

Sushi or Fish Fingers?

Food-fish Diversity, Collapsing Fish Stocks and Multi-species Fishery Management

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Abstract. Since consumers have a preference for food-fish diversity, the ongoing collapse of fish stocks is an economic problem over and above the inefficiently low resource rents caused by overfishing. We present a theoretical model of a multi-species fishery and show that (i) preferences for diversity may play an important role in the cascading collapse of fish stocks under open-access fishery, (ii) the need for regulation is higher, the stronger the preferences for diversity are, (iii) second-best optimal management of only one (or a few) species must be less strict than first-best management of the same species, and (iv) myopic regulation of one species and ignoring the economic feedback on other species may induce over-regulation of that species and depletion of other stocks that would not be depleted under full open-access. (130 words)

Keywords: marine biodiversity, sustainability, fisheries, fishery economics, multi-species fishery, product differentiation

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1 Introduction

The problem of overfishing has two sides to it. First, overfishing induces inefficiently low stocks and yields. Secondly, it leads to a decrease in the diversity of food-fish supply, as more and more fish stocks are being fished to collapse on a world-wide scale. This second aspect has attracted considerable scientific attention and public concern in the last few years Costello et al. (2008), Heal and Schlenker (2008), Pauly et al. (1998), Worm et al. (2006). From an economic perspective, the collapse of fish stocks is a problem over and above the inefficiently low stocks and yields, because many consumers like to consume a whole variety of different food fish. A prominent example is the Japanese preference for sushi, which is traditionally prepared with a clearly defined set of different species of fish and seafood.

In this paper we analyze the problem of overfishing with reference to both aspects, and study the options available to multi-species fishery management, given that consumers have preferences for food-fish diversity. Accordingly, 'preferences for diversity' mean that the utility drawn from consuming fish is higher, the more species of fish are consumed in about equal proportions. We model this through a 'love-of-variety' effect, drawing on the celebrated approach proposed by Dixit and Stiglitz Dixit and Stiglitz (1977). By contrast, the dynamics of the fish stocks is modeled in a simple way, taking no account of any ecological interactions between the different stocks. This enables us to focus on the demand-side interactions and to show that these interactions lead to interdependent harvest quantities of the different species even if ecological complexity is left unconsidered.

This approach distinguishes our paper from the previous literature on multi-species fisheries, which has mainly focused on biological interactions between different species (e.g., Conrad and Adu-Asamoah 1986, Lande et al. 2003, Quinn and Deriso 1999). Only very few studies have taken into account the economic value of diversity. Kasulo and Perrings Kasulo and Perrings (2006) focus on how biodiversity affects the effectiveness of fishing effort. They conclude that traditional

freshwater fisheries in Malawi are associated with a higher diversity of fish catches than can be observed in profit-maximizing modern fisheries. Although not explicitly considering preferences for food-fish diversity, Wilson (1982; 1985) suggests in a non-formal analysis that with relatively high cross-elasticities of demand for different species (in terms of our model 'weak' preferences for diversity) a multi-species fishery will be sustainable without any need for regulation (Wilson 1985:324). Our modeling analysis confirms this conjecture by showing that high elasticity of substitution favors the sustainability of the multi-species fishery. But the analysis of consumer preferences for diversity also generates many more important insights on multi-species fisheries and fishery management.

In Section 2 we describe the theoretical model including an arbitrary number of species of edible fish that is the basis of our analysis. In addition to open-access fishery we consider three different scenarios for multi-species fishery management: (i) first-best optimal management of all species, (ii) second-best optimal management of one species, taking into account the spill-over effects of regulation on all other species fished under open access, and (iii) myopic management of a single species ignoring the effects on other fish species. The conditions determining the dynamics of the multi-species fishery operating on the basis of open access and the three management scenarios are given in the appendix. These conditions indicate in particular that even in the absence of ecological interactions first-best optimal management of several stocks has to be coordinated.

In Section 3 we show that preference for diversity may play an important role in the observed cascading collapse of fish stocks under open access. In our view, this is a remarkable outcome since the model does not include any exogenous dynamics such as increasing demand or technical progress.

Comparing the three scenarios for fishery management in section 4, we conclude that (i) second-best management of only one species must be less strict than first-best management of the same species, and (ii) myopic regulation of one species while ignoring the economic feedback on other species is too strict in comparison with second-best management. We present a numerical example in which over-

regulation under myopic management leads to the collapse of stocks that would not collapse under full open-access.

Section 5 analyzes how different degrees in the preference for diversity affect the number and sizes of fish stocks under open access and the three management scenarios. In particular, we show that on average stronger preferences for diversity lead to less diversity of food-fish supply in equilibrium, as more stocks are fished to collapse. The intuitive reason for this is that the stronger the preferences for diversity are, the more willing consumers will be to pay for the increase in fishing costs as fish stocks decrease. Put differently, weak preferences for diversity ('fish fingers') lead to a more sustainable outcome than strong preferences for diversity ('sushi'). We also show that the need for regulation is greater, the stronger the preferences for diversity are, as the difference between the number of stocks in the first-best optimal steady state and the number of stocks in the steady state under open access increases with the preferences for diversity.

2 Preferences for diversity in a simple multi-species fishery model

The model is set up for an arbitrary initial number n_0 of fish species, labeled $i = 1, \dots, n_0$. The stock of species i is described by a simple equation of motion for its biomass x_i ¹

$$\dot{x}_{i,t} = g_i(x_{i,t}) - m_{i,t} h_{i,t}, \quad (1)$$

in which we use $g_i(x_i)$ to denote the biomass growth function, $m_{i,t}$ to denote the number of vessels, $h_{i,t}$ to denote the harvest per vessel, and $m_{i,t} h_{i,t}$ to denote the total harvest of species i , each at time t . The simplifying assumption here is that species do not interact ecologically, i.e. the natural growth rate of species i

¹Recent approaches highlight the importance of age-structured models (see Skonhøft et al. 2008, Tahvonen 2009). Though this is important, we leave age structure out of account to focus on the role of consumer preferences.

depends only on its own stock and its harvest. To be more specific, we assume that all stocks grow according to a logistic equation

$$g_i(x_{i,t}) = \rho_i x_{i,t} \left[1 - \frac{x_{i,t}}{\kappa_i} \right], \quad (2)$$

where ρ_i is the intrinsic growth rate of species i and κ_i is the carrying capacity, i.e. the biomass of species i in the absence of fishing.²

Harvest is determined by this vessel's fishing effort e_i directed at catching species i and the stock x_i . It is described by a generalized Gordon-Schaefer production function Clark (1990)

$$h_i = h_i(x_i, e_i) = \nu_i x_i^{\chi_i} e_i^{\epsilon_i}, \quad (3)$$

where ν_i is the catchability coefficient of species i and $\chi_i > 0$ is the output elasticity of the stock of species i . According to (3) and other things being equal, greater stocks lead to higher yields. Furthermore, we assume positive but decreasing returns on effort e_i , i.e. $0 < \epsilon_i < 1$. Effort is measured in units of labor.³ Beside the variable fishing costs there is a fixed cost ϕ_i for operating a vessel. That means that capital is perfectly malleable and can be used to harvest all species of fish, although the amount of capital needed may differ between species.

To set up a general equilibrium model in a simple form, we assume that each of the identical households inelastically supplies one unit of labor. The mass of households is normalized to unity such that total labor force also equals one. Rather than being employed in the fishery sector, workers can produce a numeraire commodity with constant returns to scale technology, where each unit of labor produces $\omega > 0$ units of output, i.e. the wage rate equals ω . Given the effort levels required to catch each fish species, output of the numeraire sector left for consumption is given by

$$y = \omega \left(1 - \sum_{i=1}^{n_0} m_i e_i \right) - \sum_{i=1}^{n_0} m_i \phi_i \quad (4)$$

²In the following we omit the time index for the sake of cleaner notation.

³When effort is measured in terms of number of vessels, returns on effort may be increasing. This is the case, e.g., for the North Sea herring Bjørndal and Conrad (1987). In our model, fishing effort and the number of vessels are two separate variables.

Consumer preferences are described by the utility function

$$u(v, y) = y + \gamma \frac{\eta}{\eta - 1} v^{\frac{\eta-1}{\eta}}, \quad (5)$$

where y is the consumption of the numeraire commodity and v is the consumption of fish. The parameter γ describes the weight of fish consumption in utility, while η is the demand elasticity of fish on aggregate. Consumption of fish is composed of different species, and the sub-utility of fish consumption is given by

$$v = v(q_1, \dots, q_n) = \left[\sum_{i=1}^{n_0} q_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (6)$$

where q_i is the quantity of species i consumed. The parameter σ measures elasticity of substitution in the consumption of two different species. A lower σ may be regarded as stronger ‘desirability of variety’ (see Dixit and Stiglitz (1977)) or a stronger preference for diversity. So high elasticity of substitution σ corresponds to a consumer of fish fingers, while low elasticity of substitution describes a consumer of sushi.⁴

Empirically, different species of food fish have been found to be substitutes Fousekis and Revell (2004). Accordingly, we concentrate on this case in the analysis.⁵ Concerning the demand elasticity of fish on aggregate, we concentrate on intermediate values where demand is neither very elastic nor very inelastic. Taken both together, we assume $\sigma > \eta \geq 1$ throughout the analysis. The assumption

⁴The utility function given by equations (5) and (6) is the standard way of describing preferences for diversity for example in the economics of industrial organization, international trade and New Economic Geography. For $\sigma = 1$, (6) is the Cobb-Couglas function $v = \prod_{i=1}^{n_0} q_i^{1/n_0}$. For $\sigma = 0$, the species are perfect complements; for $\sigma \rightarrow \infty$, the species are perfect substitutes in consumption. For values of σ in between, (6) describes varying degrees of substitutability/complementarity between species.

