Società italiana di economia pubblica

XXV CONFERENZA

LA FINANZA PUBBLICA NEI SISTEMI MULTILIVELLO

COORDINAMENTO, CONCORRENZA E DISCIPLINA FISCALE

Pavia, Aule Storiche dell'Università, 26 - 27 settembre 2013

IS LABOR MOBILITY A PREREQUISITE FOR AN OPTIMAL CURRENCY AREA? LABOR MOBILITY AND FISCAL POLICY IN A CURRENCY UNION

ANGELO BAGLIONI, ANDREA BOITANI, MASSIMO BORDIGNON

Is labor mobility a prerequisite for an optimal currency area?
Labor mobility and fiscal policy in a currency union

Angelo Baglioni, Andrea Boitani, Massimo Bordignon
Department of Economics and Finance, Università Cattolica del Sacro Cuore

Preliminary version: August, 2013

Abstract. *Labor mobility is commonly taken as a property of an optimal currency area. But how does that property affect the outcome of fiscal policies? We address this issue with a two country – two period model, where both asymmetric and symmetric productivity shocks may hit the countries. We show that perfect (costless) labour mobility is not necessarily welfare improving, since it prevents the national fiscal authorities from pursuing independent policies, opening the way to a coordination problem between them, which is particularly relevant when the two countries differ for their intertemporal preferences. With symmetric shocks, the federal fiscal policy can improve welfare over national policies by playing a coordinating role. With asymmetric shocks, the federal fiscal policy allows both countries to reach a higher productive efficiency; to do that, the federal government must be endowed with a federal budget, playing a stronger role than plain coordination between countries. When mobility is costly, a federal budget is needed to reach Pareto efficiency even in presence of symmetric shocks.*

Keywords: currency union, labor mobility, fiscal policy, federation.

JEL codes: E62, H77.

1 Introduction

Is (geographic) labor mobility an essential component of an optimal currency union? Since Mundell's (1961) seminal work on optimal currency area, most economists would emphatically answer yes. Mundell noted that a currency union eliminates exchange rate flexibility as a way of responding to asymmetric regional real shocks; labour mobility, by allowing workers to move from regions hit by a negative shock to regions hit by a positive one, could then work as a substitute. It then follows that a currency union without, or with not enough, labor mobility across countries or regions is deemed to fail. This view is pervasive. For instance, at the time of the introduction of the Euro, several US economists, comparing the US with the EU, criticized the European Monetary Union (EMU), precisely because this area did not satisfy the labor mobility requirement[1]. Other puzzling features of the EMU, such as the lack of a federal budget or even of a federal government, did at the time raise less concern.

However, Mundell's argument was developed without considering the role of fiscal policy and of its institutions, a point that was made as early as 1969 by Peter Kenen [2]. Furthermore, Mundell did not consider a dynamic setting. Adding these two elements might change the results. Suppose for instance that fiscal policy has some beneficial effects, say because Ricardian equivalence does not hold. Then public debt (and counter-cyclical fiscal policy) might play a positive role, for instance allowing a government to smooth private consumption across periods in response to a shock. But suppose this government is now a regional government, member of a currency union, and that this union is characterized by perfect labor mobility, as wished by the optimal currency area literature. Then, labor mobility - and in general, any kind of factor mobility- may have also some negative effects. This is because typically taxes (on any kind of income, including labor income) are everywhere organized on the basis of a *source principle* (not a residence principle); people leaving a region or a country are no longer viable to income taxation in the country of origin. But then, if workers can leave freely in each period the region, there may simply be no enough tax base or people left to pay for the government debt; the regional government may then be forced to redeem its debt[3]. Alternatively, if agents and markets correctly anticipate this potential negative effect of labor mobility on *future* public revenues, the regional

[1] For a broad survey of US economists opinion on the EMU prior to 2002, see Jonung and Drea (2009).

[2] "Fiscal and monetary policy must go hand in hand, and if there is to be an 'optimum policy mix', they should have the same domains. There should be a treasury, empowered to tax and spend, opposite each central bank" (Kenen, 1969: 45-46)

[3] The recent case of Detroit, that lost half of the population in a decade and was then forced to bankrupt, comes naturally to mind.

government might not be able to raise the desired resources to start with, so being forced to follow a sub-optimal fiscal policy. Thus, labor mobility might constraint dynamic fiscal policy; and this is bad for consumers as long as the regional government fiscal policy is welfare enhancing. Notice in contrast that a *federal* government, that is, a common government for the currency area as a whole, would not have this problem, as it could still raise taxes on individuals, wherever they decided to locate in the federation[4]. This would then suggest that even in the presence of full labor mobility, the potential advantages of a currency union could not be obtained without a federal government and a federal budget. Labor mobility and a federal budget should then be both elements of a well functioning currency union.

In the following, we set up a very simple model in order to make this intuition precise. In the model, there are two periods and two regional welfare maximizing governments, members of a currency union, that might be subject to a temporary productivity shock in the first period. The two governments may then wish to use fiscal policy (labor tax and transfers) in order to smooth private consumption across the two periods. For simplicity, we assume that public debt is always repaid, so ruling out the possibility of government bankruptcy. We first show that if the first period shock is symmetric across regions, regional labor mobility can only be damaging. By definition, in this case regional labor mobility could not generate efficiency or insurance gains, but it might force the governments to follow an inefficient intertemporal fiscal plan. Intuitively, regional governments would have to compete between them in order to attract or retain the mobile workers, so leading to inefficient fiscal choices. A federation in this case could help, by playing a coordination role, but the optimal policy would be in this case to forbid labor mobility altogether. If the shock is asymmetric across regions, labor mobility plays instead an important insurance and efficiency role, as correctly predicted by Mundell (1961). Still, we show that even in this case, competition among governments leads to inefficient level of mobility, thus making it impossible for the two countries to reap all the potential benefits in terms of increased output. Furthermore, while private consumption is equalized across regions in each period, thus providing full insurance to

[4]The federation would of course still have problems in taxing *capital income*, if this is mobile across federations, as suggested by a huge literature on tax competition. But it is a characteristic of labor income to be earned in the location where the individual works; it is generally impossible to move around labor without also moving the individual who supplies this labor.

workers, the intertemporal path of consumption across periods is not the same that the welfare maximizing governments would have chosen in the absence of mobility *and* with the extra resources generated by mobility. The solution is clearly not to reduce efficiency enhancing mobility but to reduce the competition across regional governments, a role that could only be played by a federal government. Indeed, we show that in the case of asymmetric shocks the ability of the federation to impose federal taxes is essential in order to generate Pareto improvements. Thus, interestingly, our results indeed suggest that a federal budget is needed exactly when labor mobility is potentially useful.

