Debt Portfolios*

Thomas Hintermaier[a] and Winfried Koeniger[b]

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Abstract

We provide a model with endogenous portfolios of secured and unsecured household debt. Secured debt is collateralized by durables whereas unsecured debt can be discharged in bankruptcy procedures. We show that the model matches the main quantitative characteristics of observed wealth and debt portfolios in the US and some of the observed changes over time. Furthermore, we establish two quantitative results. Firstly, modest levels of risk aversion are necessary to match observed debt portfolios. Secondly, durables do not improve consumers' access to unsecured credit, and plausible variations of durable exemptions in bankruptcy procedures have very small effects on the equilibrium.

Keywords: household debt portfolios, durables, collateral, income risk, bankruptcy.

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1 Introduction

Household debt is sizeable and has increased substantially in the last decades in the US (Dynan and Kohn, 2007), the UK (Tudela and Young, 2005) and most other European countries (Jentzsch and San José Riestra, 2006). This aggregate debt level hides substantial differences between debt types in the balance sheet of households. Most household debt is secured by durable collateral whereas some debt is unsecured and can be written off in bankruptcy procedures in the US, the UK and some, but not all, European countries. Interestingly, portfolios of these debt types differ substantially across households (see Section 2). In this paper we present a model which allows for heterogeneity across households and generates such debt portfolios endogenously.

The key new feature in our model are durables which allow for a meaningful distinction between secured and unsecured debt and thus permit us to analyze debt portfolios. We obtain heterogeneity in debt portfolios by modeling consumer choices over the life cycle, assuming uncertain labor income and incomplete markets. Consumers then cannot fully insure the labor-income risk and differ from each other as they age and experience different histories of shocks. Micro-founded heterogeneous-agent models with these characteristics have been pioneered by Deaton (1991), Aiyagari (1994) and Carroll (1997) and have attracted substantial attention in recent years.

We find that our calibrated model matches the wealth and debt portfolios in the US well and captures some of the observed changes in these portfolios since the 1980s. We establish two further quantitative results. Firstly, the coefficient of relative risk aversion needs to be "small enough" in the calibration. Otherwise consumers hold too much unsecured debt in their debt portfolio since unsecured debt can be written off in bankruptcy procedures and thus is contingent on the realization of specific states. This result relates to the literature on the equity premium puzzle (see Mehra and Prescott, 2008, and their references) in which implausibly high levels of risk aversion are needed, even in models with incomplete markets, to generate the observed large equity premium and consumer portfolios with rather small shares of positive risky assets (see, for example, Heaton and Lucas, 1997). We find instead that modest levels of risk aversion are needed to match observed debt portfolios with a small share of unsecured debt.

Secondly, we find that durables play a minor role for consumers' access to unsecured debt since few durables remain after secured debt is paid off in bankruptcy procedures. This contrasts with results in Pavan (2008) who did not explicitly distinguish between secured and unsecured debt. Pavan argues that durables serve as an informal collateral for unsecured credit so that exemptions of the durable stock in bankruptcy procedures make the supply of unsecured credit more costly. Our analysis shows that for plausible parameter values the exemption levels of durables are quantitatively irrelevant for the pricing of unsecured credit.

Our paper relates to recent research by Athreya (2002), Chatterjee, Corbae, Nakajima and Ríos-Rull (2007) and Livshits, MacGee and Tertilt (2007a) who have extended the classic heterogeneous-agent models to study unsecured debt. Importantly, these models assume that consumers only have access to unsecured debt. In this paper we relax this assumption and allow for an endogenous debt portfolio: consumers can take on secured debt like mortgages, which are collateralized by durables, *and* unsecured debt like credit-card debt. To the best of our knowledge only Athreya (2006) attempts to distinguish secured and unsecured debt but does not model durables. In his model the collateral is exogenous whereas consumers in our model endogenously accumulate durable collateral which also generates utility. This modeling of durables is closest to Fernández-Villaverde and Krueger (2005), Kiyotaki, Michaelides and Nikolov (2007) and Yang (forthcoming) who, however, do not allow for equilibrium bankruptcy and unsecured debt.[1]

Our analysis of the evolution of the debt portfolio is most closely related to the analyses of unsecured-debt trends by Athreya (2004), Livshits, MacGee and Tertilt (2007b) and Mateos-Planas (2007). The main contribution of our paper is that we explicitly model debt *portfolios*. The advantages of analyzing durables, secured and unsecured debt simultaneously are at least threefold. The first advantage is that the model has an additional margin of substitution in the debt portfolio, between secured and unsecured debt. That margin not only adds realism but also allows to distinguish between various explanations for the upward trend in unsecured debt and the bankruptcy incidence. The second advantage is more realism in a key aspect of the analysis since most of consumers' total debt holdings in the data are secured by durable collateral. Thus, a quantitative model of household debt needs to explain not only the

[1]See also Yao and Zhang (2005) for an analysis of housing and portfolio choice.

evolution of the rather small unsecured debt position but of the whole debt portfolio. This is also important for the predictions of the model concerning consumer bankruptcy because only unsecured debt can be discharged in bankruptcy proceedings. The third advantage is that the explicit modeling of durables introduces an endogenous bankruptcy cost which has been neglected in previous research. Since some of the durable is seized to satisfy creditors' claims and adjusting the durables is costly, that cost depends on the size of the consumers' durable stock and secured debt.

The rest of this paper is structured as follows. In Section 2 we present empirical facts which are instructive for our analysis. We present the model in Section 3 and study the numerical solution and calibration in Section 4. In Section 5 we apply the model to study the evolution of wealth and debt portfolios in the US. We conclude in Section 6.

2 Empirical facts

In this section we summarize the empirical facts on the wealth and debt portfolios which we attempt to explain with our model. We then briefly review the key features of US consumer bankruptcy regulation which the model shall capture.

2.1 Data

We use the Survey of Consumer Finances (SCF) 1983 and 2004 to compute facts on wealth and debt portfolios of US consumers. We have chosen these two dates because they span the time period in which detailed comparable data on consumers' wealth positions are recorded in the triennial SCF. Moreover, both years, 1983 and 2004, are after a trough in the US business cycle (1982 and 2001 according to the NBER definition) so that differences in the wealth portfolios in 1983 and 2004 reflect long-term trends rather than cyclical variation. The SCF has been widely used as it provides the most accurate information on consumer finances in the US (see Kennickell, 2003, and the references therein).

We largely follow Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) and Díaz-Giménez, Quadrini and Ríos-Rull (1997) in constructing measures for wealth and labor earnings in the US. We account for differences in household size using the equivalence scale reported in Fernández-Villaverde and Krueger (2007), Table 1, last column. To make the empirical data comparable with the data generated by the model, we normalize all variables by average net labor earnings in our sample.[2] More precisely, we use SCF data on gross labor earnings and the NBER tax simulator described in Feenberg and Coutts (1993) to construct a measure for disposable labor earnings after taxes and transfers for each household in 1983 and 2004. Arguably, after-tax rather than pre-tax earnings matter for households' consumption and portfolio decisions since some uninsurable labor earnings risk may be eliminated by redistributive taxes and transfers. More detailed information is contained in the data appendix.

We focus on households with heads between age 20 and 74, where, as in the model, we divide this age range into 15 three-year age intervals between age 20 and 65 and one last sixteenth interval between age 65 and 74. We compute sample averages for these age intervals which we then regress on a cubic polynomial of the age groups for 1983 and 2004, respectively. The resulting predictions allow us to construct smooth life-cycle profiles.

2.2 Wealth and debt portfolios

Figures 1 and 2 show how labor earnings, the wealth portfolio and bankruptcy incidence vary over the life-cycle. Each graph plots the smoothed life-cycle profiles for 1983 and 2004, respectively, and displays the 95% confidence bounds.

Figure 1 shows that labor earnings have the well-known hump shape over the life cycle where these earnings peak between age 40 and 50 at about 10% higher labor earnings than the average in the sample. The earnings profile has not changed significantly over time.

Concerning the wealth portfolio, Figure 1 shows that young consumers start their live with very little wealth. They first borrow to accumulate durables, of which a substantial part is housing, and use secured and unsecured debt to finance their non-durable and durable consumption. After age 40, significant amounts of financial assets –also financial assets net of secured and unsecured debt– are accumulated (see Fernández-Villaverde and Krueger, 2005, for documenting similar patterns of financial assets and durables in the SCF 1995).

[2] When computing the statistics in the data, we use the sampling weights provided in the SCF. The normalization by net labor earnings and the use of equivalence scales implies that normalized (aggregate) wealth is larger than the wealth to *output* ratio.

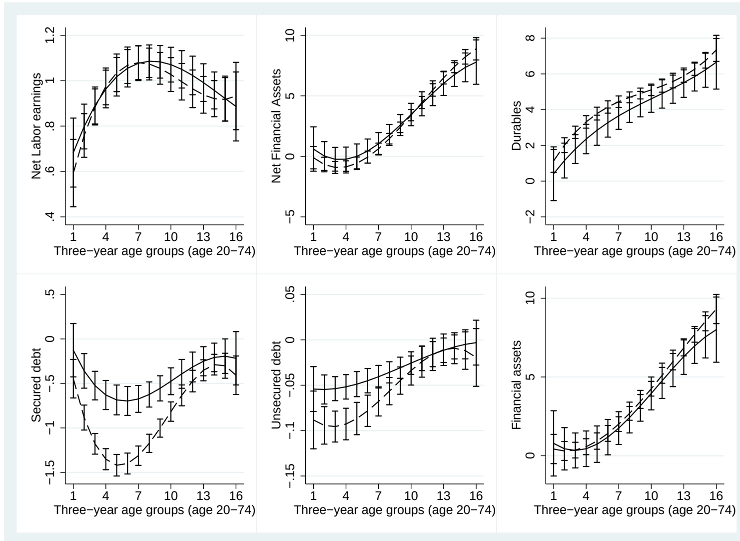

Figure 1: The wealth and debt portfolio of consumers over the life cycle in 1983 and 2004. Source: Authors' calculations based on the Survey of Consumer Finances (SCF). See the data appendix for variable definitions. Notes: Solid line: 1983 data; dashed line: 2004 data; 95% confidence intervals displayed for each data point in 1983 and 2004. The unit is the average of net labor earnings of prime-age workers in the respective sample year. Periods 1 to 15 correspond to three-year age groups between age 20 and 65. Period 16 corresponds to the last group with age 65-74.