⁵Equation (6) assumes identical elasticities of substitution for each and every pair of species. A straightforward generalization would be a nested CES function with different elasticities of substitution for different groups of species. Perhaps such a sub-utility function for fish could be parameterized with the help of a global ex-vessel fish-price database Sumaila et al. (2007). This is, however, beyond the scope of this paper.

$\sigma > 1$ seems plausible, as only in this case marginal utility of fish species i is positive even if consumption of some other species $j \neq i$ is zero.

2.1 Open access

The representative household maximizes the quasi-linear utility function (5) with sub-utility (6) for food fish subject to the budget constraint $\omega = y + \sum_{i=1}^{n_0} p_i q_i$, where p_i is the price of species i . The first-order conditions of the maximization with respect to the consumption of the different species of fish are given by

$$\gamma q_i^{-\frac{1}{\sigma}} \left[\sum_{j=1}^{n_0} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = p_i \quad \text{for } i = 1, \dots, n \quad (7)$$

Rearranging these conditions (see Appendix A.1) the functions for all species i read as follows:

$$q_i = \gamma^\eta p_i^{-\sigma} P^{\sigma-\eta}, \quad \text{with } P = \left[\sum_{j=1}^{n_0} p_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (8)$$

Obviously, demand for fish species i is decreasing in the price p_i of the species under consideration and increasing in the price index P of fish (as $\eta < \sigma$). Since the price index is an increasing function of each species' price, demand for species i is increasing in the price of the other species, as different species are substitutes.

A fisherman targeting species i maximizes profit $p_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} - \omega e_i - \phi_i$ by choosing the effort level e_i , taking both the wage rate ω and output price p_i as given. The first-order condition determines the optimal effort level:

$$e_i = \left[\frac{p_i \nu_i \epsilon_i x_i^{\chi_i}}{\omega} \right]^{\frac{1}{1-\epsilon_i}} \quad (9)$$

As long as profits are positive, new vessels will enter the business. In equilibrium, profits vanish. Using the profit-maximizing effort level (9) and the corresponding harvest per vessel in this condition, we derive the open-access minimum average cost for species i as a function of its stock (the open-access line). Since in equilibrium the price must be equal to this, we obtain

$$p_i = \frac{\phi_i^{1-\epsilon_i} \omega^{\epsilon_i}}{(1-\epsilon_i)^{1-\epsilon_i} \epsilon_i^{\epsilon_i}} \nu_i^{-1} x_i^{-\chi_i} \equiv c_i x_i^{-\chi_i}, \quad (10)$$

where c_i is the cost of fishing per unit of harvest and unit of biomass in equilibrium (cf. Appendix A.2).

When m_i vessels are fishing for species i , total harvest is $m_i h_i$. Equating supply (= total harvest) and demand (8), we obtain

$$m_i h_i = \gamma p_i^{-\sigma} \left[\sum_{j=1}^{n_0} p_j^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} = \gamma^\eta [c_i x_i^{-\chi_i}]^{-\sigma} \left[\sum_j [c_j x_j^{-\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} \quad (11)$$

Here we also have used (10) to obtain the total catch as a function of the stocks.

From Equation (11) we derive the following lemma:

Lemma 1. Under open access, the steady-state stocks of all species with $\chi_i \sigma > 1$ are strictly positive, unless species i is the only remaining species.

For the proof see Appendix A.3. □

This lemma states that no stock will be depleted under open access for which the product of the stock harvest elasticity and elasticity of substitution between different species of fish exceeds unity.⁶ Note that for the Gordon-Schaefer model, which is a single-species model where $\chi = 1$ and $\sigma = +\infty$ (as demand is assumed to be perfectly elastic), this condition is always fulfilled. However, for many species it is reasonable to assume $\chi < 1$, and it also seems reasonable to assume that elasticity of substitution between different species of fish is finite, $\sigma < +\infty$. So it is likely that some species exist for which the condition $\chi_i \sigma > 1$ is not met. Those are the species that are potentially fished to collapse under open access.

2.2 First-best optimal management

As our first management scenario we consider first-best optimal management of all species, i.e. the solution of the regulator's problem to maximize the representative household's utility by choosing intertemporally optimal harvesting paths for each species. In the following we use $H_i = m_i h_i$ to denote the aggregated harvest of all

⁶Strictly speaking, the species is depleted only in infinite time.

vessels. In Appendix A.4 we show that this problem can be written as

$$\max_{y, \{H_i\}} \int_{t=0}^{\infty} \left[y + \gamma \frac{\eta}{\eta - 1} \left[\sum_{j=1}^{n_0} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} \right] \exp(-\delta t) dt \quad (12)$$

subject to

$$\omega = y + \sum_{j=1}^{n_0} c_j H_j x_j^{-\chi_j} \quad (\text{numeraire sector}) \quad (13)$$

and (1), i.e. the fish stock dynamics for all species i .

The conditions for first-best optimal management are given in Appendix A.5.

Using these conditions we obtain the following lemma:

Lemma 2. Under first-best management, the steady-state stocks of all species with either $\chi_i \sigma > 1$ (provided species i is not the only remaining species) or $\rho_i > \delta$ are strictly positive.

For the proof see Appendix A.6. □

According to this lemma, one of two independent conditions is sufficient to guarantee a strictly positive steady-state stock under first-best optimal management. The first condition implies that it cannot be optimal to deplete a stock that would not be depleted under open access (see Lemma 1). The other condition states that it cannot be optimal to deplete a stock that has an intrinsic growth rate greater than the rate of discount. Note that Lemma 2 provides a sufficient but not necessary condition for stocks to be strictly positive in social optimum.

2.3 Second-best management

In our second management scenario we assume that the regulator can only control fisheries of some species k but takes into account the spill-overs of regulation on the harvest and stock dynamics of the other species that are assumed to be harvested under open access. For simplicity, we consider only the case where the regulator can control for only one species. The results generalize in a straightforward way to controlling for several, but less than all, species.

Harvest quantities of the non-managed species $i \neq k$ are determined by inverse demand (7) and the open-access Condition (10), i.e.

$$\gamma q_i^{-\frac{1}{\sigma}} \left[q_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = c_i x_i^{-\chi_i} \quad \text{for all species } i \neq k \quad (14)$$

Using the market-clearing condition $q_i = H_i$ for all species, the regulator's optimization problem in the second-best management setting reads as follows:

$$\max_{y, \{H_i\}} \int_{t=0}^{\infty} \left[y + \gamma \frac{\eta}{\eta-1} \left[H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} \right] \exp(-\delta t) dt \quad (15)$$

subject to (1) (the fish stock dynamics for all species i), (13), and (14) with $q_i = H_i$, i.e. the constraint that all non-managed species are harvested under open access. The conditions for second-best optimal management are given in Appendix A.7.

2.4 Myopic management

In the scenario with myopic management the regulator ignores the effects of managing species k on the stocks of all other species being harvested under an open-access setting. The optimization problem is similar to the problem of determining second-best optimal management, except that the regulator ignores the effect of management on demand for other species $i \neq k$. This means that the optimization problem is formally identical to (15), except that the regulator ignores constraint (14). The conditions for myopic management are given in Appendix A.8.

3 Preferences for diversity and cascading collapse of fish stocks under open access

As the first step in our analysis, we consider the open-access setting described in section 2.1. Equation (11) relates the total catch of species i to the stocks of the different species. Note that relationship (11) always holds, both in and off equilibrium. Since the different species are substitutes, the harvest of species i will

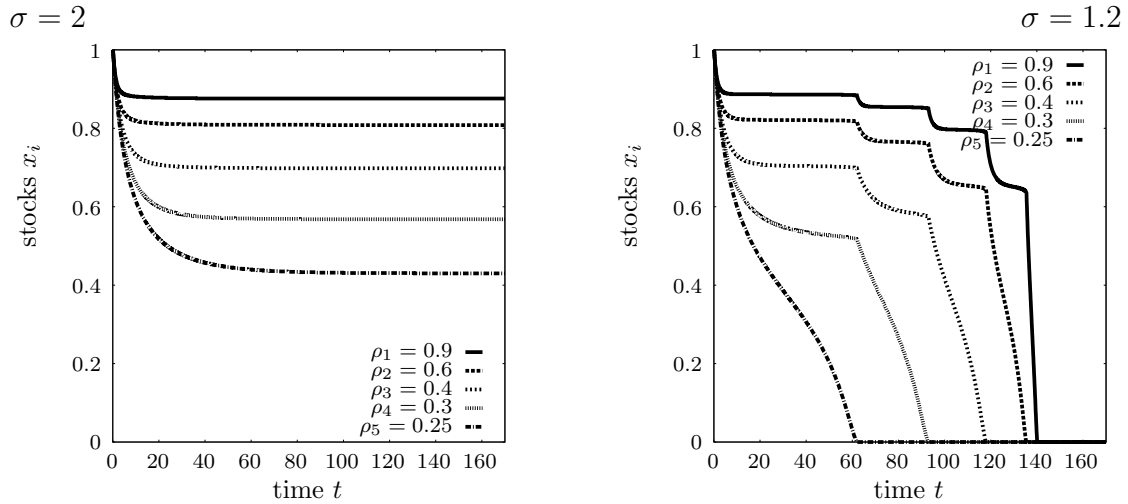


Figure 1: The stocks of five fish species with different intrinsic growth rates. Choice of parameters: $\kappa_i = 1$, $c_i = 1$, $\chi_i = 0.5$, for all $i = 1, \dots, 5$, $\eta = 1$, $\gamma = 0.6$. Left-hand figure: weak preferences for variety: $\sigma = 2$; Right-hand figure: strong preferences for variety: $\sigma = 1.2$.

increase if the stock of some other species decreases. Condition (11) in particular implies the following proposition:

Proposition 1. The collapse of one stock increases open-access harvest of all other stocks.

