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by

Johann K. Brunner*) and Susanne Pech

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Johannes Kepler University of Linz
Department of Economics
Altenberger Strasse 69
A-4040 Linz - Auhof, Austria
www.econ.jku.at

*) johann.brunner@jku.at
phone +43 (0)732 2468-8248, -9821 (fax)

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Johann K. Brunner and Susanne Pech †)

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Abstract

Inheritances create a second distinguishing characteristic of individuals, in addition to earning abilities. We incorporate this fact into an optimum income taxation model with bequests motivated by joy of giving, and show that a tax on inherited wealth is equivalent to a uniform tax on consumption plus bequests. These taxes are desirable according to an intertemporal social objective if, on average, high-able individuals inherit more wealth than low-able. We demonstrate that such a situation results as the outcome of a process with stochastic transition of abilities over generations, if all descendants are more probable to have their parent's ability rank than any other. (*JEL:* H21, H24)

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†) Department of Economics
University of Linz
Altenberger Straße 69
4040 Linz, Austria
Tel. +43/732 2468-8248
Fax: +43/732 2468 9821
johann.brunner@jku.at or susanne.pech@jku.at

1. Introduction

The tax on estate or inheritance has been a highly controversial issue for long.[1] On the political level, opponents consider it morally inappropriate to use the moment of death as a cause for imposing a tax, and stress its negative economic consequences, in particular on capital accumulation and on family business. Supporters find these consequences exaggerated and claim that a tax on bequests is desirable for redistributive reasons, contributing to "equality of opportunity".

In the academic literature, no widely accepted view on this tax seems to have evolved either. One reason for this may be that there is too little empirical knowledge of the magnitude of its effect on the economy. Another reason is that also on the theoretical level the consequences of inheritance taxation on efficiency and equity have not been worked out clearly. Indeed, we argue that studies in optimum tax theory, which provides the appropriate framework for such an analysis, have not yet succeeded in clarifying the role of this tax within the entire tax system.

The intention of this paper is to propose an optimum-taxation model, which allows a discussion of the central question: is a shift from labor income taxation to a tax on intergenerational wealth transfers a desirable means of redistribution? To answer this question, we extend the standard optimum income taxation approach in the tradition of Mirrlees (1971) to a sequence of generations and introduce a bequest motive. As the adequate version of the bequest motive we consider bequests as consumption (or joy-of

1 Specifically in the USA, there has been a heated debate on the proposal to repeal the federal estate tax permanently. In 2006 it failed the required majority narrowly in the Senate, after the House of Representatives had voted overwhelmingly for the permanent repeal. Some countries like Sweden and Singapore have just recently abolished taxation of inherited wealth, or, like Austria, phase out this tax. However, many other countries, in particular in Europe, still stick to their taxes on inheritance.

giving, see, e.g., Cremer and Pestieau 2006): the amount left to the descendants has a positive effect on the parents' utility similar to the consumption of a good.[2] Individuals differ in their earning abilities, inherited wealth increases their budget on top of their labor income; and they use their budget for consumption and bequests left to the next generation.

The essential point of our analysis is the following: inherited wealth creates a second distinguishing characteristic of individuals, in addition to earning abilities, and it is this fact which motivates the view that a tax on estates or inheritances enhances equality of opportunity. Therefore, the relevant task is to derive optimum-taxation results in a model which allows a simultaneous consideration of both the intragenerational heterogeneity in abilities and the dynamics of inequality arising from intergenerational wealth transfers. We formulate such a model which allows us to find new insights into the implications of inheritance taxation, in particular, how the welfare of different generations is affected.

Surprisingly, former contributions discussing bequest taxation in an optimum-taxation framework have not incorporated this point appropriately. Instead of concentrating on the differences caused by bequests within the generation of heirs, authors focus on the bequeathing generation and ask for the specifics of leaving bequests, as compared to other ways of spending the budget, that is, consumption of goods. Such an analysis, referring to a standard result in optimum-taxation theory (Atkinson and Stiglitz 1972, among others), leads to the question of whether preferences are separable between leisure and consumption plus bequests – then an income tax alone suffices, spending need not be taxed at all –, or

[2] Another motive would be pure altruism, where the parents' utility function has utility of the descendants as an argument. This motive leads to dynastic preferences. We do not intend to model redistribution between dynasties, but between individuals in each generation. We also leave out the strategic bequest motive as well as unintended bequests (for the latter, see Blumkin and Sadka 2003; they study estate taxation also in case of dynastic preferences).

whether leaving bequests represents a complement or a substitute to enjoying leisure.[3] We argue in the present paper that this is the inappropriate question, because the Atkinson-Stiglitz result is derived for a model where individuals only differ in earning abilities. What matters is not that bequests represent a particular use of the budget, but the fact that they transmit inequality across generations.

On the other hand, there are some papers which do pay attention to the fact that inheritances create a second distinguishing characteristic, in addition to earning abilities. However, to our knowledge this literature does not provide a unified framework for an analysis of the role of bequest taxation within an optimum tax system. Cremer et al. (2001) resume the discussion of indirect taxes, given that individuals differ in endowments (inheritances) as well as abilities and that an optimum nonlinear tax on labor income is imposed. They assume, however, that inheritances are unobservable and concentrate on the structure of indirect tax rates. Similarly, Cremer et al. (2003) and Boadway et al. (2000) study the desirability of a tax on capital income as a surrogate for the taxation of inheritances, which are considered unobservable.

In contrast to these contributions, we study a comprehensive tax system where a nonlinear tax on labor income can be combined with taxes on inherited wealth and on expenditures. Therefore, we take all these variables as being observable (only abilities are unobservable). This is indeed the basis upon which real-world tax systems, including the tax on bequests, operate. In particular, if we want to know whether the inheritance tax should be retained or abolished from a welfare-theoretic point of view, the analysis must be based on the assumption of observable initial wealth.

3 See Gale and Slemrod (2001, p.33) and Kaplow (2001), as well as Blumkin and Sadka (2003) in the context of a dynastic model.

As a starting point we consider a static model with two types of individuals, who live for one period and hold exogenously given initial wealth, which together with labor income is used for the consumption of two goods. We discuss two tax systems: (i) an optimum tax on labor income combined with a proportional (direct) tax on initial wealth, and (ii) an optimum tax on labor income combined with a proportional (indirect) tax on all consumption expenditures. We show that these two tax systems are equivalent and, moreover, that a tax on initial wealth or on consumption expenditures is desirable according to a utilitarian objective, if initial wealth increases with earning abilities. The underlying reason is that introducing these taxes allows further redistribution on top of what can be achieved through labor income taxation alone. Note that the wealth tax is lump-sum while the expenditure tax is not, but the distorting effect of the latter on labor supply can be offset by an adaptation of the labor income tax.

Then we turn to an analysis of the dynamic model, for which we choose the most parsimonious version appropriate for our purpose: there is a sequence of generations, where again each lives for one period. One of the consumption goods is now interpreted as bequests, which become the initial (= inherited) wealth of the following generation.[4] When discussing the two equivalent ways of imposing a tax (either directly on inherited wealth or indirectly on expenditures, i.e. on consumption plus bequests), we now take into account that bequests left by some generation influence the welfare of future generations. It turns out, contrary to what one expects, that introducing dynamic effects does not change anything compared to the result of the static model: that inherited wealth increases with earning abilities remains the only decisive criterion for both ways of taxation. All other

[4] We assume that bequests are not productive but represent immediate consumption possibilities for the next generation. As individuals live for one period only, there is no other saving except for the purpose of leaving bequests, and a tax on wealth transfer is equivalent to a tax on capital income. Hence we need not introduce the latter.

welfare effects – including those falling on later generations – associated with the introduction of the tax on inherited wealth (or on consumption plus bequests), are neutralized by the simultaneous adaptation of the optimum tax on labor income. Thus, we also find that the "double-counting" problem, which typically arises in models where bequests enter a social objective twice[5], does not occur in our framework.

This result has to be modified somewhat if the first instrument (a tax directly imposed on inherited wealth) is applied and if one assumes that the bequeathing individuals care for bequests net of the inheritance tax falling on the heirs. Then collecting the tax in some period will have repercussions on the bequest decision of the previous generation. This problem does not arise with an expenditure tax.

In a next step, we generalize the model to one with arbitrarily many types of individuals and with a stochastic relation between inherited wealth and earning abilities. Restricting the analysis to quasilinear preferences, we show that the results remain essentially unchanged, the crucial point for the desirability of a tax on inherited wealth (or on consumption plus bequests) being that expected inheritances increase with abilities. Finally, we provide a theoretical argument demonstrating that this is indeed plausible: we analyze a stochastic process of abilities which is built on the key assumption that all descendants are more probable to have their parent's ability rank than any other.[6] It turns out that if each parent

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5 Bequeathing causes two positive effects on the involved individuals (the donor enjoys giving, the beneficiary likes receiving), and the welfare of both appears in the social welfare function. This calls for a subsidy instead of a tax on bequests. Some authors discuss "laundering out" this double counting from the social welfare function, see, e. g., Cremer and Pestieau (2006).

6 This assumption is justified by various empirical studies which find that the children's incomes are positively correlated with those of their parents. For instance, Solon 1992 and Zimmerman 1992 both find an intergenerational correlation in income of 0.4 for the US economy.

has a descendant, to whom she leaves her bequests, this process indeed generates a distribution such that expected inheritances increase with abilities in any generation. [7]

Our work is related to contributions which study a stochastic process describing the transition of wealth over generations, and analyze the evolution of inequality. They show that, depending on the assumptions of the model, a tax on bequests may increase inequality (by reducing the role of inheritances as compensating for income shocks of the descendants, see, e.g., Becker and Tomes 1979) or decrease inequality (by redistributing wealth, see, e.g., Bossmann, Kleiber and Wälde, 2007, Davies and Kuhn 1991). In contrast to these contributions, which concentrate on inequality measures, but do not discuss welfare effects and typically assume fixed labor supply, we follow the optimum-taxation approach, which allows a combined consideration of efficiency and redistributive effects of the taxation of bequests, and we analyze its role within the tax system.

In the following Section 2 the model with two types of individuals is introduced and the results for the static as well as for the dynamic formulation are derived in turn. In Section 3 the model is generalized to more types and a stochastic relation between ability levels and inheritances. Moreover, a transition process which generates such a stochastic relation is studied. Section 4 provides concluding remarks.

7 To our knowledge, there is no direct empirical evidence on this issue. However, it has been found that earnings are positively correlated with wealth (see, e.g., Díaz-Giménez et al. 2002 for the US economy, who find a positive correlation between earnings and wealth of 0.47). This can be seen as a partial support for our result, as wealth consists of inheritances to a substantial extent (for an overview see Kessler and Masson 1989).

2. Two ways of taxing inherited wealth

We begin this Section with an analysis of a static model, which will be extended to a dynamic framework with many generations in Subsection 2.2. The economy consists of two individuals i = L, H, characterized by differing earning abilities $\omega_L < \omega_H$, and by exogenous initial endowments of (inherited) wealth e_i, i = L,H. The individuals live for one period. By supplying labor time l_i, each individual earns pre-tax income $z_i = \omega_i l_i$, i = L,H. After-tax income is denoted by x_i, which, together with initial wealth, is spent on general consumption c_i and some specific good b_i. We call the latter good bequests to be consistent with the terminology later on, though – taken literally – it makes no sense to have bequests in a static model. The individuals have common preferences, described by the concave utility function u(c,b,l), which is twice differentiable, with $\partial u / \partial c, \partial u / \partial b > 0$, $\partial u / \partial l < 0$.

2.1 A basic equivalence

In our model, the tax system consists of a tax on labor income, described implicitly by the function $\sigma: \mathbb{R} \to \mathbb{R}$, which relates gross and net income: $x = \sigma(z)$ (thus the tax is $z - \sigma(z)$), of a proportional tax τ_e on initial wealth, and of proportional taxes τ_c and τ_b on consumption and bequests, resp. Assuming that the prices of consumption and bequests are one, the budget constraint of an individual i reads:

$$(1+\tau_c)c_i + (1+\tau_b)b_i \le \sigma(z_i) + (1-\tau_e)e_i . \qquad (1)$$

Obviously, τ_e is a lump-sum tax in this case.

The budget set $B(\sigma(z_i), e_i, \tau_c, \tau_b, \tau_e)$ contains all nonnegative pairs (c_i,b_i) which fulfill the budget constraint (1). If two tax systems lead to identical budget sets for any z_i and any given e_i, then the two tax systems induce the same decisions of the individuals with respect to the choice of l_i, c_i, b_i. Therefore we call the two tax systems equivalent in this case.