Finally, we show that our results are not an artefact of assuming perfect labor mobility; very much the same normative results would occur if mobility were less than perfect, inducing costs on people who move. In this case, a coordination role by the federation could be useful even with symmetric shocks, but a federal budget is again essential to induce further Pareto improvement.

The literature related to the present paper is huge. For instance, Sala-i-Martin and Sachs (1991) already suggested that contrary to Mundell's (1961) claim, in the US is mainly the fiscal federalist system of taxes and transfers, and not labor mobility, that absorbs most of the asymmetric shocks across states, an extimation later revised and made more precise by von Hagen (1992) and others (see Kletzer and von Hagen 2001 for a survey). In Europe, before the introduction of the EMU, there was a large debate on the need to accompany the monetary union with a federal budget (e.g. MacDougall Report, 1977 and the Delors' Report, 1989), building up on the original Kenen's arguments. But as is well known the Maastricht Treaty (signed in 1991 to pave the road to the monetary union) did not consider a federal fiscal budget as a prerequisite for a monetary union and indeed imposed constraints on the fiscal policy of member countries (Sapir, 2008).

On more theoretical grounds, the role of federal fiscal policies in absorbing asymmetric shocks in a monetary union has already been discussed by a large academic literature (Kletzer, Buiter, 1997; Kletzer, 1999; Kletzer, von Hagen, 2001; Evers, 2006). Some work also focused on more specific issues - such as unemployment insurance and tax revenue sharing in Europe (von Hagen and Wyplosz, 2008) - or at evaluating whether a federal fiscal policy is stabilizing or is likely to increase the correlation among shocks (Belke and Gros, 2009a, 2009b). Finally Bofinger and Mayer (2007) already made the point that national fiscal policies are needed in order to absorb demand and/or supply shocks in the absence of federal fiscal insurance. However, none of these works discuss labour mobility in details; consequently, the interplay between labour mobility and fiscal policy that is the focus of the present paper is largely ignored.

The rest of the paper is organized as follows. Section 2 lays out the basic framework of our model. Section 3 analyzes the interplay between perfect (costless) labour mobility and fiscal policy (national and federal) in the case of symmetric shocks. Section 4 replicates the same analysis for asymmetric shocks. Section 5 considers the case of imperfect (costly) mobility. Section 6 further comments on our findings. All proofs of the propositions are in Appendix 1. Appendix 2 discusses at lenght some technical problems that we encounter in the case of full mobility for the determination of the equilibria and the way we solve them.

2 The model

In order to make our point as neatly as possible, we will consider here the simplest possible setting that allows us to develop our analysis; some extensions are discussed formally in section 5 and informally in section 6. Consider then an economy with 2 countries (regions): $i = A, B$, 2 periods: $1, 2$ (lower (UPPER) case will stand for first (second) period variables from now on) and 2 states of nature: $s = \alpha, \beta$, each occurring with probability $1/2, 1/2$. In each country, there are N workers/consumers, who are all characterized by the same utility function

(01) $$U^i = \ln(c) + \delta_i \ln(C)$$

where c (resp. C) represents workers' private consumption in the first (resp. second) period, and δ_i is the discount factor. Each worker is allowed with one unit of labour in each period that is inelastically supplied. In each county, production combines labour with some fix factor of production in order to produce an homogenous good, according to the production function

(02) $$y_{is} = \theta_{is} l_i^\gamma ; Y_{is} = \Theta L_i^\gamma$$

where l_i (resp. L_i) is the number of workers employed in the production of good y_i (resp. Y_i) in country i in the first (resp. second) period, and γ lies in the interval $0 < \gamma < 1$, implying decreasing returns to labour. θ_{is} and Θ are some positive constant. The output price is normalized to 1. As shown in (02), production in the first period is subject to a productivity shock that depends on the state of the world. In particular, we assume that in the first period, depending on the realization of s, the productivity parameter θ in region i may take either value $\underline{\theta}$ or $\overline{\theta}$ with probability 1/2, where

(03) $\underline{\theta} = \Theta(1-\varepsilon)$ and $\overline{\theta} = \Theta(1+\varepsilon)$, with $1 > \varepsilon > 0$

Notice that in line with Mundell's argument, this formulation implies that the first period shock is *temporary*; it disappears in the second. The presence of the shock induces some variance in wages and therefore in consumption in the two periods that is costly for workers/consumers, as their utility function is concave. Consumers may then wish to insure themselves against the occurrence of the shock. For convenience, we however assume that private citizens cannot save and have no access to capital markets: this is the simplest way of incorporating the imperfections of capital markets into our model. On the contrary, we assume that the national and federal governments can lend or borrow on international capital markets at some fix interest rare r. Hence, there is here a valuable role for fiscal policy, as governments can use this power to insure their citizens against the shock, for instance taxing workers in the good periods and subsiding them in bad ones. In particular, we indicate with t_{is} and T_{is} the per capita lump sum government subsidies (if > 0) or taxes (if < 0) imposed in resp. period 1 and 2 in country i on labor income[5].

Notice that as workers do not own the fixed factor and have no access to capital markets, their consumption in each period must just be equal to their labor income plus the tax/transfer; that is, $c_{is} = w_{is} + t_{is}$ and $C_{is} = W_{is} + T_{is}$, where w_{is} and W_{is} indicate the wage received by each worker in each period in region i. For simplicity, as our focus here is on labor markets and labor income taxation, we ignore the returns to the fix factor (and its owners), which with perfect competitive markets are just in each period $(1-\gamma)$ times the output; we might simply assume that these returns are completely taxed away by the governments in order to finance other un-modelled components of public expenditure, say a public good that enters separately in the utility function of consumers[6]. Note that this entails that the intertemporal budget constraint of governments requires the taxes (subsidies) raised in the second period on labor income must be enough to finance the subsidies (taxes) paid

[5] As labor supply is fixed, there is no loss of generality in assuming here that all taxes or subsidies are lump sum.

[6] As can be easily checked, taxing the fix factor returns in order to finance subsidies to workers would not change qualitatively our results, as the returns to the fixed factor also depend on the productivity shock and on workers mobility via the output level. But adding them to the picture would also raise the question of whom the owners of this factor are and their political representation, adding unnecessary complications to the discussion.

in the first period on labor income, plus the payment (the receipt) of interests.