This means that the collapse of one stock makes it more likely that other stocks will collapse too, which may provide an explanation for the cascading collapse of fish stocks that has been observed. For two different elasticities of substitution, Figure 1 shows the simulation results⁷ of the open-access dynamics for a multi-species fishery with five fish stocks with different intrinsic growth rates and identical parameters otherwise. For the case of weak preferences for diversity ($\sigma = 2$) the resulting time paths are depicted on the left-hand side of Figure 1. Starting at an equilibrium without fishing, all stocks first decline but then all reach positive, though not optimal equilibrium levels. On the right-hand side of Figure 1, we display the time paths for the case of strong preferences for diversity ($\sigma = 1.2$). We

⁷All simulations were done with MATLAB R2009a. Codes are available from the authors.

see that the least resilient species, i.e. the species with the lowest intrinsic growth rate, suffers a serious decline in stock, while initially all the other species seemingly approach positive equilibrium values. Once the least resilient species has been depleted, however, the stock of the species with the second-lowest growth rate also starts to decline sharply. After this stock has also collapsed, the same happens to the species with the next-lowest growth rate, and so on. In this example, all stocks collapse under the open-access regime. For a different parameter specification (in particular different harvesting functions) we can have both, some stocks being depleted while other stocks reach positive steady-state levels under open-access fishery.

Preferences for diversity in the open-access setting thus may provide an explanation for the cascading collapse of fish stocks that has recently been documented on a world wide scale Costello et al. (2008), Heal and Schlenker (2008), Worm et al. (2006). This is perhaps surprising, as the model does not include any exogenous dynamic driving forces such as technical progress in fishing technology or increasing overall demand for fish, which probably are also important factors in the collapse seen in reality (Skonhøft 2009, Squires and Vestergaard 2009). In this model, the cascading collapse of fish stocks is a pure outcome of the open-access transition dynamics with preferences for food-fish diversity.

4 Comparison of fishery management scenarios

Turning to fishery management, the main question is how the outcomes of the different management scenarios compare. Before we study the dynamic outcome using some numerical examples, it is insightful to analytically study the steady-states of the fishery under the different management scenarios for the simplest case: symmetric species. That is, we assume that all species have the same growth rates ($g_i(x) = g(x)$ for all i) and all species are harvested with the same technology ($c_i = c$, $\chi_i = \chi$ for all i). The assumption of symmetric species is sufficient, but not necessary for the following propositions 2 and 3. Yet, for the case of very

asymmetric species general results can not be derived.

Proposition 2. If species are symmetric with respect to ecological and harvesting parameters and if steady-state harvest under first-best optimal management is greater than harvest in the open-access equilibrium, $g(\hat{x}) \geq g(x^)$, second-best management must be less strict than first-best management, i.e. the steady-state stock of the managed species is smaller under second-best management than under first-best management.*

For the proof see Appendix A.9. □

The intuitive reason for this result is that stricter management means reducing harvest more strongly, at least if the steady-state stock of the managed species exceeds the maximum sustainable yield stock. Since the other species are substitutes for the managed species, the reduced supply of the managed species leads to increased demand for the other species. While under first-best management the harvest of all species is controlled, under second-best management this increased demand would lead to an increased fishing pressure on the non-managed species. In order to avoid this welfare-reducing effect on non-managed species, second-best management must be less strict than first-best management.

The condition $g(\hat{x}) \geq g(x^*)$ in Proposition 2 is sufficient but not necessary for the result. The numerical analysis below shows a number of examples where second-best management must be less strict than first-best management, including heterogenous species and the case that $g(\hat{x}) < g(x^*)$.

Comparing the steady states under second-best management and myopic management, we find that myopic management is always too strict.

Proposition 3. If species are symmetric with respect to ecological and harvesting parameters, myopic management is too strict compared to second-best management, i.e. the steady-state stock of the managed species is larger under myopic management than under second-best management.

For the proof see Appendix A.10. □

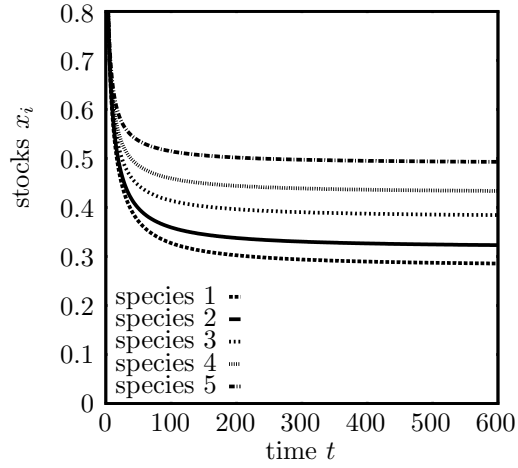
The intuitive reason for this result is that myopic management neglects the spillover

effect that increases fishing pressure on the non-managed stocks and is thus stricter than second-best management.

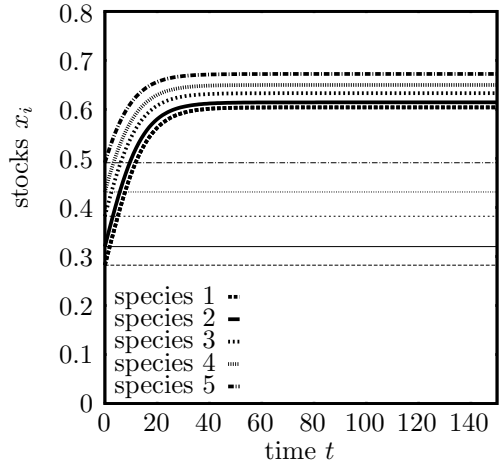
In the following, we compare the different management scenarios using two numerical examples. In both examples species differ with regard to intrinsic growth rates, while harvesting technologies and carrying capacities are the same for all species. For both examples, elasticity of substitution between species is $\sigma = 2$. Harvesting output elasticity of the stocks is $\chi = 0.44$ in the first example and $\chi = 0.33$ in the second one. The discount rate is $\delta = 0.1$.

Figure 2 shows the results for the first example. The graph top left shows the open-access dynamics starting at initial stock sizes equal to the carrying capacities. The graph top right shows the dynamics under first-best optimal management. Starting at open-access steady-state levels, which are the initial levels in all three management scenarios, all five stocks initially grow fast and reach steady-states well above the maximum sustainable yield levels. This is the case because larger stock sizes induce lower cost of harvesting. The lower graphs show the dynamics of the stocks if species 1, the species with the smallest intrinsic growth rate, is managed in a second-best optimal way (bottom left) or in a myopic way (bottom right). Under second-best management, harvest of the managed species initially decreases compared to harvest in open-access equilibrium. This leads to an increase of the managed stock. Since species are substitutes, harvest of the non-managed species increases, and the corresponding stocks decline. However, as the steady-state stock and harvest of species 1 are higher under second-best management than under open access, harvest levels for the non-managed species decline and the steady-state stocks are also above the initial open-access steady-state levels. This pattern reconfirms our analytic result from Proposition 2 establishing that second-best optimal management must be less strict than first-best optimal management. The case of myopic management of species 1 is similar, except for one notable difference: the managed species has the smallest stock level under open access, but the highest steady-state stock level under myopic management, considerably exceeding the steady-state level. This observation is in line with Proposition 3.

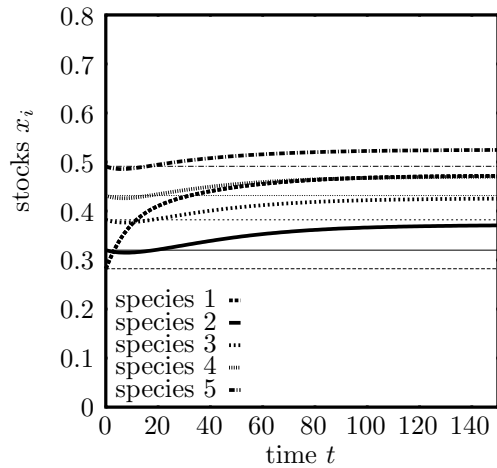
open access



first-best management



second-best management of species 1



myopic management of species 1

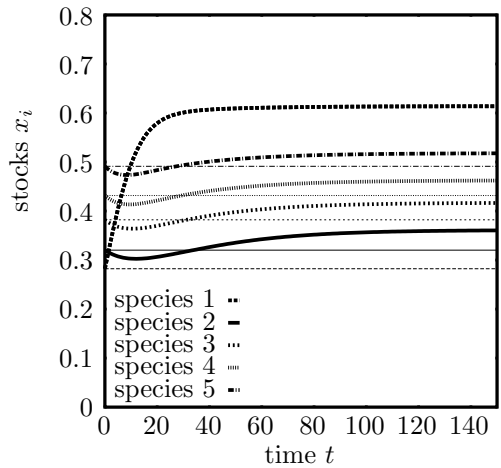


Figure 2: The stocks of five species of fish with different intrinsic growth rates, all other parameters the same: $\kappa_i = 1$, $c_i = 1$, $\chi_i = 0.44$, for all $i = 1, \dots, 5$, $\eta = 1$, $\gamma = 0.5$, $\delta = 0.1$ and an elasticity of substitution $\sigma = 2$. Intrinsic growth rates are $\rho_1 = 0.25$, $\rho_2 = 0.26$, $\rho_3 = 0.28$, $\rho_4 = 0.3$, and $\rho_5 = 0.33$. The small horizontal lines indicate the open-access equilibrium stock sizes, which are the initial stock sizes before regulation starts.