It is well known that in the absence of initial wealth a tax system consisting of an income tax plus a uniform expenditure tax is equivalent to an income tax alone. This is no longer true, if there exist wealth endowments: then there is a case for a second tax instrument, in addition to the tax on labor income. We find immediately:

Lemma 1:

(a) A tax system $(\sigma, \tau_e, \tau_c, \tau_b)$ is equivalent to a tax system $(\hat{\sigma}, \hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$, where one of $(\hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$ is zero. Moreover,

if $\hat{\tau}_e = 0$, then $\hat{\sigma} = \sigma/(1-\tau_e)$ and $\hat{\tau}_c = \dfrac{\tau_e + \tau_c}{1-\tau_e}, \hat{\tau}_b = \dfrac{\tau_e + \tau_b}{1-\tau_e}$,

if $\hat{\tau}_c = 0$, then $\hat{\sigma} = \sigma/(1+\tau_c)$ and $\hat{\tau}_b = \dfrac{\tau_b - \tau_c}{1+\tau_c}, \hat{\tau}_e = \dfrac{\tau_e + \tau_c}{1+\tau_c}$,

if $\hat{\tau}_b = 0$, then $\hat{\sigma} = \sigma/(1+\tau_b)$ and $\hat{\tau}_c = \dfrac{\tau_c - \tau_b}{1+\tau_b}, \hat{\tau}_e = \dfrac{\tau_e + \tau_b}{1+\tau_b}$.

(b) A tax system $(\sigma, \tau_e, \tau_c, \tau_b)$ with $\tau_c = \tau_b$ is equivalent to a tax system $(\hat{\sigma}, \hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$, where $\hat{\tau}_c = \hat{\tau}_b$ and either $\hat{\tau}_e = 0$ or $\hat{\tau}_c = \hat{\tau}_b = 0$. The formulas in (a) apply.

Proof: follows immediately from appropriate manipulations of the budget constraint (1).

In the following we make use of the observation, expressed in Lemma 1(b) that a tax on initial wealth is essentially the same as a uniform tax on expenditures for consumption and bequests (which in fact are a form of consumption), because the income tax can be adjusted accordingly. In particular, the uniform expenditure tax represents a kind of lump-sum tax in this framework, as does the tax on initial wealth, though expenditures are variable, while wealth is fixed.

Note that the switch to a tax system without a tax on initial wealth means that the income tax has to be reduced (net income $\sigma(z)$ is increased), while the taxes on c_i and b_i have to be increased. Similarly, a switch such that expenditures are untaxed (consider case (b)) means an increase of the income tax and of the tax on initial wealth (if $\tau_e < 1$).

The equivalence extends to the welfare effect of a marginal change of the tax system, which we discuss in an optimum income taxation framework. We introduce the indirect utility function

$$v^i(x_i, z_i, e_i, \tau_e, \tau) \equiv \max\left\{u(c_i, b_i, z_i / \omega_i) \mid (1+\tau)(c_i + b_i) \le x_i + (1-\tau_e)e_i\right\},$$

where we consider a tax system with a uniform tax rate τ on all expenditures, equivalent to the tax rate τ_e on initial wealth.

As usual, we assume that the tax authority cannot tie a tax directly with individual abilities, because they are not observable, therefore it imposes an income tax as a second-best instrument. For the determination of the latter, we take some tax rate τ and/or τ_e as fixed for the moment. In case that there are no restrictions on the functional form of the income tax, the appropriate way to determine the optimum nonlinear schedule is to maximize a social welfare function with respect to the individuals' income bundles (x,z), subject to the self-selection constraints and the resource constraints.

As is standard in optimum income taxation models, we assume that the condition of "agent monotonicity" (Mirrlees 1971, Seade 1982) holds. Define $MRS^i_{zx} \equiv -(\partial v^i/\partial z_i)/(\partial v^i/\partial x_i)$, then for any given e_i, τ_e, τ:

AM: $MRS^L_{zx} > MRS^H_{zx}$ at any vector (x,z).

As is well-known, this single-crossing condition guarantees that for any income tax function the high-able individual does not choose to earn less income than the low-able.[8]

We assume a utilitarian social welfare function with weights f_L,f_H, $f_L \geq f_H > 0$, of the two individuals, then the objective is

$$\max_{x_i, z_i} f_L v^L(x_L, z_L, e_L, \tau_e, \tau) + f_H v^H(x_H, z_H, e_H, \tau_e, \tau). \qquad (2)$$

The resource constraint reads

$$x_L + x_H \leq z_L + z_H + \tau_e\left(e_L + e_H\right) + \tau\left(c_L(\cdot) + b_L(\cdot) + c_H(\cdot) + b_H(\cdot)\right) - g, \qquad (3)$$

where g denotes the resources required by the state. $c_i(\cdot)$, $b_i(\cdot)$ are demand functions with the same arguments as $v^i(\cdot)$, i = L, H.

Moreover, we have to introduce the self-selection constraints: the government must determine the two bundles of gross and net income in such a way that no individual prefers the bundle assigned to the other. We follow the frequently made assumption of a sufficient

8 It should be noted that in the presence of initial (non-human) endowments this assumption is more critical than in the standard model à la Mirrlees: if initial wealth of the high-able individual is sufficiently larger (thus, her marginal utility of income is sufficiently lower) than that of the low-able, the former might require a larger amount of net income as a compensation for her effort to earn one more unit of gross income, than what the latter requires (even though the high able needs less additional working time for this). Such a potential problem does not occur, if we work with quasilinear preferences, as we do in Section 3.

importance of the low-able individual in the objective function (2). Then the social objective favors redistribution from the high- to the low-able individual, and one can show that only the self-selection constraint of the high-able individual is binding in the optimum and needs to be considered:

$$v^H(x_H, z_H, e_H, \tau_e, \tau) \geq v^H(x_L, z_L, e_H, \tau_e, \tau) . \tag{4}$$

Let, for given τ_e,τ, the optimum value of the social welfare function (2) subject to the constraints (3) and (4) be denoted by S(τ_e,τ), and let the Lagrange multiplier of the self-selection constraint (4) be denoted by μ. μ is positive as a consequence of the above assumption that (4) is binding in the optimum. We use the notation $\partial v^H[L]/\partial x_L > 0$ to describe marginal utility of income of the high-able individual in case of mimicking.[9] We find

Theorem 1: The welfare effect of a marginal increase of τ_e and τ, resp., reads:

(a) $$\frac{\partial S}{\partial \tau_e} = \mu \frac{\partial v^H[L]}{\partial x_L}(e_H - e_L),$$

(b) $$\frac{\partial S}{\partial \tau} = \mu \frac{\partial v^H[L]}{\partial x_L}(e_H - e_L)\frac{1-\tau_e}{1+\tau}.$$

Hence, $\partial S/\partial \tau = (\partial S/\partial \tau_e)(1-\tau_e)/(1+\tau)$ and both taxes increase social welfare, if the initial wealth of the high-able individual is larger than that of the low-able.

Proof: see Appendix.

[9] Mimicking refers to a situation where the high-able individual opts for the (x,z)-bundle designed for the low-able.

Given a larger wealth of the high-able individuals, the social objective calls for further redistribution than what is possible through an income tax alone. Such an additional redistribution can equivalently be achieved by a tax on initial wealth or on expenditures. In particular, it turns out that the justification for (uniform) indirect taxation is uniquely linked to the existence of differing wealth endowments: given these, the expenditure tax combined with an optimum income tax is indeed a lump-sum tax, being equivalent to the tax on initial wealth.

The positive effect on welfare comes from a relaxation of the self-selection constraint induced by an increase of τ_e (or τ). The intuition can be explained as follows: assume, as a first step, that after an increase of τ_e by $\Delta\tau_e$, each individual i is just compensated through an increase of net labor income x_i by $\Delta\tau_e e_i$. If $e_H > e_L$, the high-able individual experiences a larger increase of the net labor income than the less able which makes mimicking less attractive and gives slack to the self-selection constraint. As a consequence, in a second step additional redistribution from the high- to the low-able individuals becomes possible, which increases social welfare. In our model, this mechanism works as long as the social objective favors further redistribution; if the desired extent of redistribution via τ_e (or τ) is attained, the Lagrange multiplier μ becomes zero.

One may object to our model that assuming a fixed relation between (unobservable) abilities and (observable) initial wealth (or expenditures) makes an income tax not a reasonable instrument from the beginning. Namely, the tax authority can use information on initial wealth (or on expenditures) to identify individuals, and then impose a tax on abilities directly, which is first-best. In reality, however, such a method of identification is not employed, and the main reason seems to be that initial wealth (or expenditures) is not a precise indicator for earning abilities. By incorporating this idea in our model we will show in Section 3 that an accordingly modified version of Theorem 1 also holds when initial wealth is stochastic.

2.2 Taxation of inheritances in a dynamic economy

As a next step we formulate a simple intertemporal model within which we discuss the optimum taxation of inheritance. We assume that the (static) two-person economy described above represents the situation in some single period t, and we take into account that bequests (and taxes on them) affect the welfare of future generations.

In view of the results of the foregoing Subsection, the ultimate reason, why the intergenerational transfer of wealth may represent an object for taxation is that receiving inheritances creates a second distinguishing characteristic of the individuals, in addition to their earning abilities. In order to account for this, two possible instruments can be applied (in some period t):

- taxing inherited wealth e_{it} as a direct source of inequality within the receiving generation. That is, an inheritance tax τ_{et} is employed for generation t in our model.
- using a "full" expenditure tax (that is, in our terminology, a uniform tax τ_t on consumption c_{it} plus bequests b_{it}) as a surrogate taxation of unequal inherited wealth e_{it} of the bequeathing generation t.

In a static framework, these two instruments proved equivalent (and lump-sum). We now ask what can be said in an intertemporal setting, that is, when effects on future generations are taken into account. Let a series of arbitrary τ_{es},τ_s, $s \geq t$, be given (possibly zero). In some period t, the government imposes an optimum income tax and considers a change of τ_{et},τ_t. The revenues from τ_{et},τ_t run into the budget of this generation t and are redistributed through a reduced need for labor-income tax revenues.

Effects on future generations

We work with the indirect utility functions as before, now being defined as

$$v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t) \equiv \max\left\{u(c_{it}, b_{it}, z_{it} / \omega_{it}) \mid (1+\tau_t)(c_{it} + b_{it}) \leq x_{it} + (1-\tau_{et})e_{it}\right\}.$$

Inherited wealth e_{it} of an individual i of generation t is exogenous. It arises as a result of some allocation of aggregate bequests $b_{Lt-1} + b_{Ht-1}$ left by the previous generation to the individuals of generation t. For the analysis of this Section, the rules guiding this allocation need not be specified.

On the other hand, the bequests $b_{it}(\cdot)$ left by generation t represent initial wealth for the individuals of the next generation t+1 and enter their utility. Moreover, they also influence bequests left by generation t+1 and, by this, utility of generation t + 2, and so on. We take account of all these effects through a very general formulation: we simply assume that (discounted) welfare of all future generations from t+1 onwards can be described by some general (intertemporal) social welfare function $W(b_{Lt}, b_{Ht})$, which depends on the bequests left to generation t+1.[10] In order to determine the tax rates in period t, the planner must take care of how the tax rates influence future welfare, and this happens only via bequests of generation t in our model. Thus, W must be known to the planner, but it can be any suitable function.

[10] As mentioned in the Introduction, we assume a zero rate of return. However, even if there were a positive rate of return on (bequeathed) capital, its welfare effect would be included in W, and our results would remain unchanged.