We have yet to define what a government is. In the following, we will consider several possible definitions of "government". We will talk of national (regional) governments (and national policy) when decisions about t_{is} and T_{is} are only taken by the government in country i. We will talk of a federal government, when the same decisions are taken by a federal unit. In particular, we will consider two possible federal arrangements. (i) *Weak federation* (coordination): the federal government chooses local taxes but each country's intertemporal budget constraint must clear. (ii) *Strong federation* (federal budget): the federal government chooses local taxes and the *aggregate* intertemporal budget constraint must clear. The basic difference is that only the second arrangement (implicitly) allows for transfers across countries; the first just assumes that national governments can bargain among themselves and commit to Pareto efficient agreements, that we simply model here by saying that decisions are in this case directly taken by a benevolent federal government that therefore takes into account the welfare of both countries[7]. In all cases, we assume that governments are just social welfare maximizers; i.e. in different forms, they just wish to maximize the utility function of workers/consumers in (01).

The time line of the model is the following:

At the beginning of period 1, nature chooses s. Depending on who is in charge, if a federal or a national government, decisions are then taken about the tax (subsidy) t_{is} .Workers observe the realization of s and the tax (subsidy) chosen in both countries in period 1, and if mobility across countries is allowed, they then move in order to maximize their utility. Labor markets then clear in both countries and the clearing markets gross wage w_{is} are determined.

At period 2, the shock disappears**.** Governments chooses again a tax (subsidy) T_{is} . Again, workers observe these moves and decide whether moving across countries; labour markets clear and equilibrium gross wages W_{is} are set in each country.

Notice that when labor is costlessy mobile across countries in both periods, workers will have an incentive to move in each period until per capita private consumption is equalized in each country. In this sense, then, per-

[7]Thus, loosely speaking, we can think of the weak federation, as characterizing a union such as the EU or the EMU, where intergovernmental transfers either do not exist or are very small. The US (or most nations with their own currency) could instead be taken as examples of a strong federation.

fect labor mobility makes workers myopic; they do not need to plan for the future because as the future comes they can always move elsewhere. Governments instead need to plan intertemporally, because by definition, they cannot move.

In the following, we will solve the model by finding the equilibrium of the game for different hypotheses about mobility costs and for different type of government. An equilibrium is defined here as a situation where: (i) labour markets clear in both periods and in both countries; (ii) workers move across countries in order to maximize their utility, depending on mobility costs; (iii) each government sets t_{is} and T_{is} optimally, by maximizing its utility function (see below), and (iv) the intertemporal budget constraint of the public sector is satisfied (e.g. we rule out bankruptcy).

Clearly, under national fiscal policy, an equilibrium is just a Nash equilibrium in the tax rates t_{is} between the two countries, as T_{is} are determined residually in order to satisfy the intertemporal national budget.

3 Symmetric shocks

We begin by considering the case where the productivity shock is symmetric: the state α is bad and the state β is good in both countries A and B:

	α	β
A	$\underline{\theta}$	$\overline{\theta}$
B	$\underline{\theta}$	$\overline{\theta}$

In this case, labor mobility does not allow the two countries to get any efficiency gain, since there is no productivity differential between them. Hence, mobility can only be either useless or damaging. Still, it is important to study in detail this simpler case first, since, as we will show, its insights largely extend to more complicated settings.

3.1 National fiscal policies *without* labour mobility

As a benchmark case, let us begin by analyzing the outcome achieved by national fiscal policies when no labour mobility across countries is allowed. Without labor mobility, the supply of labour is N in both countries and in both periods. Moreover, the demand for labour is a function of the gross wages. Solving the model backward, we begin by deriving the clearing conditions for the labor market in periods 2 and 1 in turn. We assume perfectly competitive markets.

Period 2. The production function is $Y = \Theta L^{\gamma}$ in both countries. In each country the labour supply is $L^S = N$, so the number of people employed is equal to N.

The firm profit maximization problem is:

$$\max_L Y - WL \tag{1}$$

and the FOC is:

$$W = \Theta\gamma L^{\gamma-1} \tag{2}$$

so the labour demand function in each country is:

$$L^D = \left(\frac{\Theta\gamma}{W}\right)^{\frac{1}{1-\gamma}} \tag{3}$$

The equilibrium condition $L^S = L^D$ determines the equilibrium wage:

$$W^* = \Theta\gamma N^{\gamma-1} \tag{4}$$

Period 1. Again, in each country the labour supply is $l^S = N$.

Let us consider country A. In state α, the production function is $y_{A\alpha} = \underline{\theta} l^{\gamma}$; firm profit maximization leads to:

$$l^D_{A\alpha} = \left(\frac{\underline{\theta}\gamma}{w_{A\alpha}}\right)^{\frac{1}{1-\gamma}} \tag{5}$$

and the equilibrium wage is:

$$w_{A\alpha} = \underline{\theta}\gamma N^{\gamma-1} \equiv \underline{w} \tag{6}$$

In state β, the same procedure leads to:

$$w_{A\beta} = \overline{\theta}\gamma N^{\gamma-1} \equiv \overline{w} \tag{7}$$

The same applies to country B: $w_{B\alpha} = \underline{w}$ and $w_{B\beta} = \overline{w}$.

Because the shock is damaging, national governments may wish to insure their consumers. They do so by settting a tax profile in their own countries, by maximizing the representative consumer utility function (01), subject to the intertemporal budget constraints:

$$t_{is}\,(1+r) + T_{is} = 0 \text{ for } i = A, B \text{ and } s = \alpha, \beta \tag{8}$$

where r is the interest rate paid in the international markets. In state $s = \alpha$, substituting the budget constraint in the objective function, the governments' problem can be written as:

$$\max_{t_{i\alpha}} \ln(\underline{w} + t_{i\alpha}) + \delta_i \ln\left[W^* - t_{i\alpha}(1+r)\right] \quad \text{for } i = A, B \tag{9}$$

solving, we get the standard Euler equation[8]:

$$\frac{C^*_{i\alpha}}{c^*_{i\alpha}} = \delta_i(1+r) \quad \text{for } i = A, B \tag{10}$$

where $C^*_{i\alpha} = W^* - t^*_{i\alpha}(1+r)$ and $c^*_{i\alpha} = \underline{w} + t^*_{i\alpha}$.This condition can be solved for $t^*_{i\alpha}$:

$$t^*_{i\alpha} = \frac{W^* - \underline{w}\delta_i(1+r)}{(1+\delta_i)(1+r)} \quad \text{for } i = A, B \tag{11}$$

In state $s = \beta$, the government's problem is the same as problem (9), with $\overline{w}$ replacing $\underline{w}$. Accordingly, the optimal tax rate is

$$t^*_{i\beta} = \frac{W^* - \overline{w}\delta_i(1+r)}{(1+\delta_i)(1+r)} \quad \text{for } i = A, B \tag{12}$$

Notice that the consumption vector c^*_{is}, C^*_{is} is Pareto efficient, since it solves the optimal consumption smoothing problem of the representative agent (in each country/state of the world).