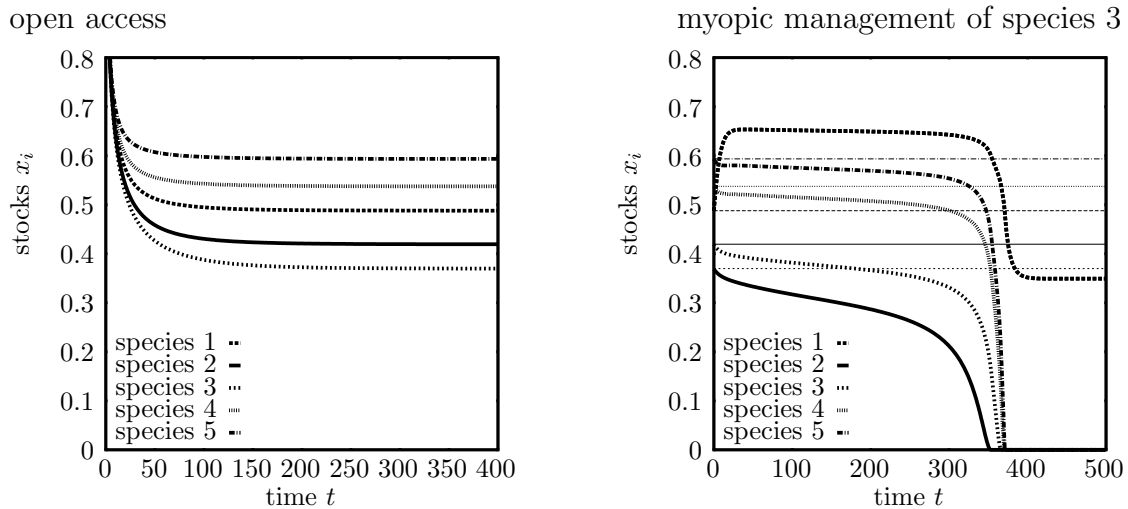


Figure 3: The stocks of five species of fish with different intrinsic growth rates under open access harvesting (diagram left) and under myopic management of species 3 (diagram right). Parameters are the same as in Figure 2, except that $\chi_i = 0.33$, and $\gamma = 0.44$.

In the example shown in Figure 3, the managed species 3 has an intermediate intrinsic growth rate and hence an intermediate open-access stock level. While the outcomes of first-best and second-best management are similar to the previous example, and hence not shown in the figure, the outcome under myopic regulation is quite different (right-hand side of Figure 3). In this case, harvest of the myopically managed species falls drastically below the open access equilibrium level, inducing a sharp increase in the harvest of all other species, including the species with the smallest intrinsic growth rate. Eventually, harvest of this species declines, reinforcing the increase in harvest of the other non-managed species. This cascade ultimately leads to the collapse of all species except for the managed one, which finally reaches a steady-state level below the initial open-access level. The reason for this perhaps counterintuitive result is that the managed fishery has to supply the whole market in the end, rather than just a fraction as at $t = 0$. Under the higher demand for this species it is optimal to harvest the stock down to a level that is below the open-access level under a comparatively low demand for this

particular species. This example shows that, through demand-side feedback, regulation of one species may have detrimental effects on other stocks that are not subject to regulation.

5 Preferences for diversity and the number and size of fish stocks

In this section we investigate how the preference for diversity, i.e. the elasticity of substitution σ , impacts on both the number and size of fish stocks in the steady states under open access and under first-best optimal management. For the analytical results in this section we concentrate on the case where species differ with respect to their intrinsic growth rates (w.l.o.g. we assume $\rho_1 < \rho_2 < \dots < \rho_n$), but are identical otherwise (i.e. $\kappa_i = \kappa$, $c_i = c$, and $\chi_i = \chi < 1$ for all $i = 1, \dots, n_0$). For simplicity, we also assume that aggregate demand elasticity for fish is equal to one, i.e. $\eta = 1$.

Proposition 4. (i) Steady-state stocks under open access are ordered according to intrinsic growth rates, $x_1^ \leq x_2^* \leq \dots \leq x_n^*$. The stronger the preferences for diversity are (i.e. the lower σ is), the larger the difference between the smallest and the largest stock will be, i.e. $d(x_n^* - x_1^*)/d\sigma \leq 0$. (ii) Steady-state stocks under first-best management are ordered according to intrinsic growth rates, $\hat{x}_1 \leq \hat{x}_2 \leq \dots \leq \hat{x}_n$. The stronger the preferences for diversity are (i.e. the lower σ is), the larger the difference between the smallest and the largest stock will be, i.e. $d(\hat{x}_n - \hat{x}_1)/d\sigma \leq 0$.*

For the proof see Appendix A.11. □

The result is illustrated in Figure 4. The intuitive reason for this result is that stronger preferences for diversity mean that consumers have a stronger desire to consume different species in similar proportions. This implies greater similarity of harvest levels for different species. This in turn leads to diverging stocks, as under harvesting pressure the stocks of less resilient species will decline more sharply

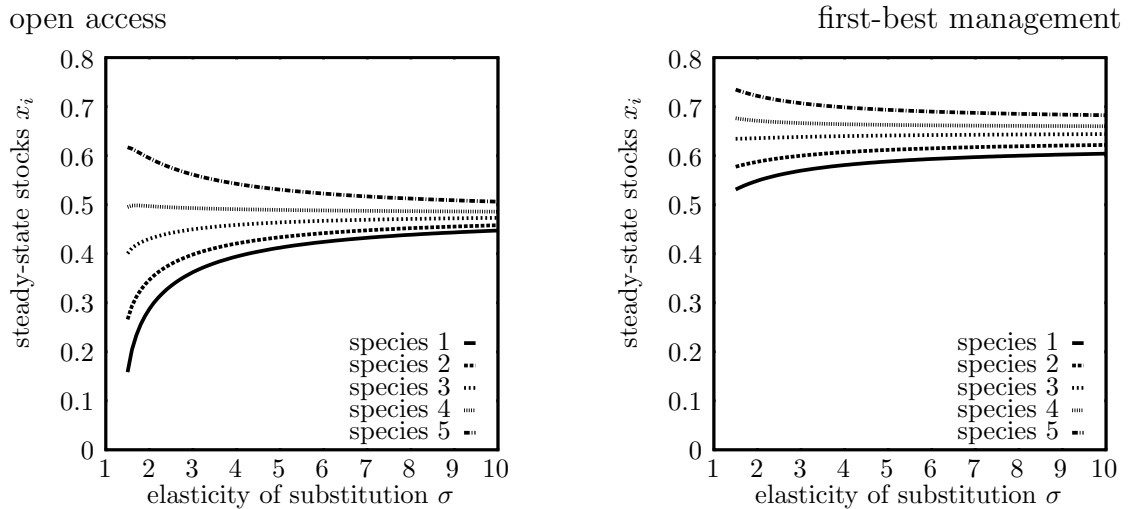


Figure 4: The steady-state stocks of five species of fish with different intrinsic growth rates, all other parameters the same: $\kappa_i = 1$, $c_i = 1$, $\chi_i = 0.7$, for all $i = 1, \dots, 5$, $\eta = 1$, $\gamma = 0.55$, $\delta = 0.1$ for different elasticities of substitution σ . Intrinsic growth rates are $\rho_1 = 0.17, \rho_2 = 0.2, \rho_3 = 0.25, \rho_4 = 0.3$, and $\rho_5 = 0.4$.

than the stocks of more resilient species. If preferences for diversity are weaker, consumers are less willing to pay a high price for scarce species.

Under strong preferences for diversity, the harvesting pressure may lead to the collapse of the least resilient stocks. More precisely we can state

Proposition 5. The stronger the preferences for diversity are (i.e. the lower σ is), the smaller will be the number n^ of stocks with strictly positive stock and harvest in the open-access steady state, i.e. $dn^*/d\sigma \geq 0$.*

For the proof see Appendix A.12. □

A similar statement can be made for the case where species are identical with respect to ecological growth parameters (ρ_i and κ_i) but differ with respect to harvesting technology, i.e. the cost parameters c_i . We can not, however, make a more general statement of the kind that, if species differ with respect to both the ecological parameters and harvesting technology, higher elasticities of substitution would always induce a higher equilibrium number of surviving stocks. In other words,

it may be the case that the equilibrium number of species rises with stronger preferences for diversity. The reason is that the effect of different ecological characteristics and the effect of different harvesting technologies may go in opposite directions.