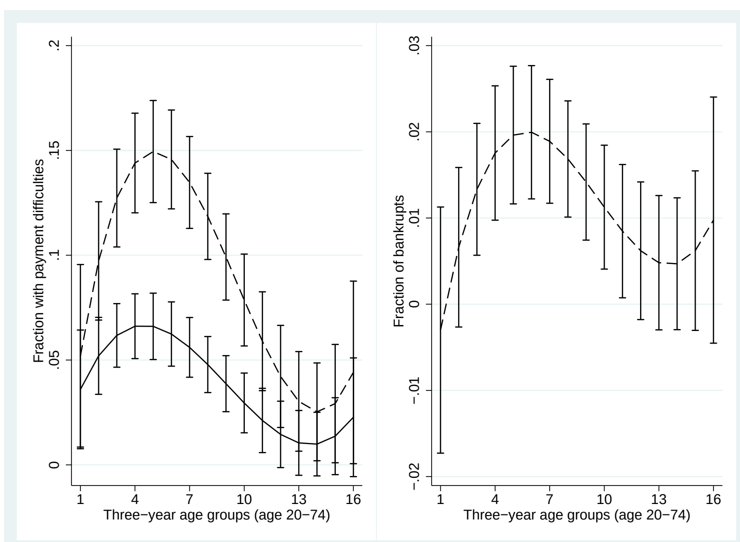

Figure 2: Payment difficulties and bankruptcy incidence over the life cycle in 1983 and 2004. Source: Authors' calculations based on the SCF. See the data appendix for variable definitions. Notes: Solid line: 1983 data; dashed line: 2004 data; 95% confidence intervals displayed for each data point in 1983 and 2004. Data for bankruptcy are not available for 1983. Periods 1 to 15 correspond to three-year age groups between age 20 and 65. Period 16 corresponds to the last group with age 65-74.

Furthermore, there is some indication that young consumers incur more debt to purchase durables in 2004 than in 1983: note that the dashed line for durables of young consumers in 2004 is above the solid line for 1983, whereas the opposite is true for net-financial assets. At the same time, young consumers hold more secured and unsecured debt in 2004 than in 1983, where the increase in secured debt is much more important in absolute terms.[3] Compared to the intensive margin, the incidence of debt is rather constant in 1983 and 2004: half of the population holds some debt, 40% hold secured debt and 20% unsecured debt.

How are these patterns for the wealth and debt portfolios associated with the incidence of bankruptcy? Since the SCF in 1983 does not contain information about bankruptcy, we construct a measure for payment difficulties which is available for both 1983 and 2004 (see the data appendix for further details). Figure 2 shows that bankruptcy incidence and our measure for payment difficulties have a very similar hump-shape over the life cycle, consistent with the evidence on bankrupts reported in Sullivan et al. (2000), Figure 2.1. Payment difficulties and bankruptcy incidence peak if consumers reach their late 30s and the increase in payment difficulties between 1983 and 2004 (and thus possibly also the bankruptcy incidence) has been most significant for consumers between age 30 and 50.[4] Complementing this with evidence of Sullivan et al. (2000) that about 0.2% of US consumers filed for bankruptcy in the beginning of the 1980s and that this number increased to 0.5% beginning of the 1990s, we conclude that there has been a significant upward trend in bankruptcy filings and payment difficulties in the US between 1983 and 2004 (see also White, 2006, and references therein).

We will calibrate preference parameters of the model to match the means of the life-cycle profiles for prime-age households between age 23 and 52 in the data. Since the questions in the SCF survey ask about income in the previous year and agents have made their consumption and portfolio choices conditional on this income, we interpret the SCF asset data as "end-of-period" information. Given our triennial periods, we thus compare the SCF data

[3]Constructing model counterparts for secured and unsecured debt in the data is not trivial. We refer to the data appendix for details.

[4]The results for payment difficulties and bankruptcy incidence, which we report for the SCF 2004, are similar to those reported in Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) for the SCF 1998. They classified households as having financial trouble if they delayed payments for more than 2 month (this was true for 6% of the households). Moreover, they report that 1.8% of the whole sample had filed for bankruptcy.

	1983 (1)	*2004* (2)
Wealth		
Durables	3.29	4.04
+ Net-financial assets	1.32	0.92
= Total net worth (fraction of average net lab. earnings)	4.61	4.96
Financial assets		
Financial assets	1.921	2.107
+ Secured debt	-0.562	-1.117
+ Unsecured debt	-0.041	-0.067
= Net-financial assets (fraction of average net lab. earnings)	1.318	0.923
Debt		
Secured debt (in % of total secured + unsecured debt above)	93.20	94.34
Payment difficulties (in % of sample size)	5.01	11.54
Bankrupt in previous year (in % of sample size)	-	1.47

Table 1: Wealth and debt portfolios of households with a head between age 23 and 52. Means of life-cycle profiles in 1983 and 2004, respectively. Source: Authors' calculation based on the SCF. Notes: Quantities are normalized by average net labor earnings in the respective sample year. Data on bankruptcy are not available in the SCF 1983.

for consumers between age 23 and 52 with the model-generated data for consumers between age 26 and 55, and use the SCF data for the group with age 20-22 as initial conditions for consumers with age 23-25 in the model simulations.

We focus on prime-age households since our model abstracts from death before age 74, as Livshits et al. (2007a), and this is a good approximation of the data only up to a certain age. Life tables for the US show that 90% of those born alive are still alive at age 55 and then have an average life expectancy of another 25 years (see the National Center of Health Statistics at http://www.cdc.gov/nchs/products/life_tables.htm). Allowing for a positive probability of death in all stages of the life cycle would unnecessarily increase the computational burden further and since debt portfolios are most relevant early in the life-cycle, as we have seen above, this simplification of our analysis seems not restrictive.[5]

For later reference, Table 1 displays the means of the life-cycle profiles of consumers between age 23 and 52 in 1983 and 2004, respectively.[6] Table 1 shows that total net worth

[5]Allowing for a positive probability of death and assuming accidental bequests, for example, would add a fixed-point problem in our numerical solution. This would be very costly given the substantial computational burden of our model.

[6]These means are not weighed by age-cell size, since the population in the model has the same size over

over the life cycle has increased by 7.6% while household debt has nearly doubled between 1983 and 2004. The fraction of debt which is secured by durables, however, has remained stable at above 90% in the sample period.

Finally, the data in the SCF 1983 and 2004 reveal some important differences in the portfolios of consumers with unsecured debt, compared to the whole sample, which we would like to capture with our model. Households with unsecured debt are younger and have smaller labor earnings than the sample mean. Moreover, they hold a smaller but non-negligible amount of durables and substantially more secured debt than the rest of the sample.

2.3 US consumer bankruptcy

Consumer bankruptcy in the US has been regulated by the Federal Bankruptcy Act of 1978 during the sample period 1983-2004 which we consider. This act contains two Chapters relevant for non-farming households. Consumers can choose to file for personal bankruptcy under either Chapter 7 or under Chapter 13. The main features of these two Chapters, which are relevant for our analysis, can be summarized as follows (see Sullivan et al., 1999, for further details).

Under Chapter 7 of the bankruptcy act, the debtor can write off his unsecured debts but must surrender all his assets except for specified exempt amounts. Most of the bankruptcy exemptions are specified in terms of durables, for example as a dollar value of housing wealth which is exempt in the bankruptcy procedure. Secured debt has to be honored, however, so that bankruptcy exemptions only apply to the durable stock which remains after servicing secured credit claims.[7]

Under Chapter 13, the debtor agrees to a repayment schedule for part or all of the debt and retains his assets. The repayment plan usually is specified for three years but can take up to five years. Importantly, the debtor cannot repay less under Chapter 13 than what creditors would get paid under Chapter 7. Hence, we focus on Chapter 7 in our model since

the life-cycle. Hence, we also assign the same weight to each age cell in the data.

[7] Our model abstracts from house price risk and negative home equity so that we do not discuss the regulation on mortgage foreclosures and bankruptcy. Data on charge-off and delinquency rates by the Federal Reserve at http://www.federalreserve.gov/releases/chargeoff/ show that real-estate loans have been essentially secure before 2007 with charge-off and delinquency rates of less than a tenth of those of other consumer loans.

it places a lower bound on the unsecured-debt claims of the creditors. This is not a strong restriction since most consumers who file for bankruptcy do so under Chapter 7 (70%) and many of the repayment plans initiated under Chapter 13 fail and are later converted into Chapter 7. If consumers file for bankruptcy under Chapter 7, they are not allowed to file for bankruptcy again in the next six years.

In the period 1983 to 2004, on which we focus when we match the model to the data, there have been some changes to the bankruptcy legislation in 1984 to make filing for bankruptcy more restrictive. In practice, however, this has had little impact on the workings of the procedure (Sullivan et al., 2000). The only significant reform took effect after the period which we consider when income testing was introduced at the end of 2005. Hence, the federal bankruptcy regulation has remained roughly unchanged in the period 1983-2004.

After describing the legal framework for consumer bankruptcy, let us refer to the main reasons for consumer bankruptcy which have been identified by Sullivan et al. (2000). Two thirds of the bankrupt consumers mention job related problems like wage cuts or unemployment. A fifth of bankrupt consumers reports health problems (multiple responses were permitted) where in 60% of these cases the implied income losses due to missed workdays, demotion or lost jobs are mentioned as the reason for bankruptcy. Further reasons for bankruptcy include divorce or the motive to "save" housing property by writing off unsecured debt. Under Chapter 7 the latter motive may be attractive if some of the housing wealth is exempt in bankruptcy procedures. Moreover, this motive seems relevant since Sullivan et al. (1999, 2000) report that more than half of bankrupt consumers are homeowners. In this paper we will focus on two reasons for bankruptcy: earnings uncertainty (which may be related to health shocks) and the motive to keep some of the durable after bankruptcy.[8] Having presented the key relevant facts, we are now ready to set up the model.

[8]We abstract from medical expense shocks to contain the computational burden given that our model has an additional endogenous state variable, durables. See Chatterjee et al., 2007, or Livshits et al., 2007a, for models with health expense shocks.

3 The model

We build on the life-cycle model of unsecured debt by Livshits et al. (2007a). We assume that the economy is populated by a large number of consumers who live for 17 periods, where each period j has a length of three years. Life begins at age 23 and the first 14 periods (until age 65) are working periods in which people receive income shocks. In the last three periods consumers are in retirement and face no uncertainty. Life ends at age 74.

Preferences. Consumers maximize expected lifetime utility

$$E_1 \Sigma_{j=1}^{J} \beta^{j-1} U(c_j, d_j)$$

where β is the discount factor and the instantaneous utility function $U(c_j, d_j)$ is non-separable in non-durable consumption c_j and durables d_j. For the quantitative application of the model we assume a CRRA utility function with relative risk aversion σ,

$$U(c_j, d_j) = \frac{\Psi(c_j, d_j)^{1-\sigma} - 1}{1-\sigma},$$

where the consumption aggregator Ψ is a Cobb-Douglas function

$$\Psi(c_j, d_j) = (c_j)^{\theta} \left(d_j + \underline{d}\right)^{1-\theta} \ .$$

The constant $\underline{d} > 0$ is assumed to be small and positive and ensures that consumers may not hold durables.

These parametric assumptions about the utility function encompass many of the previous numerical applications which we are aware of and the Cobb-Douglas consumption index is roughly in line with empirical estimates on the substitutability between durables and non-durables (see Fernández-Villaverde and Krueger, 2005, for further discussion and references).