3.2 The labour market *with* perfect labour mobility

Let us then introduce costless labour mobility. When labor mobility is allowed, the number of workers in each country might of course no longer be equal to N. As a matter of notation, we then let m (resp. M) be the number of workers moving in the first period (resp. second), with the convention that m (resp. M) > 0 if people move from A to B in the first period (resp.

[8]SOC are automatically respected as the utility function is strictly concave.

second) and m (resp. M) < 0 if people move from B to A in the first period (resp. second). Before analyzing the behavior of the fiscal authorities in this environment, we have to identify the equilibrium conditions in the labor market, taking momentarily fiscal policy as given. We focus on state $s = \alpha$; similar considerations apply to the other state of nature.

Period 2. The following system of conditions determine the equilibrium values of the gross wages and the number of people moving across countries (M), taking as given the tax/subsidy rates and the number of people that has already moved in period 1 (m):

$$\begin{aligned} \left(\frac{\Theta\gamma}{W_{A\alpha}}\right)^{\frac{1}{1-\gamma}} &= N - m - M && (13) \\ \left(\frac{\Theta\gamma}{W_{B\alpha}}\right)^{\frac{1}{1-\gamma}} &= N + m + M && (14) \\ W_{A\alpha} + T_{A\alpha} &= W_{B\alpha} + T_{B\alpha} && (15) \end{aligned}$$

where in the first two lines the demand for labour is equated to labour supply in each country; note that labour demand is a function of *gross* wages. The third line is due to the perfect labor mobility: as mobility is costless, people move until any *net* wage differential between the two countries vanishes. By solving the first two equations we get:

$$W_{A\alpha} = \Theta\gamma(N - m - M)^{\gamma-1} \tag{16}$$

$$W_{B\alpha} = \Theta\gamma(N + m + M)^{\gamma-1} \tag{17}$$

which can be substituted into the third one to get:

$$\Theta\gamma[(N - m - M)^{\gamma-1} - (N + m + M)^{\gamma-1}] + T_{A\alpha} - T_{B\alpha} = 0 \tag{18}$$

This equation implicitly defines the equilibrium number of people that move in the second period, M, as a function of the two tax/subsidy rates in the second period $T_{A\alpha}$ and $T_{B\alpha}$, and on the number of people that have moved already in period 1, m.

Period 1. By the same token, the equilibrium values of the gross wages and the number of people moving across countries in the first period, taking as given the tax/subsidy rates is given by:

$$\left(\frac{\underline{\theta}\gamma}{w_{A\alpha}}\right)^{\frac{1}{1-\gamma}} = N - m \tag{19}$$

$$\left(\frac{\underline{\theta}\gamma}{w_{B\alpha}}\right)^{\frac{1}{1-\gamma}} = N + m \tag{20}$$

$$w_{A\alpha} + t_{A\alpha} = w_{B\alpha} + t_{B\alpha} \tag{21}$$

Again, by solving the first two equations we get:

$$w_{A\alpha} = \underline{\theta}\gamma(N-m)^{\gamma-1} \tag{22}$$

$$w_{B\alpha} = \underline{\theta}\gamma(N+m)^{\gamma-1} \tag{23}$$

which can be substituted into the third one to get:

$$\underline{\theta}\gamma[(N-m)^{\gamma-1} - (N+m)^{\gamma-1}] + t_{A\alpha} - t_{B\alpha} = 0 \tag{24}$$

This equation defines implicitly the equilibrium number of people moving in the first period, m, as a function of the two tax/subsidy rates $t_{A\alpha}$ and $t_{B\alpha}$.

By using this last set of equations, it is an easy matter to establish a few interesting comparative statics results. Let $m = m(t_{A\alpha}; t_{B\alpha})$ be the number of people who moves in the first period, as derived by the arbitrage condition above. Then, we can prove (see Appendix 1):

Proposition 1 *i.* $\partial m/\partial t_{A\alpha} < 0$; *ii.* $\partial m/\partial t_{B\alpha} = -\partial m/\partial t_{A\alpha} > 0$; *iii.* $\frac{dw_{A\alpha}}{dt_{A\alpha}} = -\frac{1}{1+k} < 0$; *iv.* $\frac{dc_{A\alpha}}{dt_{A\alpha}} = \frac{k}{1+k} > 0$, *v.* $\frac{dw_{B\alpha}}{dt_{A\alpha}} = \frac{dc_{B\alpha}}{dt_{A\alpha}} = \frac{k}{1+k} > 0$, *vi.* $\frac{dc_{B\alpha}}{dt_{B\alpha}} = \frac{dc_{A\alpha}}{dt_{B\alpha}} = \frac{1}{1+k}$, *where* $k \equiv \left(\frac{N+m}{N-m}\right)^{\gamma-2} > 0$.

The proposition illustrates neatly the effect of perfect mobility in the first period. If country A raises his transfer in the first period (while B keeps its transfer unchanged), it attracts more people from (or has less people moving to) country B; as an effect, the gross wage in country A falls. Per capita consumption in country A still raises but less than the increase in $t_{A\alpha}$, because of the fall of w_{Aa}. Per capita consumption in country B raises as much as in country A, because mobility equalizes per capita consumption. Notice that how much $c_{A\alpha}$ increases following the increase in $t_{A\alpha}$ depends on

m; for instance, in the case $m = 0$ (i.e. $t_{A\alpha} = t_{B\alpha}$), $\frac{dc_{A\alpha}}{dt_{A\alpha}} = \frac{dc_{B\alpha}}{dt_{B\alpha}} = 1/2$. If $m < 0$ (meaning that country A has already attracted workers from country B), $\frac{dc_{A\alpha}}{dt_{A\alpha}} > 1/2$ as w_{Aa} falls less as more workers work in A (by concavity of the production function). Vice-versa for $m > 0$.

Very much the same results would occur in period 2, as an effect of a change in T_{Aa} or T_{Ba} except that k should be now substituted by $K \equiv \left(\frac{N+m+M}{N-m-M}\right)^{\gamma-2}$ (see the Appendix). But of course, as we discuss below, T_{ia} is not set exogenously, but it is endogenously determined at the equilibrium to satisfy the intergovernmental governments' budget constraint.

