Investment driven Mixed Firms: Partial Privatization by Local Governments

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Abstract

We analyze partial privatization by local governments, driven by investment and credit constraints, and provide a theory of mixed firms based on strategic interaction between local politicians and private shareholders. Minority participation by private investors – as empirically observed – arises endogenously in the model to prevent investment expropriation. We consider the example of water supply with perfectly inelastic demand and fix-price regulation, coupled with price discretion at a local level. Welfare maximizing local governments face a trade-off between the increase in consumers surplus and the reduction of costly public funds. This cost decreases with prices and the public ownership share. Therefore, private shareholders choose investments to keep the government share at a threshold such that the politician always sticks to the price-cap and dividends are then maximized. Through calibration and simulation, we compare investment by mixed firms and by a social planner, which is neutral with respect to finance.

Key words: corporate governance, investment expropriation, price-cap regulation, water networks.

JEL Codes: L32, G38, G32

1 Introduction

The privatization process has been extensively analyzed in the economic literature. Most attention has been devoted to privatization by central governments though also local governments have been involved in this process. However local governments have not completely privatized public firms: corporatization and partial privatization have been widespread.

As far as local governments face increasing constraints on the expansion of taxation and public expenditures, they can revert to off-budget government (Joulfaian and Marlow, 1991) by transforming municipal undertakings into local government sponsored enterprises[1]. Corporatization of public firms, where the local governments is the sole shareholder, can be the first step towards partial privatization

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[1]Such a trend was analyzed by Bennet and Dilorenzo (1982) for the US, showing that local governments have responded to tax and expenditure limitations by financing expenditures through

and the creation of a mixed firm, especially widespread in some European Countries.

Mixed firms continue to be a puzzle for economic analysis as one wonders how profit maximization goals can be consistent with the eventual pursuit of of social and political goals by public shareholders. Then theoretical questions may be raised about what the objective function of mixed firms can be. Private participation with minority stakes raises further concern, as one wonders about the rationale of private investments in firms controlled by local politicians, given the risk of investment expropriation as politicians may not be keen to profit maximization.

Empirical evidence shows that local politicians seldom loose control of mixed firms. Even when private investors hold the majoirty of shares, local governments continue to retain control and frequently a significant ownership share. In some cases private participation may be characterized by really thin (one digit) ownership share. For example, in Germany, when considering 558 municipal mixed firms with the local government as the sole public shareholder[2], one finds that in 69% of cases the local government holds a share greater or equal to 50%, in 21% of cases it still holds control with a share between 25% and 50%, and just in 9,4% of cases the local government share falls under 25% (Richter et al., 2006). The same occurs in Italy. According to a report (Mediobanca, 2015), considering a representative sample of local mixed firms, private participation is below 49% in 13 cases out of 22; Moreover, most of the firms with larger private shares are listed at the stock exchange, implying fragmentation of shareholders and control by local Governments holding minority stakes (Mediobanca, 2015, p. 42-48).

In this paper we consider the issue of minority participation of private shareholders in mixed firms, by restricting our attention to partial privatization due to the need of increasing investments. A corporation owned by a local Governments and facing credit constraints carries out partial privatization with an increase of the capital stock..The new share issue is completely subscribed by a private shareholder. We consider the specific case of network investments in the water sector, assuming price-cap regulation. The water industry is particularly suitable for our analysis because: 1) It has been characterized by partial privatization across the main European countries. 2) Privatization in this industry has been frequently motivated by the need of new investments (see Newbery, 1999 for the UK case and Bognetti and Robotti, 2007 for the Italian case[3]). 3) The risk of investment expropriation is not negligible in the water sector as asset specificity and the long life of investments may lead local politicians to devote quasi-rents to price reductions in order to protect consumers and get electoral benefits. 4)The multilevel structure of regulation in the water industry (OECD, 2011) allows some discretion to municipalities concerning final water charges.

In the framework of our model a benevolent politician can maximize local welfare either by reducing prices under the price-cap, to increase consumers surplus, or by increasing water revenues up to the maximum price, in order to increase dividends and save on costly public funds.

Given the risk of price reductions, we assume that the public corporation can as-

"off-budget enterprises." A similar phenomenon occurred in Europe, where local governments had to cope with tight fiscal policies imposed by the EU.

[2]These firms are included in the larger population of 2009 municipal firms concerning towns with more than 50,000 inhabitants.

[3]Bognetti and Robotti (2007) observe that in Italy partial privatization by local utilites involved in water supply, was accompanied by new investments in the 70% of cases.

sure a preferential treatment of private shareholders concerning the distribution of dividends, such that they can always recover their opportunity cost of capital (participation constraint). However we show that the lowest price that local politicians can afford is never chosen in equilibrium, as not only private shareholders but also the local government recover at least the opportunity cost of capital. Furthermore we show that in equilibrium the local politician can be driven to set the highest price (the price-cap) when its ownership share is high enough to imply that welfare is maximized by increasing municipal dividends rather than consumer surplus.

Therefore, private shareholders, when deciding about the amount of investment (and the increase of the corporation stock financed by them) may prefer to hold a minority share, in order to preserve the incentive for the local government to stick to the price cap and increase public dividends as much as possible. We can then conclude that the typical ownership structure of mixed firms that we empirically observe may be consistent with the strategic decisions of private shareholders and not only depend on the political will of local governments. Our result is also consistent with empirical evidence (Bortolotti and Faccio, 2004), showing that partial privatization of firms where governments do not relinquish control rights does not negatively affect market valuation as expected.

The economic literature about corporatization and mixed firm is scarce[4], with some exceptions focused on mixed oligopolies (Matsumura, 1998). In this last case the assumption is that mixed firms maximize a weighted average of the payoff of the government and its own profits, and that the weight is affected by the proportion of shares held by the government. In our opinion such an assumption does not solve the puzzle concerning the nature of mixed firms. Another contribution is due to Boycko (1996), Shleifer and Vishny (1994). In this last case politicians by assumptions pursue social goals in conflict with productive efficiency and, even when they loose control, they can try to corrupt private managers in order to persuade them to deviate from profit maximization. Even complete privatization is not exempted by this risk, leading managers in turn to bribe politicians in order to pursue profit maximization. Firm restructuring after corporatization and privatization is then likely to depend on the outcome of a bargaining process between politicians and managers. Though we simply assume that private shareholders make a take-it-or-leave-it offer to the politician concerning investments, we think that also our model could be extended to include bargaining and political economy issues, as in Boycko (1996), Shleifer and Vishny (1994).

Our result is more related to the solution of the commitment problem, considered by Perotti (2005), wondering about the rationale of gradual privatization and underpricing of shares. The author emphasizes the decisions of governments to retain large stakes in the firm, even after the loss of control, in order to commit not to expropriate private shareholders ex-post. Partial privatization and underpricing work as a signal for central governments that are interested in reducing the perceived risk of expropriation by an eventual change of public policy. According to Perotti (2005) the willingness of governments to bear the residual risk as a minority shareholder is a signal that governments are not going to redistribute value with a future policy shift. On the contrary the inability to commit by the local governments is at the center of our analysis. In our model local governments can just commit to a price rule, according to the regulatory mechanism. However given

[4]From the theoretical point of view, mixed firms should be distinguished from Public Private Partnerships (PPPs) which have been extensively analyzed in the economic literature

that such a mechanism allows the local government to choose a price lower than the price cap, value expropriation represents an ex-ante risk also in our framework. The expropriation risk is directly managed by private shareholders, that can decide on investments and strategically commit on private participation with a small ownership share, in order to make expropriation not convenient for the benevolent local politician.