Then the objective function of the planner to determine the optimum nonlinear income tax in period t reads

$$\max_{x_{it},z_{it}} \sum_{i=L,H} f_{it} v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t) + (1+\gamma)^{-1} W(b_{Lt}(\cdot), b_{Ht}(\cdot)), \tag{5}$$

where $\gamma > 0$ represents the planner's one-period discount rate. (5) is to be maximized subject to the resource constraint

$$x_{Lt} + x_{Ht} \le z_{Lt} + z_{Ht} + \tau_{et}\left(e_{Lt} + e_{Ht}\right) + \tau_t\left(c_{Lt}(\cdot) + b_{Lt}(\cdot) + c_{Ht}(\cdot) + b_{Ht}(\cdot)\right) - g_t \tag{6}$$

and to the self-selection constraint

$$v_t^H(x_{Ht}, z_{Ht}, e_{Ht}, \tau_{et}, \tau_t) \ge v_t^H(x_{Lt}, z_{Lt}, e_{Ht}, \tau_{et}, \tau_t). \tag{7}$$

Note again, that b_{Lt}, b_{Ht}, influenced by the income tax and by τ_{et},τ_t, enter welfare W of future generations.[11] We find the surprising result that this effect plays no role for the desirability of τ_{et},τ_t. Let $S^d(\tau_{et},\tau_t)$ denote the optimum value of the maximization of (5), subject to (6) and (7), and μ^d the Lagrange multiplier corresponding to the self-selection constraint (7):

Theorem 2: In a dynamic model, the welfare effect of a marginal increase of τ_{et} and τ_t, resp., in some period t, reads:

[11] To give a simple example for W: assume that all later generations consist of the two types of individuals with ability level ω_{Ls}, ω_{Hs} and in each period all bequests left by type L (H) go to type L (H) of the next generation ($e_{is} = b_{is-1}$). We define $W(b_{Lt},b_{Ht})$ as the maximum (discounted) future welfare, from t+1 onwards, for given b_{Lt}, b_{Ht}, if an optimum nonlinear income tax is imposed in each period, i.e.,

$$W(b_{Lt}, b_{Ht}) \equiv \max_{x_{is},z_{is}} \sum_{s=t+1}^{\infty} (1+\gamma)^{t+1-s} \sum_{i=L,H} f_{is} v_s^i(\cdot),$$

subject to the resource and the self-selection constraints (6) and (7), for every period s = t+1,…,∞. Note that bequests $b_{it} = e_{it+1}$ of generation t enter $v_{t+1}^i(\cdot)$.

(a) $$\frac{\partial S^d}{\partial \tau_{et}} = \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt}),$$

(b) $$\frac{\partial S^d}{\partial \tau_t} = \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt})\frac{1-\tau_{et}}{1+\tau_t}.$$

Hence, as in the static model, both taxes increase welfare, if the inheritance received by the high-able individual is larger than that received by the low-able.

Proof: see Appendix.

Thus, the dynamic character does not change anything regarding the desirability of a tax on inherited wealth or on full expenditures (i.e. on consumption plus bequests). Though the tax on inherited wealth (or full expenditure) affects (negatively) the amount of bequests left to the next generation, the same condition as in the static case applies, contrary to the intuition. The reason is the simultaneous adaptation of the optimum non-linear income tax, as can be seen from an inspection of the proof of Theorem 2. Indeed, an increase in τ_{et} or τ_t allows an increase in net income from labor which can, for each individual, be designed in such a way that all other welfare consequences of the increase of τ_{et} (or τ_t), in particular, the consequences for the subsequent generations via bequests, cancel out, except the one appearing in Theorem 2(a). The latter effect, which operates via a change of the self-selection constraint, is positive, if the high-able individual also has a higher wealth endowment, as discussed earlier.

This result may be interpreted as a rationale for the common idea that inheritance taxation serves the target of equality of opportunity. Its proponents implicitly assume that the group with the higher earning abilities also has higher inherited wealth. In the political decision it is also frequently taken as granted that taxation of bequests via an estate tax is an appropriate instrument for redistribution. However, when considered alone, an estate tax leads to a distortion of the bequest decision[12], which is avoided if all expenditures, that is, consumption plus bequests, are taxed at a uniform rate.

A specifically interesting aspect of this cancelling out of all other welfare effects is that obviously the value of the social discount rate γ – the weight of future generations – plays no direct role for the desirability of τ_{et} or τ_t (it influences the magnitude of the Lagrange multiplier μ^d). Moreover, as mentioned in the Introduction, our result shows that the well-known "double-counting" of bequests, which usually in joy-of-giving models causes a counter effect against the introduction of an estate or inheritance tax (and in fact calls for a subsidy), can be ignored as well. The point is again that in an appropriate formulation it is not the specific use of the budget for leaving a bequest which is taxed, but the initial wealth.

Repercussions on the previous generation

It must be added that up to now we have considered inherited wealth of generation t as exogenously given. That is, we have assumed that, when the inheritance and/or full expenditure tax is increased or introduced in period t, the bequest decisions of the parent generation $t-1$ are already made. Then Theorem 2 describes the effect of these taxes, and obviously the same logic applies, if in period t+1 the taxes τ_{et+1} and/or τ_{t+1} are introduced, unexpected by the previous generation t.

12 See Brunner (1997) who showed that a specific tax on bequests is desirable, if the social welfare function favors redistribution strongly enough to outweigh the distorting consequences.

As a final step of our analysis in this Section, we now ask whether something changes, if the tax authority increases or introduces the inheritance or full expenditure tax in some period t not only for that generation t but also for the subsequent generation t+1, and this is anticipated by the individuals in t. How does this affect the bequest behavior of the latter and what are the welfare consequences of the taxes in this case?

The answer to this question follows from the bequest motive in our model: bequests are regarded as some form of consumption, it is the amount left to the descendant, which per-se provides utility to the bequeathing individual. Thus, concerning the full expenditure tax, we can state, as a first result, that the introduction (or increase) of τ_{t+1}, announced already in period t, does not change anything with the above analysis. The formula of Theorem 2(b), which describes the effect of τ_t, applies – with index t+1 – in just the same way for the effect of τ_{t+1}. The reason is that the full expenditure tax in period t+1 does, by definition, not change the value of the bequest b_{it} for the bequeathing individual i of generation t, and does, therefore, not influence her bequest decision.

But the situation may be different when it comes to the direct tax on inherited wealth. Taking the bequest-as-consumption model literally, one might again argue that the anticipation of τ_{et+1} by generation t does not change anything with the formula of Theorem 2(a), because individuals simply care for what they leave as (gross) bequests to their descendants. On the other hand, however, it seems reasonable to model the bequeathing generation t as caring for net bequests, then $b_{it}^{net} \equiv b_{it}(1-\tau_{et+1})$, instead of gross bequests b_{it} [13] appears in her utility function. Such a formulation means that bequeathing individuals

[13] Note that we use the expression "gross bequests" for b_{it} from the viewpoint of the receiving generation t+1, i.e. only in reference to the inheritance tax τ_{et+1}. For the bequeathing generation, however, b_{it} is pre-tax concerning the full expenditure tax τ_t.

only pay attention to the amount going directly to the descendants; they ignore the revenues raised by τ_{et+1} (notice that these run into the public budget of the descendants' generation and reduce their income tax burden).

With this formulation, the introduction (or increase) of an inheritance tax τ_{et+1} causes a negative effect on the bequest decision of the previous generation t, which has not been considered so far. To analyze this effect, we extend the problem (5) – (7) by adding τ_{et+1} as an argument of v_t^i, c_{it} and b_{it}. Moreover, in order to see the consequences in detail, we add welfare of generation t+1 explicitly in the social objective and assume that the general welfare function $W(b_{Lt+1}(\cdot), b_{Ht+1}(\cdot))$ describes (discounted) social welfare from generation t+2 onward. Thus, the objective function for any given tax rates $\tau_{et}, \tau_t, \tau_{et+1}, \tau_{t+1}$ reads (instead of (5)):

$$\max_{x_{it}, x_{it+1}, z_{it}, z_{it+1}} \sum_{i=L,H} f_{it} v_t^i(\cdot) + (1+\gamma)^{-1} \sum_{i=L,H} f_{it+1} v_{t+1}^i(\cdot) + (1+\gamma)^{-2} W(b_{Lt+1}, b_{Ht+1}) . \qquad (8)$$

Further, a resource and a self-selection constraint for period t+1 have to be added (see (A13) – (A16) in the Appendix).

Obviously, bequests left by generation t (and influenced by τ_{et+1}) represent inheritances of generation t+1; we still need not specify the rule guiding the transfer. Let $\tilde{S}^d(\tau_{et}, \tau_t, \tau_{et+1}, \tau_{t+1})$ denote the optimum value function of the extended problem and $\tilde{\mu}_t^d, \tilde{\mu}_{t+1}^d$, $\tilde{\lambda}_{t+1}^d$ the Lagrange multipliers corresponding to the self-selection constraints (in periods t and t+1) and to the resource constraint in t+1, resp. We find by differentiation and manipulation of the Lagrangian function:

Theorem 3: In a dynamic model, where individuals care for net bequests, the welfare effect of a marginal increase of τ_{et+1} and τ_{t+1}, resp., announced in period t already, reads:

$$\text{(a)}\quad \frac{\partial \tilde{S}^d}{\partial \tau_{et+1}} = \frac{1+\tau_t}{(1-\tau_{et+1})^2}\Big[\sum_{i=L,H} -f_{it} b_{it}^{net} \frac{\partial v_t^i}{\partial x_{it}} - \tilde{\mu}_t^d \Big(b_{Ht}^{net} \frac{\partial v_t^H}{\partial x_{Ht}} - b_{Ht}^{net}[L] \frac{\partial v_t^H[L]}{\partial x_{Lt}}\Big)\Big] +$$

$$+ \tilde{\lambda}_{t+1}^d \sum_{i=L,H} \frac{\partial e_{it+1}}{\partial \tau_{et+1}} + \tilde{\mu}_{t+1}^d \frac{\partial v_{t+1}^H[L]}{\partial x_{Lt+1}}\Big[(e_{Ht+1} - e_{Lt+1}) - (1-\tau_{et+1})\Big(\frac{\partial e_{Ht+1}}{\partial \tau_{et+1}} - \frac{\partial e_{Lt+1}}{\partial \tau_{et+1}}\Big)\Big],$$

$$\text{(b)}\quad \frac{\partial S^d}{\partial \tau_{t+1}} = \mu_{t+1}^d \frac{\partial v_{t+1}^H[L]}{\partial x_{Lt+1}} (e_{Ht+1} - e_{Lt+1}) \frac{1-\tau_{et+1}}{1+\tau_{t+1}}.$$

Proof: see Appendix.

It turns out that the condition, which is decisive for the inheritance tax is more complex in this case. Still, the remarkable property that all welfare effects for later generations cancel out, arises in this context as well: on the right-hand side of Theorem 3(a) effects on generations t+2 and later do not appear.

The expression in the first square brackets in (a) shows us how the bequeathing generation t is affected. As can be seen from the first term (it is, by Roy's Lemma equivalent to $f_{it}\partial v_t^i / \partial \tau_{et+1}$), the increase of a tax τ_{et+1} on inherited wealth in period t+1 has a direct negative effect on welfare of the parent generation, which anticipates the tax. This is a result of double-counting in the social welfare function: in the present model the inheritance tax diminishes welfare of two generations, viz. t and t+1, while the revenues from the tax and their redistribution to the individuals have a positive impact only on generation t+1. The second term (multiplier $\tilde{\mu}_t^d$) shows that the increase of τ_{et+1} also affects the self-selection constraint of generation t; its sign is undetermined for arbitrary preferences[14]. (Clearly, τ_{et+1} does not change the available resources in period t, therefore the resource constraint of this period is unaffected.)

The remaining expressions on the right-hand side of Theorem 3(a) describe the welfare consequences of τ_{et+1} on the descendant generation t+1. It is decomposed into two effects: the first (multiplier $\tilde{\lambda}^d_{t+1}$) refers to the effect via the resource constraint in period t+1, as individuals of the parent generation t will adapt the amount of gross bequests left to their descendants. Resources of generation t+1 may increase or decrease, depending on the elasticity of net bequests $b^{net}_{it-1} = b_{it-1}(1-\tau_{et})$. In case of an elasticity of 1, as with Cobb-Douglas preferences over c_{it} and b^{net}_{it} (and separability with respect to labor time), gross bequests remain unchanged and the effect on the resource constraint is zero, as in the "no-anticipation"-case. The second effect (multiplier $\tilde{\mu}^d_{t+1}$) is familiar from the earlier Theorems, now augmented by the influence of τ_{et+1} on the difference between inheritances of high- and low-able individuals in period t+1. Obviously, the condition that inheritances increase with abilities now guarantees positivity of this effect only if it is not outweighed by this influence of τ_{et+1} (which may have any sign; it is again zero for Cobb-Douglas preferences[15]).

Altogether, we find that the welfare effect of an increase of the inheritance tax τ_{et+1} is diminished, if this increase is anticipated by the previous generation t and individuals care for net instead of gross bequests. A direct negative effect on the parent generation occurs, as a consequence of the fact that bequests (and, hence, their reduction through the

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14 For instance, for quasilinear preferences (introduced in Section 3) the sign is negative, because the marginal utility of net income is constant and net bequests are a normal good, i.e. $b^{net}_{Ht} > b^{net}_{Ht}[L]$.

