

# Aggregate Hours Adjustment in Frictional Labor Markets

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- The aim of this paper is to explain this pattern in the real business cycle extension of the search and matching model, which includes both the extensive and intensive margins
- In the search and matching model with both margins:
  - employment adjustment is costly and subject to frictions
  - In response to shocks, firms instantaneously increase output by raising hours per employed worker
  - over time, demand is met by increasing employment

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# Outline

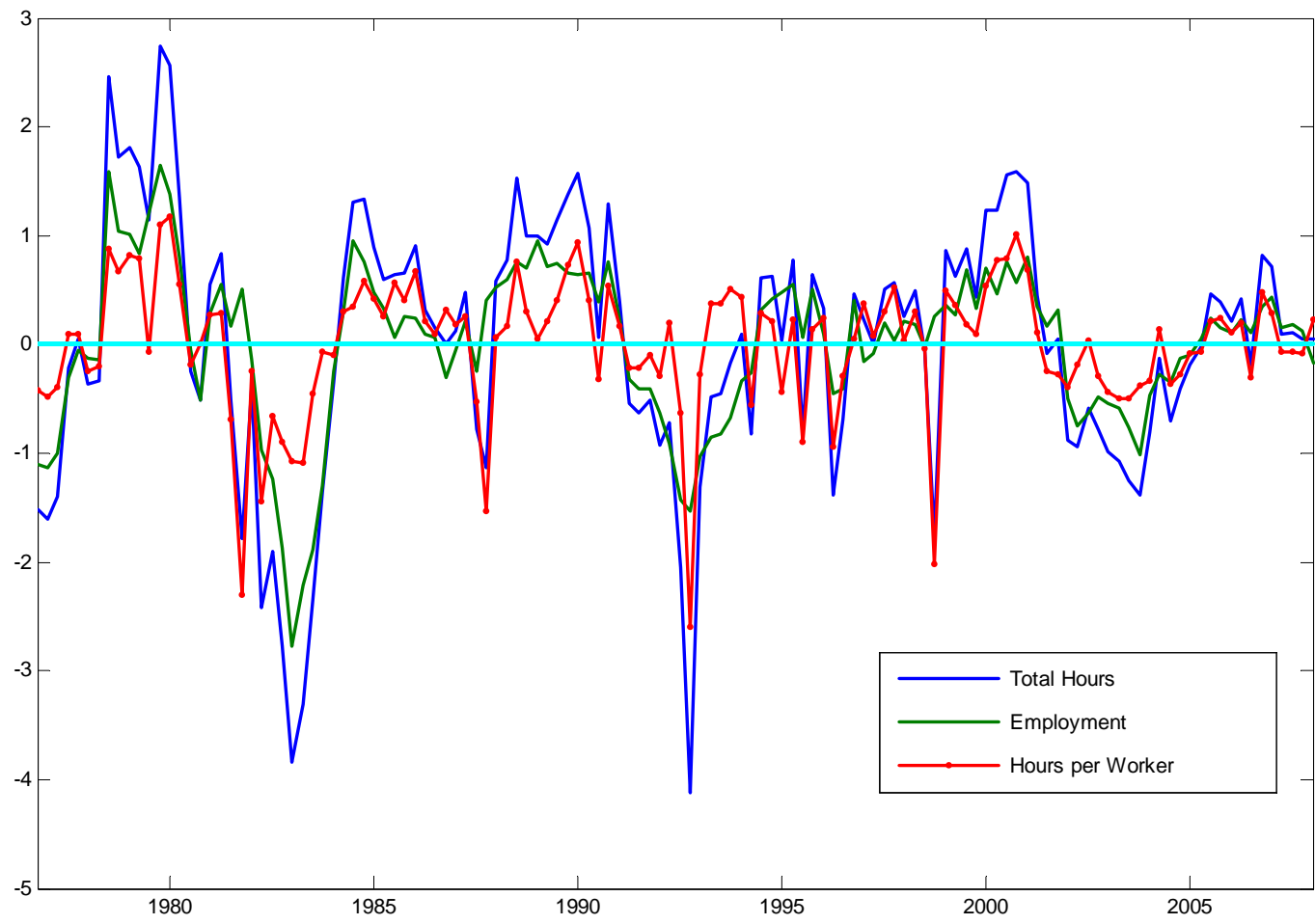
- Stylized facts
- Baseline RBC model with search frictions in the labor market
- Mechanisms and puzzles: the hours and employment margins
- Calibration and Simulation
- Exploring solutions: shocks and parameters
- Outlook and conclusions