The structure of the paper is the following. In section two we introduce the basic assumptions about demand and supply of water at the local level, network investments and price regulation. In section three we consider our assumptions about investment driven partial privatization. Equilibrium analysis is presented in section four. Investments, prices and ownership shares are then derived by considering strategic interaction between the local government and private shareholders in a multi-stage game with perfect information. In section five we consider optimal investments chosen by a social planner to be compared with equilibrium investments arising from the partial privatization process. A calibration and simulation exercise, which is also necessary to carry out the comparison, is then presented in section six. In section seven we consider investment distortions that result from the comparison between investment behavior in mixed firm and the social optimum, where finance is neutral with respect to investment decisions. Conclusions follow in section eight.

2 Investments and Price regulation in the Water Industry

Local governemnts are still involved in water supply across many countries and are frequently responsible for investments in the distribution network. Water supply in most cases is vertically integrated with distribution: a local natural monopoly with regulated prices. Another important feature of water supply is the very low price elasticity of demand, due both to the importance of water consumption and to metering problems. We can then assume that water demand is perfectly inelastic to price. Such an assumption appears to be more interesting from the analytic point of view and not far from reality, given the very low value of demand elasticity arising from empirical studies.[5] Therefore in this model market demand for water, denoted by Q^C, is perfectly inelastic up to a price $P^{\max}$,[6] and we normalize Q^C to one. Then $P^{\max}$ represents gross consumer surplus as well.

[5] Empirical studies show estimates that consistently indicate a price-inelastic demand for water. A meta-analysis of almost 300 price elasticity studies, reports a mean price elasticity of -0.41 (Dalhuisen et al., 2003). Olmstead et al. (2007) consider the bias that could be due to estimations based on linear prices. They also consider non linear tariffs with separate estimates, finding an higher value for price elasticity, though equal to -0.59. Interestingly, they fail to identify a price elasticity significantly different from zero for the uniform-price households.

[6] Motivations for a perfectly inelastic demand, up to price $P^{\max}$ – implying a discontinuity at $P^{\max}$ such that for $P > P^{\max}$, water consumption becomes zero – may also include the opportunity of switching to substitutes like the resort to water distribution carried out by tank trucks (as it happens in areas not reached by the network).

2.1 Investments

We concentrate our attention on infrastructure investments, that are needed to reduce network leaks L[7]. Water leaks generate quality losses and negative externalities which are accounted by the social damage function $D = dL$. Network investments can then reduce water leaks according to a technology characterized by decreasing returns to scale:[8]

$$L(K, i) = L_0 - 2iK + \frac{(iK)^2}{L_0}, \quad K \in \left[0, \frac{L_0}{i}\right], \tag{1}$$

being L_0 the exogenous amount of water leaks, K the amount of investments and i the efficiency of investments. The latter is a technological parameter, exogenously given and can be hydiosincratic to each water undertaking. Given existing technologies, we assume that $i \in \left[\underline{i}; \overline{i}\right]$. From (1), for $K < \frac{L_0}{i}$ we have

$$\frac{\partial L(K, i)}{\partial K} = -2i\left(1 - \frac{iK}{L_0}\right) < 0, \quad \frac{\partial^2 L(K, i)}{\partial^2 K} = -\frac{2i^2}{L_0} < 0.$$

Therefore also social damages $D = dL$ decrease when network investments K increase.

Considering that leaks could partially become endogenous to the public utility, because of the investment effects, then water supply becomes:

$$Q^S = Q^C + L(K, i). \tag{2}$$

Investment to reduce network leaks contribute to reduce operational costs. The cost of water provision is given the sum of variable costs and capital costs arising from network investments K. At this stage, we simply consider as a capital cost the fixed cost K. Let then be β the constant variable cost per unit of water provided, including both labor and energy costs[9]. Given (2), and being Q^C normalized to one, the cost function of a water undertaking can be expressed as follows:

$$C = \beta Q^S + K = \beta\left[1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right] + K.$$

Remark that network investments, by reducing water leaks, have two positive effects: on the one hand, they reduce variable costs; on the other they reduce social damages. For the sake of simplicity we consider investments in a one-period model and neglect intertemporal issues.

2.2 Price Regulation

Each water undertaking operates under the supervision of a national regulatory agency that adopts a fix-price mechanism which is independent from the organizational structure. The resort to a fix-price mechanism is due to the assumption of

[7]Water leaks in the distribution network and the related investment to detect and reduce them are one of the most important issues faced at present by the water industry. Cfr. Egenhofer et al. (2012) for an assessment concerning European countries.

[8]The assumption of decreasing return to scale appears to be rather intuitive in this case and has been already used in similar frameworks (e.g., see Chakravorty et al., 1995).

[9]The increase of water injection in the network due to water leaks implies greater pumping efforts, giving rise to an increase of energy costs. The latter can be an non negligible share of total provision costs. Additional chemicals for water treatment may also be necessary, contributing then to increase variable costs.

asymmetric information between the regulator and managers of water undertakings, as the efficiency of capital investments is private information. The regulator just knows the range $\left[\underline{i};\bar{i}\right]$ of possible values for i. Considering that variable costs are decreasing in i, to make water supply and investments feasible, even for undertakings characterized by the lowest efficiency of capital $\underline{i}$, the fix-price mechanism should allow the recovery of operational costs up to the level $\beta\left[1+L_0-2\underline{i}K+\frac{(\underline{i}K)^2}{L_0}\right]$. Therefore any water undertaking with an efficiency parameter $i>\underline{i}$ is left an information rent:

$$\beta\left[\left(2iK-\frac{(iK)^2}{L_0}\right)-\left(2\underline{i}K-\frac{(\underline{i}K)^2}{L_0}\right)\right]=\beta(i-\underline{i})K\left[2-\frac{(i+\underline{i})K}{L_0}\right]>0.$$

As, by assumption, the demand for water is perfectly inelastic: *(i)* such a rent does not create any distortions in allocative efficiency; *(ii)* a fix price-mechanism reduces to a revenue-cap.

We assume that there are no public subsides and that each water undertaking can recover the cost of water provision according to a revenue-cap $\overline{P}$ including operational costs and the cost of capital. Moreover, we assume that the regulator - in order to induce efficiency concerning the structure and cost of financing - allows any water undertakings to recover a rate of return ρ on K, being ρ the risk-adjusted cost of capital considered by the regulator for the water sector. Accordingly, the revenue-cap depends on investments K

$$\overline{P}=\beta\left[1+L_0-2\underline{i}K+\frac{(\underline{i}K)^2}{L_0}\right]+(1+\rho)K. \tag{3}$$

Assuming a one-period model, the entire amount K of non subsidized investment is recovered in the price cap.

The function $\overline{P}$ then specifies the lowest level of price that assures the supply of water with a rate of return ρ on capital, even in the worst technological case $i=\underline{i}$. As frequently regulation in the water industry implies multi-level decisions (OECD, 2011) also local governments keep control of final water charges. We avoid considering complex institutional details and simply assume that local politicians can set a price $P\leq\overline{P}$, which is consistent with fix-price regulation.

3 Investment driven partial privatization

We consider partial privatization as a transformation of a government owned corporation into a mixed joint-stock company, through an increase of the corporation stock. A new share issue completely subscribed by a private shareholder. The resort to private shareholders results from constraints on the maximum share of debt that can finance new investments. As for any amount of investments K,a maximum amount of debt $\bar{c}K$ can be obstained.

Before partial privatization the corporation stock amounts to S°. Given the amount of debt cK, (with $c\leq\bar{c}$) the residual share of investment $(1-c)K$ can be financed by the contribution ΔS° of private shareholders to the corporation stock, to get $K=cK+\Delta S^\circ$, with $\Delta S^\circ=(1-c)K$. As partial privatization implies a scarcity of financial resources for the local government, we exclude further contributions by the local government to the corporation stock. We also neglect the opportunity of self-financing through dividends not distributed to shareholders.

