.03)
opponent-lag	0.03	0	0	-0.06*
[opponent's tokens in period *t*-1]	(0.02)	(0.02)	(0.02)	(0.02)
box 1234	1.37**	0.25	2.78**	3.36**
[1 if field is 1, 2, 3, or 4]	(0.41)	(0.50)	(0.72)	(0.91)
1/*t* x *box* 1234	0.13	2.38	-2.3	-3.06
[inverse of a period trend for *box* 1234]	(1.30)	(1.84)	(2.31)	(3.01)
1/*t* x *box* 5678	-0.05	-2.39	2.09	3.07
[inverse of a period trend for *box* 5678]	(1.52)	(1.66)	(2.41)	(3.02)
Constant	22.48**	12.36**	21.45**	15.11**
	(0.67)	(0.78)	(0.84)	(0.94)
Observations	3584	3584	3584	3584
Number of subject	32	32	32	32

Robust standard errors in parentheses. * significant at 5%, ** significant at 1%
In each regression we include dummies to control for for session effects
All models include a random effects error structure, with individual subject effects

3.3. The Determinants of Payoffs: "Good Ol' Rock"

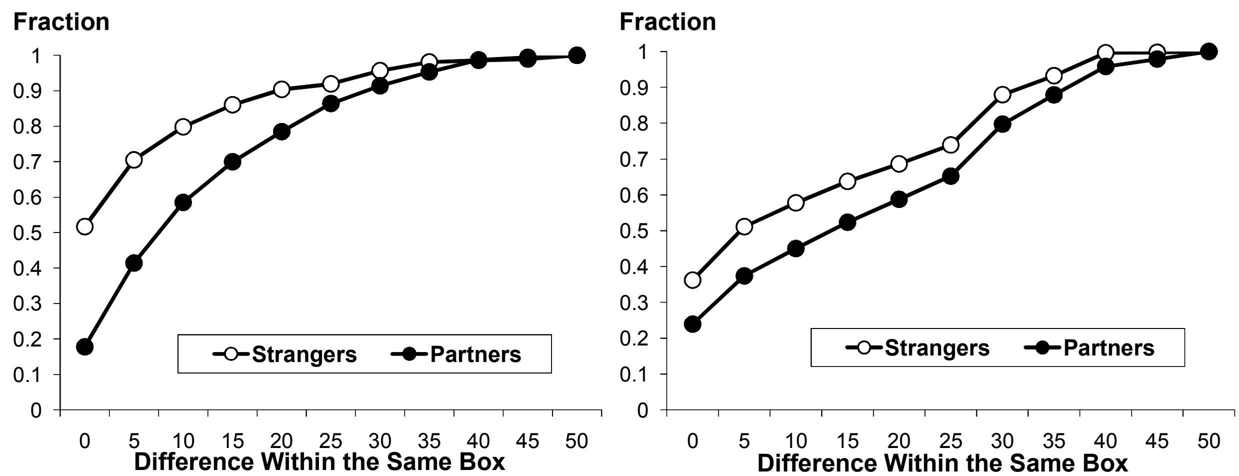
> "Lisa: Look, there's only one way to settle this. Rock-paper-scissors.
> Lisa's brain: Poor predictable Bart. Always takes `rock'.
> Bart's brain: Good ol' `rock'. Nuthin' beats that!
> Bart: Rock!
> Lisa: Paper.
> Bart: D'oh!"
> -*The Simpsons* (http://www.snpp.com/episodes/9F16.html)

In a repeated constant-sum game with equilibrium in nondegenerate mixed strategies, employing the same pure strategy in each period is not a good idea because one's opponent will eventually uncover this pattern and employ a best response. This is what happens to Bart in the episode of *The Simpsons* cited above. In the game of Rock-Paper-Scissors with Lisa, he always plays “Rock”, which Lisa recognizes from repeated play. As a result Lisa employs the best response “Paper” and always wins the game. Had Bart played Rock-Paper-Scissors against a different person each period, with his past history unobserved by his opponents, then playing his “good ol’ rock” strategy would be difficult to exploit and one would not expect Bart to perform poorly.