It is however possible to determine a lower bound n_{\min}^* on the number of stocks with strictly positive steady-state levels in open-access equilibrium. This lower bound is given by the number of species for which the condition $\chi_i \sigma > 1$ in Lemma 1 holds. Obviously, this number weakly increases with σ . Under first-best management, the lower bound \hat{n}_{\min} for the number of stocks with strictly positive steady-state levels is the number of species that fulfill either condition $\chi_i \sigma > 1$ or condition $\rho_i > \delta$. This number cannot be smaller than n_{\min}^* . Moreover, \hat{n}_{\min} weakly increases with elasticity of substitution σ , and finally the difference between \hat{n}_{\min} and n_{\min}^* increases with σ , i.e. $d(\hat{n}_{\min} - n_{\min}^*)/d\sigma \leq 0$.⁸ This result may be interpreted as saying that stronger preferences for diversity increase the need for first-best management.

In order to assess the quantitative effect of a change in σ on the number of stocks with strictly positive levels under open access and under first-best management, we have conducted a Monte-Carlo-like simulation. Using the Latin Hypercube sampling method, we randomly and independently picked the three parameters ρ_i , c_i , χ_i , and the initial stock size $x_{i,0}$ according to uniform distributions, while $\kappa_i = 1$ for all i , $\eta = 1$, $\gamma = 6.6$, and $\delta = 33\%$ are fixed.⁹ The intrinsic growth rates are drawn from the interval $\rho_i \in [0.1; 1.5]$, the cost from an interval $c_i \in [0.75; 1.25]$, and the stock elasticities from the interval $\chi_i \in [0.25; 1]$. Then by varying σ we determine the number of species with strictly positive steady-state stocks under open access and under first-best management.

⁸To prove this, let \hat{n}_1 be the number of species for which $\rho_i > \delta$, but $\chi_i \sigma \leq 1$, while n_{\min}^* is the number of species for which $\chi_i \sigma > 1$, as before. By construction of \hat{n}_1 , we obtain $\hat{n}_{\min} = \hat{n}_1 + n_{\min}^*$. Thus we have $\hat{n}_{\min} - n_{\min}^* = \hat{n}_1 \geq 0$ and hence $d(\hat{n}_{\min} - n_{\min}^*)/d\sigma = d\hat{n}_1/d\sigma \leq 0$, which is obviously the case.

⁹We have chosen a comparatively high discount rate $\delta = 33\%$ in order to trigger the collapse of some stocks in a social optimum.

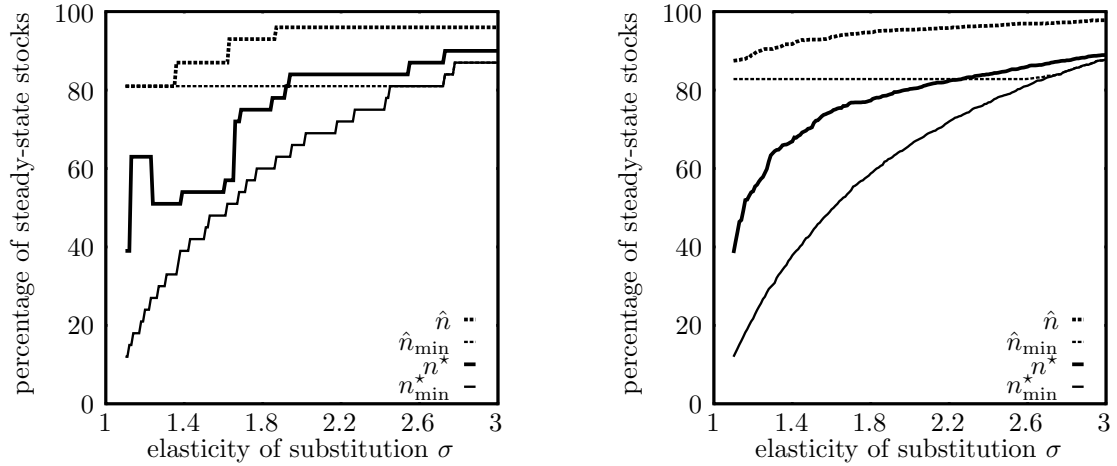


Figure 5: The simulated number of stocks in the open-access steady state (thick solid lines) and the first-best optimal steady state (dotted lines) as a function of σ for one parameter set (diagram left) and averaged over a random sample of 50 parameter sets (diagram right).

In Figure 5, the thick solid lines depict the steady-state number of stocks as a function of σ under open access, while the thick dotted lines depict the steady-state number of stocks under optimal management. The thin lines depict the corresponding lower bounds n_{\min}^* and \hat{n}_{\min} . For each of these lines the randomly chosen parameters are fixed. The left-hand diagram shows the results for one specific set of parameters, while the right-hand diagram shows the result for an average of 50 such parameter sets. As seen on the left, the steady-state number of stocks under open access does not increase monotonically in σ . The downward jump at $\sigma = 1.24$ is due to the different effects of the different parameters discussed above. Averaging over all runs, however, the relationship between σ and the steady-state number of species becomes almost monotonic, both under open access and under optimal management. Both thin lines, depicting the lower bounds n_{\min}^* on the open-access steady-state number of stocks and n_{\min}^* on the first-best steady-state number of stocks, are weakly increasing with σ , and their difference is decreasing with σ , as discussed above. As the diagram on the right-hand side of Figure 5 shows, these relationships also hold true for the actual steady-state

numbers of stocks.

6 Conclusions

In this paper we have analyzed consumers' preferences for diversity of food fish in a multi-species fishery model. Our results provide a new perspective on the economics of collapsing fish stocks and multi-species fishery management.

We have shown that consumer preferences for diversity may play an important role in the ongoing collapse of fish stocks observed. Of course, other factors also contribute to the collapse of fish stocks, in particular technical progress or other exogenous driving factors that increase the harvesting pressure on fish stocks Skonhoft (2009), Squires and Vestergaard (2009). In contrast to these explanations for the collapse of fish stocks, our analysis highlights the fact that the different fisheries are connected by demand-side interactions, which has important consequences for management.

Recent empirical studies have argued that management by individual transferable quotas (ITQ) could significantly reduce the global trend toward widespread collapse of fish stocks Costello et al. (2008), Heal and Schlenker (2008). These studies, however, fail to distinguish between stocks managed in a second-best way, taking into account spillovers to non-ITQ fisheries, and stocks managed myopically. It is likely that the database used in these studies contains at least some stocks that were managed myopically. Our analysis has shown that myopic management of some stocks via ITQs may even have caused the collapse of non-ITQ stocks, although the managed stock itself would not collapse. Empirically, such adverse effects of regulation have recently been found in the Hawaiian long line swordfish fishery, where myopic management intended to protect endangered sea turtles has led to an overall increase in sea-turtle bycatch, probably due to spillover effects Rausser et al. (2009).

The conclusion from this is that in order to prevent the world-wide collapse of fish stocks it is necessary to move from the current ITQ management of some stocks

to a coordinated (first-best) management of all fish stocks. If first-best coordinated management of all species is not possible, second-best management that takes economic feedbacks into account could still reduce the number of collapsed stocks.

Finally, we have shown that on average stronger preferences for diversity on average lead to lower diversity of food-fish supply in the long run, as more stocks are fished to collapse. Thus an increase in the preferences for diversity calls for more globally coordinated management of fish stocks. Successful multi-species fishery management that takes economic interactions of fisheries into account would contribute to solving the problem of overfishing with respect both to bringing individual fish stocks nearer to efficient levels and to reducing the overall number of stocks that are fished to collapse.

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A Appendix

A.1 Derivation of demand function (8): Taking both sides of Condition (7) to the power of $1 - \sigma$ and summing over i , we obtain

$$\sum_i^{n_0} p_i^{1-\sigma} = \left[\gamma v^{\frac{\eta-1}{\eta} - \frac{\sigma-1}{\sigma}} \right]^{1-\sigma} v^{\frac{\sigma-1}{\sigma}} = \left[\gamma v^{-\frac{1}{\eta}} \right]^{1-\sigma} \quad (16)$$

$$\Leftrightarrow v = \gamma^\eta \left[\sum_i^{n_0} p_i^{1-\sigma} \right]^{-\frac{\eta}{1-\sigma}} = \gamma^\eta P^{-\eta} \quad (17)$$

where P is the price index for fish. Substituting into (7) and rearranging leads to the demand functions (8).

A.2 Cost of fishing: Using (10) in (9) we obtain equilibrium harvesting effort

$$e_i = \frac{\epsilon_i}{1 - \epsilon_i} \frac{\phi_i}{\omega} \quad (18)$$

Substituting into the harvesting function (3) we derive equilibrium harvest as a function of the species' biomass: $h_i = \nu_i x_i^{\chi_i} \left[\frac{\epsilon_i}{1 - \epsilon_i} \frac{\phi_i}{\omega} \right]^{\epsilon_i}$. Total cost of fishing is $\omega e_i + \phi_i = \phi_i \epsilon_i / (1 - \epsilon_i) + \phi_i = \phi_i / (1 - \epsilon_i)$. Hence cost per unit of harvest is

$$[\omega e_i + \phi_i] h_i^{-1} = \frac{\phi_i}{1 - \epsilon_i} \nu_i^{-1} x_i^{-\chi_i} \left[\frac{\epsilon_i}{1 - \epsilon_i} \frac{\phi_i}{\omega} \right]^{-\epsilon_i} = \frac{\phi_i^{1-\epsilon_i} \omega^{\epsilon_i}}{(1 - \epsilon_i)^{1-\epsilon_i} \epsilon_i^{\epsilon_i}} \nu_i^{-1} x_i^{-\chi_i} \quad (19)$$

A.3 Proof of Lemma 1: We show that for a sufficiently small initial size the stock x_i would increase under open access. For this purpose, we show that growth $g_i(x_i) - n_i h_i$ increases with x_i at $x_i = 0$ under the conditions of the proposition.