## Data

- Measures of hours worked and its components: average hours per worker times employment divided by the civilian labor force
- Establishment survey: 1964:1 to 2007:4
  - based on payroll data
  - employment numbers fairly exact; hours per worker not so much
- Household survey: 1976:1 to 2007:4
  - based on survey responses
  - both series imprecise, but cover all workers



Hours and Employment, Establishment Survey (red: hours per worker, blue: total hours, green: employment)



Hours and Employment, Household survey

# The data

Table 1: Measures of Hours Worked

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	Standard Deviation (%)			Correlation
	Total Hours	Employment	Hours	(N,H)
Data Set 1	1.55	1.28	0.42	0.53
Data Set 2	1.19	0.74	0.61	0.57

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Table 2: Business Cycle Statistics

Standard Deviation (%)				
	U	V	V/U	W
	7.71	9.36	16.76	0.67
	H	TH	I	Y
	0.30	1.10	4.86	1.41
Correlation				
	(U,V)	(H,W)	(H,TH)	(H,Y)
	-0.93	0.72	0.71	0.72

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- Total hours about as volatile as output
- Hours per worker explain 33% to 50% of total hours variation
- Employment and hours are positively correlated
  
- High volatilities of vacancies, unemployment, and tightness
- Wages and hours are less volatile than output
- Hours positively correlated with wages, total hours, and output.

## The model

- RBC model with search and matching frictions in the labor market: Merz (1995), Andolfatto (1996), Den Haan et al. (2000)
- Aggregate household with continuum of workers, utility from bundle of consumption goods and hours worked, perfect risk sharing
- Monopolistically competitive firms with flexible prices, capital accumulation and investment adjustment costs

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- Aggregate household with continuum of workers, utility from bundle of consumption goods and hours worked; perfect risk sharing
- Monopolistically competitive firms with flexible prices, capital accumulation and investment adjustment costs
- Frictional labor market: matching function, hiring time-consuming
- Nash bargaining over wages and hours
- Shocks: technology, markup, labor supply, intertemporal preference, investment-specific technology, matching

## Households

- Welfare of household

$$\mathcal{W}(N_{it}) = \max_{C_{it}} E_{\tau} \sum_{t=0}^{\infty} \beta^t \zeta_t \left[ \frac{C_{it}^{1-\sigma} - 1}{1-\sigma} - \chi_t N_{it} \frac{H_{it}^{1+\mu}}{1+\mu} \right]$$

- Budget constraint

$$W_t H_{it} N_{it} + (1 - N_{it})b + D_{it} + r_t K_{it-1} = T_t + I_{it} + C_{it}$$

- Capital accumulation (rented to firms)

$$K_{it} = (1 - \delta)K_{it-1} + \psi_t I_{it} \left( 1 - S \left( \frac{I_{it}}{K_{it}} \right) \right)$$

- Consumption aggregate

$$C_{it} = \left( \int_0^1 C_{it,j}^{\frac{\epsilon_t-1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t-1}}$$

- Yields: Euler equation, Q-theoretical equation, goods demand

## Firms

- Present value of profits

$$\mathcal{J}^j(N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \left( \frac{P_{jt}}{P_t} \right)^{1-\epsilon} Y_t - W_{jt} N_{jt} H_{jt} - r_t K_{jt} - c(V_{jt}) \right],$$

- Demand for product

$$Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\epsilon_t} Y_t,$$

- Production function (capital rented from households)

$$Y_{jt} = A_t K_{jt}^{1-\alpha} (H_{jt} N_{jt})^\alpha,$$

- Employment evolution ( $\rho$  constant)

$$N_{jt} = (1 - \rho) [N_{jt-1} + V_{jt-1} q(\theta_{t-1})],$$

- Matching function  $M_t = V_t q(\theta_t) = m_t U_t^\xi V_t^{1-\xi}$ , with  $\theta_t = V_t/U_t$

## Bargaining over wage and hours

- Wages and hours chosen to maximize

$$\left( \frac{1}{\lambda_t} \frac{\partial \mathcal{W}_t(N_t)}{\partial N_t} \right)^\eta \left( \frac{\partial \mathcal{J}_t(N_t)}{\partial N_t} \right)^{1-\eta}$$