Labor earnings. Labor earnings are given by

$$y_j = \phi_j \lambda_j,$$

where ϕ_j is the stochastic productivity of the household in period j of the life cycle and λ_j is the deterministic labor endowment which is hump-shaped over the life cycle.

Assets. Consumers hold portfolios of secured debt $a^s \leq 0$, unsecured debt $a^u < 0$, risk-free financial assets $a^u \geq 0$ and durables d. Secured debt is backed by durables as collateral and bears an interest rate r^s. Risk-free financial assets $a^u \geq 0$ earn interest r^a. We assume that there is a borrowing spread, $r^s > r^a$, due to a fixed cost of financial intermediation. This borrowing spread ensures that the amount of secured debt or risk-free assets is well determined (see Yao and Zhang, 2005, for further discussion). We further assume that the cost of intermediation is larger for unsecured debt so that the interest rate for unsecured debt is at least $\underline{r^u} > r^s > r^a$. As we discuss further when we calibrate the model, this is a common assumption which is realistic.

Unsecured debt is not backed by durables and we allow consumers to discharge unsecured debt in bankruptcy procedures. Since creditors price the possibility of bankruptcy, the interest rate on unsecured debt consists of the base rate $\underline{r^u}$ and a risk premium. We present the pricing of unsecured debt by financial intermediaries in detail below.

Adjustment costs. Whereas financial assets a^s and a^u can be adjusted costlessly by the consumer, we assume that the adjustment of durables d is costly. Since the most important component of durables in our model is housing, the costs can be thought of as moving costs or fees for real estate agents. This assumption generates realistic lumpy investment patterns for durables. Moreover, it makes the distinction between durables and non-durables in our model more meaningful as adjustment costs are one key difference between these two types of goods. Similar to Díaz and Luengo-Prado (forthcoming), we specify the costs as

$$\alpha(d_j, \iota_j) = \begin{cases} c_f^+ d_j \text{ if } \iota_j > 0 \\ c_f^- d_j \text{ if } \iota_j < 0 \\ 0 \qquad \text{if } \iota_j = 0 \end{cases},$$

where ι_j denotes durable investment and the adjustment cost is allowed to be asymmetric.

Timing within the period. Given net-financial assets $a_j \equiv a_j^s + a_j^u$ and durables d_j at the beginning of the period, consumers choose non-durable consumption c_j, their financial assets $q_j^s a_{j+1}^{*s}$, $q_j^k a_{j+1}^{*u}$ and durables $q^d d_{j+1}^*$. The prices are $q_j^s = 1/(1+r_j^s)$, $q_j^k = 1/(1+r_j^k)$ with $r_j^k = r_j^a$ if $a_{j+1}^{*u} > 0$ and $r_j^k = r_j^u$ otherwise, and $q^d = 1/(1-\delta)$. We attach asterisks to the portfolio choices since they may differ from the realized values due to possible bankruptcy. Note that writing the choices above in a discounted way simplifies the recursive formulation of the problem.

After the consumption and portfolio decisions, the consumers enjoy utility before the interest for the financial assets accrues and the durable depreciates. Then uncertain income is drawn before agents decide whether to declare bankruptcy. This determines the net financial assets a_{j+1} and durables d_{j+1} in the next period. We now characterize the constraints for the consumer choices and the bankruptcy procedure in more detail before we formulate the recursive problem.

Collateral constraint. The amount of secured debt of the consumer is bounded by the collateral constraint. Since federal law prevents garnishment of wage income in bankruptcy procedures, only durables net of adjustment costs can be used as collateral to secure debt. Hence, we specify the collateral constraint as

$$a_{j+1}^{*s} \geq -\min(\mu, 1 - c_f^-) d_{j+1}^* \ , \tag{1}$$

where μ is the exogenous maximum loan-to-value ratio imposed by the financial regulator. If $\mu < 1 - c_f^-$, the access to secured debt is more constrained than necessary to guarantee certain repayment in the presence of adjustment costs.

Budget constraint. The consumer's budget constraint is

$$q_j^s a_{j+1}^{*s} + q_j^k a_{j+1}^{*u} + c_j + \underbrace{q^d d_{j+1}^* - d_j}_{q^d \iota_j} + \ \alpha(d_j, \iota_j) \leq \ a_j + y_j \ , \ k = a, u \ . \tag{2}$$

Bankruptcy. At the time of bankruptcy filing the consumer is obliged by law to reveal his financial status to the bankruptcy judge. In particular, the judge knows durables d_{j+1}^* and the composition of financial debt, $a^{*s}_{j+1} < 0$ and $a^{*u}_{j+1} < 0$. Since *secured* debt has priority and needs to be paid irrespective of specified durable exemption levels, the bankruptcy judge first uses the durables to repay all secured debt. The durables which remain for repaying unsecured debt are

$$d^{\text{left for unsecured}} = (1 - c^{-}_{f})d^{*}_{j+1} + a^{*s}_{j+1} \ .$$

The judge then determines the maximum amount which could be divested from the remaining durables, given the exemption level $d^{\dagger}$ specified in the bankruptcy regulation. That amount is

$$\iota^{\text{max net divestment}} = -\max\left\{(1 - c^{-}_{f})d^{\text{left for unsecured}} - d^{\dagger}, 0\right\} \ .$$

The durables used to repay unsecured debt are then equal to that maximum amount or less if the outstanding amount of unsecured debt is smaller:

$$d^{\text{to unsecured}} = \min\left\{-\iota^{\text{max net divestment}}, -a^{*u}_{j+1}\right\} \ .$$

The durables which remain for the consumer after the bankruptcy procedure are

$$d^{B} = (1 - c^{-}_{f})d^{\text{left for unsecured}} - d^{\text{to unsecured}}$$

and the consumer starts fresh without debt.

The evolution of the assets can thus be summarized as

$$d_{j+1} = \begin{cases} d^{*}_{j+1} \equiv (1-\delta)d_j + \iota_j & \text{if no bankruptcy} \\ d^{B}(a^{*s}_{j+1}, a^{*u}_{j+1}, d^{*}_{j+1}) & \text{if bankruptcy} \end{cases} \ , \tag{3}$$

$$a_{j+1} = \begin{cases} a^{*}_{j+1} \equiv a^{*s}_{j+1} + a^{*u}_{j+1} & \text{if no bankruptcy} \\ 0 & \text{if bankruptcy} \end{cases} \ . \tag{4}$$

The pricing of unsecured debt. The price of unsecured debt is determined by perfectly competitive financial intermediaries which observe current income y_j, the portfolio $\nu_j \equiv (a^{*s}_{j+1}, a^{*u}_{j+1}, d^{*}_{j+1})$ and the age j of the consumer. The intermediaries price unsecured debt forming expectations about future income draws and the possibility of bankruptcy. There is no cross-subsidization across consumers so that consumers with different portfolios, age or income state may receive a different interest quote.

Defining the probability of default as $\pi_j(\nu_j, y_j)$, the zero-profit condition implies that the price for unsecured debt is given by

$$
\begin{aligned}
q_j^u(\nu_j, y_j) = & (1 - \pi_j(\nu_j, y_j))\, \overline{q^u} \\
& + \pi_j(\nu_j, y_j)\overline{q^u}\frac{d^{\text{to unsecured}}(\nu_j)}{\left|a_{j+1}^{*u}\right|},
\end{aligned}
\tag{5}
$$

where $\overline{q^u} = 1/(1+\underline{r^u})$. If the probability of bankruptcy $\pi_j(\nu_j, y_j) = 0$ or no unsecured debt is discharged if consumers file, $d^{\text{to unsecured}}(\nu_j) = \left|a_{j+1}^{*u}\right|$, then there is no risk premium on unsecured debt: $q_j^u(\nu_j, y_j) = \overline{q^u}$.

The recursive formulation with optimal default. Substituting the budget constraint (2) in the Bellman equation of the consumer with age j, we get

$$
\begin{aligned}
& V_j(a_j, d_j, y_j) = \\
\max_{a_{j+1}^{*s}, a_{j+1}^{*u}, d_{j+1}^{*}} & \Big\{ U(\underbrace{a_j + y_j + d_j - q_j^s a_{j+1}^{*s} - q_j^j a_{j+1}^{*u} - q^d d_{j+1}^{*} - \alpha(d_j, \iota_j)}_{c_j}, d_j) \\
& + \beta E \max[V_{j+1}(a_{j+1}^{*}, d_{j+1}^{*}, y_{j+1}), V_{j+1}^{B}(0, d^B, y_{j+1}) - \psi] \Big\},
\end{aligned}
\tag{6}
$$

where V_j is the value function if the consumer has not filed for bankruptcy, V_j^B is the value function if the consumer has filed for bankruptcy, ψ is an exogenous utility cost of bankruptcy and E is the expectation operator.

Note that there are four costs of bankruptcy in the consumer problem (6). Firstly, consumers have to pay adjustment costs for forced durable sales in the bankruptcy procedure. Secondly, consumers bear a cost for foregone durable utility. Both of these costs are implicit in the different arguments for durables and financial assets in the value functions V_{j+1} and V_{j+1}^B in (6).

Thirdly, consumers cannot declare bankruptcy again in the period following a bankruptcy procedure. This is consistent with the US bankruptcy law which forbids consumers to file for bankruptcy six years after a previous bankruptcy procedure. Since a period has a length of three years in our model, we assume that no bankruptcy can be declared for one period for which we define a different value function V_j^B:

$$V_j^B(a_j, d_j, y_j) = \max_{a_{j+1}^{*s},, a_{j+1}^{*u}, d_{j+1}^{*}} \left[U(c_j, d_j) + \beta E V_{j+1}(a_{j+1}^{*}, d_{j+1}^{*}, y_{j+1}) \right] . \quad (7)$$

Note that we do not need to assume that consumers are excluded from credit markets after bankruptcy. This assumption is often imposed in models with unsecured debt to make bankruptcy costly enough. Since we have endogenous bankruptcy costs related to durables, we do not need this assumption which is at odds with empirical evidence on consumer borrowing after bankruptcy procedures.

Finally, we allow for an exogenous bankruptcy cost ψ which can be interpreted as psychological pain or stigma (see Athreya, 2004). This permits us to relate our model to the literature but since this cost has no economic content we set $\psi = 0$ in our benchmark calibration.

The Bellman equations (6) and (7) together with the equations (3), (4) for the evolution of assets and the constraints (1), (2), $d_{j+1}^{*} \geq 0$ and $a_{j+1}^{*s} \leq 0$ complete the recursive formulation.

Equilibrium definition. A recursive competitive equilibrium is characterized by the policy functions for non-durable consumption $c_j(a, d, y)$, the portfolio $a_{j+1}^{*s}(a, d, y)$, $a_{j+1}^{*u}(a, d, y)$, $d_{j+1}^{*}(a, d, y)$ and optimal default so that for given prices $\{r^a, r^s\}$ of risk-free assets and secured debt:

(i) the envelope of value functions $V_j(a, d, y)$ and $V_j^B(a, d, y)$ attains its maximal value,

(ii) the pricing scheme for unsecured debt $q_j^u(\nu, y)$ satisfies the zero-profit condition (5), with default probabilities $\pi_j(\nu_j, y_j)$ being determined by optimal default.