3.3 National fiscal policies

Consider now fiscal policy, starting with the case where such a policy is run at the national level. Labor mobility introduces an additional constraint into the optimization problem faced by the fiscal authority, namely the fact that now net wages must be equated across the two countries (see equations 15 and 21), so their consumers must all share the same consumption path over time, even if their intertemporal preferences differ. This is the reason why labor mobility is not only useless in this context, but it is also potentially harmful, since countries might be forced to follow a suboptimal consumption smoothing plan.

As a preliminary step, we have to define the objective function of national governments. With labor mobility, the residents in a country might no longer coincide with its citizens. In principle, we could then image that a welfarist government could follow either a *nationality* principle or a *residence* principle (or a combination of the two). Under the first principle, a government maximizes the utility of its own citizens, regardless of where they live. Under the second one, a government maximizes the utility of those people resident in the country, regardless if they are its citizens or not. Which hypothesis is more convincing depends on the institutional context. In the EMU area, for instance, living and working in an European country does not give a person the right to vote in that country; people still vote in their country of origin. Hence, if governments are welfarist because they want to get the votes of the people who are eligible to vote, a plausible justification, the nationality principle would seem more reasonable for this context. Following this rule, the government of, say, country A, who has $m \geq 0$ people living in country B in period 1 and $(m + M) \geq 0$ people living in country B in period 2, would then maximize the following objective function (in state α):

$$(N-m)\ln(w_{A\alpha}+t_{A\alpha})+m\ln(w_{B\alpha}+t_{B\alpha})+$$
$$+(N-m-M)\delta_A\ln\left[W_{A\alpha}+T_{A\alpha}\right]+(m+M)\delta_A\ln\left[W_{B\alpha}+T_{B\alpha}\right]$$

Notice however, by inserting the labor market equilibrium conditions (15) and (21), that under costless mobility, this objective function boils down to the following:

$$N[\ln(w_{A\alpha}+t_{A\alpha})+\delta_A\ln(W_{A\alpha}+T_{A\alpha})]$$

and the same reasoning applies to country B[9]. Intuitively, each government knows that because of costless mobility, in equilibrium the per capita consumption of each consumer in each period must be the same in both countries.

Thus, under the nationality principle, labor mobility does not change the structure of each government preferences. It does change the budget constraints, however. In fact, assuming that a country cannot discriminate between national and not national residents in terms of labor taxation[10] and cannot tax not resident workers, these now become:

$$(N-m)t_{A\alpha}\,(1+r)+(N-m-M)T_{A\alpha}=0 \tag{25}$$

$$(N+m)t_{B\alpha}\,(1+r)+(N+m+M)T_{B\alpha}=0 \tag{26}$$

Notice that in the equations m, $T_{A\alpha}$, and M are all functions of the tax/subsidies selected by both countries in the *first* period, as $T_{A\alpha}$ and therefore M, need to adjust in order to guarantee the budget constraint of each country in the second, and labor markets equilibrium conditions must also be respected in the both periods. This makes each budget constraint above a potentially very complex function of the first period tax rates.

In Appendix 2, we analyze this function in detail, proving that

Lemma 2 *i.* $sign(dM/dt_{is})=-sign(dm/dt_{is})$ *ii.* $|dM/dt_{is}|>|dm/dt_{is}|$

[9] As is easy to check, the resident principle and the nationality principle would in general not coincide as the number of people resident in each period with costless mobility would be different, and as immigrants would carry their time preferences with them.

[10] This would seem implicit in any federation, however loose, that adopts a common currency. It is for example the case in the EMU, where discriminatory taxation is explicitly forbidden by the European treaties.

In order to satisfy the dynamic budget constraint, $dM/dt_{A\alpha}$ needs to have the opposite sign and being larger in magnitude than $dm/dt_{A\alpha}$ (the same for $dM/dt_{B\alpha}$). Intuitively, under perfect mobility, if country A raises his subsidy in period 1, it attracts workers from B. This increases $A's$ public deficit in period 1, that will have to be repaid with interests in period 2. This in turn means that taxes in period 2 have to be larger than without labor mobility, and higher taxes will push several workers to emigrate to B, up to the point where labor supply falls so much that gross wage in A raises enough to compensate for the higher taxation.

The dependence of m and M on first period taxation has clearly also effects on the rate of substitution between first and second period consumption. In particular, using the Lemma above, in Appendix 2 we prove

Proposition 3 *i. For* $m = 0$, $dC_{Aa}/dt_{A\alpha} = \frac{-(1+r)}{2}$. *ii. For* $m \neq 0$, *but not "too" large,* $dC_{ia}/dt_{i\alpha} < 0$ *and* $d^2C_{ia}/dt_{i\alpha}^2 < 0$.*iii. For* $m \neq 0$ *and large, sign* $dC_{ia}/dt_{i\alpha}$ *uncertain.*

Notice that point (i) in Proposition 2 implies that in the special case where $t_{A\alpha} = t_{B\alpha}$ (that implies $m = M = 0$), the slope of the intertemporal budget constraint is the same that without labor mobility. In fact, invoking Proposition 1, in the case $t_{A\alpha} = t_{B\alpha}$ 1 euro of transfer more in period 1 in country A, increases per capita consumption in that country by 1/2 euro in period 1, and by Proposition 2 reduces per capita consumption in period 2 by $\frac{-(1+r)}{2}$ euro. Hence, in this special case, a country can still trade consumption between the two periods at the same price, $(1 + r)$. When $t_{A\alpha} \neq t_{B\alpha}$, but the two taxes are not that different so that m is small, $dC_{ia}/dt_{i\alpha}$ is certainly negative. This implies, again invoking Proposition 1, that the rate of substitution between consumption today and consumption tomorrow is still negative, but its exact slope depends on m, that is on the difference between first period tax rates. In particular, it can be shown the $\left|\frac{dc_{Aa}}{dC_{Aa}}\right| > (<)(1 + r)$ as $t_{A\alpha} > (<)t_{B\alpha}$; because of labor mobility, it is more costly, in term of future consumption, increases consumption today for country A when $t_{A\alpha}$ increases above $t_{B\alpha}$, because a much larger number of workers will then leave the country in period 2. Finally, it can even be shown that for $t_{B\alpha}$ much larger than $t_{A\alpha}$, $dC_{ia}/dt_{i\alpha}$ might even become positive (see Appendix 2), implying that an increase in $t_{A\alpha}$ in the first period would result an increase in the consumption in *both* periods. But of course, this part of the dynamic budget constraint would never be observed because in this case country A would have an incentive to increase $t_{A\alpha}$ up to the point where $dC_{ia}/dt_{i\alpha}$ becomes negative and equal to the benefit for country A of increasing consumption in period 1.