In the context of our analysis, a subject playing a pure strategy or some other strategy with high serial correlation in the repeated game would be expected to perform more poorly under the partners protocol than under the strangers protocol. Under the strangers protocol, independent randomization in the selection of matches in each period and the fact that the identity of a subject’s current opponent cannot be attached to specific past actions before current play, make it difficult for opponents to detect patterns of play that would be quickly exploited under the partners protocol. Of course, the extent to which high serial correlation in a subject’s strategy may be exploited by opponents in the strangers protocol relies in a complicated way on several factors. Certainly, the number of periods of play is important. If the number of periods is sufficiently large relative to the number of subjects in each role, one would expect repetition of a single action to be eventually detected. Due to the randomization in matching, the extent to which this repetition will be exploited depends on the strategies of other subjects playing the same role in a given session. If these subjects play equilibrium mixed strategies, one would expect the aggregate behavior of subjects in this role to make it difficult to detect a single player’s deviation to a pure strategy in a timely fashion. However, once detected one would expect such a deviation to be exploited, since the game is constant sum and, if all players conform to the equilibrium, opponents are indifferent to pure strategies in the support of their equilibrium mixed strategies. On the other hand, if other subjects in the same role also deviate from equilibrium behavior by employing strategies with high serial correlation, the payoff that a given player receives from playing a pure strategy (or otherwise employing high serial correlation) would be expected to depend in a complicated way on the nature of these deviations. Learning on the part of opponents may end up benefiting or harming a given player, depending on how a given player's strategy deviates from the aggregate play of subjects in the same role. .

Figure 3.5 – Cumulative Distribution of Differences in Auction

(Player 1 – left panel, Player 2 – right panel)



Indeed, we found that, within each box, the "good ol' rock" strategy was frequently employed by subjects under the strangers protocol in the auction treatment.[9] Specifically, we found that a number of players maintained a constant within-box allocation over different periods and earned substantial payoffs. This type of behavior was significantly reduced when we employed the partners protocol. Support for this finding can be found in Figure 3.5 which displays the cumulative distribution of the absolute differences between allocations within the same box in periods t and t+1 in the auction treatment. When paired with strangers, for

[9] Since pure strategies are octuples, our use of "good 'ol rock" in this context refers to a constant allocation to a given box over time and not to a constant octuple over time. As a general rule, when subjects maintained a constant within-box allocation across time for one or more of the eight boxes, they did so over a strict subset of the boxes and varied the within-box allocation of other boxes.

approximately 50% of the time periods subjects in the role of player 1 do not change their allocation from t to t+1 (the difference within the same box is 0). For subjects in the role of player 2, this percentage is around 35%. However, when paired up with the same partner, the percentage goes down to around 20% for both players. Note that the empirical CDF of these within-box absolute differences under the partners protocol first order dominates the corresponding CDF under the strangers protocol.

Table 3.4 – Determinants of Payoff (Strangers)

Treatments	Lottery	Lottery	Auction	Auction	Auction	Auction
Dependent variable,	(1)	(2)	(3)	(4)	(5)	(6)
payoff	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2
between-boxes	-0.46*	-1.00**	0.78**	1.40**	2.31**	2.84**
[difference between 8 fields in period *t*]	(0.20)	(0.16)	(0.20)	(0.17)	(0.43)	(0.44)
between^2-boxes					-0.68**	-0.61**
[difference squared between 8 fields in period *t*]					(0.22)	(0.18)
within-boxes	0.22	0.13	-0.03	0.05	0.29	0.11
[difference within the same field in periods *t* and *t*-1]	(0.17)	(0.14)	(0.13)	(0.07)	(0.27)	(0.25)
within^2-boxes					-0.12	-0.01
[difference squared within the same field in periods *t* and *t*-1]					(0.10)	(0.07)
1/*t*	-0.33	0.38	1.04*	0.29	1.37**	0.26
[inverse of a period trend]	(0.51)	(0.48)	(0.43)	(0.38)	(0.41)	(0.39)
Constant	5.09**	3.33**	4.92**	0.79**	4.53**	0.22
	(0.16)	(0.15)	(0.19)	(0.24)	(0.18)	(0.28)
Observations	448	448	448	448	448	448
Number of subject	32	32	32	32	32	32