Having presented the model and its recursive formulation we now solve the model numerically and calibrate it to match wealth and debt portfolios in the US.

4 Calibration and numerical results

The discrete nature of the bankruptcy decision and the presence of non-convex adjustment costs imply that we cannot use numerical algorithms which rely on the differentiability of the value function and the first-order conditions to solve the model. Thus, we discretize portfolio choices and specify an equi-spaced grid for $a^s \in [-5; 0]$ and $a^u \in [-2; 10]$ in terms of per-period average-income equivalents with 60 and 142 gridpoints, respectively. Choosing equi-spaced grids ensures that consumers remain on the endogenous grid of possible values for $a = a^s + a^u$ with 201 gridpoints where $a \in [-7; 10]$. We then specify the grid for the second endogenous state variable $d \in [0; 10]$ with 70 gridpoints where the grid for d is chosen to include the bankruptcy exemption value $d^\dagger$ and the values of d implied by the grid for secured debt a^s and the collateral constraint (1). The bounds of the grid are chosen such that consumers are not spuriously constrained by them in the simulations. Finally, we allow for 5 Markov states of the stochastic component of labor earnings. With this specification of the grid the model is solved in 4–5 hours on a PC of the current computing vintage using Fortran code.

4.1 Numerical algorithm

We start with the last period J. In that period the consumer sells all assets to consume them before death. We compute the available resources, with and without filing for bankruptcy, on the state space $A \times D \times Y$ and calculate the value functions V_{J-1} and V^B_{J-1}.[9] The functions allow us to determine the set of choices and future income states for which consumers declare bankruptcy, i.e., $V^B_{J-1} - \psi > V_{J-1}$. We then compute the price of unsecured debt for all income states and *feasible* choices before we solve the maximization problem of the consumer to determine the *optimal* choices. We continue with analogous computations for the previous period $J-2$ and so on until the beginning of life.

We use the model solution to simulate a population of 10,000 consumers whose initial exogenous and endogenous states at the beginning of life are determined in the following way.

[9] Since the amount of durables after bankruptcy would in general fall off the discretized grid, we convexify it by a weighted combination of the two neighboring gridpoints, with weights depending on their relative distances.

The stochastic income component is randomly drawn from the stationary income distribution and the initial conditions for durables and net-financial assets are drawn from the sample distribution of consumers with age 20-22 in the SCF, applying the sampling weights provided in the SCF.[10]

4.2 Calibration

We now discuss the calibration of the income process and other parameters.

4.2.1 The income process.

We calibrate the life-cycle income profile similar to Livshits et al. (2007a). Recall that labor earnings are given by

$$y_j = \phi_j \lambda_j,$$

where ϕ_j is the stochastic productivity of the household in period j of the life cycle and λ_j is the deterministic labor endowment which is hump-shaped over the life cycle. We calibrate the deterministic component using the income means for the age groups reported in Figure 1.

We assume that the stochastic component follows a five-state Markov chain. For calibrating this component we purge net labor earnings of life-cycle effects focussing on households with a head between 23 and 52 years of age. For this sample we regress net labor earnings on an age polynomial and compute the quintile means of the residual distribution around the mean income in the SCF 1983. This results in

$$\phi_{1983} = [0.28,\ 0.59,\ 0.85,\ 1.19,\ 2.09].$$

We approximate the distribution as log-normal, $\log \phi_{1983} \sim \mathcal{N}(-0.21, 0.42)$, where the mean of ϕ_{1983} is normalized to 1. Interestingly, this variance is similar to the variance of the logarithm of after-tax labor earnings in CEX data reported in Krueger and Perri (2006), Figure 1. We then assume an AR(1) process with first-order correlation of 0.9 which is in the

[10] We discard four observations of the initial conditions in 1983 and three observations in 2004 since they would imply an empty budget set.

range of commonly assumed values for the persistence of income shocks. We use Tauchen's (1986) method to discretize the income process and compute the transition matrix for the triennial periods as

$$\Gamma_{1983} = \begin{bmatrix} 0.5645 & 0.3191 & 0.0833 & 0.0294 & 0.0037 \\ 0.1965 & 0.3736 & 0.2240 & 0.1574 & 0.0485 \\ 0.0901 & 0.2884 & 0.2526 & 0.2486 & 0.1203 \\ 0.0365 & 0.1863 & 0.2300 & 0.3115 & 0.2357 \\ 0.0052 & 0.0575 & 0.1245 & 0.2930 & 0.5198 \end{bmatrix}.$$

Although the Markov chain with five states approximates the log-normally distributed AR(1) process very well, we implement a bias correction which ensures that the discrete Markov chain implies *exactly* the same mean and variance.[11] The productivity of households in the first period of life is drawn from the stationary distribution

$$\pi_{1983} = [0.1728, 0.2532, 0.1888, 0.2086, 0.1766].$$

4.2.2 Benchmark parameters.

Table 2 displays the parameter values which we use for our numerical solution. For the preference parameters we assume that the aggregator of durable and non-durable consumption goods is Cobb-Douglas which is roughly in line with empirical estimates on the substitutability between durables and non-durables (see Fernández-Villaverde and Krueger, 2005, for further discussion and references). We set $\underline{d}= 0.01$, a small and quantitatively negligible value, which allows consumers to hold no durable. We calibrate the remaining preference parameters β, θ and σ to match wealth and portfolio statistics in the data.

For the technology parameters we assume that durables depreciate at an annual rate of $\delta = 0.02$ which is a good approximation for consumer durables which mostly consist of housing. The adjustment costs are specified symmetrically for upward and downward adjustments and are assumed to equal 5% of the stock, consistent with typical fees charged by real-estate brokers in the US (Díaz and Luengo-Prado, forthcoming). These adjustment

[11]The idea is to choose the standard deviation which we use to compute the transition matrix so that the implied standard deviation of the Markov chain is exactly equal to the one in the data.

	Parameters		
Preferences	σ	1	
	β	0.8947	annual: 0.9636
	θ	0.712	
	$\underline{d}$	0.01	
Technology	δ	0.06	annual: 0.02
	$c_f^+, c_f^-,$	0.05	
	μ	0.85	
Bankruptcy	$d^\dagger$	0.25	annual: 0.75
	ψ	0	
Interest rates	r^a	0.1249	annual: 0.04
	r^s	0.1412	annual: 0.045
	$\underline{r^u}$	0.2597	annual: 0.08

Table 2: Benchmark parameters for the numerical solution

costs *per se* would imply that the consumer can use at most 95% of the durable stock to secure debt. In 1983 the terms of consumer credit have been more restrictive, however, with loan-to-value ratios of 85% (see the historical data of the Federal Reserve Statistical Release, Table G.19). Thus, we impose this additional restriction by choosing $\mu = 0.85$.

The parameters for the bankruptcy procedure are set as follows. We assume that the value of the exempt durables amounts to three quarters of average annual labor earnings which shall approximate the homestead exemption in the US although there is significant variation across US states (Athreya, 2006). As we will discuss further below, the size of the exemption has little effect on our results in strong contrast to Athreya (2006) or Pavan (2008) who do not analyze durables, secured and unsecured debt jointly. We assume that there is no ad-hoc utility cost of declaring bankruptcy in our benchmark calibration, $\psi = 0$, as our model is rich enough to allow for economically meaningful and realistic bankruptcy cost.

As in Livshits et al. (2007a), we assume a small-open economy and set the annual risk-free lending rate to 4%. We assume a small transaction cost for secured debt so that the secured borrowing rate is 4.5% and a larger transaction cost for unsecured debt so that the unsecured borrowing rate without the risk premium $\underline{r^u}$ is 8%. These interest rates are consistent with the evidence on interest spreads in Davis, Kubler and Willen (2006), historical interest-rate

Variable	*SCF data 1983* (1)	*Model 1983* (2)
Durables (as fraction of net lab. earnings)	3.29	3.29
Net-financial assets (as fraction of net lab. earnings)	1.32	1.33
Secured debt (as fraction of net lab. earnings)	-0.56	-0.51
Unsecured debt (as fraction of net lab. earnings)	-0.04	-0.03
Financial assets (as fraction of net lab. earnings)	1.92	1.87
Bankrupt (in % of sample)*	0.2	0

Table 3: Averages of the life-cycle profiles in the data and the model. Source: Authors' calculations based on the SCF and the model. Notes: *Data on bankruptcy are not available in the SCF 1983. Statistics from Sullivan et al. (2000).

data of the Federal Reserve (Table H.15) and the assumptions on interest spreads in Athreya (2006) and Livshits et al. (2007a).

It remains to discuss how we calibrate the preference parameters β, θ and σ. We calibrate these parameters to match the average total wealth of the life cycle profile and its components durables, financial assets and household debt. Table 3 shows that for $\beta = 0.96$, $\theta = 0.71$ and $\sigma = 1$ the model matches the data targets well. These parameters are within the range of commonly used values and further imply that the expenditure for non-durable consumption is 5.5 times the expenditure for durables for prime-age consumers. This is only slightly below the long-run average 6.2 for the US (Fernández-Villaverde and Krueger, 2005).

Furthermore, our model matches the fraction of debtors (50%) and secured debtors (40%) in the data and generates half the amount of unsecured debtors (10%) observed in the SCF data. The calibration predicts no bankruptcy in 1983, consistent with the very few bankruptcy cases reported in the 1980s (Sullivan et al., 2000). Finally, our calibration reproduces the empirical facts that consumers with unsecured debt are younger, have smaller labor earnings than the sample mean, hold a smaller but non-negligible amount of durables and substantially more secured debt than the rest of the sample. We now discuss the model mechanisms that drive these results in more detail.

4.3 Life-cycle profiles

After describing the calibration and the good match of averages over the life-cycle, we now present the implications of our calibration for the life-cycle profiles of consumers in more detail. These have not been targeted directly by our calibration and thus give a further indication of the model's fit of the data.

Figure 3 displays the life-cycle profiles of prime-age consumers with age 23-55 for the main variables of interest. This corresponds to the triennial periods 2 to 12 in our model. In the figures, the profiles generated by the model are solid whereas the data profiles are dashed. The profiles generated by the model are averages of the simulated population of 10,000 consumers who, at the beginning of period 2, start with a random draw from the SCF-data distribution of durables and net-financial assets across consumers aged 20-22 (period 1). This is why durable and net-financial assets are identical in the graphs at the beginning of period 2. Based on these initial conditions, consumers make their choices which determine the wealth and debt portfolio in period 3 so that we compare the model with the data for periods 3 to 12 (age 26-55) for all variables of interest. Recall that we focus on prime-age consumers since our model abstracts from death before the end of life and this is a good approximation of the data only up to a certain age.