15 Given that the rule guiding how gross bequests are allocated to generation t+1 does not depend on τ_{et+1}. Then unchanged gross bequests b_{Lt} and b_{Ht} mean unchanged gross inheritances e_{Lt+1} and e_{Ht+1}.

inheritance tax) appear twice in the social welfare function, while the repayment of the tax revenues (the reduction of the income tax) occurs only once.

Theorem 2(b) states that, as already discussed above, anticipation does in no way change the condition which is decisive for the desirability of the full expenditure tax τ_{t+1}. Let us finally mention an obvious implication of the bequest-as-consumption motive: taxes introduced in some period never have repercussions on generations living more than one period earlier, even if individuals care for net bequests.

3. Taxation of inheritances in a stochastic framework

As already mentioned, an objection against the models of Section 2 could be that with a fixed one-to-one relation between abilities and inherited wealth it is possible to identify individuals by their inherited wealth or by their expenditures (given these are observable) and to impose a first-best tax. In reality, no tax authority follows this strategy, because the relation between inherited wealth (or expenditures) and skills is not fixed, but stochastic. In order to capture this issue, we now assume that inherited wealth is random and prove a stochastic version of Theorem 2, where still a positive relation between inherited wealth and abilities is decisive. Furthermore, we offer a theoretical argument for the plausibility of such a relation: it results as the outcome of a stochastic process of abilities, if a mild condition on the probabilities relating the possible realizations of the child's ability to the parent's ability holds.

In order to make the model tractable, we assume in this Section that the utility function (identical for all individuals) is quasilinear, i.e., $u(c,b,l) = \varphi(c,b) + \psi(l)$, where $\varphi : \mathbb{R}^2 \to \mathbb{R}$ is concave and linear-homogeneous with $\partial\varphi/\partial c > 0, \partial\varphi/\partial b > 0$, and

$\psi : \mathbb{R} \to \mathbb{R}$ is strictly concave with $\psi' < 0$. One observes immediately that for quasilinear utility the following statements hold for indirect utility and demand:[16]

(q1) $\partial v / \partial x = \rho /(1+\tau)$. ρ is a constant for any ability ω and any x, z. $\partial v / \partial e = \rho(1-\tau_e)/(1+\tau)$.

(q2) $\partial b / \partial z = \partial c / \partial z = 0$. Demand is independent of gross income and labor time.

(q3) $c = \alpha_c(x+(1-\tau_e)e)/(1+\tau)$ and $b = \alpha_b(x+(1-\tau_e)e)/(1+\tau)$. α_c, α_b are the constant shares of consumption and bequests in the available budget, after correcting for τ, with $\alpha_c+\alpha_b = 1$. For later use, we define $\tilde{\alpha} \equiv \alpha_b /(1+\tau)$, $\hat{\alpha} \equiv \alpha_b(1-\tau_e)/(1+\tau)$.

The most important consequence of (q1) is that the self-selection constraint is independent of income effects, that is, of inheritances (see (11) later on).

We generalize the model by introducing n (not just two) different types of individuals, characterized by their earning abilities $\omega_{it} > 0$, i = 1,…,n, with $\omega_{it} < \omega_{i+1t}$ in period t.

3.1. A stochastic relation between ability levels and inheritances

Let some tax rates τ_{et}, τ_t (possibly zero) be given in period t. At the beginning of this period the planning tax authority determines the optimum tax on labor income (that is, the optimum bundles x_{it}, z_{it}, i = 1, ..., n) and decides whether a change of the tax rates τ_{et}, τ_t (or their introduction) is desirable.

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[16] For simplicity we drop the indices referring to the types and periods, because the statements hold for individuals of any ability level ω in any period.

When making the decision, the planner knows the ability levels $\omega_{1t}, \ldots, \omega_{nt}$ of the individuals of generation t period, but cannot identify individuals. Moreover, we assume that the planner knows the aggregate amount of bequests, e_t^{agg}, left to the generation t in total (no uncertainty on aggregate resources in period t exists). There is, however, only a stochastic relation between the ability level and the amount of inheritance an individual receives. Thus, the planner cannot, even when the realization of inheritances is known, infer the ability type of the receiving individual. (Nor is identification possible from the expenditures of an individual.)

More formally, we assume that there exists a (finite) number k of ways of how the aggregate amount e_t^{agg} may be distributed to the individuals of generation t, where each specific allocation j, j = 1,…k, occurs with probability κ_{jt} (with $\kappa_{1t} + ... + \kappa_{kt} = 1$) and transfers e_{it}^j to individual i, with $e_{1t}^j + ... + e_{nt}^j = e_t^{agg}$. The possible realizations and their probabilities are known.

Facing uncertainty, the planner wants to maximize expected social welfare in period t. With $f_{1t} > f_{2t} > ... > f_{nt} > 0$ being the weights of the different types in the social objective[17], the problem to determine the optimum income tax (that is, the bundles x_{it}, z_{it}) reads, for given τ_{et}, τ_t:

$$\max_{x_{it}, z_{it}} \sum_{j=1}^{k} \left(\sum_{i=1}^{n} f_{it} v_t^i (x_{it}, z_{it}, e_{it}^j, \tau_{et}, \tau_t) \right) \kappa_{jt} + (1+\gamma)^{-1} \sum_{j=1}^{k} W(b_{1t}^j, ..., b_{nt}^j) \kappa_{jt}, \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{it} \leq \sum_{i=1}^{n} z_{it} + \tau_{et} \sum_{j=1}^{k} \left(\sum_{i=1}^{n} e_{it}^j \right) \kappa_{jt} + \tau_t \sum_{j=1}^{k} \left(\sum_{i=1}^{n} (c_{it}^j + b_{it}^j) \right) \kappa_{jt} - g_t, \tag{10}$$

[17] Note that with quasilinear preferences the marginal utility of income is identical for all individuals; therefore a utilitarian objective with equal weights would not imply downward redistribution of income.

$$\frac{\rho}{1+\tau_t}(x_{it}-x_{i-1t}) \geq \psi(\frac{z_{i-1t}}{\omega_i})-\psi(\frac{z_{it}}{\omega_i}), \quad i = 2, ..., n. \tag{11}$$

Here c_{it}^j, b_{it}^j denote consumption of individual i and bequests left by her, in case that allocation j of inheritances is realized. Moreover, similar to the formulation in Subsection 2.2, W describes how future social welfare is influenced by the bequests of generation t. We have assumed that only the self-selection constraints (11) for the respective higher-able individuals are relevant in the optimum.[18] This is justified, if the social objective implies downward redistribution, which follows from our assumption $f_{it} > f_{i+1t}$.

We have to check, whether this problem is well defined, that is, whether it can be solved by the planner without knowing the actual realization of the inheritances. For this, the constraints (10) and (11) must be independent of the actual realization. As the b_{it}^j do not appear in the self-selection constraints (11) (due to the consequence (q1) of quasilinear utility, as already mentioned), the required independence is clearly fulfilled for these constraints. Moreover, exchanging the order of summation in the resource constraint (10) and using the property (q3) of quasilinear utility, it can be written as

$$\sum_{i=1}^{n} x_{it} \leq \sum_{i=1}^{n} z_{it} + \tau_{et} e_t^{agg} + \frac{\tau_t}{1+\tau_t}[\sum_{i=1}^{n} x_{it} + (1-\tau_{et})e_t^{agg}] - g_t. \tag{10'}$$

Thus, the resource constraint is independent of the particular realization of the inheritances as well. Only the aggregate amount of inheritances matters, which we assume to be known. This proves

[18] It is well-known that only the self-selection constraints of pairs of individuals with adjacent ability levels need to be considered.

Lemma 2: The optimum bundles (x_{it}, z_{it}), i = 1, ..., n of problem (9) – (11) can be determined independently of the particular realization of individual inheritances e_{it}.

To derive the following theorem, we need the assumption that W has some "quasilinear property", namely that, given any i, the derivatives $\partial W / \partial b_{it}^j$ are independent of j. In other words, the marginal welfare effect of an increase of an individual's bequests on the welfare of future generations is constant and is, in particular independent of the specific realization of inheritances received by generation t. This is obviously fulfilled, if W is a discounted sum of future expected social welfare (see footnote 11), with quasilinear individual utility in each period.

Let now $S^r(\tau_{et},\tau_t)$ be the optimum value of (9) subject to (10) and (11), for given τ_{et}, τ_t, and let $\overline{e}_{it}$ denote the expected value of the inheritances e_{it}^j which individual i of generation t receives. As the criteria for a change (or the introduction) of taxes on inheritances and/or full expenditures we find

Theorem 4: With stochastic inheritances, the welfare effect of a marginal increase of τ_{et} and τ_t, resp., in some period t, reads:

(a) $$\frac{\partial S^r}{\partial \tau_{et}} = \frac{\rho}{1+\tau_t} \sum_{i=2}^{n} \mu_i^r (\overline{e}_{it} - \overline{e}_{i-1t}),$$

(b) $$\frac{\partial S^r}{\partial \tau_t} = \frac{(1-\tau_{et})\rho}{(1+\tau_t)^2} \sum_{i=2}^{n} \mu_i^r (\overline{e}_{it} - \overline{e}_{i-1t}).$$

Proof: see Appendix.

Thus, we arrive at a direct stochastic analogon of Theorem 2, referring to expected values instead of deterministic inheritances. A sufficient (but not necessary) condition for the desirability of a tax on inheritances (or on full expenditures) is that the order of expected inheritances is the same as the order of earning abilities, because then the right-hand sides of (a) and (b) are positive.[19]

3.2. An intertemporal model with stochastic transition of abilities

In this Subsection we provide a theoretical argument for the plausibility of the sufficient condition of Theorem 4. We do so by studying a stochastic process which determines how the relation between abilities and inherited wealth evolves over time. The essential elements of the process we consider are the following:

(P1) In each period t there exists the same number n of individuals with identical quasilinear utility, as introduced at the beginning of Section 3. They differ in their earning abilities, with order $0 < \omega_{1t} < \omega_{2t} < \ldots < \omega_{nt}$.[20]

(P2) Each individual has a single descendant to whom she leaves all her bequests.

(P3) The order of ability levels of the descendants can be any permutation of the order of the parent individuals' abilities.

(P4) In each period t the identical permutation, where each individual's ability is ranked just as her parent's ability (in period t – 1), has a higher probability p_{Et} than any

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19 One can show that a sufficient *and* necessary condition for the desirability of these taxes is that the social marginal valuation of individual i's income (including its value for all future generations via bequests), i.e., $[f_{it}\rho + (1+\gamma)^{-1}\alpha_b \partial W / \partial b_{it}]/(1+\tau_t)$), is negatively correlated with expected inheritance $\bar{e}_{it}$. This result is obtained by solving (A22) – (A24) in the Appendix for μ_i^r and using this expression together with the definition of the covariance in the RHS's of (a) and (b).

20 Thus, we allow any change of the ability levels, e.g., they could grow by some common growth rate.

other permutation. All other permutations occur with the same probability p_t, with $(n!-1)p_t = 1 - p_{Et}$.

(P1) – (P4) seem to be reasonable properties. In particular – as mentioned in footnote 6 – there is much empirical evidence indicating a positive correlation between children's and parents' earning abilities, which we capture by property (P4).[21] Note that the process has the well-known property of "regression to the mean" in the following sense: if we consider a parent with ability rank i in the upper half (i > (n + 1)/2), then the descendant's ability has a higher probability to rank below i than above i.[22] An analogous relation holds for a parent with ability rank i < (n +1)/2.

In addition, we assume that in each period t a tax system exists, consisting of taxes on labor income, inheritance and full expenditure (all possibly zero). Individuals earn gross income z_{it} and net income x_{it} and choose c_{it}, b_{it}.

Generally, the transfer of wealth over generations and the stochastic nature of how abilities are linked to inheritances in each generation generate a very complex process, whose properties are difficult to analyze. The reason is that in each period the amount which an individual receives as inheritance depends on the combination of ability level and inheritance that characterized her parent, and the inheritance of the latter in turn depended on the combination characterizing the grandparent and so on. Thus, the number of possible combinations grows rapidly over time.

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21 An alternative way would be to assume that the probability of a descendant's ability level having the same rank as the parent's is higher than the probability of having any other rank. This would imply our assumption of a higher probability of the identical permutation.

22 As for the descendant any rank $j \neq i$ has the same probability $(n-1)!p_t$, the probability that her rank is lower than i is $(i-1)(n-1)!p_t$ for her, while that of a higher rank is $(n-i)(n-1)!p_t$. $i > (n+1)/2$ implies $i - 1 > n - i$. See also the proof of Lemma 3 below.