Using (11), we get

$$\begin{aligned}
& \lim_{x_i \rightarrow 0} \frac{\partial}{\partial x_i} \left\{ \rho_i x_i \left[1 - \frac{x_i}{\kappa_i} \right] - \gamma^\eta [c_i x_i^{-\chi_i}]^{-\sigma} \left[\sum_j^{n_0} [c_j x_j^{-\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} \right\} \\
&= \lim_{x_i \rightarrow 0} \left\{ \rho_i \left[1 - 2 \frac{x_i}{\kappa_i} \right] - \chi_i \sigma \gamma^\eta c_i^{-\sigma} x_i^{\chi_i \sigma - 1} \left[\sum_j^{n_0} [c_j x_j^{-\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} \right. \\
&\quad \left. - \chi_i (\eta - \sigma) \gamma^\eta c_i^{-2\sigma} x_i^{2\chi_i \sigma - \chi_i - 1} \left[\sum_j^{n_0} [c_j x_j^{-\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1} - 1} \right\} \\
&\geq \lim_{x_i \rightarrow 0} \left\{ \rho_i \left[1 - 2 \frac{x_i}{\kappa_i} \right] - \chi_i \sigma \gamma^\eta c_i^{-\sigma} x_i^{\chi_i \sigma - 1} [c_j x_j^{-\chi_j}]^{\eta - \sigma} \right\} = \rho_i > 0 \quad (20)
\end{aligned}$$

A.4 Setting up the regulator's optimization problem: The regulator chooses consumption of the numeraire y and of each fish species q_i , harvesting effort e_i , and number of vessels n_i so as to maximize the present value of utility. Using (6) in (5) the problem can be written as

$$\max_{y, \{q_i, e_i, n_i\}} \int_{t=0}^{\infty} \left[y + \gamma \frac{\eta}{\eta - 1} \left[\sum_{j=1}^{n_0} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} \right] \exp(-\delta t) dt \quad (21)$$

subject to $q_i = n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i}$, (1), and (4). Plugging in (4) and using the co-state variables π_i for the Constraints $q_i = n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i}$ and μ_i for Constraints (1) the current-value Hamiltonian reads

$$\begin{aligned}
\mathcal{H} = & \gamma \frac{\eta}{\eta - 1} \left[\sum_{i=1}^{n_0} q_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \omega \left(1 - \sum_{i=1}^{n_0} n_i e_i \right) - \sum_{i=1}^{n_0} n_i \phi_i \\
& + \sum_{i=1}^{n_0} \pi_i [n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} - q_i] + \sum_{i=1}^{n_0} \mu_i [g_i(x_i) - n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i}] \quad (22)
\end{aligned}$$

The necessary conditions for optimal effort and number of vessels are

$$\frac{\partial \mathcal{H}}{\partial e_i} = 0 \quad \Leftrightarrow \quad [\pi_i - \mu_i] \epsilon_i n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i - 1} = \lambda \omega n_i \quad (23)$$

$$\frac{\partial \mathcal{H}}{\partial n_i} = 0 \quad \Leftrightarrow \quad [\pi_i - \mu_i] \nu_i x_i^{\chi_i} e_i^{\epsilon_i} = \lambda \omega e_i + \lambda \phi_i \quad (24)$$

Both conditions hold for all species i . Using (23) in (24), we derive the optimal effort levels $e_i = \epsilon_i \phi_i / ((1 - \epsilon_i) \omega)$. Evidently, the effort per vessel in the open-access setting (Equation 18) is equal to the optimal effort level. To determine

the optimal transitional dynamics toward the steady state, we reformulate the regulator's optimization problem by considering total harvest $H_i = n_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i}$ as the decision variable. Using $e_i = \epsilon_i \phi_i / ((1 - \epsilon_i) \omega)$, total cost of fishing species i is

$$\begin{aligned} \omega n_i e_i + \phi_i n_i &= \frac{\omega H_i}{\nu_i x_i^{\chi_i} e_i^{\epsilon_i - 1}} + \frac{\phi_i H_i}{\nu_i x_i^{\chi_i} e_i^{\epsilon_i}} = \left[\frac{\omega}{\nu_i \left[\frac{\epsilon_i \phi_i}{(1 - \epsilon_i) \omega} \right]^{\epsilon_i - 1}} + \frac{\phi_i}{\nu_i \left[\frac{\epsilon_i \phi_i}{(1 - \epsilon_i) \omega} \right]^{\epsilon_i}} \right] \frac{H_i}{x_i^{\chi_i}} \\ &= [\epsilon_i + 1 - \epsilon_i] \frac{\omega^{\epsilon_i} \phi_i^{1 - \epsilon_i}}{\nu_i (1 - \epsilon_i)^{1 - \epsilon_i} \epsilon_i^{\epsilon_i}} \frac{H_i}{x_i^{\chi_i}} = c_i \frac{H_i}{x_i^{\chi_i}} \end{aligned} \quad (25)$$

Substituting this and the market-clearing condition $q_i = H_i$ into (21) and rearranging leads to the optimization problem (12).

A.5 Conditions for first-best optimal management: The current-value Hamiltonian for the regulator's optimization problem is

$$\mathcal{H} = \gamma \frac{\eta}{\eta - 1} \left[\sum_{i=1}^{n_0} H_i^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\eta - 1}{\eta} \frac{\sigma}{\sigma - 1}} + \omega - \sum_{i=1}^{n_0} c_i H_i x_i^{-\chi_i} + \mu_i [g_i(x_{i,t}) - H_i]$$

The first-order conditions for the optimal management of the fishery read

$$\frac{\partial \mathcal{H}}{\partial H_i} = 0 \quad \Leftrightarrow \quad \gamma q_i^{-\frac{1}{\sigma}} \left[\sum_{j=1}^{n_0} q_j^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\eta - 1}{\eta} \frac{\sigma}{\sigma - 1} - 1} = c_i x_i^{-\chi_i} + \mu_i \quad (26)$$

$$\frac{\partial \mathcal{H}}{\partial x_i} = \delta \mu_i - \dot{\mu}_i \quad \Leftrightarrow \quad \chi_i c_i H_i x_i^{-\chi_i - 1} + \mu_i g'_i(x_i) = \delta \mu_i - \dot{\mu}_i \quad (27)$$

where both conditions (26) and (27) hold for all species i . From (26) and (13) we derive the total harvest of species i as a function of the shadow prices μ_i :

$$H_i = \gamma^\eta (c_i x_i^{-\chi_i} + \mu_i)^{-\sigma} \left[\sum_j^{n_0} (c_j x_j^{-\chi_j} + \mu_j)^{1 - \sigma} \right]^{-\frac{\sigma - \eta}{\sigma - 1}} \quad (28)$$

Note that (28) holds both in and off the steady state. In a steady state, the shadow price of fish does not change, i.e. $\dot{\mu}_i = 0$. Using this in (27), we obtain the steady-state shadow price of the stock of species i

$$\mu_i = c_i x_i^{-\chi_i} \frac{\chi_i g_i(x_i)}{x_i [\delta - g'_i(x_i)]} = c_i x_i^{-\chi_i} \frac{\chi_i \rho_i [1 - x_i / \kappa_i]}{\delta - \rho_i [1 - 2 x_i / \kappa_i]} \quad (29)$$

The social cost of harvesting one unit of species i , reflected by $c_i x_i^{-\chi_i} + \mu_i$, equals the current marginal cost of harvest plus the present value of additional future harvesting cost due to the marginally smaller fish stock.

A.6 Proof of Lemma 2: With a similar argument as in the proof of Lemma 1 (section A.3), we can show that if $\chi_i \sigma > 1$ the optimal steady-state harvest of species i is smaller than natural growth for small stock sizes of species i . This is the case because the optimal shadow price of species i is proportional to the open-access price $c_i x_i^{-\chi_i}$. It remains to be shown that $\rho_i > \delta$ is a sufficient condition for a strictly positive steady-state stock. Consider the steady-state shadow price of the stock i (Equation 29). If $\rho_i > \delta$, this shadow price diverges to $+\infty$ as the stock approaches $\underline{x}_i = \frac{\kappa_i}{2} \left(1 - \frac{\delta}{\rho_i}\right)$ from above. Hence the optimal steady-state stock must be greater than \underline{x}_i with $\underline{x}_i > 0$ whenever $\rho_i > \delta$.