Figure 3 shows that the model profiles match the data profiles quite well. Non-durable consumption has the standard shape over the life cycle and is upward sloping for consumers in prime-age. Since non-durable consumption is not reported in the SCF, there is no data counterpart in the graph but it is comforting that the predicted increase of non-durable consumption with age is of a similar size as in the literature (see, for example, Fernández-Villaverde and Krueger, 2005). More interestingly, the model matches the main characteristics of the profiles of durables and net financial assets. Whereas durable holdings increase with age, net financial assets first decrease with age and then increase after age 30. The decrease in net-financial assets at the beginning of life is due to the financing of durables with secured debt. The typical consumer accumulates durables financing part of this investment with debt. That debt is mostly secured and at the beginning of life about a third of consumers is at the collateral constraint. For our calibrated parameters these are the consumers who also take on more expensive unsecured debt which equals up to 6% of average population

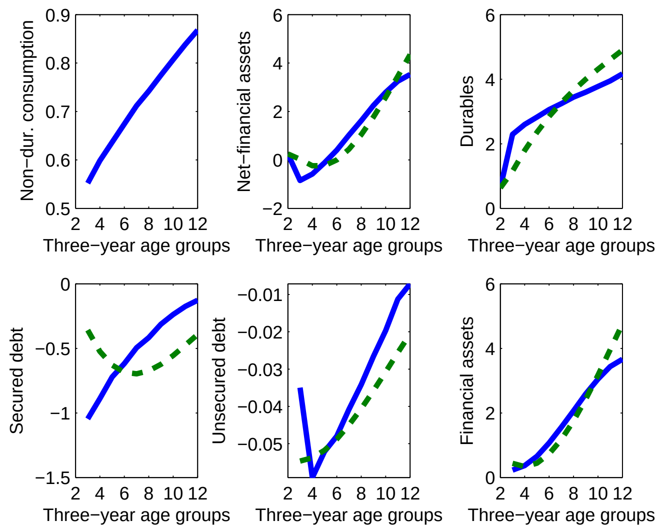

Figure 3: Life-cycle profiles predicted by the model (solid graph) and the data (dashed graph) for prime-age consumers with age 23-55. Source: Authors' calculations based on the model and the Survey of Consumer Finances (SCF). See the data appendix for variable definitions. Notes: no data on non-durable consumption are available in the SCF. Solid line: model prediction; dashed line: data. The unit is the average of net labor earnings of prime-age workers. Periods 2 to 12 correspond to three-year age groups between age 23 and 55.

labor earnings over the life cycle. As the labor earnings of consumers grow on average over the life cycle, consumers repay their debts as they age and eventually start to accumulate financial assets. On average, consumers have a positive net-financial asset position after age 35. Since unsecured debt is much more costly than secured debt, consumers first repay their unsecured debts and eventually also their secured debts. Whereas few consumer hold unsecured debt after age 40, substantially more consumers hold secured debt at later stages of their life cycle.

Figure 3 further shows that, quantitatively, the model predicts too much durable accumulation at the beginning of life compared with the data and too little accumulation thereafter. Since a substantial part of the durable investment is financed by secured debt, this implies that young consumers hold more secured debt in the model than in the data. The profiles of unsecured debt and risk-free assets instead are matched more closely by the model. Overall, the model fits the data quite well given that we abstract from geographical mobility or household formation which, in reality, may delay the purchase of housing or other durables

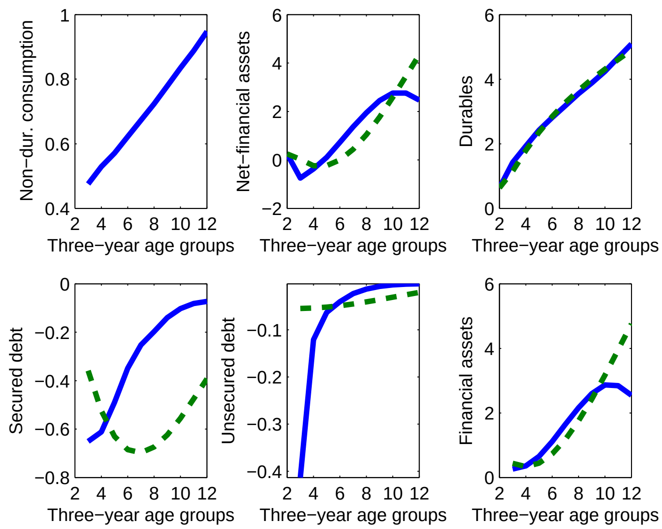

Figure 4: Recalibrated life-cycle profiles for a relative risk aversion $\sigma = 5$ with $\beta = 0.975$, $\theta = 0.672$ and annual $\underline{r^u}$=0.18. Source: Authors' calculations based on the model and the Survey of Consumer Finances (SCF). See the data appendix for variable definitions. Notes: no data on non-durable consumption are available in the SCF. Solid line: model prediction; dashed line: data. The unit is the average of net labor earnings of prime-age workers. Periods 2 to 12 correspond to three-year age groups between age 23 and 55.

early in life.

We now use our model of debt portfolios to make two quantitative points. Firstly, we find that risk aversion needs to be "small enough" to replicate the observed dominance of secured debt in the debt portfolio. Secondly, the role of durables as informal collateral for unsecured debt is very limited and thus the amount of durables which is exempt in bankruptcy procedures is not important quantitatively. We now discuss each of these two points in more detail.

4.4 Risk aversion and unsecured debt

When recalibrating the model for higher levels of risk aversion, say 2 or 5, we have found that the model predicts too little secured debt over the life cycle and much too high levels of unsecured debt at the beginning of life. More specifically, unsecured debt is predicted to be too high at the beginning of life by a factor 5 and 10 for a coefficient of relative risk aversion of 2 and 5, respectively.[12] This is illustrated in Figure 4 for a coefficient of relative risk aversion $\sigma = 5$. In the figure we also increase the base rate for unsecured debt $\underline{r^u}$ to an annual rate of 0.18 for illustration purposes. This very high unsecured-debt premium helps to contain the amount of unsecured debt for young consumers but still is not high enough for the model to match the data well.

This result relates to the literature on the equity premium puzzle (Mehra and Prescott, 2008) in which implausibly high levels of risk aversion are needed, even in models with incomplete markets, to generate the observed large equity premium. A related finding in this literature is that with a large equity premium implausibly high levels of risk aversion are needed to generate realistic consumer portfolios with rather small shares of positive risky *assets* (Heaton and Lucas, 1997). In our model with *debt* portfolios, however, high risk aversion worsens the fit of the model with the data. The intuition is that high risk aversion makes the state-contingency of unsecured debt more attractive. Hence, consumers are willing to pay the additional interest spread for unsecured debt in order to have the option to write off unsecured debt in bankruptcy proceedings if a bad income shock occurs. This implies that for higher risk aversion some consumers hold unsecured debt although they are *not* at the collateral constraint and could take on more secured debt. Hence, the behavior of the typical consumer described above, who takes on unsecured debt only if at the collateral constraint, is not general and depends on the level of risk aversion.

More generally, our results suggest that the parameter of risk aversion is not sufficient to match both debt and asset portfolios if one would also allow for risky assets in the model. Since our model abstracts from risky assets and focuses on debt portfolios, we prefer the risk aversion of 1 in our benchmark calibration.

[12] Note that few durables at the beginning of life make bankruptcy less costly in terms of adjustment costs and foregone utility. With non-separable utility, durables also affect the variability of the marginal utility derived from consumption which matters for the insurance value of unsecured debt. An increase of relative risk aversion σ makes non-durables and durables more complementary in the instantaneous utility function and thus reduces the variation in the marginal utility of non-durable consumption for consumers with more durables. This makes the insurance value of unsecured debt relatively more attractive for young consumers with few durables.

4.5 Durables and the pricing of unsecured debt

Compared with the literature a new feature of our model is the joint analysis of durables, secured and unsecured credit. Thus, we now discuss the role of durables for the pricing of unsecured debt in more detail. Previous research by Pavan (2008) has argued that durables serve as an informal collateral for unsecured credit so that exemptions of the durable stock in bankruptcy procedures make the supply of unsecured credit more costly. Our analysis instead shows that for plausible parameter values the exemption levels of durables in bankruptcy procedures are essentially irrelevant for the pricing of unsecured credit.

The intuition for this quantitative result is as follows. What matters for the pricing of unsecured debt is the durable stock which remains *after* repaying all secured debt in the bankruptcy procedure, $d^{\text{left for unsecured}}$ defined in section 3. This remaining durable stock is very small for plausible parameter values since most, if not all, consumers are at the collateral constraint if they take on unsecured debt. Indeed, consumers at the collateral constraint have a positive durable stock after repaying all secured debt only if a binding loan-to-value ratio μ restricts secured credit below the amount available with adjustment costs. In terms of the parameters of our model, it is necessary that $\mu < 1 - c_f^-$. Plausible parameter values for μ and c_f^- imply, however, that only a small amount of the durable stock, less than 10%, can be used as informal collateral for unsecured debt. It follows that, for the exemption level $d^\dagger = 0.75$ in our benchmark calibration, durables serve as informal collateral for unsecured debt if consumers hold a durable stock larger than 7.5!

Figure 5 illustrates the point that durables only serve as informal collateral for unsecured credit if the durable stock is large. The figure plots the price for unsecured debt in our benchmark calibration as a function of unsecured debt for different values of the durable stock. Note that the pricing function is the same for a durable stock of 1.5 or 6. For $d > 7.5$, durables serve as collateral for unsecured credit so that prices for unsecured credit increase with durables and thus the unsecured borrowing rate decreases.

The bottom line is thus that durables serve as informal collateral for unsecured credit only for consumers with large amounts of durables. In our model simulations, and in the data, however, the typical consumer who holds unsecured debt holds *less* durables than the

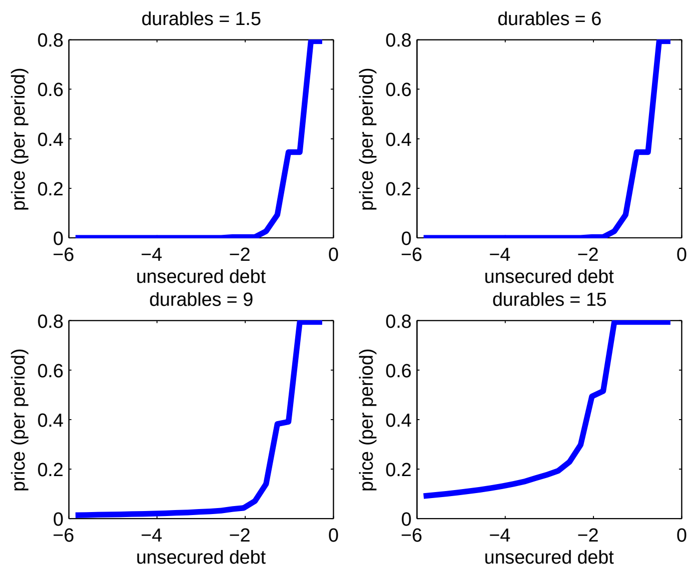

Figure 5: The price of unsecured debt as a function of unsecured debt for different durable wealth. Source: Authors' calculations based on the model. Notes: Prices are per three-year period. The unit of unsecured debt is the average of net labor earnings of prime-age workers.

population average and most of the times less than 7.5.[13] To be sure, empirically plausible changes of the exemption level matter for the pricing of unsecured debt, but they do so for values of durables at which consumers do not find it optimal to hold unsecured debt: a large durable stock financed with secured credit makes the state-contingency of unsecured debt less attractive since bankruptcy becomes more costly. In this case, a larger durable collateral needs to be sold which implies more transaction costs and forgone utility from durable consumption. Note that the small durable stock which remains *after* repaying all secured debt in the bankruptcy procedure also implies that the motive to "save" durables by writing off unsecured debt is not important quantitatively.