Robust standard errors in parentheses. * significant at 5%, ** significant at 1%
In each regression we include dummies to control for for session effects
All models include a random effects error structure, with individual subject effects

Another interesting result under the two different protocols is the effect of randomization on payoffs. We find that deviations from equilibrium behavior by employing either greater dispersion of resources across boxes in the lottery treatment or less dispersion across boxes (or within each box, across time) in the auction treatment are associated with lower payoffs. To show this, we estimate random effects models of the following form:

$$payoff_{it} = \beta_0 + \beta_1 between\text{-}boxes_{it} + \beta_2 between\text{^}2\text{-}boxes_{it} + \beta_3 within\text{-}boxes_{it} +$$
$$+ \beta_4 within\text{^}2\text{-}boxes_{it} + \beta_5(1/t) + \textstyle\sum_h \beta_{6h} S_h + u_i + \varepsilon_{it} ,$$

where *payoff* is player *i*'s payoff in a period *t*. The *between-boxes* variable is defined as the absolute difference between the tokens allocated to a specific box and the mean across all boxes.

So, for player 1 (player 2) this variable indicates how far the allocation to a specific box is from 25 (15). The *between^2-boxes* variable is defined as the square of the difference between the tokens allocated to a specific box and the mean. The *within-boxes* variable is defined as the absolute difference between the tokens allocated to the same box in periods *t* and *t*-1. The *within^2-boxes* variable is defined as the square of the difference between the tokens allocated to the same box in period *t* and period *t*-1.

The estimation results for treatments which used the strangers protocol are shown in Table 3.4. The *between-boxes* coefficient in specifications (1) and (2) is negative and significant. This indicates that in the lottery treatment, under the partners matching protocol, a deviation from the mean allocation of 25 (15) significantly decreases player 1's (2's) payoff. This finding indicates that there is a strong incentive for subjects to converge to the equilibrium allocation. In the auction treatment the opposite effect takes place. The *between-boxes* coefficient in specifications (3) and (4) is positive and significant, indicating that by deviating from the mean, players 1 and 2 earn significantly higher payoffs. However, it misleading to infer that a player would obtain a higher payoff by increasing the magnitude of the *between-boxes* variable to the extreme. If true, then the best strategy for player 1 would be to allocate 200 to one box and 0 to the other boxes. To control for the fact that too much deviation from the mean can be harmful, in specifications (5) and (6), we include the *between^2-boxes* and the *within^2-boxes* variables. As in specifications (3) and (4), the *between-boxes* coefficient is still positive and significant. However, the *between^2-boxes* coefficient is negative, indicating that some deviation from the mean increases payoff while excessive deviation decreases payoff. The *within-boxes* and *within^2-boxes* variables are not significant in any of the specifications, indicating that the "good ol' rock" strategy does not affect the payoff and thus can be optimal under the strangers protocol.

Table 3.5 reports the estimation results under the partners protocol. An interesting contrast with Table 3.4 is reflected in the estimation of specifications (3) through (6). Because of the partners protocol, the "good ol' rock" strategy does not work well in the auction treatment. This is reflected in specifications (3) and (4) by the significant *within-boxes* coefficient. Randomizing allocations within the same box across periods significantly increases the payoff for both players. Specifications (5) and (6) confirm that large variation in allocations, even within box, has a negative effect on payoff.

Table 3.5 – Determinants of Payoff (Partners)

Treatments	Lottery	Lottery	Auction	Auction	Auction	Auction
Dependent variable,	(1)	(2)	(3)	(4)	(5)	(6)
payoff	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2
between-boxes	-0.53*	-0.71**	0.11	0.64**	0.87*	0.57
[difference between 8 fields in period *t*]	(0.25)	(0.19)	(0.15)	(0.17)	(0.40)	(0.35)
between^2-boxes					-0.27	0.15
[difference squared between 8 fields in period *t*]					(0.18)	(0.13)
within-boxes	0.39*	0.02	0.31*	0.24*	1.18**	1.08**
[difference within the same field in periods *t* and *t* -1]	(0.19)	(0.17)	(0.13)	(0.10)	(0.28)	(0.23)
within^2-boxes					-0.24**	-0.23**
[difference squared within the same field in periods *t* and *t* -1]					(0.07)	(0.06)
1/*t*	-0.73	0.74	0.45	-0.05	0.47	-0.17
[inverse of a period trend]	(0.49)	(0.49)	(0.41)	(0.39)	(0.38)	(0.39)
Constant	5.12**	3.23**	5.24**	1.41**	4.67**	0.99**
	(0.20)	(0.18)	(0.18)	(0.22)	(0.20)	(0.26)
Observations	448	448	448	448	448	448
Number of subject	32	32	32	32	32	32