After partial privatization, the corporation stock increase to $S = S^\circ + (1-c)K$. Therefore the share of the local government reduces to $p = \frac{S^\circ}{S}$, so that the public and private shares are, respectively,

$$p = \frac{S^\circ}{S^\circ + (1-c)K}, \qquad (1-p) = \frac{(1-c)K}{S^\circ + (1-c)K}. \tag{4}$$

Under mixed ownership, we assume that local government holds the majority of shares, i.e. $p \geq 0.51$and therefore has the power to choose final water prices. Given this governance constraint, the (approximate) upper bound on the the increase of corporation stocks due to private investors is $\Delta S^\circ = (1-c)K \leq 0.96\, S^\circ$.

The financing support of private shareholders entitles them to become pivotal concerning the amount of investments, given the constraint on the maximum amount of debt. therefore we assume that the investment decision pertains to private shareholders, once public shareholders have decided about the amount of debt. Through the investment choice, private shareholders are able to affect the ownership structure of the company and then also the local government share (cfr. (4)). The benevolent politician retains the right to choose the amount of debt and the final price, to maximize social welfare. Price regulation grants a rate of return ρ on firm assets, including both new investments K and the pre-existing corporation stock S°:

$$\overline{P}^C = \beta \left(1 + L_0 - 2\underline{i}K + \frac{(\underline{i}K)^2}{L_0}\right) + (1+\rho)K + \rho S^\circ. \tag{5}$$

Local politicians can set a final price $P < \overline{P}^C$, provided that such a lower price maximizes local welfare. Therefore private shareholders run the ex-ante risk that their public partner will not maximize the corporation profits in case welfare maximization leads politician to choose $P < \overline{P}^C$ instead of $\overline{P}^C$. We assume that, in order to attract private investors, the local government should assure them a minimum profit π (a participation constraint), sufficient to recover at least the opportunity cost of capital, independently of the price choice:

$$\pi = (1-c)\alpha\rho K, \quad c \leq \bar{c}, \tag{6}$$

where ρ is the rate of return set by the regulator and α results from the correction of ρ in order to account for the opportunity cost of capital; we then assume that $\alpha < 1$.

More generally, the profit of the mixed firm will be

$$\Pi^C = P - \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right) - K - crK, \tag{7}$$

where crK represents the cost of debt, given the rate of interest r. From (6) we obtain

$$\Pi^C \geq \pi = (1-c)\alpha\rho K.$$

As private stockholders are assured a minimum profit, total dividends U are shared between private (U_{1-p}) and public (U_p) stockholders as follows:

$$\begin{array}{lcl} U_{1-p} & = & \max\left\{(1-p)\Pi^C; (1-c)\alpha\rho K\right\}, \\ U_p & = & \Pi^C - U_{1-p}. \end{array} \tag{8}$$

It is possible to define the minimum price P_0 which a local water undertaking can afford. It will be such to recover the variable costs, the cost of debt (where $r < \rho$), and the opportunity cost of capital for private investors. One can then notice that by setting $P = P_0$, the local politician may give up its rights to cash its dividends (including the opportunity cost of capital of the local government stock S°):

$$P_0 = \beta \left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0} \right) + K + (cr + \alpha\rho(1-c))K.$$

Actually, if $P = P_0$ then $U_p = 0$, as $\Pi^C = U_{1-p} = (1-c)\alpha\rho K$.

If prices increase above P_0, profit growth assures a share of profits also to the local government, though still assuring that the participation constraint be satisfied. Then we can define a price interval $(P_0, P_S]$, with

$$P_S = \left(\frac{1-c}{1-p}\,\alpha\rho + 1 + cr \right) K + \beta \left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0} \right),$$

where P_S is the price level such that, for $P = P_S$ profits are distributed as follows:

$$\begin{aligned} (1-p)\Pi^C &= (1-c)\alpha\rho K \\ U_{1-p} &= (1-c)\alpha\rho K, \\ U_p &= \Pi^C - U_{1-p} = \alpha\rho S^\circ. \end{aligned}$$

Then at $P = P_S$, the local government obtains a rate of return on assets equal to the opportunity cost of capital granted to private shareholders. For any $P > P_S$, the dividends cashed by private stockholders exceed their participation constraint π and are equal to $U_{1-p} = (1-p)\Pi^C \neq (1-c)\alpha\rho K$. Likewise, the local government will get $U_p = p\Pi^C$. In this last case, if $P = \overline{P}^C > P_S$ the regulator grants a rate of return on assets $\rho > \alpha\rho$.

4 Equilibrium Analysis

We assume that borrowing, investment and pricing decisions are taken sequentially, given symmetric information between the politician and private shareholders and common knowledge about the pricing rule derived from theregulatory mechanism, such that the local politician can set $P \leq \overline{P}$ Strategic interaction can be represented as a sequential three stage game with perfect information. The timing is the following:

1. In the first stage the benevolent politician decides the optimal share of debt c^* by welfare maximization, given the constraint $c \leq \bar{c}$.

2. In the second stage (privatization stage), private shareholders act as a Stackelberg leader with respect to the price-maker politician, by choosing the optimal amount of K, accounting for the price rule and welfare maximization by the politician. At this stage, we assume that private investors will dispose of all the bargaining power and make a take-it or leave-it offer to the politician concerning the amount of investment K, accounting for the share of investment cK to be financed by debt.

3. In the third stage (price-setting stage), given the amount of K previously chosen and the resulting ownership shares p, $(1-p)$ – which depend on S°, K and c – the local government will maximize welfare by choosing $P \leq \overline{P}^C$, according to the price rule which is common knowledge.

The game can then be solved by backward induction. Firstly we solve the third stage to determine the welfare maximizing price for any amount of K previously chosen by the private manager and any share of debt cK. Then we solve the second stage, where the private manager commits to a level of K, taking into account the subsequent pricing choice of the politician. Finally, we consider the welfare maximizing level of debt in the first stage, given the constraint $c \leq \bar{c}$ and according to the choice of the benevolent politician.

III stage In case of mixed ownership local welfare is given by net consumer surplus, minus environmental damages, plus the social gain related to the amount of dividends distributed to the local government. Actually, these dividends will accrue to the public budget as a tax reduction (or as an increase of expenditure without any tax increase). By assumption, we do not include in social welfare the gains of private shareholders, to the extent that they are external to the local community. In the specific case of water provision such an assumption can capture the fact that private firms involved in the business of water provision (through partial or complete privatization) are mainly multinational firms. Then, the welfare function can be expressed as follows:

$$W^C = P^{\max} - P - d\left[L_0 - 2iK + \frac{(iK)^2}{L_0}\right] + (1+\lambda)U_p. \tag{9}$$

Where λ is the marginal cost of public funds, considering that the dividends cashed by the local government U_p accrue to the budget of the local public administration. Then the maximization problem of the local government is:

$$\begin{aligned}
\max_P W^C &= \max_P \left\{P^{\max} - P - d\left(L_0 - 2iK + \frac{(iK)^2}{L_0}\right) + (1+\lambda)U_p\right\}, \\
\text{s.t. } P \leq \overline{P}^C &= \beta\left(1 + L_0 - 2\underline{i}K + \frac{(\underline{i}K)^2}{L_0}\right) + (1+\rho)K + \rho S^\circ, \\
P \geq P_0 &= \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right) + K + (cr + \alpha\rho(1-c))K,
\end{aligned}$$

where U_p is given by (8) and $p = \frac{S^\circ}{S^\circ + (1-c)K}$.