5 The evolution of wealth and debt portfolios

After characterizing the benchmark solution for 1983, we now investigate whether our model can explain some of the observed changes in wealth and debt portfolios in the period 1983-

[13]This result is robust for substantially smaller exemption levels such as $d^{\dagger} = 0.25$, for example, in which case durables would serve as informal collateral for $d > 2.5$.

2004. This also allows us to compare our model with the previous literature which has focused on explaining the increase in unsecured debt and bankruptcy filings in models without durables and secured debt (see, for example, Athreya, 2004, and Livshits et al., 2007b). We consider the following changes:

- A fall of the real interest rate and the borrowing spread for unsecured debt.
- An increase in labor earnings risk.
- An increase of the loan-to-value ratio.

After decomposing the effect of these three changes, we discuss whether the following two changes may help to explain the evolution of debt and wealth portfolios:

- A change in the utility cost of declaring bankruptcy frequently referred to as stigma (see Gross and Souleles, 2002, or Fay, Hurst and White, 2002). The qualitative implications of changes in stigma are mainly of interest to compare our model with the previous literature.
- An appreciation of durable prices.

The consensus based on models with unsecured debt (but without durables and secured debt) is that the improvement in the technology of financial intermediaries, which reduced the unsecured borrowing spread, quantitatively explains some of the observed increase in unsecured debt and bankruptcy filings (Athreya, 2004; Livshits et al., 2007b). We reconsider these findings and investigate whether they are consistent with the observed composition of the debt portfolio in terms of secured and unsecured debt. This is not obvious since consumers in our model have an additional margin of substitution between secured and unsecured debt and the borrowing rate has fallen for both types of debt. We now briefly discuss the calibration of the considered changes in the period 1983-2004.

Interest rates in 2004. Evidence by Caporale and Grier (2000) and Caballero, Farhi and Gourinchas (2008) indicates that the real interest rate in the US has fallen by 1-2 percentage points since 1983. We thus reduce r^a from 4% to 3%. Moreover, historical data from the

Federal Reserve (Table H.15) suggests that the borrowing spread between interest rates on secured debt and rates on treasury bills has remained roughly constant so that we set r^s equal to 3.5%, keeping the spread for the secured borrowing rate constant. Finally, we reduce the spread for the unsecured borrowing rate by 2 percentage points, setting $\underline{r^u}$ to 5% in 2004.[14]

The income process for 2004. We use the deterministic component of the life-cycle income profile in 2004, as displayed in Figure 1. Since the income means by age group change very little, the new deterministic component for 2004 will not affect the model predictions very much. Concerning the stochastic component, we use the same procedure detailed above where we compute the income quintile means in the SCF 2004 to specify the income grid as

$$\phi_{2004} = [0.10,\ 0.39,\ 0.74,\ 1.22,\ 2.53]$$

and approximate the income distribution in the SCF 2004 by $\log \phi_{2004} \sim \mathcal{N}(-0.34, 0.68)$. As before, this variance is similar to the variance of the logarithm of after-tax labor earnings in CEX data reported in Krueger and Perri (2006), Figure 1. Using Tauchen's method the transition matrix for 2004 is given by

$$\Gamma_{2004} = \begin{bmatrix} 0.5988 & 0.3701 & 0.0285 & 0.0025 & 0.0001 \\ 0.0890 & 0.5164 & 0.2798 & 0.1013 & 0.0135 \\ 0.0177 & 0.2947 & 0.3594 & 0.2554 & 0.0728 \\ 0.0034 & 0.1335 & 0.2994 & 0.3661 & 0.1976 \\ 0.0002 & 0.0247 & 0.1269 & 0.3410 & 0.5072 \end{bmatrix}$$

and the stationary distribution is

$$\pi_{2004} = [0.0782, 0.2910, 0.2645, 0.2279, 0.1384].$$

[14] The spreads reported in Davis, Kubler and Willen (2006), Table 1, do not provide much support for a change in the overall spread for unsecured borrowing. As we will see below, this is consistent with the fall in intermediation costs which we consider. The overall interest on unsecured credit is the sum of the base rate $\underline{r^u}$ and the endogenous risk premium where the fall in the base rate may be offset by the increase in the average risk premium.

The loan-to-value ratio in 2004. Historical data of the Federal Reserve Statistical Release, Table G.19, show that the loan-to-value ratios have increased by about 10 percentage points. Hence, we set $\mu = 0.95$ in 2004. Since the adjustment cost parameter $c_f^- = 0.05$, this implies that durables do not serve as informal collateral for unsecured debt if unsecured borrowers are at the collateral constraint. In this case, no durables are left after paying adjustment costs and secured debt in the bankruptcy procedure.

Changes in stigma. We have set the utility cost of bankruptcy $\psi = 0$ in our benchmark calibration for 1983. The advantage of our model with durables is that we can afford to neglect such ad-hoc costs in our calibration. Since there has been a debate on whether a decrease in stigma in the last decades is the cause of the higher unsecured debt and bankruptcy incidence, we check how the model solution changes for a higher ψ to compare the qualitative findings of our model with the literature. We illustrate the effect of changes in the parameter ψ on the model predictions for 2004. Since the size of the change in ψ is arbitrary and we are just interested in the qualitative response of the model equilibrium, we increase ψ from 0 to 1. As can be seen in the Bellman equation (6) this shifts the continuation value under bankruptcy down compared with the continuation value under repayment.

The appreciation of durable prices. Most of the durable stock in our model consists of real estate and data of Davis and Heathcote (2007) reveal that prices of homes, consisting of land and residential structures, have increased by 1-2% per annum in the period 1983-2004. The size of the price growth depends on whether years in the new millenium are included which have witnessed much stronger price growth. We have abstracted from such price changes so far and investigate the effect on the wealth and debt portfolio if consumers anticipate a higher durable-price appreciation of 1% per annum. We describe in the appendix how we introduce price changes into the model.

Variable	*Model 1983*	r^a=3%	*(2) and* r^s=3.5%	*(3) and* $\underline{r}^u$=5%	*(4) and* μ=0.95	*(5) and stoch.*	*(6) and determ.*	*(7) and initial cond. = Model 2004*	*Data 2004*
		Interest rates			*LTV*	*Income process*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Durables	3.29	3.54	3.66	3.66	3.73	3.69	3.63	3.65	4.04
Net-financial assets	1.33	0.36	-0.06	-0.19	-0.34	0.46	0.55	0.57	0.92
Secured debt	-0.51	-0.55	-0.88	-0.94	-1.02	-0.66	-0.64	-0.64	-1.12
Unsecured debt	-0.03	-0.03	-0.05	-0.11	-0.11	-0.05	-0.05	-0.05	-0.07
Financial assets	1.87	0.94	0.87	0.86	0.79	1.15	1.22	1.25	2.11
Bankrupt (in % of sample)*	0	0	0	0	0	0.3	0.3	0.3	1.47

Table 4: The evolution of wealth and debt portfolios. Source: Authors' calculations based on the model. Notes: Quantities are normalized by average net labor earnings in the respective year. Columns (2) to (8) implement changes between 1983 and 2004 sequentially. For example, the change of the stochastic part of the labor income process in column (5) is added on top of all the changes already implemented in columns (2) to (4). Note that unsecured debt, secured debt and financial assets do not add up to net-financial assets if the bankruptcy incidence is positive. Net-financial assets are a stock after bankruptcy and unsecured debt, secured debt and financial assets are choices before bankruptcy.

5.1 Results

Table 4 displays the results when we implement the changes between 1983 and 2004 sequentially. For example, column (3) shows the effect of the fall in the secured-borrowing rate *and* the lending rate. The main findings are summarized as follows.

The fall of the real interest rate in column (2) reduces financial assets but has little effect on consumer debt or the bankruptcy incidence. Thus, net financial assets fall substantially. Since accumulation of risk-free assets becomes relatively less attractive and the user cost of durables falls, durables increase.

If we also lower the interest rate for *secured* debt in column (3), secured debt increases by 57% which is more than half of the increase observed in the data. Since unsecured debt is much more expensive than secured debt in column (3), unsecured debt only increases slightly although more consumers are at the collateral constraint. Thus, secured debt becomes more important in the debt portfolio and the wealth portfolio shifts further towards durables.

The smaller costs of financial intermediation for *unsecured* debt considered in column (4) have no significant effect on the wealth portfolio of financial assets and durables. Also the incidence of bankruptcy remains negligible. Most interestingly, both secured and unsecured consumer debt increase. Secured debt increases since it is less expensive to be collateral constrained when unsecured debt is cheaper. Thus, compared with Athreya (2004) and Livshits et al. (2007b), the results of columns (3) and (4) illustrate the importance of capturing the adjustment of the whole debt portfolio, that is secured *and* unsecured debt, when analyzing the quantitative effects of interest rate changes.

A higher loan-to-value ratio in column (5) increases secured debt and durables, although the size of the effect is rather modest since the collateral constraint is binding only for a fraction of consumers who are mostly at the beginning of the life cycle.