Bargaining yields a wage equation

$$W_t H_t = \eta \varphi_t \alpha \frac{Y_t}{N_t} + (1 - \eta) \left( b + \frac{C_t^\sigma \chi_t H_t^{1+\mu}}{(1 + \mu)} \right) + \theta_t \eta c$$

and an hours equation

$$C_t^\sigma \chi_t H_t^{1+\mu} = \varphi_t \alpha^2 Y_t / N_t$$



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- Insert hours equation into wage equation

$$W_t H_t = \left( \eta + (1 - \eta) \frac{\alpha}{1 + \mu} \right) \varphi_t \alpha \frac{Y_t}{N_t} + (1 - \eta) b + \theta_t \eta c$$

## Wages and job creation

- Job creation condition

$$\frac{c}{q(\theta_t)} = (1 - \rho)E_t\beta_{t+1} \left[ \varphi_{t+1}\alpha\frac{Y_{t+1}}{N_{t+1}} - W_{t+1}H_{t+1} + \frac{c}{q(\theta_{t+1})} \right]$$

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- Inserting the wage

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into job creation condition

$$\begin{aligned} \frac{c}{q(\theta_t)} = (1 - \rho) E_t \beta_{t+1} & \\ & \left[ (1 - \eta) \left( 1 - \frac{\alpha}{1 + \mu} \right) \varphi_{t+1} \alpha \frac{Y_{t+1}}{N_{t+1}} - (1 - \eta) b \right. \\ & \left. - \left( \theta_{t+1} \eta c - \frac{c}{q(\theta_{t+1})} \right) \right] \end{aligned}$$

- Analyse the responsiveness of return to posting vacancies to changes in expected variables

## Why the model cannot explain labor market dynamics

- Linearized job creation condition ( $\hat{\theta}_t = \hat{V}_t - \hat{U}_t$ )

$$\begin{aligned} \xi \hat{\theta}_t &= E_t \hat{\beta}_{t+1} \\ &+ \frac{q(\theta)}{c} \Phi E_t \left[ \hat{\varphi}_{t+1} + \hat{Y}_{t+1} - \hat{N}_{t+1} \right] \\ &- (1 - \rho) \beta [\eta q(\theta) \theta - \xi] E_t \hat{\theta}_{t+1} \end{aligned}$$

- with  $\Phi = (1 - \rho) \beta (1 - \eta) \left( 1 - \frac{\alpha}{1 + \mu} \right) \alpha \varphi \frac{Y}{N}$
- Expected value of  $\hat{\theta}_{t+1}$  is multiplied by  $\eta q(\theta) \theta - \xi$ .

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- with  $\Phi = (1 - \rho) \beta (1 - \eta) \left( 1 - \frac{\alpha}{1 + \mu} \right) \alpha \varphi \frac{Y}{N}$
- Expected value of  $\hat{\theta}_{t+1}$  is multiplied by  $\eta q(\theta) \theta - \xi$ .
  - For plausible calibrations, this is close to zero. E.g.  $\eta = 0.5$ ,  $\xi = 0.4$ , and job finding rate  $\theta q(\theta) = 0.8$
  - When  $\eta = 0$ , then future labor market tightness is not offset by coinciding wage increase. (Hall and Shimer)
- Expected values of  $\hat{\varphi}_{t+1}$ ,  $\hat{Y}_{t+1}$ , and  $\hat{N}_{t+1}$  are multiplied by  $q(\theta)/c$

## Why the model cannot explain labor market dynamics

- Steady state job creation condition, simplified

$$\frac{c}{q(\theta)} = B [(1 - \eta) [x - b] - \theta\eta c]$$

or

$$\frac{c}{q(\theta)} (1 + B\theta q(\theta)\eta) = B (1 - \eta) [x - b]$$

with  $x = \left(1 - \frac{\alpha}{1+\mu}\right) \varphi \alpha \frac{Y}{N}$  and  $B = \frac{(1-\rho)\beta}{1-(1-\rho)\beta}$ .

- All depends on  $x - b$ . If  $x$  close to  $b$ , then  $c/q(\theta)$  very small.
- Shimer calibrated  $b$  to be about 0.5 which is about half  $x$ .
- Hagedorn and Manovskii argue that  $b$  is large, close to  $x$ . Then Mortensen-Pissaridies model can explain labor market volatilities.