The key observation, which allows us to derive a clear-cut result on the long-run stochastic properties of the distribution of inherited wealth and earning abilities, as introduced above, is the following: assume that in some starting period 0 there is no initial wealth. With quasilinear preferences, each individual with ability level ω_{i0} leaves bequests $b_{i0} = \tilde{\alpha}_0 x_{i0}$ (remember property (q3) at the beginning of Section 3; we add a period index to indicate that $\tilde{\alpha}_t$ depends on the tax rate of the respective period) to her descendant with some ability level ω_{j1}. The latter in turn, for whom $b_{i0}(1-\tau_{e1}) = e_{j1}(1-\tau_{e1})$ is part of the budget, bequeaths an amount $\hat{\alpha}_1 b_{i0} = \hat{\alpha}_1\tilde{\alpha}_0 x_{i0}$ out of b_{i0} to her descendant[23] (with some ability level ω_{m1}), who again leaves $\hat{\alpha}_2\hat{\alpha}_1\tilde{\alpha}_0 x_{i0}$ out of it, and so on.[24]

That is, each net income x_{i0} initiates an own series of bequests, which can, given quasilinear utility, be described by a simple formula. Obviously, this observation can be generalized to later periods: out of the net incomes x_{it} of that period, each generation t initiates a new series, which we call a *bequest series*, denoted by β_t. β_t consists of the elements β_{it}^{s}, where i indicates the ability level of the first bequeathing individual and s denotes the receiving generation, thus $\beta_{it}^{t+1} = \tilde{\alpha}_t x_{it}$ and $\beta_{it}^{s+1} = \hat{\alpha}_s \beta_{it}^{s}$ for $s \geq t+1$. One finds immediately that each bequest series vanishes in the course of time, as all $\hat{\alpha}_t < 1$. Note also that the ability levels of the receiving individuals of any generation t' > t do not influence the value of subsequent β_{it}^{s}, s > t'.

23 In addition, of course, the individual of type ω_{j1} also bequeaths $\tilde{\alpha}_1 x_{j1}$ out of her own net income.

24 Here we have assumed that bequeathing individuals care for gross bequests b_{it}. If they anticipate the next period's inheritance tax and care for net bequests $b_{it}(1 - \tau_{et+1})$, the respective definitions of $\tilde{\alpha}_t$ and $\hat{\alpha}_t$ continue to hold, but with a different value of the parameter α_b, which now depends on the inheritance tax τ_{et+1} of the next period.

From the perspective of a receiving individual in some period s, her inheritance is the sum of what she receives through all bequest series β_t initiated by earlier generations. We first study the joint evolution of a single bequest series and of the earning abilities.

Let $P_i(\beta_{jt}^s)$ denote the probability that individual i in period s receives the bequest initiated by individual j in period t < s. The following relations between the probabilities characterizing the distribution of inheritances are derived from the properties (P1) - (P4):

Lemma 3: For any i,j = 1,…,n, i $\neq$ j, and any s > t, the inequalities

(a) $P_i(\beta_{it}^s) > P_i(\beta_{jt}^s)$,

(b) $P_i(\beta_{it}^s) > P_i(\beta_{it}^{s+1})$, $P_i(\beta_{jt}^s) < P_i(\beta_{jt}^{s+1})$,

are fulfilled.

Proof: see Appendix.

In any later period, an individual has a higher probability of receiving the bequests left initially by a parent with identical ability rank than of receiving the bequests of any other parent. However, (b) tells us that the difference between these probabilities becomes smaller with any further transition. That is, in the course of time, the elements of a bequest series become more equally distributed within a generation of heirs. On the other hand, this equalization occurs for lower and lower values of the transfers in a bequest series, as each series β_t diminishes with $\hat{\alpha}_s < 1$. What dominates the inheritances received by some generation are the bequest series initiated by rather recent generations, which are more unequally distributed.

A consequence of the properties of the wealth transfer as described above is that for any bequest series the order of expected values of inheritances coincides with the order of ability levels, if in the initial period net incomes rise with abilities. Let $E_i[\beta_t^s]$ denote the expected value of the inheritance received by an individual with ability ω_{is} in period s > t from the bequest series β_t.

Lemma 4: Assume that $x_{it} < x_{i+1t}$. Then for any $s > t$ and any bequest series β_t, $E_i[\beta_t^s] < E_{i+1}[\beta_t^s]$ for all $i = 1,...,n-1$. $E_i[\beta_t^s] \le E_{i+1}[\beta_t^s]$ holds if $x_{it} \le x_{i+1t}$.

Proof: see Appendix.

Note that the condition $x_{it} \le x_{i+1t}$ is always fulfilled, if preferences have the property AM (see Subsection 2.1). $x_{it} = x_{i+1t}$[25] may occur, if the income tax function is not smooth.

As the inheritances received by the individuals of some generation s are the sum of what they get out of all the bequest series β_t initiated by earlier generations, we arrive at the desired relation between expected inheritances $\overline{e}_{is}$ and ability levels ω_{is}:

Theorem 5: Assume that in period 0 there is no initial wealth and $x_{it} < x_{i+1t}$ for at least one $t < s$. Then $\overline{e}_{is} < \overline{e}_{i+1s}$.

Proof: see Appendix.

[25] This possibility is called "bunching" in the literature on optimum income taxation, see, e.g. Guesnerie and Seade (1982) or Brunner (1989) for a finite economy.

Theorem 5 allows us to formulate a definite result on the desirability of an inheritance and/or full expenditure tax in our model. We consider an economy developing according to the stochastic process described by (P1) – (P4), where in each period a tax system may exist. Then, in some period s, the planner chooses an optimum nonlinear income tax and thinks of a change of the tax rates τ_{es}, τ_s. She aims at maximizing present and (discounted) future welfare and knows the aggregate amount of inheritances received by generation s, and its possible allocations. Given that the downward self-selection constraints are binding, (9) – (11) is the relevant optimization problem and we find:

Theorem 6: Assume that in period 0 there is no initial wealth and $x_{it} < x_{i+1t}$ for at least one $t < s$ and one $i \in \{1,\ldots, n-1\}$. Then in period s an increase of the taxes on inheritance and/or on full expenditures, combined with an optimum nonlinear income, increases social welfare.

Proof: Combine Theorems 4 and 5.

A shift from income taxation to inheritance (or full expenditure) taxation allows further redistribution, except the extreme case that in all prior periods the income tax is designed in such a way that all individuals choose the same gross (and net) income.[26] Note further that Theorem 6, as far as the inheritance tax is concerned, rests on the assumption that decisions of the prior generation are already made, when the increase of τ_{et} is announced (compare the discussion of Theorem 3).

[26] This extreme situation does not occur, if in some period $t < s$ an optimum nonlinear income tax is imposed, because, as is well known, the latter requires "no bunching at the top". See Guesnerie and Seade (1982), Brunner (1989).

4. Conclusion

Bequests create wealth differences within the generation of heirs. Drawing on this observation, which is central to the equality-of-opportunity argument, we have clarified the role of inheritance taxation in an optimum-taxation framework with a bequest-as-consumption motive. In particular, we have worked out how different generations are affected by this tax. More generally, our results shed new light on the role of indirect taxes as well as of a tax on inherited wealth in combination with an optimum nonlinear income tax. The two main messages are the following:

First, in a static setting it is desirable, according to a utilitarian social objective, to shift some tax burden from labor income to initial wealth, if initial wealth increases with earning abilities. From a theoretical point of view, this result is a consequence of the information constraint which motivates income taxation in the Mirrlees-model: if the tax authority could observe individual earning abilities, it would impose the tax directly on these, as a (differentiated) first-best instrument. Given that this is impossible, it seems natural, then, that the authority can improve the tax system by use of information (i.e. imposing a tax) on inherited wealth (in addition to information on income), in case that it is observable and correlated with abilities. (In fact, if the correlation were negative, wealth should be subsidized.) Equivalently, a uniform tax on consumption plus bequests is also appropriate for this purpose.

Secondly, this result remains unchanged in a dynamic model in which the social welfare function accounts for effects on future generations: these effects cancel out when the optimum labor income tax is adapted accordingly. This is the final result for the case that a uniform tax on consumption plus bequests is imposed, as a surrogate for a tax on inherited wealth. In case that inheritances are taxed directly, an additional effect hast to be observed: if the parent individuals care for net instead of gross bequests (and anticipate the tax falling on the recipients of the wealth transfer in the next generation), then the bequest decision of the previous generation is affected and a further welfare effect arises, which is negative, because of "double-counting" of bequests.

Obviously, for the second message the assumption of the joy-of-giving motive for leaving bequests is important. With this motive, individuals care for the amount they leave to their descendants (and possibly for its reduction through an inheritance tax). However, they do not care for which purpose the descendants use their inheritance, nor, in particular, to which extent the descendants are subjected to a tax when they use the inherited amount for own consumption as well as for bequests in favor of a further generation. This is a reasonable standard assumption; it implies that a uniform tax on consumption and bequests produces no negative effects for the parent generation.

Finally, we have demonstrated that the results on the taxation of inheritances remain essentially valid, if there is a stochastic instead of a deterministic connection between abilities and inheritances: taxation is desirable, if expected inheritances of more able individuals are larger. Moreover, such a situation was shown to arise as the outcome of a stochastic process in which the descendants' ability ranks are more likely to be the same as their parents' ranks than any other.

Throughout this paper we have assumed that earning abilities are exogenous. In reality, of course, they depend on human capital investments, which are financed out of the parents' budget, as are inheritances of non-human capital. Given that both increase with the budget, this provides an additional argument for the positive relation between abilities and inherited wealth within the generation of heirs.

When investigating the welfare consequences of the taxation of inheritances, we confined our analysis to a uniform tax on consumption plus bequests and to a proportional tax on inherited wealth, and proved that, in principle, they are equivalent. We did not consider the possibility that a differentiation of tax rates according to the type of expenditures might increase welfare further, as it does in the Atkinson-Stiglitz model. Moreover, also the welfare consequences of other tax schedules, for instance a linear (instead of a nonlinear) income tax or a nonlinear tax on inheritances, deserve further analysis.

Appendix

Proof of Theorem 1

(a) The Lagrangian to the maximization problem (2) – (4) reads

$$\begin{aligned}L &= f_L v^L(x_L, z_L, e_L, \tau_e, \tau) + f_H v^H(x_H, z_H, e_H, \tau_e, \tau) - \\ &- \lambda\Big(x_L + x_H - z_L - z_H - \tau_e\big(e_L + e_H\big) - \tau\big(c_L(\cdot) + b_L(\cdot) + c_H(\cdot) + b_H(\cdot)\big) + g\Big) + \\ &+ \mu\Big(v^H(x_H, z_H, e_H, \tau_e, \tau) - v^H(x_L, z_L, e_H, \tau_e, \tau)\Big)\end{aligned}$$

which gives us the first-order condition with respect to x_L, x_H, i = L,H (we use the abbreviation $v^H[L] \equiv v^H(x_L, z_L, e_H, \tau_e, \tau)$):

$$f_L \frac{\partial v^L}{\partial x_L} - \lambda + \lambda\tau(\frac{\partial c_L}{\partial x_L} + \frac{\partial b_L}{\partial x_L}) - \mu \frac{\partial v^H[L]}{\partial x_L} = 0\,, \tag{A1}$$

$$f_H \frac{\partial v^H}{\partial x_H} - \lambda + \lambda\tau(\frac{\partial c_H}{\partial x_H} + \frac{\partial b_H}{\partial x_H}) + \mu \frac{\partial v^H}{\partial x_H} = 0\,. \tag{A2}$$

Using the Envelope Theorem we get for the optimal value function $S(\tau_e, \tau)$

$$\begin{aligned}\frac{\partial S}{\partial \tau_e} &= f_L \frac{\partial v^L}{\partial \tau_e} + f_H \frac{\partial v^H}{\partial \tau_e} + \lambda(e_L + e_H) + \lambda\tau(\frac{\partial c_L}{\partial \tau_e} + \frac{\partial b_L}{\partial \tau_e} + \frac{\partial c_H}{\partial \tau_e} + \frac{\partial b_H}{\partial \tau_e}) + \\ &+ \mu(\frac{\partial v^H}{\partial \tau_e} - \frac{\partial v^H[L]}{\partial \tau_e}).\end{aligned} \tag{A3}$$