A.7 Conditions for second-best management: The current-value Hamiltonian for the regulator's optimization problem is

$$\begin{aligned} \mathcal{H} = & \gamma \frac{\eta}{\eta - 1} \left[H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} + \omega - \sum_{i=1}^{n_0} c_i H_i x_i^{-\chi_i} + \sum_{i=1}^{n_0} \mu_i [g_i(x_{i,t}) - H_i] \\ & + \sum_{i \neq k} \lambda_i \left\{ c_i x_i^{-\chi_i} - \gamma H_i^{-\frac{1}{\sigma}} \left[H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} \right\} \end{aligned}$$

The first-order conditions for the second-best management of the fishery with respect to H_k and x_k read

$$\gamma H_k^{-\frac{1}{\sigma}} \left[\sum_{j=1}^{n_0} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = c_k x_k^{-\chi_k} + \mu_k - \sum_{i \neq k} \lambda_i \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right) H_k^{-\frac{1}{\sigma}} c_i x_i^{-\chi_i}}{H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}}} \quad (30)$$

$$\chi_k c_k H_k x_k^{-\chi_k - 1} + \mu_k g'_k(x_k) = \delta \mu_k - \dot{\mu}_k \quad (31)$$

where Condition (30) has been simplified using (14). Conditions (30) and (31) must hold for the managed species k . In addition, for the non-managed species $i \neq k$ the following first-order conditions with respect to H_i and x_i must hold for all species $i \neq k$

$$c_i x_i^{-\chi_i} = c_i x_i^{-\chi_i} + \mu_i - \lambda_i \frac{c_i x_i^{-\chi_i}}{\sigma H_i} - \sum_{l \neq k} \lambda_l \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right) H_l^{-\frac{1}{\sigma}} c_i x_i^{-\chi_i}}{H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}}} \quad (32)$$

$$\delta \mu_i - \dot{\mu}_i = \chi_i c_i H_i x_i^{-\chi_i - 1} + \mu_i g'_i(x_i) - \lambda_i \chi_i c_i x_i^{-\chi_i - 1} \quad (33)$$

Since in a steady state we have $\dot{\mu}_i = 0$ for all species, we obtain

$$\mu_k = c_k x_k^{-\chi_k} \frac{\chi_k g_k(x_k)}{x_k [\delta - g'_k(x_k)]} \quad (34)$$

for the managed species, just as in the first-best, and

$$\mu_i = c_i x_i^{-\chi_i} \frac{\chi_i g_i(x_i) - \lambda_i}{x_i [\delta - g'_i(x_i)]} \quad (35)$$

for the non-managed species. Plugging this into (32), we find

$$c_i x_i^{-\chi_i} \frac{\chi_i g_i(x_i) - \lambda_i}{x_i [\delta - g'_i(x_i)]} = \lambda_i \frac{c_i x_i^{-\chi_i}}{\sigma H_i} + \sum_{l \neq k} \lambda_l \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right) H_l^{-\frac{1}{\sigma}} c_i x_i^{-\chi_i}}{H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}}} \quad (36)$$

$$\sum_{l \neq k} \lambda_l \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right) H_l^{-\frac{1}{\sigma}}}{H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}}} = \frac{\chi_i g_i(x_i) - \lambda_i}{x_i [\delta - g'_i(x_i)]} - \frac{\lambda_i}{\sigma g_i(x_i)} \quad (37)$$

A.8 Conditions for myopic management: The current-value Hamiltonian for the regulator's optimization problem is

$$\mathcal{H} = \gamma \frac{\eta}{\eta - 1} \left[H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} + \omega - \sum_{i=1}^{n_0} c_i H_i x_i^{-\chi_i} + \sum_{i=1}^{n_0} \mu_i [g_i(x_{i,t}) - H_i]$$

The first-order conditions for the optimal management of the fishery with respect to H_k and x_k read

$$\gamma H_k^{-\frac{1}{\sigma}} \left[\sum_{j=1}^{n_0} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = c_k x_k^{-\chi_k} + \mu_k \quad (38)$$

$$\chi_k c_k H_k x_k^{-\chi_k - 1} + \mu_k g'_k(x_k) = \delta \mu_k - \dot{\mu}_k \quad (39)$$

These conditions are formally identical with conditions (26) and (27) for the first-best optimal management, with two notable differences. First, they hold only for the managed species. Secondly, under myopic management the open-loop solution of conditions (26) and (27) would lead to time-inconsistent management. Hence (38) and (39) have to be solved as the closed-loop optimal reaction of the regulator to the (changing) prices of all other species.

A.9 Proof of Proposition 2: For identical species, the condition for first-best steady-state management becomes

$$\left[\gamma c \hat{x}^{-\chi} \left[1 + \frac{\chi g(\hat{x})}{\hat{x} [\delta - g'(\hat{x})]} \right] \right]^\eta n_0^{-\frac{\sigma-\eta}{\sigma-1}} = g(\hat{x}) \quad (40)$$

$$\Leftrightarrow \gamma g(\hat{x})^{-\frac{1}{\sigma}} \left[g(\hat{x})^{\frac{\sigma-1}{\sigma}} n_0 \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}-1} = c \hat{x}^{-\chi} \left[1 + \frac{\chi g(\hat{x})}{\hat{x} [\delta - g'(\hat{x})]} \right] \quad (41)$$

The right-hand side (RHS) of this equation is monotonically decreasing with \hat{x} ,

$$\begin{aligned} \frac{d}{d\hat{x}} c \hat{x}^{-\chi} \left[1 + \frac{\chi g(\hat{x})}{\hat{x} [\delta - g'(\hat{x})]} \right] \\ = -\chi c \hat{x}^{-\chi-1} \left[1 + \frac{\chi g(\hat{x})}{\hat{x} [\delta - g'(\hat{x})]} \right] - c \hat{x}^{-\chi} \frac{\chi \frac{\rho}{\kappa} (\delta + \rho)}{[\delta - g'(\hat{x})]^2} < 0 \end{aligned} \quad (42)$$

The left-hand side (LHS) is monotonically decreasing with \hat{x} for $\hat{x} < x^{\text{MSY}}$ and monotonically increasing with \hat{x} for $\hat{x} > x^{\text{MSY}}$, i.e.

$$\frac{d}{d\hat{x}} \gamma g(\hat{x})^{-\frac{1}{\sigma}} \left[g(\hat{x})^{\frac{\sigma-1}{\sigma}} n_0 \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}-1} = -\frac{1}{\eta} \gamma g(\hat{x})^{-\frac{1}{\eta}-1} n_0^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}-1} g'(\hat{x}) \quad (43)$$

For $\hat{x} \rightarrow \kappa$, the LHS diverges to infinity, while the RHS falls to $c \kappa^{-\chi}$. Hence, at the steady state, the slope of the RHS is smaller than the slope of the LHS.

The open-access condition for symmetric species is

$$[\gamma c \hat{x}^{-\chi}]^\eta n_0^{-\frac{\sigma-\eta}{\sigma-1}} = g(\hat{x}) \quad (44)$$

Turning to second-best, Condition (37) becomes

$$\begin{aligned} (n_0 - 1) \lambda \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma} \right) g(x_o)^{-\frac{1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} &= \frac{\chi g(x_o) - \lambda}{x_o [\delta - g'(x_o)]} - \frac{\lambda}{\sigma g(x_o)} \\ \lambda \left[\frac{(n_0 - 1) \left(\frac{1}{\eta} - \frac{1}{\sigma} \right) g(x_o)^{-\frac{1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} + \frac{1}{x_o [\delta - g'(x_o)]} + \frac{1}{\sigma g(x_o)} \right] &= \frac{\chi g(x_o)}{x_o [\delta - g'(x_o)]} \\ \lambda \left[1 + \frac{x_o [\delta - g'(x_o)]}{g(x_o)} \right] \left[\frac{(n_0 - 1) \left(\frac{1}{\eta} - \frac{1}{\sigma} \right) g(x_o)^{\frac{\sigma-1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} + \frac{1}{\sigma} \right] &= \chi g(x_o) \end{aligned} \quad (45)$$

The term in brackets on the LHS is positive, as we see from slight rearrangement

$$1 - \underbrace{\frac{x_o [g'(x_o) - \delta]}{g(x_o)}}_{<1 \text{ (concavity of } g(x))} \underbrace{\frac{\frac{1}{\sigma} g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) \frac{1}{\eta} g(x_o)^{\frac{\sigma-1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}}}_{<1 \text{ } (\sigma > 1, \eta \geq 1)} > 0$$

Since the RHS is also positive, we conclude that $\lambda > 0$. For identical species Condition (30) becomes

$$\begin{aligned} & \gamma g(x_m)^{-\frac{1}{\sigma}} \left[g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} \\ & + (n_0 - 1) \lambda \frac{\left(\frac{1}{\eta} - \frac{1}{\sigma} \right) g(x_m)^{-\frac{1}{\sigma}} c x_o^{-\chi}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} = c x_m^{-\chi} \left[1 + \frac{\chi g(x_m)}{x_m [\delta - g'(x_m)]} \right] \end{aligned} \quad (46)$$

Rearranging, we obtain

$$\begin{aligned} & \left[1 + \frac{(n_0 - 1) \lambda \left(\frac{1}{\eta} - \frac{1}{\sigma} \right) c x_o^{-\chi}}{\left[g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}}} \right] \times \\ & \left[\frac{n_0 g(x_m)^{\frac{\sigma-1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} \right]^{1 - \frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}} \times \\ & \gamma g(x_m)^{-\frac{1}{\sigma}} \left[n_0 g(x_m)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = c x_m^{-\chi} \left[1 + \frac{\chi g(x_m)}{x_m [\delta - g'(x_m)]} \right] \end{aligned} \quad (47)$$

We now show that under the conditions of the proposition each of the first two factors on the LHS of this equation are larger than one for $x_m = \hat{x}$. This demonstrates that the solution x_m of (47) must be smaller than the solution \hat{x} of (41), as the slope of the RHS of Condition (47) is larger than the slope of the LHS.

First, for $x_m = \hat{x}$ we have $g(x_m) > g(x_o) > g(x^*)$, since from the comparison of

$$c x_o^{-\chi} = \gamma g(x_o)^{-\frac{1}{\sigma}} \left[g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} - 1} \quad (48)$$

with (44) we conclude that $x_o = x^*$ for $g(x_m) = g(x^*)$, $x_o < x^*$ for $g(x_m) < g(x^*)$, and $x_o > x^*$ for $g(x_m) > g(x^*)$. Hence for $x_m = \hat{x}$, we have

$$\frac{n_0 g(x_m)^{\frac{\sigma-1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} > \frac{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}}{g(x_m)^{\frac{\sigma-1}{\sigma}} + (n_0 - 1) g(x_o)^{\frac{\sigma-1}{\sigma}}} = 1 \quad (49)$$

Second, $1 - \frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1} > 0$, as $\sigma > \eta$. Third, $\lambda > 0$ and $1/\eta > 1/\sigma$, as $\sigma > \eta$.