## Closing the model

- Symmetric equilibrium: individual choices equal aggregates.
- Aggregate output

$$Y_t = C_t + I_t + cV_t$$

- Markup

$$\varphi_t = \frac{\epsilon_t - 1}{\epsilon_t}$$

- Rental rate of capital

$$r_t = (1 - \alpha)\varphi_t \frac{Y_t}{K_{t-1}}$$

- Investment dynamics

$$\begin{aligned} Q_{it} &= E_t \beta_{t+1} [Q_{it+1}(1 - \delta) + r_{t+1}] \\ 1 &= Q_{it} \psi_t \left[ \left( 1 - \Phi \left( \frac{I_{it}}{I_{it-1}} \right) \right) - \Phi' \left( \frac{I_{it}}{I_{it-1}} \right) \frac{I_{it}}{I_{it-1}} \right] \\ &\quad + E_t \beta_{t+1} Q_{it+1} \psi_{t+1} \Phi' \left( \frac{I_{it+1}}{I_{it}} \right) \left( \frac{I_{it+1}}{I_{it}} \right)^2 \end{aligned}$$

# Calibration

Table 3: Calibration

<b>Parameter</b>	<b>Value</b>	<b>Definition</b>
$\beta$	0.99	Discount Factor
$\sigma$	1	Intertemporal substitution
$\mu$	1	labor supply elasticity
$b$	0.7	outside option of workers
$\eta$	0.5	bargaining power
$\rho$	0.1	job destruction rate
$\xi$	0.5	matching elasticity
$m$	0.4	match efficiency
$c$	0.05	cost of vacancies
$\epsilon$	11	demand elasticity
$\alpha$	0.67	labor share
$\delta$	0.025	depreciation
$s$	0.5	investment adjustment



# Simulations

Table 4: Business Cycle Statistics: Alternative Shocks

	Standard Deviation (rel. to GDP)							Correlation			
	U	V	$\theta$	H	TH	W	I	(U,V)	(H,N)	(H,W)	(H,Y)
Data	7.71	9.36	16.76	0.30	1.10	0.67	4.86	-0.93	0.53	0.72	0.72
Tech.	0.39	0.46	0.75	0.30	0.32	0.76	3.14	-0.54	0.63	0.70	0.84
Demand	0.97	1.14	1.85	1.17	1.22	1.37	3.99	-0.52	0.79	0.92	0.94
Leisure	0.39	0.46	0.75	1.26	1.28	0.42	3.13	-0.56	0.79	-0.76	0.97
Discount	0.40	0.47	0.81	0.73	0.93	0.73	7.36	-0.73	0.58	-0.29	0.74
Invest.	0.28	0.32	0.60	0.85	0.86	0.82	6.83	-0.92	0.30	-0.42	0.65

## Simulations

- No shock can generate realistic unemployment or vacancy volatility
- Almost all variation is due to hours per worker
- Only investment specific shock: realistic Beveridge curve
- Leisure, intertemporal, and investment shock: negative hours-wage correlation
- Matching shock delivers volatilities, but gets comovement wrong
- However, hours-employment correlation about right.

## Robustness

- Results obtained under baseline calibration with elastic labor supply and moderate outside option of workers
- lower labor supply elasticity should reduce hours variation
- higher unemployment benefit should increase labor market volatility
- lower bargaining power should increase volatility

Table 5: Business Cycle Statistics: Robustness

	Standard Deviation (rel. to GDP)							Correlation			
	U	V	$\theta$	H	TH	W	I	(U,V)	(H,N)	(H,W)	(H,Y)
Data	7.71	9.36	16.76	0.30	1.10	0.67	4.86	-0.93	0.53	0.72	0.72
Labor Supply											
$\mu = 5$	0.04	0.05	0.09	0.09	0.09	1.05	3.03	-0.99	0.58	0.69	0.78
$\mu = 0$	0.81	5.28	5.60	0.55	0.64	0.49	3.24	-0.31	-0.62	-0.76	-0.27
Benefit											
Bargaining											

Table 6: Business Cycle Statistics: Robustness

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<u>Benefit</u>											
$b = 0.95$	1.92	4.45	5.90	0.25	0.57	0.41	2.85	-0.64	-0.57	-0.53	-0.35
$b = 0.40$	0.05	0.06	0.11	0.30	0.30	0.89	3.15	-0.97	0.60	0.62	0.81
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Table 7: Business Cycle Statistics: Robustness