Robust standard errors in parentheses. * significant at 5%, ** significant at 1%
In each regression we include dummies to control for for session effects
All models include a random effects error structure, with individual subject effects

4. Conclusions

This paper represents a first attempt at experimentally investigating the classic Colonel Blotto game employing two popular contest success functions: the auction and lottery CSFs. Under the lottery treatment, the equilibrium prediction is that each player should divide their resources equally across all battlefields. The experimental results support this prediction. Moreover, deviations from equilibrium behavior result in lower payoffs. Under the auction treatment, equilibrium requires that the disadvantaged player stochastically allocate zero resources to a subset of battlefields and the advantaged player allocate random, but positive, resource levels across the battlefields. Again, the data support this theoretical prediction and deviations from equilibrium behavior in the form of strategies exhibiting low dispersion of allocations across battlefields at a point in time or within a battlefield over time are associated with lower payoffs.

Due to the constant-sum nature of the game, we examined both partners and strangers matching protocols. The choice of matching protocol has significant effects on subject behavior under the auction treatment. Under the strangers protocol subjects are prone to "hot box" and "good 'ol rock" strategies. In the former winning a box in a period encourages the subject to allocate more resources to that box in the next period. In the latter, independent randomization across periods is replaced with strategies exhibiting high within-box serial correlation of allocations. In fact, under the strangers protocol subjects often allocate exactly the same level of the resource to a given box across periods. Occurrence of both the "hot box" and "good ol' rock" strategies significantly diminishes under the partners protocol. To our knowledge, this is the first study to explicitly recognize such effects of the strangers protocol in constant sum games. Our results signal the need for further analyses of these issues.

The Colonel Blotto game is one that is easy to understand yet analytically quite challenging. Because of its compelling structure as a prototype model of strategic multi-dimensional resource allocation, the game has been utilized in many real-life applications, such as military conflicts, advertising resource allocation, political campaigns, and research and development portfolio selection. Although researchers have been grappling with an analytical solution to the game since Borel (1921), and a complete characterization of the set of equilibria under the auction treatment is still an open question, it took only one hour for subjects who were unfamiliar with this game to exhibit behavior consistent with equilibrium. Players' marginal distributions and payoffs conformed to what we know must be true of *all* equilibria in the Colonel Blotto game (Roberson, 2006)

The success of experimental results in strongly supporting existing theory in this computationally challenging game is very encouraging. It also suggests that experiments can be used extensively to provide guidance for other theoretically challenging problems arising in related games. This remains a promising avenue for the future research.

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Appendix

GENERAL INSTRUCTIONS

This is an experiment in the economics of strategic decision making. Various research agencies have provided funds for this research. The instructions are simple. If you follow them closely and make careful decisions, you can earn an appreciable amount of money.

The experiment will proceed in three parts. Each part contains decision problems that require you to make a series of economic choices which determine your total earnings. The currency used in Part 1 of the experiment is U.S. Dollars. The currency used in Parts 2 and 3 of the experiment is francs. Francs will be converted to U.S. Dollars at a rate of **X** francs to **1** dollar. At the end of today's experiment, you will be paid in private and in cash. **16** participants are in today's experiment.

It is very important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation.

At this time we proceed to Part 1 of the experiment.

INSTRUCTIONS FOR PART 1
YOUR DECISION

In this part of the experiment you will be asked to make a series of choices in decision problems. How much you receive will depend partly on **chance** and partly on the **choices** you make. The decision problems are not designed to test you. What we want to know is what choices you would make in them. The only right answer is what you really would choose.