A.10 Proof of Proposition 3: For identical species the condition for myopic management is

$$\gamma g(x_m)^{-\frac{1}{\sigma}} \left[n_0 g(x_m)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta} \frac{\sigma}{\sigma-1}-1} = c x_m^{-\chi} \left[1 + \frac{\chi g(x_m)}{x_m [\delta - g'(x_m)]} \right] \quad (50)$$

We compare this condition with the corresponding condition for second-best optimal management (47). As we have shown in A.9, the first factor on the LHS of the second equation is larger than one. Hence the x_m that solves (50) is larger than the x_m that solves (47).

A.11 Proof of Proposition 4: We first prove the statement on open-access equilibrium and then turn to the statement on the first-best management scenario. The first step of the proof consists in proving the following lemma:

Lemma 3. Under the assumptions of Proposition 4, the open-access equilibrium stock sizes are ordered according to intrinsic growth rates, i.e.

$$x_1^* < x_2^* < \dots < x_n^* \quad (51)$$

Proof. Given the assumptions of the proposition, total harvest H_i^* of species i is (by Equation 11)

$$H_i^* = \frac{\gamma}{c} \frac{x_i^{\chi\sigma}}{\sum_j^{n_0} x_j^{\chi(\sigma-1)}} \quad (52)$$

For the stable equilibrium stock, open-access harvest must increase more strongly with stock size than natural growth, i.e. the stable equilibrium is characterized by the condition

$$\frac{\partial H_i^*}{\partial x_i^*} > \rho_i \left[1 - 2 \frac{x_i^*}{\kappa} \right] \quad (53)$$

We now show the intermediate result (51) by differentiating the equilibrium condition $\rho_i x_i^* [1 - x_i^*/\kappa] = H_i^*$ with respect to ρ_i . This yields

$$x_i^* \left[1 - \frac{x_i^*}{\kappa} \right] = \left[-\rho_i \left[1 - 2 \frac{x_i^*}{\kappa} \right] + \frac{\partial H_i^*}{\partial x_i^*} + \sum_{j \neq i} \frac{\partial H_i^*}{\partial x_j^*} \frac{\partial x_j^*}{\partial x_i^*} \right] \frac{dx_i^*}{d\rho_i}$$

By Condition (53), the sum of the first two terms in brackets on the RHS of this condition is positive. From Condition (52) we have $\partial H_i / \partial x_j < 0$. Differentiating the equilibrium condition $\rho_i x_i^* [1 - x_i^* / \kappa] = H_i^*$ implicitly with respect to x_j and applying Condition (53) again, we obtain $\partial x_j / \partial x_i < 0$. We thus have $dx_i^* / d\rho_i > 0$, i.e. the larger the intrinsic growth rate, the larger is the steady-state stock. \square

Next we prove the following lemma:

Lemma 4. Under the assumptions of Proposition 5, the following holds: For all $\hat{\sigma}$, there exists a species j such that if we increase σ in a neighborhood of $\hat{\sigma}$, less of all species $i \leq j$ will be harvested under open access.

Proof. Differentiating (52) with respect to σ leads to

$$\frac{\partial H_i}{\partial \sigma} = H_i \chi \left[\ln x_i - \frac{\sum_{j=1}^{n_0} x_j^{\chi(\sigma-1)} \ln x_j}{\sum_{j=1}^{n_0} x_j^{\chi(\sigma-1)}} \right] \quad (54)$$

The last term in brackets is independent of the species under consideration, being the natural logarithm of a weighted geometric mean of fish stocks:

$$\frac{\sum_{j=1}^{n_0} x_j^{\chi(\sigma-1)} \ln x_j}{\sum_{j=1}^{n_0} x_j^{\chi(\sigma-1)}} = \frac{\ln \left[\prod_{j=1}^{n_0} x_j^{x_j^{\chi(\sigma-1)}} \right]}{\sum_{j=1}^{n_0} x_j^{\chi(\sigma-1)}}$$

The natural logarithm of the stock size of some species must be smaller than this weighted geometric mean. Thus the right hand side of (54) is negative for these species. Now let j denote the species with the largest stock that is smaller than the weighted geometric mean. For all species i with $i \leq j$, the RHS of equation (54) is still negative, which concludes the proof. \square

Turning to first-best management, we proceed analogously. The first step of the proof consists in proving the following lemma:

Lemma 5. Under the assumptions of Proposition 4, the first-best steady-state stock sizes are ordered according to intrinsic growth rates, i.e.

$$\hat{x}_1 < \hat{x}_2 < \dots < \hat{x}_n \quad (55)$$

Proof. Let $\pi_i = c \hat{x}_i^{-\chi_i} \left[1 + \frac{\rho_i (1 - \frac{\hat{x}_i}{\kappa})}{\delta - \rho_i (1 - 2 \frac{\hat{x}_i}{\kappa})} \right]$. Given the assumptions of the proposition, total harvest \hat{H}_i of species i is (by Equation 11)

$$\hat{H}_i = \gamma \frac{\pi_i^{-\sigma}}{\sum_j \pi_j^{1-\sigma}} \quad (56)$$

Since $d\pi/d\hat{x}_i < 0$ (cf. Equation 42), it is straightforward to show that H_i is monotonically increasing in \hat{x}_i for $\hat{x}_i > \frac{\kappa}{2} \left(1 - \frac{\delta}{\rho_i} \right)$. Hence, for the optimal steady state (see section A.9) we obtain

$$\frac{\partial \hat{H}_i}{\partial \hat{x}_i} > \rho_i \left[1 - 2 \frac{\hat{x}_i}{\kappa} \right] \quad (57)$$

Also, it is straightforward to show that $\partial H_i / \partial \rho_i < 0$.

We now show the intermediate result (55) by differentiating the equilibrium condition $\rho_i \hat{x}_i [1 - \hat{x}_i / \kappa] = \hat{H}_i$ with respect to ρ_i . This yields

$$\hat{x}_i \left[1 - \frac{\hat{x}_i}{\kappa} \right] = \left[-\rho_i \left[1 - 2 \frac{\hat{x}_i}{\kappa} \right] + \frac{\partial \hat{H}_i}{\partial \hat{x}_i} + \sum_{j \neq i} \frac{\partial \hat{H}_i}{\partial \hat{x}_j} \frac{\partial \hat{x}_j}{\partial \hat{x}_i} \right] \frac{d\hat{x}_i}{d\rho_i} + \frac{\partial \hat{H}_i}{\partial \rho_i}$$

By Condition (57), the sum of the first two terms in brackets on the RHS of this condition is positive. From Condition (56) we have $\partial H_i / \partial x_j < 0$. Differentiating the equilibrium condition $\rho_i \hat{x}_i [1 - \hat{x}_i / \kappa] = \hat{H}_i$ implicitly with respect to x_j , and applying Condition (57) again, we obtain $\partial x_j / \partial x_i < 0$. We thus have $dx_i^* / d\rho_i > 0$, i.e. the larger the intrinsic growth rate, the larger is the first-best steady-state stock. \square

Next we prove the following lemma:

Lemma 6. *Under the assumptions of Proposition 5, the following holds: For all $\hat{\sigma}$, there exists a species j such that if we increase σ in a neighborhood of $\hat{\sigma}$, less will be harvested of all species $i \leq j$ under first-best management.*

Proof. Differentiating (56) with respect to the σ leads to

$$\frac{\partial H_i}{\partial \sigma} = H_i \left[-\ln \pi_i + \frac{\sum_{j=1}^{n_0} \pi_j^{1-\sigma} \ln \pi_j}{\sum_{j=1}^{n_0} \pi_j^{1-\sigma}} \right] \quad (58)$$

The last term in brackets is independent of the species under consideration, being the natural logarithm of a weighted geometric mean of shadow prices,

$$\frac{\sum_{j=1}^{n_0} \pi_j^{1-\sigma} \ln \pi_j}{\sum_{j=1}^{n_0} \pi_j^{1-\sigma}} = \frac{\ln \left[\prod_{j=1}^{n_0} \pi_j^{\pi_j^{1-\sigma}} \right]}{\sum_{j=1}^{n_0} \pi_j^{1-\sigma}}$$

The natural logarithm of the shadow price of some species must be larger than this weighted geometric mean. Thus the RHS of (58) is negative for these species. Now let j denote the species with the smallest shadow price that is larger than the weighted geometric mean. For all species i with $i \leq j$, the RHS of equation (58) is still negative. Noting that, as in the proof of Lemma 55, one can show $\pi_1 < \pi_2 < \dots < \pi_n$, this concludes the proof. \square

A.12 Proof of Proposition 5: We first consider the case $\sigma\chi \leq 1$. If σ decreases, more of the species with the smallest equilibrium stock will be harvested (Lemma 4). Hence this species' equilibrium stock decreases, as Condition (53) applies for the (stable) equilibrium. With further decreasing σ , i.e. increasing harvest, the stock size will further decrease until the equilibrium eventually becomes unstable and the stock collapses. In the remaining case $\sigma\chi > 0$, the statement of the proposition holds trivially, since then no species will be depleted under open access (Lemma 1).