Write now the welfare function, using (8) for U_p, and (1) for the losses

$$W^C = P^{\max} - P - d\,L(i,K) + (1+\lambda)\begin{cases} \Pi^C - (1-c)\alpha\rho K, & \text{if } \Pi^C \leq \frac{1-c}{1-p}\alpha\rho K, \\ p\,\Pi^C, & \text{if } \Pi^C > \frac{1-c}{1-p}\alpha\rho K. \end{cases}$$

where $\Pi^C \leq \frac{1-c}{1-p}\alpha\rho K$, for $P \leq P_S$ and $\Pi^C > \frac{1-c}{1-p}\alpha\rho K$ for $P > P_S$. (cfr. the previous section)

Since $\frac{\partial \Pi}{\partial P} = 1$ we get

$$\frac{\partial W^C}{\partial P} = \begin{cases} -1 + (1+\lambda) = \lambda, & \text{if } P \leq P_S, \\ -1 + (1+\lambda)p, & \text{if } P > P_S. \end{cases} \tag{10}$$

Lemma 1 *A welfare maximizing politician will never select $P < P_S$.*

Any price increase has a twofold effect on social welfare: 1) A reduction of consumers' surplus; 2) An increase in dividends accruing to local governments and then to tax-payers. Since when $P \leq P_S$ any increase in price is cashed by the local government (as private shareholders stick to their participation constraint), the social benefit due to fiscal gains more than compensate the loss in consumer surplus. Actually, a marginal increase of prices leads to a marginal fiscal benefit $(1+\lambda)$, while the marginal cost is a reduction of consumer surplus. Therefore, social welfare is increasing with price (as $\lambda > 0$) and the politician never finds it optimal to fix a price lower than P_S. Also notice that it is never optimal for the local government to give up its right to a remuneration of its share of the corporation stock in order to benefit consumers. (As shown at the end of Section 3, when $P = \overline{P}^C$, $U_p = \alpha\rho S^\circ$, then for $P \geq P_S$, $U_p \geq \alpha\rho S^\circ$.)

Instead, for $P > P_S$ any profit increase is shared between the local government and the private shareholder in proportion to the ownership shares p and $(1-p)$ respectively, and the welfare effect of a price increase depends on the relationship between p and λ. More precisely, thanks to (4),

$$\frac{\partial W^C}{\partial P} = (1+\lambda)p - 1 \begin{cases} > 0, & \text{if } p > \frac{1}{1+\lambda}, \Longrightarrow P^C = \overline{P}, \\ < 0, & \text{if } p < \frac{1}{1+\lambda}. \Longrightarrow P^C = P_S. \end{cases}$$

The greater the local government stock share and the greater the marginal cost of public funds, the more it is likely that the benefits of a price increase for tax-payers can overcome the cost for final consumers, leading the benevolent politician to choose the maximum price $\overline{P}^C$, in order to use revenues within the public budget. On the contrary, with a lower ownership share and low marginal cost of public funds, the government would find it better to minimize prices to the benefits of consumers and set $P^C = P_S$.

As $p = \frac{S^\circ}{S^\circ + (1-c)K}$, the above price rule will depend on the value of K, to be chosen by private shareholders in the second stage: for $K \leq \frac{S^\circ \lambda}{1-c}$ the politician will increase the price to the maximum allowed level $\overline{P}^C$, whereas for $K > \frac{S^\circ \lambda}{1-c}$ welfare is decreasing with prices, as soon as the price is larger than P_S, implying that $P = P_S < \overline{P}^C$

$$\begin{cases} P^C = \overline{P}^C, & \text{if } p = \frac{S^\circ}{S^\circ+(1-c)K} > \frac{1}{1+\lambda}, \quad \text{i.e. } K < \frac{S^\circ \lambda}{1-c}, \\ P^C = P_S, & \text{if } p = \frac{S^\circ}{S^\circ+(1-c)K} < \frac{1}{1+\lambda}, \quad \text{i.e. } K > \frac{S^\circ \lambda}{1-c}. \end{cases} \tag{11}$$

II stage The previous price rule is common knowledge and prices increase with the local government share of the corporation stock and the marginal cost of public funds. To the extent that the local government share p, depends both on the share of debt (to be decided by the politician) and on the amount of investment (to be decided by the private shareholders), we can notice that the private investor, through his choice of K can strategically affect the choice of P by the benevolent politician. In what follows, we firsly analyze the choice of K, by private shareholders interested in the maximization of their profit share.

Given the politician's price selection rule (11), the private shareholder will maximize the private dividends in excess over its opportunity cost $(1-c)\alpha\rho K$:

$$V = \begin{cases} (1-p)\left[\beta(i-\underline{i})K\left(2-\frac{(i+\underline{i})K}{L_0}\right)+(\rho-rc)K+\rho S^\circ\right]-(1-c)\alpha\rho K, & \text{if } K \leq S^\circ\frac{\lambda}{1-c}, \\ 0, & \text{if } K > S^\circ\frac{\lambda}{1-c}. \end{cases} \tag{12}$$

where in the first line, the squared brackets include the profit of the mixed firm for $P^C = \overline{P}^C$, while in the second line $V = 0$ and the price is $P^C = P_S$. One can check that with $P^C = \overline{P}^C$, the profits of the mixed firm depends on the information rent left to the local water undertaking plus the net return on the new investment and the return on the value of asset existing before the increase of the capital stock. Then the function V is composed by a positive branch and an identically null branch. Therefore, to maximize V, private shareholders will select $K \leq S^\circ\frac{\lambda}{1-c}$. Actually, $K \leq S^\circ\frac{\lambda}{1-c}$ assures that the investment is sufficiently low to imply that the increase of the capital stock $(1-c)K$ is such to keep the ownership share of the local government (i.e. its dividend share) higher enough to lead the welfare maximizing politician to choose $P^C = \overline{P}^C$ in the third stage.

Beyond $K \leq S^\circ\frac{\lambda}{1-c}$, one should also consider the technological constraint $\frac{L_0}{i}$ and the governance constraint $p \geq 0.51$, to be stated as $K \leq \frac{0.96S^\circ}{1-c}$. Then, by substituting (4) into (12), it is possible to show (Appendix II) that V is strictly increasing in K, for $0 \leq K \leq \min\left\{\frac{\lambda S^\circ}{1-c}, \frac{L_0}{i}, \frac{0.96S^\circ}{1-c}\right\}$, and being $V = 0$ for $K > S^\circ\frac{\lambda}{1-c}$, the global maximum is then $K^C = \min\left\{\frac{\lambda S^\circ}{1-c}, \frac{L_0}{i}, \frac{0.96S^\circ}{1-c}\right\}$.

Now we can distinguish three cases according to which constraint is binding:

1. $K^C = \frac{\lambda S^\circ}{1-c}$; (ownership constraint) in this case it is straightforward to obtain the equilibrium ownership shares as: $p = \frac{1}{1+\lambda}$, $1-p = \frac{\lambda}{1+\lambda}$ which just depend on λ.[10]

2. $K^C = \frac{L_0}{i}$; (technological constraint) this case is more likely to occur the higher the value of the efficiency of capital is.

3. $K^C = \frac{0.96S^\circ}{1-c}$; (governance constraint) the private shareholder finds it profitable to increase K (together with its ownership share $(1-p)$) until $(1-p) = 0.49$. Please notice that in this last case $\lambda \geq 0.96$.[11], therefore this last constraints is not likely to be binding, implying that in most cases $(1-p) < 0.49$.

I stage Let us consider the decision of the politician about the optimal share of debt c,

$$\max_c W^C = \max_c \left\{P^{\max} - P - d\left[L_0 - 2iK + \frac{(iK)^2}{L_0}\right] + (1+\lambda)U_p\right\},$$
$$\text{s.t. } c \leq \overline{c}.$$

[10]For example, with $\lambda = 0.01$ the ownership share of the local government into the mixed firms will be around 77%.

[11]Such an high value for λ appears to be at odds with empirical findings. According to them λ is not expected to be greater than 0.3 (Snow and Warren, 1996).