For each line in the table in the next page, please state whether you prefer option A or option B. Notice that there are a total of **15 lines** in the table but just **one line** will be randomly selected for payment. Each line is equally likely to be chosen, so you should pay equal attention to the choice you make in every line. After you have completed all your choices a token will be randomly drawn out of a bingo cage containing tokens numbered from **1 to 15**. The token number determines which line is going to be paid.

Your earnings for the selected line depend on which option you chose: If you chose option A in that line, you will receive **$1**. If you chose option B in that line, you will receive either **$3** or **$0**. To determine your earnings in the case you chose option B there will be second random draw. A token will be randomly drawn out of the bingo cage now containing twenty tokens numbered from **1 to 20**. The token number is then compared with the numbers in the line selected (see the table). If the token number shows up in the left column you earn $3. If the token number shows up in the right column you earn $0.

Are there any questions?

Participant ID _________

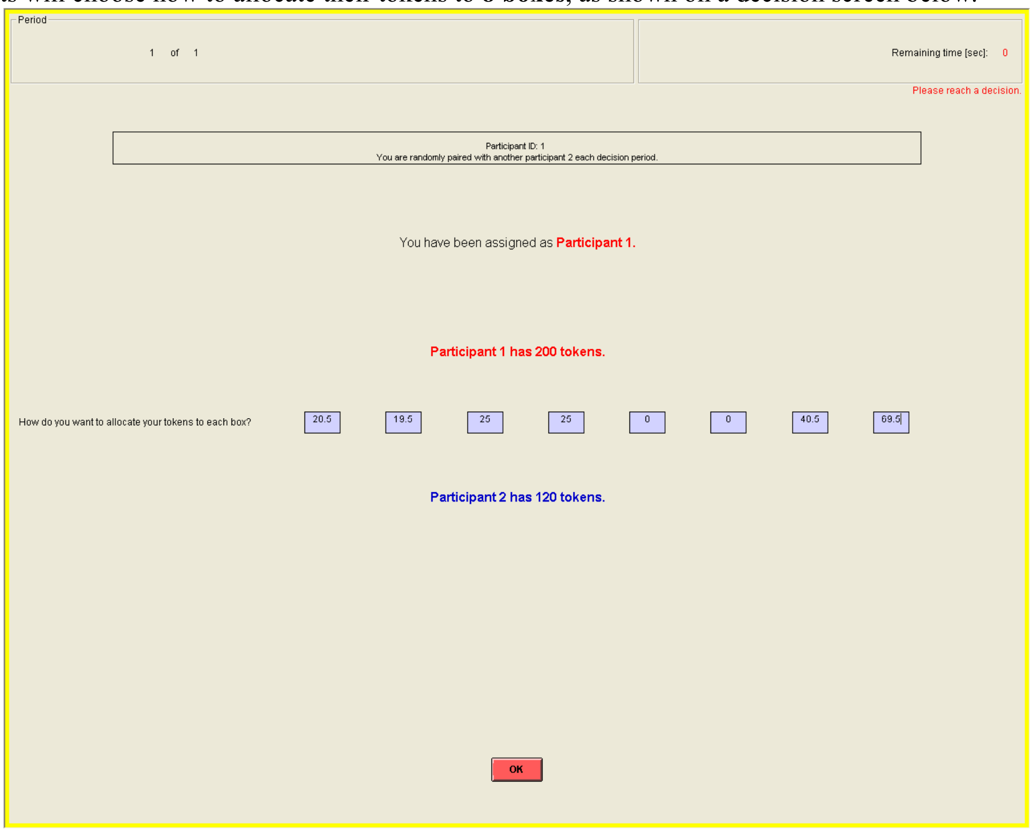
Decision no.	Option A	Option B		Please choose A or B
1	**$1**	**$3** never	**$0** if 1,2,3,4,5,6,7,8,9,10,11,12,13, 14,15, 16,17,18,19,20	
2	**$1**	**$3** if 1 comes out of the bingo cage	**$0** if 2,3,4,5,6,7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
3	**$1**	**$3** if 1 or 2 comes out	**$0** if 3,4,5,6,7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
4	**$1**	**$3** if 1,2, or 3	**$0** if 4,5,6,7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
5	**$1**	**$3** if 1,2,3,4	**$0** if 5,6,7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
6	**$1**	**$3** if 1,2,3,4,5	**$0** if 6,7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
7	**$1**	**$3** if 1,2,3,4,5,6	**$0** if 7,8,9,10,11,12,13,14,15, 16,17,18,19,20	
8	**$1**	**$3** if 1,2,3,4,5,6,7	**$0** if 8,9,10,11,12,13,14,15, 16,17,18,19,20	
9	**$1**	**$3** if 1,2,3,4,5,6,7,8	**$0** if 9,10,11,12,13,14,15, 16,17,18,19,20	
10	**$1**	**$3** if 1,2,3,4,5,6,7,8,9	**$0** if 10,11,12,13,14,15,16,17,18,19,20	
11	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10	**$0** if 11,12,13,14,15,16,17,18,19,20	
12	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11	**$0** if 12,13,14,15,16,17,18,19,20	
13	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12	**$0** if 13,14,15,16,17,18,19,20	
14	**$1**	**$3** if 1,2,3,4,5,6,7,8,9,10 11,12,13	**$0** if 14,15,16,17,18,19,20	
15	**$1**	**$3** if 1,2,3,4,5,6,7,8,9,10 11,12,13,14	**$0** if 15,16,17,18,19,20	