3.3.1 Nash equilibria

The government in each country sets the optimal tax/subsidy rates by solving the following problem:

$$\max_{t_{i\alpha},T_{i\alpha}} \ln(w_{i\alpha}+t_{i\alpha})+\delta_i \ln\left[W_{i\alpha}+T_{i\alpha}\right] \quad \text{for } i=A,B \tag{27}$$

subject to its own budget constraint and to the mobility constraints defined in the previous section. In a Nash equilibrium, it also takes as given the fiscal choices of the other country. One potential difficulty in solving this problem is that is no obvious that the problem is still convex after mobility has been taken into account. As is well known by optimal taxation theory, this is not guaranteed by strict concavity of the objective function, once the optimal reactions of agents to taxation are taken into account. In our case, as we saw above, the problem is that the interplay between mobility and budget constraint makes the intertemporal budget constraint of government at points potentially not convex. In Appendix 2, we nevertheless show that the government optimizing problem is still well behaved at the equilibrium, provided that the elasticity of m to tax differentials is not too high [11]. Assuming this to be the case, and solving governments problem we get at a Nash equilibrium the following first order conditions must be jointly satisfied[12]:

$$\frac{C^n_{A\alpha}}{c^n_{A\alpha}}=-\delta_A\frac{dT_{A\alpha}}{dt_{A\alpha}}\frac{K}{k}\frac{1+k}{1+K} \text{ and } \frac{C^n_{B\alpha}}{c^n_{B\alpha}}=-\delta_B\frac{dT_{B\alpha}}{dt_{B\alpha}}\frac{1+k}{1+K} \tag{28}$$

where

$$\frac{dT_{A\alpha}}{dt_{A\alpha}}=-(1+r)\frac{\left[(N-m)-t_{A\alpha}\frac{dm}{dt_{A\alpha}}\right]}{(N-m-M)-T_{A\alpha}\frac{dM}{dT_{A\alpha}}} \tag{29}$$

[11] For $\gamma\rightarrow 0$, or $\gamma\rightarrow 1$, the problem has no solution as m will became infinitely elastic to the tax rate differentials.

[12] To increase the readibility of the formulas we express these conditions as function of T_{is}, without solving explicitly for the latter. See Appendix 2 for an alternative formulation.

$$\frac{dT_{B\alpha}}{dt_{B\alpha}} = -(1+r)\frac{\left[(N+m) - t_{B\alpha}\frac{dm}{dt_{A\alpha}}\right]}{(N+m+M) - T_{B\alpha}\frac{dM}{dT_{A\alpha}}} \tag{30}$$

are derived by applying the implicit function theorem to the budget constraints (25) and (26) respectively.

By continuity of the reaction functions, it is easy to show that a Nash equilibrium generally exists. A different matter is to establish whether this equilibrium is also unique. In Appendix 2 we provide conditions for this to be the case, although the complexity of the formulas does not allow for an easy interpretation of these conditions. Fortunately, our point here does not rely on the uniqueness of the Nash equilibrium and our statements below can be thought as referring to any possible Nash equilibria.

Comparing the FOC above with the corresponding conditions in absence of labor mobility, it is clear that labor mobility in general distorts the optimal consumption path chosen by the two governments. Hence, we would not expect the Nash equilibria to be in general Pareto efficient. Still, in the special case where $\delta_A = \delta_B = \delta$, we can prove the following result.

Proposition 4 *If $\delta_A = \delta_B = \delta$, $t^n_{is} = t^*_s$ (for $s = \alpha, \beta$ and $i = A, B$) is a Nash equilibrium.*

where the suffix "n" indicates that we are considering the optimal tax rates at the Nash equilibrium. Notice that if $t^n_{is} = t^*_s$, $m = M = 0$, and $\frac{dT_{A\alpha}}{dt_{A\alpha}} = \frac{dT_{B\alpha}}{dt_{B\alpha}} = -(1+r)$; substituting in the FOC above we get $\frac{C^n_{i\alpha}}{c^n_{i\alpha}} = \frac{C^*_{i\alpha}}{c^*_{i\alpha}} = \delta(1+r)$ for $i = A, B$, which is the optimal Euler condition for the case without mobility. Hence, if the time preferences are the same across countries, labor mobility is without consequences; at the symmetric Nash equilibrium, the same Pareto efficiency allocations would result, exactly because no consumer would move at the equilibrium[13].

The intuition for this result is quite simple. Suppose that starting from the allocation $t^n_{is} = t^*_s$ country A considers raising the subsidy (or reducing the tax) by 1 euro in the first period. As we show in Proposition 1, (at $m = 0$) this would increase the consumption of his citizens by 1/2 euro in

[13] There could be other Nash equilibria of course, but a symmetric Nash equlibrium with identical countries would seem to be a focal point where countries' expectations would converge.

the first period, because, as a consequence of labor mobility, the subsidy would be perfectly shared with the consumers of country B. Normally, this should lead country A to set the subsidy at an inefficient lower level, as country A does not care for the benefits accruing to consumers of country B. However, in the second period, as we showed in Proposition 2, because of perfect labor mobility, the additional cost of the increased subsidy would also be shared by the consumers in country B, imposing only a cost equal to $(1+r)/2$ euro to the consumers living in A. But this is exactly the optimal trade off between consumption in the two periods for country A at $t_{is}^{n} = t_{s}^{*}$, which means that at t_{s}^{*} country A is indifferent between raising (or reducing) the transfer by 1 euro or stick at t_{s}^{*}. Hence, t_{s}^{*} is a Nash equilibrium. The argument also provides an intuition on why the Nash equilibrium is in this case Pareto efficient; it is as if perfect labor mobility leads the two countries to perfectly endogeneize the consequences of their actions, so eliminating one of the potential source of inefficiency of the Nash equilibria[14].

There is also a second, simpler, argument that can be used to prove the same result (and it is indeed used in the proof of the Proposition in the Appendix). Suppose on the contrary that t_{s}^{*} is not a Nash equilibrium; than, one of the two countries, say country A, must have a feasible deviation from this equilibrium that is beneficial for country A. But because of labor mobility and common preferences across countries, if this deviation is beneficial for country A it must also be beneficial for country B. But then the deviation by A would represent a Pareto improvement with respect to the consumption allocation defined by t_{s}^{*}, contradicting the fact that this allocation is Pareto efficient.