Considering that the solution of this problem depends on P, to be chosen by the politician in the third stage, for any K chosen in the second stage by the private manager, one can show (check Appendix III) that the assumption that the opportunity cost of equity is greater than the cost of debt, $\alpha\rho > r$, ensures that the politician always finds it convenient to choose the maximum share of debt, $c = \overline{c}$, both in case $P = P_S$ and in the case $P = \overline{P}^C$.[12]

The results of equilibrium analysis are summarized in the following proposition.

Proposition 2 *The sequential game with perfect information has a sub-game perfect Nash equilibrium characterized as follows: In the first stage, the benevolent politician chooses the maximum share of debt $cK = \overline{c}K$ regardless of K and P. In the second stage, private shareholders choose a level of investment $K^C \leq \frac{\lambda S^\circ}{1-\overline{c}}$. In the third stage the local government will charge $P = \overline{P}^C$*

The equilibrium level of investment K^C will be equal to $\frac{\lambda S^\circ}{1-\overline{c}}$ provided the technological constraint, $K^C \leq \frac{L_0}{i}$, and the governance constraint, $K^C \leq \frac{0.96S^\circ}{1-c}$, are satisfied, i.e. $\frac{\lambda S^\circ}{1-\overline{c}} \leq \min\left\{\frac{L_0}{i}, \frac{0.96S^\circ}{1-c}\right\}$. As shown in the calibration and simulation section, and considering a value of $\lambda \geq 0.96$,the governance constraint is never binding. Therefore equilibrium investments are likely to be determined either by the ownership constraint or by the technological constraint. In the latter case the equilibrium amount of investment is even lower with respect to the one implied by the ownership constraint. Therefore finance provided by private shareholders is further reduced, together with their ownership share.

5 Social Optimum

In order to find a benchmark for the analysis of investment distortions due to partial privatization decisions, we consider welfare maximization by a benevolent social planner, which is perfectly informed about the efficiency of investments $i \in \left[\underline{i}; \overline{i}\right]$.

[12]The intuition about welfare being strictly increasing in c - for any P - can be explained as follows. Let us consider firstly the case of $P = P_S$. In this case $U_p = \alpha\rho S^\circ$ (cfr. (8)), the variation of c affects welfare only through its effect on P_S, which can be conveniently written as $P_S = S^\circ\alpha\rho + (1-c)K\alpha\rho + K + crK + \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right)$. One can easily check that an increase of c has two opposite effects on P_S. On the one hand, it increases the cost of capital in proportion to r; on the other hand, by reducing the private shareholders contribution to the corporation stock, it reduces their ownership share and the minimum dividends to be granted to them in proportion to $\alpha\rho$. If the second effect more than compensates the first, due to $\alpha\rho > r$, then any increase of c leads to a price reduction and thereby to an increase of welfare. Now let us consider the case of $P = \overline{P}^C$. The regulator rewards all the amount of capital at a rate of ρ, regardless of the financial source (cfr. (11)), so a variation of c affects welfare only through the effect on the local government dividends U_p, as they are devoted to the reduction of distortionary taxes. By considering that $U_p = p\Pi$, with $\Pi = \overline{P}^C - \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right) - K - crK$, one can check that an increase of c has two opposite effects on the dividends cashed by the local government, and thereby on welfare. On the one hand, it increases the share of profits gained by the local government through, p, which in turn leads to an increase of U_p, a reduction of distortionary taxation and, thereby, to a welfare increase. On the other hand, any increase of c reduces the amount of corporation profits in proportion to r, leading to a lower reduction of distortionary taxation which negatively affects welfare. Notice that $\rho > r$ is a sufficient condition for making the first effect greater than the second one, and thereby letting social welfare to increase in c.

We assume that investments are financed by revenues net of variable costs (self-financing) and by public funds T, raised through non-distortionary taxation. Social welfare is given by the sum of gross consumer surplus $P^{\max}$, minus revenues P, minus the social damages dL due to water leaks, minus the social cost of taxation T; plus profits, given by

$$\Pi = P - \beta \left[1 + L_0 - 2iK + \frac{(iK)^2}{L_0} \right] - K + T. \tag{13}$$

Therefore, social welfare can be expressed as

$$\begin{aligned} W &= P^{\max} - P - d \left(L_0 - 2iK + \frac{(iK)^2}{L_0} \right) - T + \Pi = \\ &= P^{\max} - \beta - (\beta + d) \left(L_0 - 2iK + \frac{(iK)^2}{L_0} \right) - K, \end{aligned}$$

and the maximization problem of the social planner is

$$\begin{aligned} &\max_K W = \max_K \left\{ P^{\max} - \beta - (\beta + d) \left(L_0 - 2iK + \frac{(iK)^2}{L_0} \right) - K \right\} \\ &\text{s.t. } K \geq 0, K < \frac{L_0}{i}. \end{aligned}$$

The level of optimal investment K^* shall satisfy the first order condition

$$2i(\beta + d) \left[1 - \frac{(iK^*)}{L_0} \right] = 1. \tag{14}$$

On the left side, the marginal benefits of investments are given by the reduction of variable costs and social damages due to the reduction of water leaks caused by investments. On the right side the marginal cost of investments, i.e. one Euro to be raised indiferrently either by market revenues or by non-distortionary taxation. The optimal level of K^* is

$$K^* = \frac{L_0}{i} \left(1 - \frac{1}{2i(\beta + d)} \right) < \frac{L_0}{i}.$$

Notice that, investments are independent both from T and P, since with non-distortionary taxation and perfectly inelastic demand, it is indifferent to finance investments by public subsidies or by increasing prices. Therefore, the social optimum is neutral with respect to financial structure.

6 Calibration exercise

Given the expression of optimal investments chosen by the social planner and those arising in equilibrium with partial privatization, it is difficult to consider the issue of investment distortions just from the theoretical point of view. We expect however, that in most real situations, with credible parameter values, this distortion can be well defined. For this reason, in this section we calibrate the model to a plausible real case. Due to the difficulty to retrieve real data, we use a couple of studies to find the parameter values, and to cross check the consistence of some common parameter estimates. Calibration data are obtained from South-West France (Garcia and Thomas, 2003) and Norway (Venkatesh, 2012). Total demand is normalized to 1, so every water quantity is rescaled accordingly; we assume that, without any investment, water leaks would amount to 40% of the total demand, i.e. $L_0 = 0.4$. Monetary values in millions of Euros, M€.

	minimum	maximum	average
total demand (Mm^3)	0.012	3	0.40
leaks (Mm^3)	0.001	10	0.15
total variable cost (M€)	0.010	2	0.25
damages (€/Mm^3)	–	–	0.10

Table 1: Extrapolated data from Garcia and Thomas (2003), approximated figures. The original costs were in FRF and has been transformed to current Euros applying a deflator.

France Table 1 presents the main figures from the French case. We use the average values for calibration. The variable cost is equal to $\beta(1 + L)$, therefore $\beta \simeq 0.18$.

Norway The study presents many data from which we can infer technical and economic values. We use the extrapolated data concerning: total demand, leaks volumes, rehabilitation cost, avoided leaks, cost savings. Monetary values are converted to Euros and considered in current terms. We obtain the following values: $i \simeq 0.15$, variable costs can be estimated $0.568\,\mathrm{M€/Mm^3}$, therefore, in our normalized example, $\beta \simeq 0.227$.