The higher labor income risk in column (6) has effects on the wealth and debt portfolio which are opposite to the effects of lower interest rates, previously analyzed in columns (2) to (4). Due to the precautionary saving motive, higher labor income risk increases financial assets while the effect on durables is small, as in Nakajima (2005), for example. More labor income risk also reduces consumer debt where most of the change in debt (in absolute terms) occurs for secured debt. Interestingly, higher labor income risk also increases the

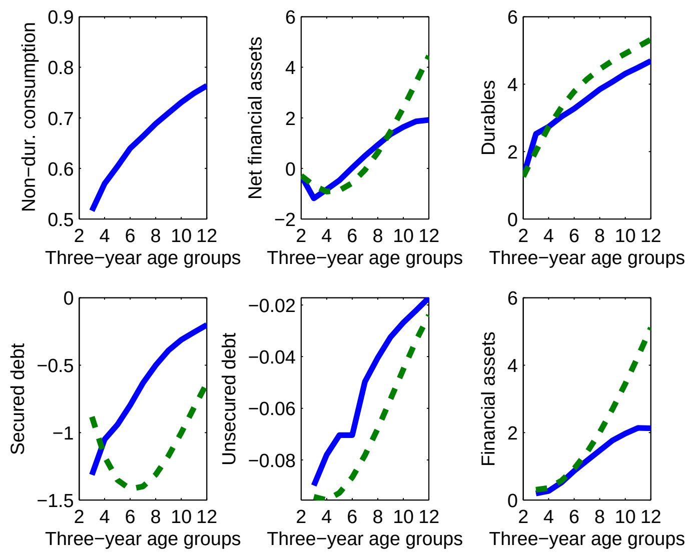

Figure 6: Life-cycle profiles predicted by the model in 2004 (solid graph) and the 2004 data (dashed graph) for prime-age consumers with age 23-55; Source: Authors' calculations based on the model and the Survey of Consumer Finances (SCF). Note: no data on non-durable consumption are available in the SCF.

bankruptcy incidence. This is because the lowest quintile mean in the SCF 2004, which we use to calibrate the lowest income state, is smaller and thus more "catastrophic" than in 1983. In fact, there would be no bankruptcy if we held the income grid constant at the 1983 values and implemented the change of the income process entirely by adjusting the probability matrix of the Markov chain. Although this shows that the results on consumer bankruptcy depend on details of the income process, we use the data as our guidance for specifying the income grid so that it is not arbitrarily chosen. Quite interestingly, we find that the smaller lowest income state in 2004 also makes the state-contingency of unsecured debt more attractive. As for the case of higher risk aversion discussed in the previous section, some consumers take on unsecured debt even if they are not at the collateral constraint and could borrow more at the secured-borrowing rate.

In columns (7) and (8) we add the rather small change in the deterministic labor income profile and the initial conditions for durable and net-financial assets according to the SCF 2004. Both changes do not affect the results much. Column (8) then shows the overall model prediction for 2004 which can be compared with the data statistics reported in column (9).

The model qualitatively predicts the changes in the wealth and debt portfolios correctly but for the decrease of financial assets which have increased in the data instead. Quantitatively, the model predicts about half of the increase in durables and twice the observed decrease in net-financial assets, mostly because the model does not capture the increase in financial assets for the quantitative changes which we have studied. Concerning consumer debt, the model explains a fourth of the observed increase in secured debt and a third of the increase in unsecured debt and bankruptcy filings.[15] In terms of the fraction of debtors, the model continues to match the fraction of debtors rather well although it predicts a modest increase by 10 percentage points which is not observed in the data. Concerning the price of debt, the model predicts a substantial increase in the dispersion of unsecured interest rates, with a standard deviation of 0.09 in 2004 compared with no dispersion in 1983. This model prediction is consistent with the increase in the dispersion of borrowing rates documented by Edelberg (2006) and Athreya, Tam and Young (2008). Interestingly, the average risk premium on unsecured debt is 3.3 percentage points so that the fall of the unsecured borrowing rate without the risk premium $\underline{r^u}$ is fully compensated in the 2004 prediction by the increase of the average risk premium. Hence, as mentioned above, our model predictions do not contradict the rather constant interest rates for unsecured debt over time reported in Davis et al. (2006). Finally, the model predicts that those consumers filing for bankruptcy on average hold unsecured debt that amounts to three quarters of average labor earnings which is in the ballpark of statistics reported by Sullivan et al. (1999).

Figure 6 and 7 compare the life-cycle profiles predicted by the model for 2004 (the solid graphs) with the data (the dashed graphs). The figures show that the predictions of the model are better for consumers at the beginning of the life-cycle and less satisfactory for financial assets, secured debt levels and the bankruptcy incidence of consumers in their 40s and 50s, even if we adjust the observed bankruptcy incidence (the dashed graph displayed in Figure 7) by a factor of 2/3 as only 2/3 of the bankruptcies in the data are job related. Consumer bankruptcy in our life-cycle model with durables is difficult to generate since unsecured debt is held mostly by young collateral-constrained consumers who accumulate

[15] Since the only source of risk in our model is due to fluctuations in labor income and only 2/3 of bankruptcy filings are job related, we have to downward adjust the observed 1.5% for bankruptcy filings to a target of 1%.

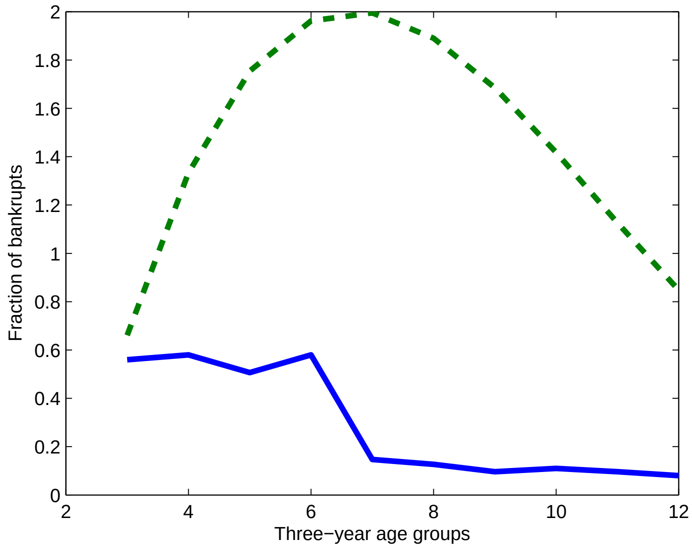

Figure 7: Life-cycle profiles of bankruptcy predicted by the model in 2004 (solid graph) and the 2004 data (dashed graph) for prime-age consumers with age 23-55; Source: Authors' calculations based on the model and the Survey of Consumer Finances (SCF).

durables. Once these consumers hold significant amounts of durables, bankruptcy is quite costly since all durables that are used as collateral and all remaining durables above the exemption level are seized.

Importantly, our quantitative exercise to predict changes in the period 1983-2004 allows us to compare our model further to the previous literature. We find that a fall in interest rates which reduces the cost of borrowing may explain some of the increase in consumer debt but not much of the increase in bankruptcy filings. This result is similar to the life-cycle model with expense shocks, but without durables and secured debt, of Livshits et al. (2007b). In the infinite-horizon framework of Athreya (2004) instead bankruptcy filings are more elastic to changes in interest rates (see also Mateos-Planas, 2007).

An important value added of our analysis is that we distinguish between secured and unsecured debt. Whereas cheaper borrowing rates imply more unsecured debt in Livshits et al. (2007b) and Athreya (2004), and thus possibly also more bankruptcy incidence, our analysis has shown that it is hard to generate a substantial increase in bankruptcies if both secured and unsecured debt become cheaper which is necessary if one wants to match the rather constant share of secured debt in the debt portfolio observed in the data.

5.2 Further potential explanations for the evolution of wealth and debt portfolios

Before we conclude we discuss two further candidate explanations for the changes in the wealth and debt portfolios.

Changes in the utility costs of bankruptcy. The utility costs of bankruptcy are captured by the parameter ψ in our model which is often called stigma in the literature. Livshits et al. (2007b) argue that a fall in stigma may explain some of the upward trend in bankruptcy incidence whereas Athreya (2004) highlights the importance of supply-side responses which tighten access to unsecured credit. If we increase ψ from 0 to 1 starting from column (8) in Table 4, we find that the equilibrium remains nearly unchanged. In accordance with Livshits et al. (2007b) we find that both unsecured debt and the bankruptcy incidence fall. Hence, a fall in the utility costs of bankruptcy costs may have contributed to some of the increase in unsecured debt and bankruptcy incidence. A problem of this explanation is, however, that a possible quantitative change in the utility cost cannot be measured directly and assuming an arbitrary change of ψ is not attractive from a methodological point of view.

The appreciation of durable prices. Column (2) in Table 5 displays the results if we add the deterministic durable-price appreciation of 1% per annum to all the other changes used for the model prediction for 2004 in column (8) of Table 4. For convenience, we report that prediction again in column (1) of Table 5.

The results in column (2) of Table 5 show that an anticipated 1% durable-price increase shifts the wealth portfolio towards durables and the debt portfolio towards secured debt. Quantitatively, the change of durables predicted by the model is 130% of the observed change and the change of secured debt predicted by the model is 63% of the observed change. Whereas higher anticipated durable-price appreciation helps the model to match changes in durables and secured debt, it worsens the model predictions for changes in unsecured debt and financial assets. Quantitatively, this is most important for financial assets which are

Variable	*Model 2004*	*Durable-price appreciation* (1) and 1% p.a. price increase	*Data 2004*
	(1)	(2)	(3)
Durables	3.65	4.30	4.04
Net-financial assets	0.57	0.07	0.92
Secured debt	-0.64	-0.87	-1.12
Unsecured debt	-0.05	-0.04	-0.07
Financial assets	1.25	0.97	2.11
Bankrupt (in % of sample)	0.3	0.2	1.47

Table 5: The effect of durable-price appreciation on wealth and debt portfolios. Source: Authors' calculations based on the model. Notes: Quantities are normalized by average net labor earnings. Column (2) adds a 1% appreciation of durable prices to all the changes already implemented in column (1). Note that unsecured debt, secured debt and financial assets do not add up to net-financial assets if the bankruptcy incidence is positive. Net-financial assets are a stock after bankruptcy and unsecured debt, secured debt and financial assets are choices before bankruptcy.

predicted to be only half of their observed size (compare columns (2) and (3)).

How could one further improve the predictions of the model? Our quantitiative experiments reported in Table 4 have shown that financial assets are particularly sensitive to the fall in the lending rate. Indeed, the model would be able to match the level of financial assets in 2004 much better if we kept the lending rate constant (and the secured-borrowing rate half percentage point above the lending rate) but otherwise allowed for all other changes implemented in column (2), Table 5. In that case, however, the model would predict no increase in secured debt between 1983 and 2004. This suggests that a substantial fall in the spread between the secured-borrowing rate and the lending rate, holding the lending rate constant, would be necessary for the model to predict the changes of financial assets *and* debt over time. In the data, however, the spread between the secured-borrowing rate and lending rate is small already in 1983 and there is no significant downward trend in that spread in the time period 1983-2004. Since our model underpredicts financial assets mostly for consumers after age 40, it thus seems more promising in future research to extend the model to analyze whether changes in health risk or longevity risk in the period 1983-2004 boost the accumulation of financial assets at later stages of the life cycle.

6 Conclusion

We have set up and studied a model in which consumers hold portfolios of secured and unsecured debt. We have shown that this model explains the main characteristics of wealth and debt portfolios in the US and some of the observed changes over time. Our results also show for plausible parameter values that durables and durable exemptions, which are an important part of US bankruptcy regulation, matter very little for the consumers' access to unsecured credit.