These arguments also explain why on the contrary Nash equilibria when $\delta_A \neq \delta_B$ cannot be Pareto efficient; perfect labor mobility would still lead the two countries to internalize the consequences of their actions, but as sec-

[14]Indeed, one may wonder if there exist other Nash equilibria with identical countries still charaterized by the same efficient level of consumption in the two periods, but different tax rates. Say, country A setting a subsidy in the first period that would enforce however the Pareto efficient consumption vector c_{is}^{*}, C_{is}^{*} in the two countries and country B replying with $t_{B\alpha} = 0$. The answer is no. This is so because with different tax rates m would be different from zero, which would also imply that the slope of the budget constraint would not be equal to $1 + r$. Thus, even if country A managed to choose $t_{A\alpha}$ so as to get c_{is}^{*} in the first period with $m \neq 0$ (see Proposition 1), it could not afford C_{is}^{*} in the second period, because this allocation would violate its budget constraint. This argument also proves that if asymmetric Nash equilibria exist with identical countries, they must be Pareto inefficient.

ond time consumption is now evaluated differently by the different countries, unilateral beneficial deviation for a country is now possible at the original allocations. In particular, it is easy to show that with different time preferences the first best tax rates defined by t^*_s cannot longer be sustained as a Nash equilibrium under perfect mobility. These allocations would entail different tax rates in the first period, and therefore induce labor mobility that would violate the intergovernmental budget constraint of the governments. Formally,

Proposition 5 *If $\delta_A \neq \delta_B$, at the Nash equilibrium it is either $t^n_{As} \neq t^n_{Bs}$ or $T^n_{As} \neq T^n_{Bs}$ or both (for $s = \alpha, \beta$), and it is $m \neq 0$ and $M \neq 0$. The equilibrium is not Pareto efficient.*

At a Nash equilibrium, with different time preferences the two national governments will then set different tax/subsidy rates. However, people will move and gross wages adjust, so that the time path of consumption will be the same across the two countries. As a consequence, the two countries are forced to follow a consumption path different from the Pareto efficient path obtained in absence of labor mobility. Formally:

$$\frac{C^n_{A\alpha}}{c^n_{A\alpha}} = \frac{C^n_{B\alpha}}{c^n_{B\alpha}}, \ \frac{C^n_{A\alpha}}{c^n_{A\alpha}} \neq \frac{C^*_{A\alpha}}{c^*_{A\alpha}}, \ \frac{C^n_{B\alpha}}{c^n_{B\alpha}} \neq \frac{C^*_{B\alpha}}{c^*_{B\alpha}}$$

In Appendix 1, we further prove:

Proposition 6 *$\delta_A \leq \delta_B$ at a Nash equilibrium implies $t^n_A \geq t^n_B$ when the shock in the first period is negative.*

Intuitively, the two countries still attempt to push the consumption path toward their most preferred allocation by selecting different tax rates in the first period, but they are frustrated at the equilibrium. The basic effect of mobility is to get the two taxes in the first period closer than they would have been in absence of mobility. But whatever the choices of the two countries at the different Nash equilibria, the conclusion remain the same: different time preferences necessarily imply that these equilibria are Pareto inefficient; i.e. mobility is welfare damaging.

3.4 Federal fiscal policy

Let us now then turn to federal fiscal policy. Recall that a *federal* government is defined here as a fiscal authority which sets the tax/subsidy rates in *both* countries, after observing the state of nature (in period 1). Moreover, we distinguish between a weak federation (coordination), where each government's intertemporal budget constraint must clear, and a strong federation (federal budget), where the aggregate intertemporal budget constraint must clear. Consider first the latter case.

The federal government maximizes the following social welfare function:

$$\max_{t_{A\alpha},t_{B\alpha},T_{A\alpha},T_{B\alpha}} \ln(w_{A\alpha}+t_{A\alpha})+\delta_A \ln\left[W_{A\alpha}+T_{A\alpha}\right]+\ln(w_{B\alpha}+t_{B\alpha})+\delta_B \ln\left[W_{B\alpha}+T_{B\alpha}\right] \tag{31}$$

subject to the following *aggregate* budget constraint:

$$(N-m)t_{A\alpha}\,(1+r)+(N+m)t_{B\alpha}\,(1+r)+(N-m-M)T_{A\alpha}+(N+m+M)T_{B\alpha}=0 \tag{32}$$

where again gross wages and mobility in both periods are determined by the equilibrium conditions in the labor market (see equations 16 - 18 and 22 - 24).

Again, we assume that even after inserting the reaction function of the individuals in the objective functions, the problem is still globally convex so that it entails a unique solution (see again the discussion in the Appendix 2).

The FOCs for this problem can be written as follows:

$$\frac{C^f_{A\alpha}}{c^f_{A\alpha}}=\overline{\delta}\frac{(N-m)(1+r)+\frac{dm}{dt_A}\left[(1+r)(t_{B\alpha}-t_{A\alpha})+T_{B\alpha}-T_{A\alpha}\right]}{(N-m-M)+\frac{dM}{dT_{A\alpha}}(T_{B\alpha}-T_{A\alpha})}\frac{K}{k}\frac{1+k}{1+K} \tag{33}$$

$$\frac{C^f_{B\alpha}}{c^f_{B\alpha}}=\overline{\delta}\frac{(N+m)(1+r)+\frac{dm}{dt_A}\left[(1+r)(t_{A\alpha}-t_{B\alpha})+T_{A\alpha}-T_{B\alpha}\right]}{(N+m+M)+\frac{dM}{dT_{A\alpha}}(T_{A\alpha}-T_{B\alpha})}\frac{1+k}{1+K} \tag{34}$$

where $\overline{\delta}\equiv\frac{\delta_A+\delta_B}{2}$ is the average discount factor.

By perfect labor mobility, per capita consumption levels in each country in each period must be equalized; that is, that the LHS of the equations above must be the same. This implies that the RHS of the two equations must also be the same. Going through, it is clear that there is only one solution to the above problem that satisfies both equations simultaneously. This is for the federal government to set the same tax/subsidy rate in both countries in each period, that also implies that no one moves at the equilibrium.