Summing up, we obtain reasonably close values for β. Moreover, we can set $i \simeq 0.15$ and $d \simeq 0.04$,[13] corresponding to $0.1\,€/\mathrm{m}^3$. Therefore, we consider a "current" setting with parameters values

$$\begin{array}{llll} P^{\max} = 8; & L_0 = 0.4; & \lambda = 0.065; & \rho = 0.06. \\ E = 24; & \beta = 0.227; & \bar{c} = 0.5; & = \\ \overline{T} = 0.05; & i = 0.15; & \alpha = 0.9; & = \\ S^\circ = 4; & \underline{i} = 0.074925; & r = 0.03; & = \end{array}$$

Starting from this reference settings, we explore a wide set of scenarios, for i ranging from 0.075 to 1.8 and β from 0.016 to 3.178. All cases are evaluated for λ from 0.01 to 0.3 (Snow and Warren, 1996). This means that, for robustness check, we extend the parameter values well beyond a reasonably realistic range. For purposes of concision, we present what we consider more interesting and illustrative; the full results are available upon request. Figure 1 shows the optimal investment chosen by a social planner compared with those arising from partial privatization. The values

[13] Estimation of social damages due water leaks appears to be quite difficult. To the best of our knowledge, there are no references in the literature, and evaluating the value of wasted water may be an arbitrary exercise. Our estimation is then based on the the sum of: 1) Environmental costs due to the increase of energy use implied by greater pumping effort. This part of the damage can be evaluated by resorting to carbon prices or more generally to carbon values. 2) The value of a tax imposed by the French government on all water users to finance a national fund devoted to investments in water supply. According to Dore et al. (2004) this tax was set at a rate of FF $0.105/\mathrm{m}^3$ in 1992.

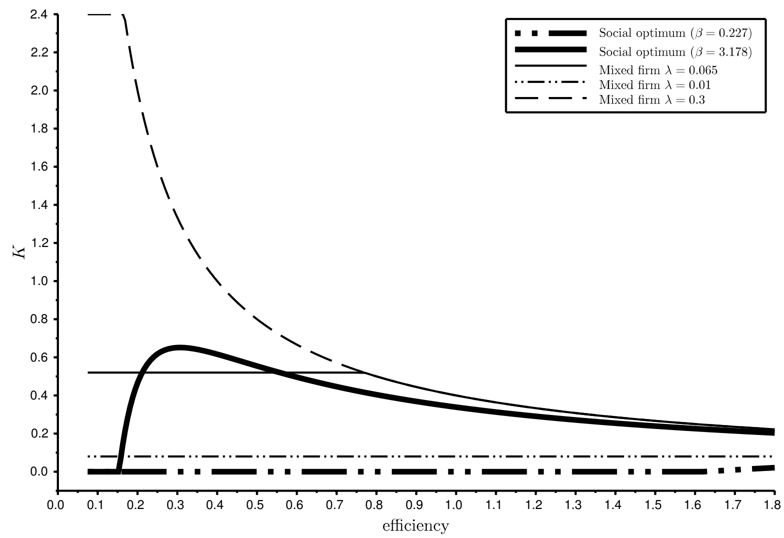

Figure 1: Optimal investments. Mixed firm for low/regular/high cost of public funds (social optimum not affected by this variable). Thick lines: social optimum for current/high variable cost (mixed firm optimum not affected). The horizontal axis offset is intended to facilitate the readability, showing when social optimum investments are null.

are plotted with respect to the efficiency parameter i, in the cases of current/high variable cost and low/regular/high cost of public funds. We highlight that, for the mixed firm, the flat portions of the curves correspond to the case where the ownership constraint is binding. Instead the decreasing portions of the curves is due to the technological constraint which is binding. On the contrary, for the social optimum case, the investment is flat only when investments are null.

7 Investment Distortions

In this section we compare investments arising from partial privatization with investment in the social optimum. We consider various values for the technological and financial parameters. Theoretical analysis will be supported by numerical simulations, which will be helpful in assessing not only the profile, but also the size of investment distortions.

In case of partial privatization investments may be the result of a strategic choice aimed to affect ownership shares in order to avoid investment expropriation by the politician unless ownership constraints are not binding and the investment choice turn out to be selected according to technological parameters. The optimal investment are given by $K^C = \min\left\{\frac{\lambda S^\circ}{1-c}, \frac{L_0}{i}, \frac{0.96 S^\circ}{1-c}\right\}$. The numerical comparison is shown in Figure 1, assuming $\lambda < 0.96$ (i.e. $\frac{\lambda S^\circ}{1-c} < \frac{0.96 S^\circ}{1-c}$, then the governance constraint $\frac{0.96S^\circ}{1-c}$ is never binding).

We can conclude that the investment is larger with partial privatization with respect to the social optimum when both the ownership constraint and the governance constraints are not binding and $K^C = \frac{L_0}{i} < \frac{\lambda S^\circ}{1-c} < \frac{0.96S^\circ}{1-c}$. In this last case private shareholders are lead to expand investment as much as possible, though K^C decreases when efficiency increases (as in Figure 1). When lower values of efficiency would lead to greater values of K^C, then $K^C = \frac{\lambda S^\circ}{1-c} < \frac{L_0}{i}$. Then the ownership constraint becomes binding and, accordingly, K^C becomes flat as it is independent from i (investments cannot grow too much otherwise the ownership share of the local government falls and private shareholders risk investment expropriation). Moreover, given high values of variable costs β (to be coupled with lower values of the efficiency parameter), we can also observe that $K^C < K^*$, i.e. if the ownership constraint is binding we could observe underinvestment in mixed firms (check again Figure 1). In this last case, even if the technological parameters are such to require greater investments to reduce private and social costs, the strategic behavior of private shareholders leads to a lower amount of K^C

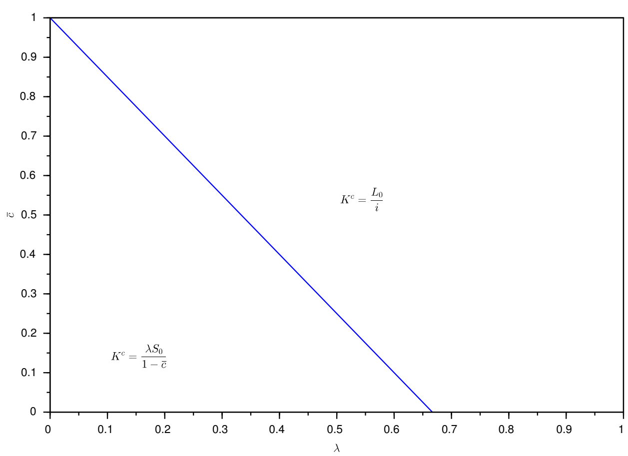

Figure 2: Optimal investment K^C for corporatization, with respect to λ and $\bar{c}$. The line has equation $\bar{c} = 1 - \frac{S^\circ i}{L_0}\lambda$. The efficiency parameter i is set to its current value 0.15, and it is assumed that $\lambda < 0.96$.

Furthermore in Figure 2 one can check that, still assuming $\lambda < 0.96$, the optimal solution $K^C = \frac{\lambda S^\circ}{1-c}$ is more likely to occur, the higher λ is and the lower the constraint on debt $\bar{c}$ is. When the latter increases (for example $\bar{c} > 0.5$), the constraint on the ownership share of the politician (to be satisfied by a greater level of investments and leading to $K^C = \frac{\lambda S^\circ}{1-c}$) becomes less and less important (the greater part of new investments are financed by debt, so that an increase of K affects ownership shares to a lesser extent, thereby relaxing the constraint $K^C = \frac{\lambda S^\circ}{1-c}$) and private shareholders can expand investments as much as possible so that just the (zero-leaks) technological constraint $K^C = \frac{L_0}{i}$ becomes effective (as shown in Figure 2). Still in this latter case the ownership share of private shareholders is likely to lead them to hold minority stakes, given that most part of the new investments are financed by debt.