We use $\partial v^i/\partial \tau_e = -e_i\, \partial v^i/\partial x_i$, $\partial v^H[L]/\partial \tau_e = -e_H\, \partial v^H[L]/\partial x_L$, $\partial c_i/\partial \tau_e = -e_i\, \partial c_i/\partial x_i$, $\partial b_i/\partial \tau_e = -e_i \partial b_i/\partial x_i$, compute $f_i\, \partial v^i/\partial x_i$, i = L,H, from (A1) and (A2) and transform, thus, (A3) to

$$\frac{\partial S}{\partial \tau_e} = \mu \frac{\partial v^H[L]}{\partial x_L}(e_H - e_L)\,. \tag{A4}$$

(b) We determine

$$\frac{\partial S}{\partial \tau} = f_L \frac{\partial v^L}{\partial \tau} + f_H \frac{\partial v^H}{\partial \tau} + \lambda(c_L + b_L + c_H + b_H) + \\ + \lambda\tau(\frac{\partial c_L}{\partial \tau} + \frac{\partial b_L}{\partial \tau} + \frac{\partial c_H}{\partial \tau} + \frac{\partial b_H}{\partial \tau}) + \mu\frac{\partial v^H}{\partial \tau} - \mu\frac{\partial v^H[L]}{\partial \tau}. \tag{A5}$$

The individual i's budget equation can be written as $c_i + b_i = B_i$, where $B_i \equiv (x_i + (1-\tau_e)e_i)/(1+\tau)$. Thus, $\partial c_i/\partial\tau = (\partial c_i/\partial B_i)(\partial B_i/\partial\tau) = -(c_i + b_i)\partial c_i/\partial x_i$ (use $\partial B_i/\partial\tau = -(x_i + (1-\tau_e)e_i)/(1+\tau)^2 = -(c_i + b_i)/(1+\tau)$) and $\partial c_i/\partial x_i = \partial c_i/\partial B_i/(1+\tau)$); equivalently $\partial b_i/\partial\tau = -(c_i + b_i)\partial b_i/\partial x_i$. Substituting these terms, together with $\partial v^i/\partial\tau = -(c_i + b_i)\partial v^i/\partial x_i$, $\partial v^H[L]/\partial\tau = -(c_H[L] + b_H[L])\partial v^H[L]/\partial x_L$ (where $c_H[L]$, $b_H[L]$, resp., denotes consumption and bequests of individual H, having L's gross and net income), and with (A1),(A2) into (A5) yields

$$\frac{\partial S}{\partial \tau} = \mu\frac{\partial v^H[L]}{\partial x_L}\big((c_H[L] + b_H[L]) - (c_L + b_L)\big). \tag{A6}$$

Inserting the (transformed) budget equations of individual H when mimicking and of individual L, i.e., $c_H[L] + b_H[L] = (x_L + (1-\tau_e)e_H)/(1+\tau)$ and $c_L + b_L =$ $= (x_L + (1-\tau_e)e_L)/(1+\tau)$ into (A6), we obtain the formula of Theorem 1(b). QED

Proof of Theorem 2

(a) From the Lagrangian to the optimization problem (5) – (7) we derive the first-order conditions with respect to x_{Lt}, x_{Ht}, where λ^d, μ^d are the multipliers corresponding to the resource constraint and to the self-selection constraint, resp.:

$$f_{Lt}\frac{\partial v_t^L}{\partial x_{Lt}} + (1+\gamma)^{-1}\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial x_{Lt}} - \lambda^d + \lambda^d\tau_t(\frac{\partial c_{Lt}}{\partial x_{Lt}} + \frac{\partial b_{Lt}}{\partial x_{Lt}}) - \mu^d\frac{\partial v_t^H[L]}{\partial x_{Lt}} = 0, \tag{A7}$$

$$f_{Ht}\frac{\partial v_t^H}{\partial x_{Ht}}+(1+\gamma)^{-1}\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial x_{Ht}}-\lambda^d+\lambda^d\tau_t(\frac{\partial c_{Ht}}{\partial x_{Ht}}+\frac{\partial b_{Ht}}{\partial x_{Ht}})+\mu^d\frac{\partial v_t^H}{\partial x_{Ht}}=0. \tag{A8}$$

The derivative of the optimum-value function S^d with respect τ_{et} is found by differentiating the Lagrangian:

$$\begin{aligned}\frac{\partial S^d}{\partial \tau_{et}}&=f_{Lt}\frac{\partial v_t^L}{\partial \tau_{et}}+f_{Ht}\frac{\partial v_t^H}{\partial \tau_{et}}+(1+\gamma)^{-1}(\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial \tau_{et}}+\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial \tau_{et}})+\lambda^d(e_{Lt}+e_{Ht})+\\&+\lambda^d\tau_t(\frac{\partial c_{Lt}}{\partial \tau_{et}}+\frac{\partial b_{Lt}}{\partial \tau_{et}}+\frac{\partial c_{Ht}}{\partial \tau_{et}}+\frac{\partial b_{Ht}}{\partial \tau_{et}})+\mu^d(\frac{\partial v_t^H}{\partial \tau_{et}}-\frac{\partial v_t^H[L]}{\partial \tau_{et}}).\end{aligned} \tag{A9}$$

By use of the formulas below (A3), (A9) can be transformed to

$$\begin{aligned}\frac{\partial S^d}{\partial \tau_{et}}&=-f_{Lt}e_{Lt}\frac{\partial v_t^L}{\partial x_{Lt}}-f_{Ht}e_{Ht}\frac{\partial v_t^H}{\partial x_{Ht}}+(1+\gamma)^{-1}(-e_{Lt}\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial x_{Lt}}-e_{Ht}\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial x_{Ht}})+\\&+\lambda^d(e_{Lt}+e_{Ht})+\lambda^d\tau_t[-e_{Lt}(\frac{\partial c_{Lt}}{\partial x_{Lt}}+\frac{\partial b_{Lt}}{\partial x_{Lt}})-e_{Ht}(\frac{\partial c_{Ht}}{\partial x_{Ht}}+\frac{\partial b_{Ht}}{\partial x_{Ht}})]-\\&-\mu^d e_{Ht}(\frac{\partial v_t^H}{\partial x_{Ht}}-\frac{\partial v_t^H[L]}{\partial x_{Lt}}).\end{aligned} \tag{A10}$$

Multiplying (A7), (A8) by e_{Lt}, e_{Ht}, resp., and substituting into (A10) gives us the formula of Theorem 2(a).

(b) Differentiating the Lagrangian of problem (5) - (7) with respect to τ_t gives:

$$\begin{aligned}\frac{\partial S^d}{\partial \tau_t}&=f_{Lt}\frac{\partial v_t^L}{\partial \tau_t}+f_{Ht}\frac{\partial v_t^H}{\partial \tau_t}+(1+\gamma)^{-1}(\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial \tau_t}+\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial \tau_t})+\\&+\lambda^d[c_{Lt}+b_{Lt}+c_{Ht}+b_{Ht}+\tau_t(\frac{\partial c_{Lt}}{\partial \tau_t}+\frac{\partial b_{Lt}}{\partial \tau_t}+\frac{\partial c_{Ht}}{\partial \tau_t}+\frac{\partial b_{Ht}}{\partial \tau_t})]+\\&+\mu^d(\frac{\partial v_t^H}{\partial \tau_t}-\frac{\partial v_t^H[L]}{\partial \tau_t}).\end{aligned} \tag{A11}$$

By use of the formulas below (A5), (A11) can be transformed to

$$\begin{aligned}\frac{\partial S^d}{\partial \tau_t} = \sum_{i=L,H} \{ & -f_{it}(c_{it}+b_{it})\frac{\partial v_t^i}{\partial x_{it}} - (1+\gamma)^{-1}((c_{it}+b_{it})\frac{\partial W}{\partial b_{it}}\frac{\partial b_{it}}{\partial x_{it}} + \\ & + \lambda^d[c_{it}+b_{it}-\tau_t(c_{it}+b_{it})(\frac{\partial c_{it}}{\partial x_{it}}+\frac{\partial b_{it}}{\partial x_{it}}]\} - \\ & -\mu^d(c_{Ht}+b_{Ht})\frac{\partial v_t^H}{\partial x_{Ht}} + \mu^d(c_{Ht}[L]+b_{Ht}[L])\frac{\partial v_t^H[L]}{\partial x_{Lt}}).\end{aligned} \tag{A12}$$

Multiplying (A7), (A8) by $(c_{Lt}+b_{Lt})$, $(c_{Ht}+b_{Ht})$, resp., and substituting into (A12) gives us

$$\frac{\partial S^d}{\partial \tau_t} = \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}}(c_{Ht}[L]+b_{Ht}[L]-c_{Lt}-b_{Lt}),$$

or, as shown in the proof of Theorem 1(b), the formula of Theorem 2(b). QED

Proof of Theorem 3

(a) If individuals care for net bequests, indirect utility of an individual i of generation t depends also on τ_{et+1}:

$$\begin{aligned} & v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t, \tau_{et+1}) \equiv \\ & \qquad \max\left\{u(c_{it}, b_{it}^{net}, z_{it}/\omega_{it}) \mid (1+\tau_t)(c_{it}+b_{it}^{net}/(1-\tau_{et+1})) \le x_{it}+(1-\tau_{et})e_{it}\right\}\end{aligned}$$

Obviously, consumption $c_{it}(\cdot)$, net bequests $b_{it}^{net}(\cdot)$ and gross bequests $b_{it}(\cdot) = b_{it}^{net}(\cdot)/(1-\tau_{et+1})$ depend on the same arguments as $v_t^i(\cdot)$. Moreover, gross inheritances $e_{it+1}(\cdot)$ are endogenous, they result from bequests of generation t via some (unspecified) rule and depend on the same arguments as $b_{Lt}(\cdot)$ and $b_{Ht}(\cdot)$.

When determining taxes for the periods t and t+1, the tax authority has to observe the resource and the self-selection constraints for these periods:

$$\sum_{i=L,H} x_{it} \leq \sum_{i=L,H} [z_{it} + \tau_{et}e_{it} + \tau_t(c_{it}(\cdot) + b_t^{net}(\cdot)/(1-\tau_{et+1}))] - g_t, \tag{A13}$$

$$\sum_{i=L,H} x_{it+1} \leq \sum_{i=L,H} [z_{it+1} + \tau_{et+1}e_{it+1}(\cdot) + \tau_{t+1}(c_{it+1}(\cdot) + b_{t+1}^{net}(\cdot)/(1-\tau_{et+2}))] - g_{t+1}, \tag{A14}$$

$$v_t^H(x_{Ht}, z_{Ht}, e_{Ht}, \tau_{et}, \tau_t, \tau_{et+1}) \geq v_t^H(x_{Lt}, z_{Lt}, e_{Ht}, \tau_{et}, \tau_t, \tau_{et+1}), \tag{A15}$$

$$\begin{aligned} &v_{t+1}^H(x_{Ht+1}, z_{Ht+1}, e_{Ht+1}(\cdot), \tau_{et+1}, \tau_{t+1}, \tau_{et+2}) \geq \\ &\qquad v_{t+1}^H(x_{Lt+1}, z_{Lt+1}, e_{Ht+1}(\cdot), \tau_{et+1}, \tau_{t+1}, \tau_{et+2}). \end{aligned} \tag{A16}$$

Using the Envelope Theorem we get for the optimum value function $\tilde{S}^d(\tau_{et}, \tau_t, \tau_{et+1}, \tau_{t+1})$ of the maximization problem (8), (A13) – (A16) ($\tilde{\lambda}_t^d$, $\tilde{\lambda}_{t+1}^d$, $\tilde{\mu}_t^d$, $\tilde{\mu}_{t+1}^d$ are the Lagrange multipliers corresponding to (A13) – (A16)):

$$\begin{aligned} \frac{\partial \tilde{S}^d}{\partial \tau_{et+1}} = &\sum_{i=L,H} f_{it} \frac{\partial v_t^i}{\partial \tau_{et+1}} + (1+\gamma)^{-1} \sum_i f_{it+1} \frac{\partial v_{t+1}^i}{\partial \tau_{et+1}} + (1+\gamma)^{-2} (\sum_i \frac{\partial W}{\partial b_{it+1}} \frac{\partial b_{it+1}}{\partial \tau_{et+1}}) + \\ &+ \tilde{\lambda}_t^d \tau_t \sum_i (\frac{\partial c_{it}}{\partial \tau_{et+1}} + \frac{b_{it}^{net}}{(1-\tau_{et+1})^2} + \frac{1}{(1-\tau_{et+1})} \frac{\partial b_{it}^{net}}{\partial \tau_{et+1}}) + \\ &+ \tilde{\lambda}_{t+1}^d \sum_i (e_{it+1} + \tau_{et+1} \frac{\partial e_{it+1}}{\partial \tau_{et+1}}) + \tilde{\lambda}_{t+1}^d \tau_{t+1} \sum_i (\frac{\partial c_{it+1}}{\partial \tau_{et+1}} + \frac{1}{(1-\tau_{et+2})} \frac{\partial b_{it+1}^{net}}{\partial \tau_{et+1}}) + \\ &+ \tilde{\mu}_t^d (\frac{\partial v_t^H}{\partial \tau_{et+1}} - \frac{\partial v_t^H[L]}{\partial \tau_{et+1}}) + \tilde{\mu}_{t+1}^d (\frac{\partial v_{t+1}^H}{\partial \tau_{et+1}} - \frac{\partial v_{t+1}^H[L]}{\partial \tau_{et+1}}). \end{aligned} \tag{A17}$$