INSTRUCTIONS FOR PART 2

YOUR DECISION

The second part of the experiment consists of **15** decision-making periods. At the beginning of each period, you will be randomly and anonymously placed into a group which consists of **two participants**: participant 1 and participant 2. At the beginning of the first period you will be randomly assigned either as **participant 1** or as **participant 2**. You will remain in the same role assignment throughout the entire experiment. So, if you are assigned as participant 1 then you will stay as participant 1 throughout the entire experiment.

Each period, participant 1 will receive **200** tokens and participant 2 will receive **120** tokens. Both participants will choose how to allocate their tokens to **8 boxes**, as shown on a decision screen below.



Participant 1 can allocate any number of tokens between **0** and **200** (including 0.5 decimal points) to each box. The total number of tokens in all boxes must sum to **200** or the computer will not accept the decision of participant 1. Similarly, participant 2 can allocate any number of tokens between **0** and **120** (including 0.5 decimal points). The total number of tokens in all boxes must sum to **120** or the computer will not accept the decision of participant 2.

YOUR EARNINGS

After each participant has made his or her decisions, your earnings for the period are calculated. Your period earnings are proportional to the number of boxes you win. For each box you win you will receive **1 franc**.

Your earnings = Number of boxes you won × 1 franc

So, if you win all 8 boxes, you will receive 8 francs for this period. If you do not win any of the boxes, you will receive 0 francs. Francs will be converted to U.S. Dollars at a rate of __X__ francs to __1__ dollar. Your conversion rates are your private information. All conversion rates for participant 1 are equal and all conversion rates for participant 2 are equal. However, the conversion rates are different for participants 1 and 2. Notice that the more francs you earn, the more dollars you earn. What you earn depends partly on your decision and partly on the decision of the other participant with whom you are paired.

The more tokens you allocate to a particular box, the more likely you are to win that box. The more tokens the other participant allocates to the same box, the less likely you are to win that box. Specifically, for each **token** you allocate to a particular box you will receive **10 lottery tickets**. At the end of each period the computer **draws randomly** one ticket among all the tickets purchased by you and the other participant in your group. The owner of the drawn ticket wins the box and receives 1 franc for that box. Thus, your chance of winning a particular box is given by the number of tokens you allocate to that box divided by the total number of tokens you and the other participant allocate to that box.

$$\text{Chance of winning a box} = \frac{\text{Number of tokens you allocate to that box}}{\text{Number of tokens you allocate + Number of tokens the other participant allocates to that box}}$$

In case both participants allocate zero to the same box, the computer will randomly chose a winner of that box. Therefore, each participant has the same chance of winning the box.

Example of the Random Draw

This is a hypothetical example used to illustrate how the computer makes a random draw. Let's say participant 1 and participant 2 allocate their tokens to eight boxes in the following way.