8 Conclusions

In this paper we have provided a theory of partial privatization and mixed firms assuming that local governments are run by benevolent politician which maximize local welfare. Private shareholders are entitled to decide the amount of investment as they provide finance to the firm through an increase of the capital stock that complements finance by debt. Private shareholders, in order to avoid investment expropriation, choose investments with the aim of affecting the ownership share of local government. The latter is then led to maximize welfare by raising prices up to the price-cap in order to maximize dividends that contribute to the municipal budget and reduce the cost of public funds.

Such a theory can explain persistent minority participation of private shareholders in mixed firms controlled by local governments, as shown by endogenous ownership shares resulting in our model. Actually we empirically observe that in many cases the ownership share of local governments can exceed the threshold which may be necessary to hold control. Our result can then explain the rationale of private participation with such a low share.

If the investment choice is driven by strategic financial decisions, both overinvenstment and underinvestment with respect to the social optimum can result. Considering that investment outcomes in the social optimum are neutral with respect to finance but affected by cost and efficiency drivers, investments distortions in mixed firms may be actually due to the strategic behavior of private investors, motivated in turn by the fear of investment expropriation. As long as local governments can increase their resort to debt the ownership constraints arising from the strategic behavior of private investors become less and less binding, affecting then the sign and size of the investment distortions. Actually, for a given marginal cost of public funds, an increasing resort to debt can lead to an increase of investments, as far as the threshold which matters for strategic decisions is increasing in the share of debt.

Further extensions of our model may consider bargaining between private shareholders and local governments, concerning the amount of investments, following results by Shleifer and Vishny (1994). It could also be interesting to explore the case of non benevolent politicians with a private agenda, to be satisfied either by corruption or by taking care of voting behavior by their local constituency. For example politicians may give more weight to consumer surplus than to the value of dividends, when accounting for the electoral benefits deriving from a lower price of water. Then one can test if our theory is robust with respect to the introduction of political economy issues into the model.

References

Bennet, J. T. and Dilorenzo, T. J. (1982). Off-Budget Activities of Local Government: The Bane of the Tax Revolt. *Public Choice*, 1982, 39(3), 333-342.

Bognetti, G. and Robotti, L. (2007). The Provision of Local Public Services through Mixed Enterprises: the Italian Case. *Annals of Public and Cooperative Economics*, 78(3), 415-437.

Bortolotti, B., and Faccio, M. (2004). Reluctant privatization. Fondazione ENI Enrico Mattei, NOTA DI LAVORO 130.2004.

Boycko, M., Shleifer, A. and Vishny, R. W. (1996). A theory of privatisation. *The Economic Journal*, 106: 309-319.

Cavaliere, A., Maggi, M. and Stroffolini, F. (2015). Investments in Water Networks: Regulation and Political Economy. *mimeo.*

Chakravorty, U., Hochman E. and Zilberman, D. (1995). A Spatial Model of Optimal Water Conveyance. *Journal of Environmental Economics and Management*, 29(1), 25-41.

Dalhuisen, J. M., Florax, R. J. G. M., de Groot, H. L. F. and Nijkamp, P. (2003). Price and Income Elasticities of Residential Water Demand: A Meta-Analysis. *Land Economics*, 79(2), 292-308.

Dore, M. H. I., Kushner, J. and Zumer, K. (2004). Privatization of water in the UK and France: what can we learn? *Utilities Policy*, 12(1), 41-50.

Egenhofer, C., Alessi, M., Teusch, J. and Nunez-Ferrer, J. (2012). Which Economic Model for a Water-Efficient Europe? *CEPS Task Force Reports*, Available at SSRN: `http://ssrn.com/abstract=2181382`

Joulfaian, D. and Marlow, M. L. (1991). The relationship between on-budget and off-budget government. *Economic Letters*, 35(3), 307-310.

Garcia, S. and Thomas, A. (2003). Regulation of Public Utilities under Asymmetric Information. *Environmental and Resource Economics*, 26, 145–162.

Matsumura, T. (1998). Partial privatization in mixed duopoly. *Journal of Public Economics*, 70(3), 473-483.

Mediobanca (2015). Economia e finanza delle principali società partecipate dai maggiori Enti locali (2006-2013),www.mbres.it/it/publications/companies-owned-local-entities

Newbery, D. M. (1999). Privatization, restructuring, and regulation of network industries. Cambridge: MITPress.

Noll, R. G., Shirley, M. M. and Cowan, S. (2000). Reforming urban water systems in developing countries. In: Krueger, A. O. Economic policy reform: The second stage. The University of Chicago Press, Chicago and London, 2000: 243-289.

Noll, R. G. (2002). The economics of urban water systems. In: Shirley, M. M. (Ed.) Thirsting for efficiency: The economics and politics of urban water system reform The World Bank, Washington, 2002: 43-64.

OECD (2011). Water Governance in OECD Countries: a Multi-Level Approach. OECD Studies on Water, OECD Publishing `\http://dx.doi.org/10.1787/9789264119284-en`

Olmstead, S., Hanemann, W. M. and Stavins, R. N. (2007). Water demand under alternative price structures. *Journal of Environmental Economics and Management*, 54(2), 181-198.

Perotti, E., (1995). Credible Privatization. *American Economic Review*, 85(4), 847-859.

Richter, P., Edeling, T. and Reichard, C. (2006). Kommunale Betriebe in größeren Städten. In: Kilian, W., Richter, P. and Trapp, J. H. (Eds.). Ausgliederung und Privatisierung in Kommunen. Empirische Befunde zur Struktur kommunaler Aufgabenwahrnehmung. Berlin, 2006: 56-84.

Shleifer, A. and Vishny, R. W. (1994). "Politicians and Firms". *Quarterly Journal of Economics* 109, 133-150.

Snow, A. and Warren, R. S. (1996). The Marginal Welfare Cost of Public Funds: Theory and Estimates. *Journal of Public Economics*, 61(2), 289-305.

Venkatesh, G., (2012). Cost-benefit analysis – leakage reduction by rehabilitating old water pipelines: Case study of Oslo (Norway). *Urban Water Journal*, 9(4), 277-286.

Appendix I

In this appendix we study the monotonicity of the function $U_{1-p}$with respect to K.

$$U_{1-p} = \begin{cases} (1-p)\left[\beta(i-\underline{i})K\left[2-\frac{(i+\underline{i})K}{L_0}\right]+(\rho-rc)K+\rho S^\circ\right]-(1-c)\alpha\rho K, & \text{if } K \le S^\circ\frac{\lambda}{1-c} \\ 0, & \text{if } K > S^\circ\frac{\lambda}{1-c} \end{cases}$$

When $K \le S^\circ\frac{\lambda}{1-c}$ the complete form of U_{1-p} is

$$U_{1-p} = \frac{(1-c)K}{S^\circ+(1-c)K}\left[\beta(i-\underline{i})K\left[2-\frac{(i+\underline{i})K}{L_0}\right]+(\rho-rc)K+\rho S^\circ\right]-(1-c)\alpha\rho K$$

which can be written as

$$U_{1-p} = \frac{(1-c)K}{S^\circ+(1-c)K}\left[\beta(i-\underline{i})K\left[2-\frac{(i+\underline{i})K}{L_0}\right]+(\rho-rc-(1-c)\alpha\rho)K+\rho(1-\alpha)S^\circ\right]$$

where the two factors are positive:

$$\frac{(1-c)K}{S^\circ+(1-c)K} = 1-p > 0,$$

$$\beta(i-\underline{i})K\left[2-\frac{(i+\underline{i})K}{L_0}\right] > 0 \text{ if } K < 2\frac{L_0}{i+\underline{i}}, \text{ where } 2\frac{L_0}{i+\underline{i}} > 2\frac{L_0}{i+i} = \frac{L_0}{i},$$

$$(\rho-rc-(1-c)\alpha\rho) = \rho[1-(1-c)\alpha]-rc\underbrace{>}_{\rho>r}r[1-(1-c)\alpha-c] = r(1-\alpha)(1-c) > 0,$$

$$\rho(1-\alpha)S^\circ > 0$$

The first factor $\frac{(1-c)K}{S^\circ+(1-c)K}$ has a positive derivative $\frac{(1-c)S^\circ}{[S^\circ+(1-c)K]^2}$, whereas the second factor has the derivative

$$2\beta(i-\underline{i})+\rho-rc-(1-c)\alpha\rho-2\beta\frac{i^2-\underline{i}^2}{L_0}K,$$

which is positive when

$$K < \frac{2\beta(i-\underline{i})+\rho-rc-(1-c)\alpha\rho}{2\beta\frac{i^2-\underline{i}^2}{L_0}} = \frac{L_0}{i+\underline{i}}+\frac{\rho-rc-(1-c)\alpha\rho}{2\beta(i^2-\underline{i}^2)}.$$