Differentiating the individual budget constraint of an individual i with respect to τ_{et+1} we obtain

$$\frac{\partial c_{it}}{\partial \tau_{et+1}} + \frac{b_{it}^{net}}{(1-\tau_{et+1})^2} + \frac{1}{(1-\tau_{et+1})} \frac{\partial b_{it}^{net}}{\partial \tau_{et+1}} = 0. \tag{A18}$$

For shorter notation we introduce net inheritances $e_{it+1}^{net}(\cdot) \equiv e_{it+1}(\cdot)(1-\tau_{et+1})$, with $\partial e_{it+1}^{net}/\partial \tau_{et+1} = -e_{it+1} + (1-\tau_{et+1})\partial e_{it+1}/\partial \tau_{et+1}$, thus $\partial e_{it+1}/\partial \tau_{et+1} - \partial e_{it+1}^{net}/\partial \tau_{et+1} = e_{it+1} +$ $+\tau_{et+1}\partial e_{it+1}/\partial \tau_{et+1}$. Further, we have:

$\partial v_t^i / \partial \tau_{et+1} = -\Big((1+\tau_t) b_{it}^{net} / (1-\tau_{et+1})^2\Big) \partial v_t^i / \partial x_{it}$ (use Roy's Lemma),

$\partial v_{t+1}^i / \partial \tau_{et+1} = (\partial e_{it+1}^{net} / \partial \tau_{et+1})(\partial v_{t+1}^i / \partial x_{it+1})$, $\partial c_{it+1} / \partial \tau_{et+1} = (\partial e_{it+1}^{net} / \partial \tau_{et+1})(\partial c_{it+1} / \partial x_{it+1})$,

$\partial b_{it+1}^{net} / \partial \tau_{et+1} = (\partial e_{it+1}^{net} / \partial \tau_{et+1})(\partial b_{it+1}^{net} / \partial x_{it+1})$, $\partial b_{it+1} / \partial x_{it+1} = (\partial b_{it+1}^{net} / \partial x_{it+1}) / (1-\tau_{et+2})$.

By use of these formulas and of (A18), (A17) can be transformed to

$$\begin{aligned}
\frac{\partial \tilde{S}^d}{\partial \tau_{et+1}} &= \frac{1+\tau_t}{(1-\tau_{et+1})^2} \sum_{i=L,H} -f_{it} b_{it}^{net} \frac{\partial v_t^i}{\partial x_{it}} + (1+\gamma)^{-1} \sum_i f_{it+1} \frac{\partial v_{t+1}^i}{\partial x_{it+1}} \frac{\partial e_{it+1}^{net}}{\partial \tau_{et+1}} + \\
&+ (1+\gamma)^{-2} \sum_i \frac{\partial W}{\partial b_{it+1}} \frac{\partial b_{it+1}}{\partial x_{it+1}} \frac{\partial e_{it+1}^{net}}{\partial \tau_{et+1}} + \tilde{\lambda}_{t+1}^d \sum_i \Big(\frac{\partial e_{it+1}}{\partial \tau_{et+1}} - \frac{\partial e_{it+1}^{net}}{\partial \tau_{et+1}}\Big) + \\
&+ \tilde{\lambda}_{t+1}^d \tau_{t+1} \sum_i \Big(\frac{\partial c_{it+1}}{\partial x_{it+1}} + \frac{\partial b_{it+1}}{\partial x_{it+1}}\Big) \frac{\partial e_{it+1}^{net}}{\partial \tau_{et+1}} + \tilde{\mu}_t^d \frac{1+\tau_t}{(1-\tau_{et+1})^2} \Big(-\frac{\partial v_t^H}{\partial x_{Ht}} b_{Ht}^{net} + \\
&+ \frac{\partial v_t^H[L]}{\partial x_{Lt}} b_{Ht}^{net}[L]\Big) + \tilde{\mu}_{t+1}^d \Big(\frac{\partial v_{t+1}^H}{\partial x_{Ht+1}} - \frac{\partial v_{t+1}^H[L]}{\partial x_{Lt+1}}\Big) \frac{\partial e_{it+1}^{net}}{\partial \tau_{et+1}}.
\end{aligned} \tag{A19}$$

Finally, we derive the first-order conditions from the Lagrangian to the maximization problem (8) and (A13) – (A16) with respect to x_{Lt+1}, x_{Ht+1} (we use again that $\partial b_{it+1} / \partial x_{it+1} = (\partial b_{it+1}^{net} / \partial x_{it+1}) / (1-\tau_{et+2})$):

$$\begin{aligned}
&(1+\gamma)^{-1} f_{Lt+1} \frac{\partial v_{t+1}^L}{\partial x_{Lt+1}} + (1+\gamma)^{-2} \frac{\partial W}{\partial b_{Lt+1}} \frac{\partial b_{Lt+1}}{\partial x_{Lt+1}} - \tilde{\lambda}_{t+1}^d + \\
&+ \tilde{\lambda}_{t+1}^d \tau_{t+1} \Big(\frac{\partial c_{Lt+1}}{\partial x_{Lt+1}} + \frac{\partial b_{Lt+1}}{\partial x_{Lt+1}}\Big) - \tilde{\mu}_{t+1}^d \frac{\partial v_{t+1}^H[L]}{\partial x_{Lt+1}} = 0,
\end{aligned} \tag{A20}$$

$$\begin{aligned}
&(1+\gamma)^{-1} f_{Ht+1} \frac{\partial v_{t+1}^H}{\partial x_{Ht+1}} + (1+\gamma)^{-2} \frac{\partial W}{\partial b_{Ht+1}} \frac{\partial b_{Ht+1}}{\partial x_{Ht+1}} - \tilde{\lambda}_{t+1}^d + \\
&+ \tilde{\lambda}_{t+1}^d \tau_{t+1} \Big(\frac{\partial c_{Ht+1}}{\partial x_{Ht+1}} + \frac{\partial b_{Ht+1}}{\partial x_{Ht+1}}\Big) + \tilde{\mu}_{t+1}^d \frac{\partial v_t^H}{\partial x_{Ht}} = 0.
\end{aligned} \tag{A21}$$

Multiplying (A20) by $\partial e_{Lt+1}^{net} / \partial \tau_{et+1}$ and (A21) by $\partial e_{Ht+1}^{net} / \partial \tau_{et+1}$, resp., and substituting into (A19), gives us the formula of Theorem 3(a).

(b) Follows immediately from the fact that indirect utility $v_t^i(\cdot)$ of an individual i of generation t - even if she cares for net bequests - does not depend on τ_{t+1}, neither do net bequests $b_{it}^{net}(\cdot)$ nor consumption $c_{it}(\cdot)$. QED

Proof of Theorem 4

(a) From the Lagrangian to the problem (9), (10'), (11), we derive the first-order conditions for the optimum x_{it}, i = 1,...,n, where $\lambda^r, \mu_i^r, i = 2,...,n$, are the multipliers corresponding to the resource constraint and the self-selection constraints, respectively (remember that $\partial v / \partial x = \rho/(1+\tau_t)$, $\partial c / \partial x = \alpha_c/(1+\tau)$, $\partial b / \partial x = \alpha_b/(1+\tau)$ and $\alpha_c + \alpha_b = 1$):

$$\frac{f_{1t}\rho}{1+\tau_t} + (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{1t}^j}\kappa_{jt} - \lambda^r + \lambda^r\frac{\tau_t}{1+\tau_t} - \frac{\mu_2^r\rho}{1+\tau_t} = 0, \tag{A22}$$

$$\begin{aligned}&\frac{f_{it}\rho}{1+\tau_t} + (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{it}^j}\kappa_{jt} - \lambda^r + \lambda^r\frac{\tau_t}{1+\tau_t} + \frac{\mu_i^r\rho}{1+\tau_t} - \\ &-\frac{\mu_{i+1}^r\rho}{1+\tau_t} = 0, \qquad\qquad i = 2,...,n-1,\end{aligned} \tag{A23}$$

$$\frac{f_{nt}\rho}{1+\tau_t} + (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{nt}^j}\kappa_{jt} - \lambda^r + \lambda^r\frac{\tau_t}{1+\tau_t} + \frac{\mu_n^r\rho}{1+\tau_t} = 0. \tag{A24}$$

Next we consider the derivative of the Lagrangian with respect to τ_{et}:

$$\frac{\partial S^r}{\partial \tau_{et}} = \sum_{j=1}^{k}\sum_{i=1}^{n} f_{it}\frac{\partial v_t^i}{\partial \tau_{et}}\kappa_{jt} + (1+\gamma)^{-1}\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{\partial W}{\partial b_{it}^j}\frac{\partial b_{it}^j}{\partial \tau_{et}}\kappa_{jt} + \lambda^r e_t^{agg} - \lambda^r\frac{\tau_t}{1+\tau_t}e_t^{agg}. \tag{A25}$$

Using $\partial v_t^i(\cdot, e_{it}^j, \cdot)/\partial \tau_{et} = -e_{it}^j \partial v_t^i / \partial x_{it} = -e_{it}^j \rho/(1+\tau_t)$ and $\partial b_{it}^j / \partial \tau_{et} = -e_{it}^j \alpha_b/(1+\tau_t)$, (A25) reads

$$\frac{\partial S^r}{\partial \tau_{et}} = -\frac{\rho}{1+\tau_t}\sum_{i=1}^{n} f_{it}\bar{e}_{it} - (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{i=1}^{n}\bar{e}_{it}\frac{\partial W}{\partial b_{it}} + \lambda^r e_t^{agg} - \lambda^r\frac{\tau_t}{1+\tau_t}e_t^{agg}. \tag{A26}$$

Here we have used the property that $\partial W / \partial b_{it}^{j}$ is assumed independent of j, as mentioned in the text (we write $\partial W / \partial b_{it}$). Using this property again in (A22) – (A24) and multiplying each equation by the appropriate $\overline{e}_{it}$ gives

$$-\frac{f_{1t}\rho}{1+\tau_t}\overline{e}_{1t} = (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\overline{e}_{1t}\frac{\partial W}{\partial b_{1t}} - \lambda^r \overline{e}_{1t} + \lambda^r \frac{\tau_t}{1+\tau_t}\overline{e}_{1t} - \frac{\mu_2^r \rho}{1+\tau_t}\overline{e}_{1t}, \tag{A27}$$

$$\begin{aligned} -\frac{f_{it}\rho}{1+\tau_t}\overline{e}_{it} &= (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\overline{e}_{it}\frac{\partial W}{\partial b_{it}} - \lambda^r \overline{e}_{it} + \lambda^r \frac{\tau_t}{1+\tau_t}\overline{e}_{it} + \frac{\mu_i^r \rho}{1+\tau_t}\overline{e}_{it} - \\ &\quad - \frac{\mu_{i+1}^r \rho}{1+\tau_t}\overline{e}_{it}, \qquad i = 2,\ldots,n-1, \end{aligned} \tag{A28}$$

$$-\frac{f_{nt}\rho}{1+\tau_t}\overline{e}_{nt} = (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\overline{e}_{nt}\frac{\partial W}{\partial b_{nt}} - \lambda^r \overline{e}_{nt} + \lambda^r \frac{\tau_t}{1+\tau_t}\overline{e}_{nt} + \frac{\mu_n^r \rho}{1+\tau_t}\overline{e}_{nt}. \tag{A29}$$

Substituting (A27) – (A29) into (A26) and observing that, by assumption

$$\sum_{i=1}^{n}\overline{e}_{it} = \sum_{i=1}^{n}\sum_{j=1}^{k} e_{it}^{j}\kappa_{jt} = \sum_{j=1}^{k}\kappa_j \sum_{i=1}^{n} e_{it}^{j} = e_t^{agg},$$

gives us the formula of Theorem 4(a).