In future research it would be interesting to relax some of the assumptions which we have made to contain the computational burden. Allowing for health risk or longevity risk may improve the predictions of the model for financial assets at later stages of the life cycle. Furthermore, modeling durable ownership *and* rental contracts, geographical mobility shocks or household formation may improve the predictions for the life-cycle profile of secured debt if consumers postpone some of their durable purchases to later ages.

Appendices

Data appendix

This data appendix describes how we construct data counterparts for the wealth and debt portfolio as well as labor earnings in the model, using data from the Survey of Consumer Finances (SCF). We construct all variables for the full SCF sample and then apply the sample-selection criteria mentioned below.

Gross labor income is the sum of wage and salary income. As in Budría Rodríguez et al. (2002) we add a fraction of the business income where this fraction is the average share of labor income in total income in the SCF. *Disposable labor income* is computed using the NBER tax simulator. We use the programs by Kevin Moore provided on http://www.nber.org/~taxsim/ to construct disposable labor earnings for each household in the SCF 1983 and 2004. Following the standardized instructions on the NBER website, we feed the following required SCF data into the NBER tax simulator: the US state (where available, otherwise we use the average of the state tax payments across states), marital status, number of dependents, taxpayers above age 65 and dependent children in the household, wage income, dividend income, interest and other property income, pensions and gross social security benefits, non-taxable transfer income, rents paid, property tax, other itemized deductions, unemployment benefits, mortgage interest paid, short and long-term capital gains or losses. We then divide the resulting federal and state income tax payments as well as federal insurance contributions of each household by the household's gross total income in the SCF. This yields the implicit average tax rate for each household in 1983 and 2004. The mean of that average tax rate for consumers in the SCF is 24% in 1983 and 23% in 2004. Finally, we use the average tax rate of each household in 1983 and 2004 to compute household disposable labor income as (1 - household average tax rate) * household gross labor income (including taxable transfers) and then add non-taxable transfers.

When constructing data counterparts for the wealth and debt portfolio of each household in the model, it is useful to refer to the following stylized household balance sheet:

Household balance sheet

Assets	Liabilities
Durables (housing, vehicles)	Gross debt secured by durables
	Durable equity
Gross financial assets	Gross unsecured debt
	Other equity

Durables are defined as the sum of the value of homes, residential and non-residential property and vehicles. These are the most important durable items which can be used as collateral in real-world debt contracts.

Gross secured debt is defined as the sum of mortgage and housing debt, other lines of credit and debt written against residential and nonresidential property or vehicles.

The difference between the value of durables and gross secured debt is the *durable equity* held by the household.

Gross financial assets are defined as the sum of assets besides the durables defined above. This is the sum of money in checking accounts, savings accounts, money-market accounts, money-market mutual funds, call accounts in brokerages, certificates of deposit, bonds, account-type pension plans, thrift accounts, the current value of life insurance, savings bonds, other managed funds, other financial assets, stocks and mutual funds, owned non-financial business assets, jewelry, antiques or other small durable items not included in the durable definition above.

Gross unsecured debt is defined as all debt besides the gross secured debt defined above. This consists of credit-card debt, non-auto consumer loans and other financial debt.

The difference between the gross financial assets and gross unsecured debt is the *other equity* held by each household.

Net worth is then defined as the sum of durable equity and other equity.

We still need to define the data counterparts for unsecured debt, secured debt and financial assets in the model. These counterparts are not equal to the gross positions since many households in the data hold debt and financial assets at the same time which cannot occur in the model. In order match the SCF data to the model, we consolidate the data at the household level so that households indeed either hold debt or financial assets. We proceed in the following way:

Unsecured debt is zero for households with nonnegative other equity and equals other equity if other equity is negative. *Secured debt* for households whose other equity is negative is set equal to their gross secured debt and their *financial assets* are set to zero.

For households who hold positive amounts of other equity we then consolidate these positions with gross secured debt to obtain the corresponding measures as follows.

Secured debt is zero for households whose sum of gross secured debt and positive amounts of other equity is positive. Otherwise secured debt equals gross secured debt net of positive amounts of other equity.

Financial assets are zero for households whose sum of gross secured debt and positive amounts of other equity is negative. Otherwise financial assets equal positive amounts of other equity net of gross secured debt.

Net financial assets are the sum of *financial assets*, *secured debt* and *unsecured debt*.

It remains to describe how we classify households as bankrupt or having payment difficulties.

Payment difficulties: The SCF 1983 does not contain direct information on consumer bankruptcy. However, consumers were asked in 1983 *and* 2004 whether they "had had a request for credit turned down by a particular lender or creditor in the past few years, or had been unable to get as much credit as [they] had applied for." Moreover, they were asked whether they "had not applied for credit because [they] thought [they] would be turned down. [They were] asked for what reasons [they] thought [they] would be turned down on the most recent occasion when this occurred." We classify households as having payment difficulties if they answer to either of these two questions that they were turned down because of "credit records/history from other institutions; other loans or charge accounts; previous payment records or bankruptcy." Of course, this measure of payment difficulties is far from perfect but it allows us to look at time trends in payment difficulties in the SCF since there is no information about bankruptcy filings in the SCF 1983. Interestingly, the trend of this measure of payment difficulties is similar to measures of Sullivan et al. (1999, 2000) who used administrative data on bankruptcy filings in 10 judicial districts in 1981 and 16 districts in 1991.

Bankruptcy: We classify a household as bankrupt in the SCF 2004 if the household head or husband/wife/partner have filed for bankruptcy in the last year.

Sample selection criteria: We focus on consumers between age 20-74 to construct life-cycle profiles. In order to contain the effect of outliers on the means for each three-year age cell over the life cycle, we drop observations if gross labor income is negative (4/11 observations in 1983/2004 are deleted), net worth is smaller than -1.2 in terms of the population average of disposable labor income in the respective year (additional 5/19 observations in 1983/2004 are deleted) and gross unsecured debt is larger than 9 in terms of the population average of disposable labor income (additional 7/2 observations in 1983/2004 are deleted). After constructing the smoothed life-cycle profiles, we further restrict our attention to the life-cycle profiles of prime-age households between age 23 and 52 when matching the model to the data, for reasons discussed further in the main text.

Numerical solution of the model with durable-price appreciation

In this appendix we show how we solve the model with changes in durable prices. For computational purposes it is convenient to express the problem in terms of durables since this allows us to keep the grids constant over the life cycle in the numerical solution.

We start from the problem with constant prices

$$\max_{(a^s_{j+1},a^u_{j+1},d_{j+1})} \quad \sum_{j=1}^{J} \beta^{j-1} \frac{(c_j^{\theta} d_j^{1-\theta})^{1-\sigma} - 1}{1-\sigma} \tag{8}$$

subject to

$$a^s_{j+1} \geq -\min(\mu, 1-c_f^-)\ d_{j+1}, \tag{9}$$

$$q^s_j a^s_{j+1} + q^j_j a^u_{j+1} + c_j + q^d d_{j+1} - d_j + \alpha(d_j, \iota_j) \leq \ a_j + y_j, \quad j = a, u, \tag{10}$$

$$a^s_{j+1} + a^u_{j+1} = a_{j+1}\ . \tag{11}$$

We introduce prices p_j for durables and assume a constant factor of price change

$$\Pi = \frac{p_{j+1}}{p_j}\ .$$

The problem with changing prices is then defined as follows. (8) is unchanged, while the collateral constraint and the budget constraint include prices for durables:

$$a^s_{j+1} \geq -\min(\mu, 1-c_f^-)\ p_{j+1} d_{j+1}, \tag{12}$$

$$q^s_j a^s_{j+1} + q^k_j a^u_{j+1} + c_j + q^d p_{j+1} d_{j+1} - p_{j+1} d_j + p_{j+1}\alpha(d_j, \iota_j) \leq \ a_j + y_j, \quad k = a, u\ . \tag{13}$$

Dividing (12) and (13) by p_{j+1}, we get

$$\frac{a^s_{j+1}}{p_{j+1}} \geq -\min(\mu, 1-c_f^-) d_{j+1}, \tag{14}$$

$$q_j^s \frac{a_{j+1}^s}{p_{j+1}} + q_j^k \frac{a_{j+1}^u}{p_{j+1}} + \frac{c_j}{p_j} \frac{p_j}{p_{j+1}} + q^d d_{j+1} - d_j + \alpha(d_j, \iota_j) \leq \frac{a_j}{p_j} \frac{p_j}{p_{j+1}} + \frac{y_j}{p_j} \frac{p_j}{p_{j+1}}, \quad k = a, u \ . \tag{15}$$

Defining variables expressed in units of durables,

$$\widetilde{a_j^k} = \frac{a_j^k}{p_j}, \ k = s, a, u \,, \quad \widetilde{c_j} = \frac{c_j}{p_j}, \quad \widetilde{y_j} = \frac{y_j}{p_j} \ ,$$

the constraints (14) and (15) can be expressed in these transformed variables to become

$$\widetilde{a_{j+1}^s} \geq - \min(\mu, 1 - c_f^-) d_{j+1}, \tag{16}$$

$$q_j^s \widetilde{a_{j+1}^s} + q_j^j \widetilde{a_{j+1}^u} + \widetilde{c_j} \frac{1}{\Pi} + q^d d_{j+1} - d_j + \alpha(d_j, \iota_j) \leq \widetilde{a_j} \frac{1}{\Pi} + \widetilde{y_j} \frac{1}{\Pi}, \quad j = a, u, \tag{17}$$

and similarly

$$\widetilde{a_{j+1}^s} + \widetilde{a_{j+1}^u} = \widetilde{a_{j+1}} \ . \tag{18}$$

To complete the formulation of the problem in units of durables, we need to express c_j in terms of $\widetilde{c_j}$. By definition we have

$$c_j = \widetilde{c_j} p_j = \widetilde{c_j} p_0 \Pi^j \ .$$

Substituting for c_j in (8) we obtain

$$\sum_{j=1}^{J} \beta^j \frac{((\widetilde{c_j} p_0 \Pi^j)^\theta d_j^{1-\theta})^{1-\sigma} - 1}{1 - \sigma} \tag{19}$$

or after some algebra

$$p_0^{\theta(1-\sigma)} \sum_{j=1}^{J} (\beta \Pi^{\theta(1-\sigma)})^{j-1} \frac{(\widetilde{c_j}^\theta d_j^{1-\theta})^{1-\sigma} - 1}{1 - \sigma} + \text{constant} \tag{20}$$

Maximization of (20) is equivalent to

$$\max_{(\widetilde{a^s_{j+1}},\widetilde{a^u_{j+1}},d_{j+1})} \quad \sum_{j=1}^{j} \widetilde{\beta}^{j-1} \frac{(\widetilde{c_j}^{\theta} d_j^{1-\theta})^{1-\sigma} - 1}{1-\sigma}, \tag{21}$$

where

$$\widetilde{\beta} = \beta \Pi^{\theta(1-\sigma)} \ .$$

Combined with the constraints (16) and (17) as well as (18) in units of durables this allows for the standard recursive formulation.

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