This show that an analytic conclusion about the monotonicity of U_{1-p} cannot be easily found. A sufficient condition is that for $K < \frac{L_0}{i+\underline{i}}$, the function U_{1-p} is strictly increasing in K. However, we obtained an interesting result by simulation as follows:

- We randomly generate 100,000 parameter sets in the ranges, $\underline{i} = 0.074925$;

$$\begin{array}{lll} S^\circ \in (1,20); & i \in (\underline{i}, \underline{i}+2); & \alpha \in (0.1,1); \\ L_0 \in (0.1,20); & \lambda \in (0.001,0.35); & r \in (0.001,0.1); \\ \beta \in (0.01,3); & \bar{c} \in (0.01,1); & \rho \in (r/\alpha, r/\alpha+0.1). \end{array}$$

- We computed the K° that maximizes U^C_{1-p} in the range $K \in \left(0, 2\frac{L_0}{i}\right)$.

- We computed the number of times when $K^\circ < \frac{S^\circ \lambda}{1-c}$ and $K^\circ < \frac{L_0}{i}$. We found that this number is 0.

Therefore, we are reasonably sure that the function U^C_{1-p} is strictly increasing for $K \in \left(0, \frac{S^\circ \lambda}{1-c}\right)$.

9 Appendix II

In this appendix we formally prove that in case of corporatization social welfare is strictly increasing in c for any P.

1. $P = P_S$. By substituting and $U_p(P_S) = \alpha\rho S^\circ$ (see eq 8)) in (9) we get the maximization problem

$$\max_c W^C = \max_c \left\{ P^{\max} - P_S - d\left[L_0 - 2iK_m + \frac{(iK)^2}{L_0}\right] + (1+\lambda)\alpha\rho S^\circ \right\}$$
$$\text{s.t. } P_S = \left(\frac{1-c}{1-p}\alpha\rho + 1 + cr\right)K + \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right)$$
$$c \le \bar{c}.$$

By substitution of $(1-p) = \frac{(1-c)K}{S^\circ + (1-c)K}$ in P_S we obtain

$$P_S = S^\circ \alpha\rho + (1-c)K\alpha\rho + K + crK + \beta\left(1 + L_0 - 2iK + \frac{(iK)^2}{L_0}\right),$$

and differentiating W^C w.r.t. P_S, we get

$$\frac{\partial W^C}{\partial c} = -\frac{\partial P_S}{\partial c} = (\alpha\rho - r)K > 0 \text{ for } \alpha\rho > r.$$

2. $P^C = \overline{P}^C$, then the welfare maximization problem is

$$\max_c W^C = \max_c \left\{ P^{\max} - \overline{P}^C - d\left[L_0 - 2iK + \frac{(iK)^2}{L_0}\right] + (1+\lambda)p(c,K)\Pi^C\left(\overline{P}^C, K\right) \right\}$$
$$\text{s.t. } \overline{P}^C = \beta\left(1 + L_0 - 2\underline{i}K + \frac{(\underline{i}K)^2}{L_0}\right) + (1+\rho)K + \rho S^\circ$$
$$\Pi^C(\overline{P}^C) = \beta(i - \underline{i})K\left[2 - \frac{(i+\underline{i})K}{L_0}\right] + (\rho - cr)K + \rho S^\circ$$
$$p(c,K) = \frac{S^\circ}{S^\circ + (1-c)K}.$$

By differentiating w.r.t. to c, we get

$$\begin{aligned}\frac{\partial W}{\partial c} &= \frac{\partial p}{\partial c}(1+\lambda)\Pi^C(\overline{P}^C, K) + p(c,K)(1+\lambda)\frac{\partial \Pi^C(\overline{P}^C, K)}{\partial c} = \\ &= \frac{Kp(c,K)}{S^\circ + (1-c)K}(1+\lambda)\Pi^C(\overline{P}^C, K) - p(c,K)(1+\lambda)rK.\end{aligned}$$

By giving prominence to $(1+\lambda)Kp(c,K)$ and by substition of $\Pi^C(\overline{P}^C, K)$, we get

$$\frac{\partial W}{\partial c} = \frac{\beta K(i-\underline{i})}{S^\circ + (1-c)K_m}\left[2 - \frac{(i+\underline{i})K}{L_0}\right] + \frac{(\rho - cr)K + \rho S^\circ}{S^\circ + (1-c)K} - r$$

Notice that

$$\frac{(\rho - cr)K + \rho S^\circ}{S^\circ + (1-c)K} - r > 0 \Longrightarrow \frac{\partial W}{\partial c} > 0 \Longrightarrow c = \overline{c}.$$

By solving we obtain

$$\frac{(\rho - cr)K + \rho S^\circ}{S^\circ + (1-c)K} - r = \frac{(\rho - r)(S^\circ + K)}{S^\circ + (1-c)K} > 0.$$

Appendix III

9.1 Social Optimum

The network losses function is

$$L = L(i,K) = L_0 - 2iK + \frac{(iK)^2}{L_0},\ K \leq \frac{L_0}{i}, \tag{15}$$

where L_0 is the initial amount of losses, and $\frac{L_0}{i}$ is the level of investments reducing the losses to 0. We indicate explicitly the dependence on i because in subsequent models this will be useful.

The consumer surplus S, the producer profits Π and the welfare maximization problem are

$$\begin{aligned} S &= P^{\max} - P - d\,L(i,K) - T \\ \Pi &= P - \beta\,[1 + L(i,K)] - K + T \\ \max_{K,P,T} W &= \max_{K,P,T}\{S + \Pi\} = \max_{K,P,T}\{P^{\max} - (\beta + d)L(i,K) - K - \beta\} \\ \text{s.t. } K &\geq 0,\ K \leq \frac{L_0}{i},\ P \geq 0,\ P \leq P^{\max}, \\ \Pi &= P - \beta\,[1 + L(i,K)] - K + T \geq 0, \end{aligned} \tag{16}$$

We remark that the objective function does not depend on P and T because the demand is perfectly inelastic and taxation is non-distortionary ($\lambda = 0$). Therefore, we can only obtain the optimal investment K^* from

$$\begin{aligned} \frac{\partial W}{\partial K} &= -(\beta + d)\left(-2i + \frac{i^2}{L_0}K\right) - 1 = 0 \\ K_1 &= \frac{L_0}{i}\left(1 - \frac{1}{2i(\beta + d)}\right). \end{aligned}$$

Remark that $K_1 < \frac{L_0}{i}$, so the technical constraint $K \leq \frac{L_0}{i}$ is never binding. Moreover, if $2i(\beta + d) < 1$, i.e. if for $K = 0$ the marginal benefit of investment for is lower than its marginal cost, then $K^* = 0$. Therefore the optimal investment in the first best case is

$$K^* = \max\left\{\frac{L_0}{i}\left(1 - \frac{1}{2i(\beta + d)}\right), 0\right\}$$