(b) The proof of Theorem 4(b) is analogous. QED

Proof of Lemma 3

(a) The proof is by induction, where we also show: for any s > t, $P_i(\beta_{it}^s)$ is the same for all i and $P_i(\beta_{jt}^s) = P_i(\beta_{kt}^s)$ for all $j,k \neq i$. Consider the first generation of heirs after the beginning of a bequest series (set s = t+1). There are (n – 1)! permutations that have the property that the descendant of an individual with ability rank i has the same rank. One of these permutations is the identical, which has probability p_{Et+1}, while the others have probability p_{t+1}, therefore

$$P_i(\beta_{it}^{t+1}) = p_{Et+1} + [(n-1)!-1]p_{t+1}\,. \tag{A30}$$

Analogously, there are (n – 1)! permutations with the property that a descendant with rank i has a parent of some rank $j \neq i$. All these permutations have probability p_{t+1}, thus

$$P_i(\beta_{jt}^{t+1}) = (n-1)!\,p_{t+1}. \tag{A31}$$

Using the definitions (A30) and (A31), one checks immediately that indeed

$$\begin{aligned} P_i(\beta_{it}^{t+1}) + (n-1)P_i(\beta_{jt}^{t+1}) &= p_{Et+1} + [(n-1)!-1]p_{t+1} + (n-1)(n-1)!\,p_{t+1} \\ &= p_{Et+1} + n!\,p_{t+1} - p_{t+1} = 1, \end{aligned}$$

where the latter equality follows from property (P4). The inequality $P_i(\beta_{it}^{t+1}) > P_i(\beta_{jt}^{t+1})$ is equivalent to $p_{Et+1} - p_{t+1} > 0$, which is guaranteed again by (P4). Moreover, from the RHS's of (A30) and (A31), resp., it is immediate that $P_i(\beta_{it}^{t+1})$ is the same for all i, and $P_i(\beta_{jt}^{t+1}) = P_i(\beta_{kt}^{t+1})$ for all $j,k \neq i$.

Next, assume that $P_i(\beta_{it}^{s}) > P_i(\beta_{jt}^{s})$, $P_i(\beta_{it}^{s}) = P_j(\beta_{jt}^{s})$ and $P_i(\beta_{jt}^{s}) = P_i(\beta_{kt}^{s})$ (which obviously implies $P_i(\beta_{jt}^{s}) = P_j(\beta_{it}^{s})$, $i \neq j$) hold for some arbitrary s. To see that then all three relations also hold for s+1, we note that for the transition from generation s to s+1, there are two ways for a type-i individual to receive, in period $s+1$, the bequest left by an identically ranked individual in the initial period t < s: either from the type-i individual in period s (who has received the i-bequest with probability $P_i(\beta_{it}^{s})$) or from some other (type-j) individual in period s (who has received the i-bequest with probability $P_j(\beta_{it}^{s})$). Therefore (remember the considerations above)

$$P_i(\beta_{it}^{s+1}) = P_i(\beta_{it}^{s})[p_{Es+1} + ((n-1)!-1)p_{s+1}] + (n-1)P_i(\beta_{jt}^{s})(n-1)!\,p_{s+1}. \tag{A32}$$

Analogously, the three ways for a type-i individual in period $s+1$ to receive the bequest left by some type-j individual in period t < s are: either from the type-i individual in period s or from the type-j individual in period s or from any other individual ($\neq$ i,j) in period s. Therefore

$$\begin{aligned} P_i(\beta_{jt}^{s+1}) = P_i(\beta_{jt}^{s})[p_{Es+1} + ((n-1)!-1)p_{s+1}] + P_i(\beta_{it}^{s})(n-1)!p_{s+1} + \\ + (n-2)P_i(\beta_{jt}^{s})(n-1)!p_{s+1}. \end{aligned} \tag{A33}$$

Using the definitions (A32) and (A33), we obtain, by appropriate grouping,

$$\begin{aligned} &P_i(\beta_{it}^{s+1}) + (n-1)P_i(\beta_{jt}^{s+1}) = \\ &\quad = P_i(\beta_{it}^{s})p_{Es+1} + P_i(\beta_{it}^{s})p_{s+1}[(n-1)!-1+(n-1)(n-1)!] + P_i(\beta_{jt}^{s})p_{Es+1}(n-1) + \\ &\quad\quad + P_i(\beta_{jt}^{s})p_{s+1}(n-1)[(n-1)! + (n-1)!-1+(n-2)(n-1)!] \\ &\quad = P_i(\beta_{it}^{s})[p_{Es+1} + p_{s+1}(n!-1)] + P_i(\beta_{jt}^{s})(n-1)[p_{Es+1} + p_{s+1}(n!-1)], \end{aligned}$$

which is equal to 1, as $p_{Es+1} + (n!-1)p_{s+1} = 1$ and $P_i(\beta_{it}^{s}) + (n-1)P_i(\beta_{jt}^{s}) = 1$.

Now, straightforward transformations show that $P_i(\beta_{it}^{s+1}) > P_i(\beta_{jt}^{s+1})$ is equivalent to $P_i(\beta_{it}^{s})(p_{Es+1} - p_{s+1}) > P_i(\beta_{jt}^{s})(p_{Es+1} - p_{s+1})$, which holds, because $P_i(\beta_{it}^{s}) > P_i(\beta_{jt}^{s})$ and $p_{Es+1} > p_{s+1}$. Moreover, $P_i(\beta_{it}^{s+1})$ is the same for all i and $P_i(\beta_{jt}^{s+1})$ is the same for any i,j, because the RHS's of (A32) and (A33) are the same for all i,j, resp. This completes the proof of (a).

(b) Note from (A32) that $P_i(\beta_{it}^{s+1})$ is a convex combination of $P_i(\beta_{it}^{s})$ and $P_i(\beta_{jt}^{s})$, because the sum of the coefficients of $P_i(\beta_{it}^{s})$ and $P_i(\beta_{jt}^{s})$ is

$$p_{Es+1} - p_{s+1} + (n-1+1)(n-1)!p_{s+1} = p_{Es+1} + (n!-1)p_{s+1} = 1.$$

Thus, $P_i(\beta_{it}^s) > P_i(\beta_{jt}^s)$ implies $P_i(\beta_{it}^s) > P_i(\beta_{it}^{s+1})$. Finally, $P_i(\beta_{jt}^s) < P_i(\beta_{jt}^{s+1})$ follows from $P_i(\beta_{it}^s) + (n-1)P_i(\beta_{jt}^s) = 1$ and $P_i(\beta_{it}^{s+1}) + (n-1)P_i(\beta_{jt}^{s+1}) = 1$. QED

Proof of Lemma 4

Remember from the main text that a bequest series β_t, initiated in t as $b_{it} = \tilde{\alpha}_t x_{it}$, leads to net inheritances Γx_{it} in period s with $\Gamma \equiv \tilde{\alpha}_t \prod_{s'=t+1}^{s-1} \hat{\alpha}_{s'}$.

Therefore $E_i[\beta_t^s] < E_{i+1}[\beta_t^s]$ is equivalent to

$$P_i(\beta_{it}^s)\Gamma x_{it} + \sum_{j\neq i} P_i(\beta_{jt}^s)\Gamma x_{jt} < P_{i+1}(\beta_{i+1t}^s)\Gamma x_{i+1t} + \sum_{m\neq i+1} P_{i+1}(\beta_{mt}^s)\Gamma x_{mt}$$

and further to (remember from the Proof of Lemma 3 that $P_i(\beta_{it}^s)x_{it}$ is the same for all i = 1,…,n and that $P_i(\beta_{jt}^s)x_{jt} = P_{i+1}(\beta_{mt}^s)x_{mt}$ for j ≠ i, m ≠ i + 1)

$$P_i(\beta_{it}^s)x_{it} + P_i(\beta_{jt}^s)x_{i+1t} < P_i(\beta_{it}^s)x_{i+1t} + P_i(\beta_{jt}^s)x_{it}.$$

The validity of the latter relation follows from $P_i(\beta_{it}^s) > P_i(\beta_{jt}^s)$ and $x_{it} < x_{i+1t}$. By the same logic, $x_{it} \leq x_{i+1t}$ implies $E_i[\beta_t^s] \leq E_{i+1}[\beta_t^s]$. QED

Proof of Theorem 5

The inheritances of an individual i in period s can be written as being the sum of all bequest series initiated in periods t < s. That is,

$$\overline{e}_{is} = E_i[\sum_{t=0}^{s-1} \beta_t^s] = \sum_{t=0}^{s-1} E_i[\beta_t^s].$$

Therefore, we conclude from Lemma 4: as $E_i[\beta_t^s] \leq E_{i+1}[\beta_t^s]$ due to $x_{it} \leq x_{i+1t}$ for all i = 1,… n – 1, $\overline{e}_{is} < \overline{e}_{i+1s}$, if $x_{it} < x_{i+1t}$ for at least one t < s. QED

References

Atkinson, A.B. and J.E. Stiglitz (1972), The Structure of Indirect Taxation and Economic Efficiency, Journal of Public Economics 1, 97-119.

Barro, R. (1974), Are Government bonds net wealth?, Journal of Political Economy 82, 1095-1117.

Becker, G. S and N. Tomes (1979), An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility, Journal of Political Economy 87, 1153-1189.

Blumkin T. and E. Sadka (2003), Estate Taxation with intended and accidental bequests. Journal of Public Economics 88, 1-21.

Boadway R., M. Marchand and P. Pestieau (2000), Redistribution with Unobservable Bequests: A Case for Taxing Capital Income. Scandinavian Journal of Economics 102(2), 253-267.

Bossmann, M., Ch. Kleiber and K. Wälde (2007), Bequests, taxation and the distribution of wealth in a general equilibrium model, Journal of Public Economics 91, 1247-1271.

Brunner, J. K. (1989), Theory of equitable taxation, Berlin et al.: Springer.

Brunner, J.K. (1997), Optimal Taxation of Income and Bequests, Working paper 9722, Department of Economics, University of Linz, Austria.

Cremer H., P. Pestieau and J.-Ch. Rochet (2001), Direct versus Indirect Taxation: The Design of the Tax Structure Revisited. International Economic Review 42(3), 781-799.

Cremer H., P. Pestieau and J.-Ch. Rochet (2003), Capital income taxation when inherited wealth is not observable. Journal of Public Economics 87, 2475-2490.

Cremer H. and P. Pestieau (2006), Wealth transfer taxation: a survey of the theoretical literature. In: Kolm, S.-Ch. And J. M. Ythier (eds.), Handbook of Economics of Giving, Altruism and Reciprocity, Vol. 2, North-Holland.

Davies, J. B. and P. J. Kuhn (1991), A dynamic model of redistribution, inheritance, and inequality, Canadian Journal of Economics 24, 324-344.

Díaz-Giménez J., V. Quadrini, J.-V. Ríos-Rull and S.B. Budriá Rodriguez (2002), Updated Facts on U.S. Distributions of Earnings, Income and Wealth, Federal Reserve Bank of Minneapolis Review, Vol. 26, No. 3, 2-35.

Gale, W.G. and J. Slemrod (2001), Overview. In: Gale, W.G., J.R. Hines Jr. and J. Slemrod (eds.), Rethinking Estate and Gift Taxation, Brookings Institution Press, Washington D.C.

Guesnerie, R. and J. Seade (1982), Nonlinear Pricing in a Finite Economy, Journal of Public Economics 17, 157-179,

Kaplow, L. (2001), A Framework for Assessing Estate and Gift Taxation. In: Gale, W.G., J.R. Hines Jr. and J. Slemrod (eds.), Rethinking Estate and Gift Taxation, Brookings Institution Press, Washington D.C.

Kessler D. and A. Masson (1989), Bequest and Wealth Accumulation: Are Some Pieces of the Puzzle Missing? Journal of Economic Perspectives 3, 141-152.

Mirrlees, J. (1971), An exploration in the theory of optimum income taxation. Review of Economic Studies 38, 175-208.

Seade, J. (1982), On the sign of the optimum marginal income tax. Review of Economic Studies, 49, 637-643.

Solon, G. (1992), Intergenerational Income Mobility in the United States, American Economic Review 82, 393-408.

Zimmerman, D.J. (1992), Regression Toward Mediocrity in Economic Stature, American Economic Review 82, 409-429.