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$b = 0.40$	0.05	0.06	0.11	0.30	0.30	0.89	3.15	-0.97	0.60	0.62	0.81
Bargaining											
$\eta = 0.99$	0.26	1.20	1.40	0.26	0.38	0.64	2.97	-0.68	-0.11	0.32	0.43
$\eta = 0.01$	1.69	1.78	3.46	0.32	0.34	0.38	2.85	-0.99	0.76	0.57	0.87

## Understanding hours variation

- Optimal hours choice

$$H_t = \left( \varphi_t \alpha^2 \frac{Y_t C_t^{-\sigma}}{N_t \chi_t} \right)^{\frac{1}{1+\mu}}$$

- For simplicity assume that there is no capital so that  $C_t = Y_t = A_t N_t H_t$  :

$$H_t^{\mu+\sigma} = \varphi_t A_t^{1-\sigma} \frac{1}{\chi_t N_t^\sigma}$$

- The less employment  $N$  responds to an initial shock, the more will hours  $H$  be moving
- Movements in  $N$  will reduce response of hours
- As employment rises over time, hours per worker will fall
- The more volatility in  $N_t = 1 - U_t$ , the lower is volatility in  $H_t$ , and the more realistic comoments

## A More Formal Empirical Approach

- Calibration and simulation analysis shows that no single specific shock nor parameterization can resolve the various puzzles.
- However, each specification is able to match selected statistics
- Likelihood-based estimation of the full model delivers a weighting scheme that reconciles the different directions
- Consequently, we estimate the model on data for unemployment, vacancies, hours, output, and investment using Bayesian methods
  - prior: based on baseline calibration
  - shocks: technology, mark-up, disutility, matching, investment



## Results

- Estimation algorithm resolves the tension in the model in the following way:
  - low hours elasticity  $\mu$   $\rightarrow$  match relative hours volatility
  - low worker bargaining power  $\eta$  and benefit  $b$   $\rightarrow$  high incentive for vacancy creation
  - important role of mark-up and matching shocks
- Estimation results are at odds with typical parameter choices and shock processes used in calibration studies
- Estimation reveals advantage of systems approach of taking models to the data

Posterior Estimates

		Prior	Posterior	
		Mean	Mean	90% Interval
Relative Risk Aversion	$\sigma$	1.00	0.60	[0.48, 0.72]
Labor Supply Elasticity	$\mu$	1.00	4.92	[4.38, 5.48]
Elast. of Matching	$\xi$	0.30	0.22	[0.10, 0.34]
Scaling Factor Matching Function	$m$	0.70	0.73	[0.53, 0.92]
Elast. of Vacancy Cost	$\psi$	1.00	4.96	[3.90, 6.03]
Vacancy Creation Cost	$\kappa$	0.05	0.05	[0.03, 0.06]
Bargaining Power	$\eta$	0.50	0.00	[0.00, 0.01]
Worker Outside Option	$b$	0.40	0.09	[0.06, 0.12]
Separation Rate	$\rho$	0.10	0.12	[0.08, 0.17]
Elasticity of Demand	$\epsilon$	11.00	9.80	[8.11, 11.3]
Input Elasticity	$\alpha$	0.67	0.64	[0.56, 0.72]
Investment Adjustment Elasticity	$s$	10.00	6.04	[3.31, 9.56]

Variance Decompositions

	Technology	Markup	Lab.Supply	Matching	Inv.Specific
<i>U</i>	0.11	0.00	0.00	0.88	0.00
<i>V</i>	0.50	0.21	0.00	0.28	0.00
<i>H</i>	0.02	0.35	0.31	0.05	0.25
<i>Y</i>	0.72	0.02	0.02	0.17	0.06
<i>I</i>	0.23	0.05	0.00	0.00	0.72

## Conclusions

- Attempt to match stylized facts about hours and employment when there are search and matching frictions in the labor market
- In the data, 30% to 50% of total hours variation is due to variation in hours per worker.

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- In the data, 30% to 50% of total hours variation is due to variation in hours per worker.
- We find that the model predicts almost all variation in total hours is due to variation in hours per worker.
- None of the included shocks can bring the labor market variables closer to the data.
- The calibrations that raise labor market volatilities lead to counter-factual correlation between hours and employment.

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- We find that the model predicts almost all variation in total hours is due to variation in hours per worker.
- None of the included shocks can bring the labor market variables closer to the data.
- The calibrations that raise labor market volatilities lead to counter-factual correlation between hours and employment.
- Estimation basically imposes real wage rigidity, but requires atypical parameter values
- Further work: contemporaneous hiring, overtime and effort, wage setting assumptions