Box	Participant 1	Participant 2	Chance of winning the box for Participant 1	Chance of winning the box for Participant 2
1	20.5	15	20.5/(20.5+15) = 0.58	15/(20.5+15) = 0.42
2	19.5	15	19.5/(19.5+15) = 0.57	15/(19.5+15) = 0.43
3	25	10	25/(25+10) = 0.71	25/(25+10) = 0.29
4	25	10	25/(25+10) = 0.71	25/(25+10) = 0.29
5	0	0	0.50	0.50
6	0	40	0/(0+40) = 0.00	40/(0+40) = 1.00
7	40.5	15.5	40.5/(40.5+15.5) = 0.72	15.5/(40.5+15.5) = 0.28
8	69.5	14.5	69.5/(69.5+14.5) = 0.83	14.5/(69.5+14.5) = 0.17
Total	200	120		

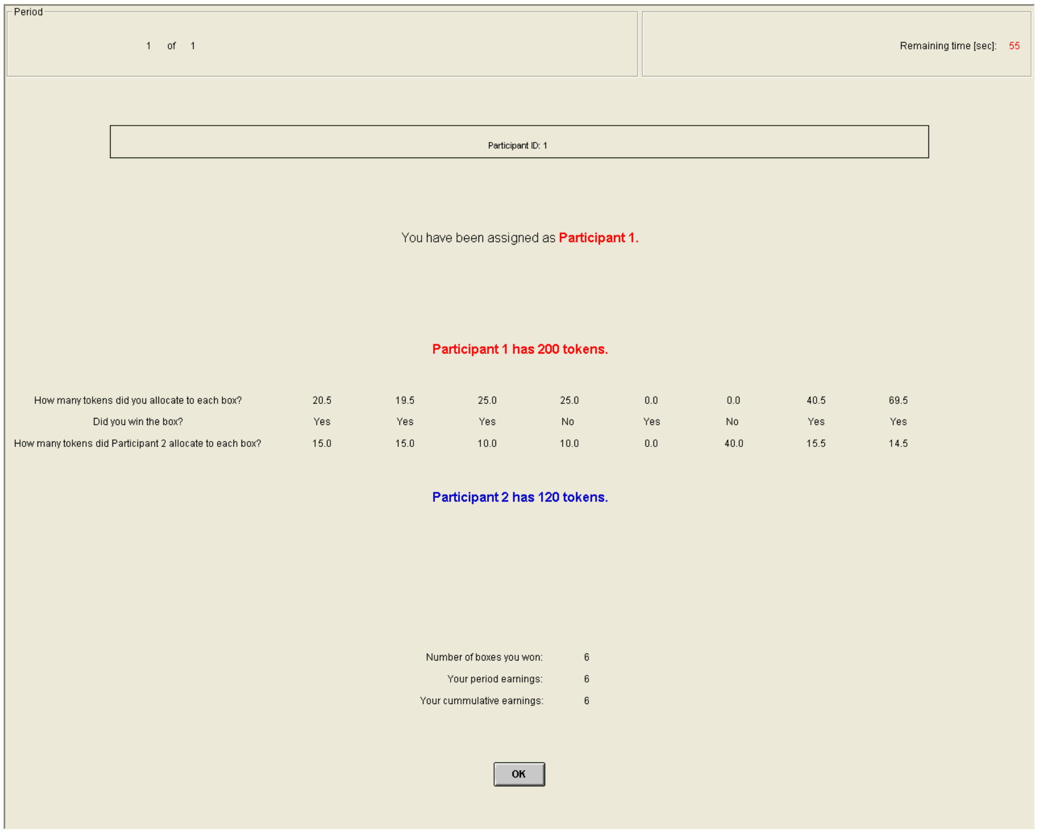
Participant 1 allocates 20.5 tokens to box 1, 19.5 tokens to box 2, 25 tokens box 3, 25 tokens to box 4, 0 tokens to box 5, 0 tokens to box 6, 40.5 tokens to box 7, and 69.5 tokens to box 8 (total of 200 tokens). Participant 2 allocates 15 tokens to box 1, 15 tokens to box 2, 10 tokens to box 3, 10 tokens to box 4, 0 tokens to box 5, 40 tokens to box 6, 15.5 tokens to box 7, and 14.5 tokens to box 8 (total of 120 tokens). Therefore, the computer will assigns lottery tickets to participant 1 and to participant 2 according to their allocation of tokens.

For example, in box 1, the computer will assign 205 lottery tickets to participant 1 and 150 lottery tickets to participant 2. Then the computer will randomly draw **one lottery ticket out of 355** (205+150). As you can see, participant 1 has a **higher chance** of winning box 1: 20.5/(20.5+15) = 0.58. Participant 2 has **lower chance** of winning box 1: 15/(20.5+15) = 0.42.

Similarly, in box 6, the computer will assign 0 lottery tickets to participant 1 and 400 lottery tickets to participant 2. Then the computer will randomly draw **one lottery ticket out of 400** (0+400). As you can see, participant 1 has no chance of winning box 6: 0/(0+40) = 0.0. Therefore, participant 2 will win box 6 for sure: 40/(0+40) = 1.0.

After all participants allocate their tokens and press the OK button, the computer will make a random draw for each box separately and independently. Note that you can never guarantee that you will win a particular box. However, by increasing your allocation to that box, you can increase your chance of winning that box. The random draw made by the computer will decide which boxes you win. Then the computer will calculate your period earnings based on how many boxes you won.

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, your period earnings, and your cumulative earnings are reported on the outcome screen as shown below. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading.



IMPORTANT NOTES

At the beginning of the first period you will be randomly assigned either as participant 1 or as participant 2. You will remain in the same role assignment throughout the entire experiment. Each consecutive period you will be randomly re-paired with another participant of opposite assignment. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

At the end of the experiment you will convert your cumulative earnings into a payment in U.S. dollars. Your conversion rates are your private information. All conversion rates for participant 1 are equal and all conversion rates for participant 2 are equal. However, the conversion rates are different for participants 1 and 2.

Are there any questions?

INSTRUCTIONS FOR PART 3

The third part of the experiment consists of **15** decision-making periods. As in the previous part 2 of the experiment, you will be placed into a group which consists of **two participants**: participant 1 and participant 2. Your assignment as participant 1 or participant 2 will be the same as it was in the previous part 2 of the experiment.

Each period, participant 1 will receive **200** tokens and participant 2 will receive **120** tokens. Both participants will choose how to allocate their tokens to **8 boxes**. Participant 1 can allocate any number of tokens between **0** and **200** (including 0.5 decimal points) to each box. The total number of tokens in all boxes must sum to **200** or the computer will not accept the decision of participant 1. Similarly, participant 2 can allocate any number of tokens between **0** and **120** (including 0.5 decimal points). The total number of tokens in all boxes must sum to **120** or the computer will not accept the decision of participant 2.

After each participant has made his or her decisions, your earnings for the period are calculated. Your period earnings are proportional to the number of boxes you win. For each box you win you will receive **1 franc**.

Your earnings = Number of boxes you won × 1 franc

The main difference from part 2 is that the computer will choose the winner of each box in the following way. The participant who allocates more tokens than the other participant to a particular box wins that box with certainty. So, if participant 1 allocates 30 tokens to a particular box while participant 2 allocates 29.5 tokens to the same box then the computer will chose participant 1 as the winner of that box. In case both participants allocate the same amount to the same box, the computer will always chose participant 1 as a winner of that box. In case both participants allocate zero to the same box, the computer will always chose participant 1 as a winner of that box.

After all participants allocate their tokens and press the OK button, the computer will determine the winner of each box and will calculate your period earnings based on how many boxes you won.

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, your period earnings, and your cumulative earnings are reported on the outcome screen. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading.

IMPORTANT NOTES

Your assignment as participant 1 or participant 2 will be the same as it was in the previous part 2 of the experiment. You will remain in the same role assignment throughout the entire experiment. Each consecutive period you will be randomly re-paired with another participant of opposite assignment. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

The participant who allocates more tokens than the other participant to a particular box wins that box with certainty.

At the end of the experiment you will convert your cumulative earnings into a payment in U.S. dollars. Your conversion rates are your private information. All conversion rates for participant 1 are equal and all conversion rates for participant 2 are equal. However, the conversion rates are different for participants 1 and 2.

Are there any questions?