Proposition 7 *The federal government sets* $t_{is}^f = t_s^f$ *and* $T_{is}^f = T_s^f$*, for* $s = \alpha, \beta$ *and* $i = A, B$*. The equilibrium number of people moving is* $m = M = 0$*. In both countries the consumption path is:*

$$\frac{C_{is}^f}{c_{is}^f} = \overline{\delta}(1+r) \qquad \text{for } s = \alpha, \beta \text{ and } i = A, B. \tag{35}$$

Intuitively, as mobility is useless, the federal government simply eliminates it by setting the same tax rates everywhere in each period, and it takes into account the different time preferences of the two countries by setting the time path of consumption according to the *average* time preference. Notice that because of the aggregate budget constraint, the federal government could in principle enforce compensating lump sum transfers across countries and let countries set different tax rates. But this would be costly, and any rate because of mobility, the desires of at least one country would be frustrated at the equilibrium as the consumption path would still have to be the same for both countries. Hence, the optimal solution is to eliminate mobility altogether and to find a compromise between the two different time preferences.

In the special case where $\delta_A = \delta_B = \delta = \overline{\delta}$, $t_{is}^f = t_{is}^n = t_s^*$ (for $s = \alpha, \beta$ and $i = A, B$) and the federal solution coincides with the national solutions and be also Pareto efficient. When preferences differ, the federal government solutions are also constrained by mobility and cannot replicate the optimal solutions $t_{is}^f = t_{is}^n = t_s^*$. But because the federal solution takes into account the utility function of both countries, we can safely conclude that in the presence of different time preferences the federal solution represents however at least a *potential* Pareto improvement with respect to any Nash equilibrium, meaning that the winner at the federal solution could in principle compensate the loser and still being better off [15]. Finally, notice that the allocation

[15] It cannot be an *actual* Pareto improvement, as at least one country would be made worse off at the federal solution with respect to the Nash equilibrium. This is so because both the federal solution and the Nash equilibria lie in between the optimal choices of the countries in absence of mobility. Hence, the country whose preferred allocation is closer to the Nash equilibrium would necessarily be worse off at the federal solution.

achieved by the federal government under the assumption of a strong federation, could be implemented also by the weak form of federation. This is so because mobility is zero, and therefore by construction not only the aggregate budget constrain, but also the budget constraint of each country must be satisfied at the federal solution. We then conclude that in the presence of symmetric shocks, coordination is enough to reach Pareto improving solutions and there is no need of a federal budget.

Still, notice that even the federal solution falls short of the first best solutions that would be reached with no mobility. Hence, when shock are symmetric, the optimal policy is still to forbid mobility altogether.

4 Asymmetric shocks

Let us then consider the case where shocks are asymmetric. In this case, the possibility to move across countries for workers plays a positive efficient role. Suppose for example that country A is hit by a negative productivity shock and country B by a positive one. These shocks introduce a positive wedge between the wages in B and A, since they increase and decrease the labour productivity in the two countries respectively. As a consequence people will move from A to B until the wage wedge vanishes, and in doing so they make the average productivity of labour across the two countries go up. Therefore there is a potential efficiency gain from labor mobility. Full mobility also plays an insurance role; by moving in the lucky countries, workers of the countries hit by the negative shock can insure themselves against the occurrence of negative shock in their country. Still, a remarkable result of the analysis below is that the federal fiscal policy enables all consumers to enjoy the full efficiency gain they can get from labor mobility, while this is not true with national fiscal policies.

4.1 Assumptions and game structure

All the assumptions of the model and the structure of the game remain unchanged, except for the following:

Productivity shock (transitory) in period 1:

	α	β
A	$\underline{\theta}$	$\overline{\theta}$
B	$\overline{\theta}$	$\underline{\theta}$

The two countries are now hit by opposite productivity shocks in period one.

4.2 The labour market *with* labour mobility

Let us see first how labor market equilibrium conditions change with asymmetric shocks. We focus on state $s = \alpha$; the analysis for $s = \beta$ is the same, with the productivity shock reversed in the two countries.

Period 2. The analysis of the previous section still applies, since in the second period the productivity shocks vanish, so the two countries share the same productivity parameter Θ. Therefore the equilibrium conditions of the labor market are unaffected by the asymmetry of the productivity shocks, which hit the two countries only in period one.

Period 1. The equilibrium conditions in the labor market are affected in the first period. In particular, equation (20) becomes:

$$\left(\frac{\overline{\theta}\gamma}{w_{B\alpha}}\right)^{\frac{1}{1-\gamma}} = N + m \tag{36}$$

since country B is hit by a positive productivity shock. As a consequence, the equilibrium wage in this country is now:

$$w_{B\alpha} = \overline{\theta}\gamma(N+m)^{\gamma-1} \tag{37}$$

and the equilibrium condition (24) is modified as follows:

$$\gamma[\underline{\theta}(N-m)^{\gamma-1} - \overline{\theta}(N+m)^{\gamma-1}] + t_{A\alpha} - t_{B\alpha} = 0 \tag{38}$$

from which the equilibrium number of people moving in the first period m is implicitly defined. Notice that now m does not only depend on the difference between $t_{A\alpha}$ and $t_{B\alpha}$; people now move from A to B even if this difference were zero. Still, we can again prove:

$$\frac{dm}{dt_{A\alpha}} = \frac{1}{\gamma(\gamma-1)[\underline{\theta}(N-m)^{\gamma-2} + \overline{\theta}(N+m)^{\gamma-2}]} < 0 \text{ and } \frac{dm}{t_{B\alpha}} = -\frac{dm}{t_{A\alpha}} \tag{39}$$

Easy computations show that the derivatives of the equilibrium wages relative to the tax/subsidy rates are still written as in equations (68), with the only difference that k is now defined by:

$$k \equiv \frac{\overline{\theta}}{\underline{\theta}}\left(\frac{N+m}{N-m}\right)^{\gamma-2} \tag{40}$$

By modifying k accordingly, **Proposition 1** still holds.

4.3 Federal fiscal policy

In the case of asymmetric shocks, it is more useful to start the analysis by considering the federal solution first. Consider the strong federation first. The federal government's problem is still defined by the objective function (31) together with the aggregate budget constraint (32). Solving the problem, the FOCs are still given by equations (33) and (34), taking into account that the derivative $\frac{dm}{dt_{A\alpha}}$ is now given by (39) and k by (40)[16].

The next proposition shows (and the Appendix proves) that the federal fiscal policy that we examined in the previous section still solves the federal government's problem, even with asymmetric shocks. The federal government sets the same tax/subsidy rate in both countries and both countries follow the same consumption path, determined by the average discount factor.

Proposition 8 *The federal government sets $t^f_{is} = t^f_s$ and $T^f_{is} = T^f_s$, for $s = \alpha, \beta$ and $i = A, B$. The equilibrium gross wage in period 1 is $w^f > \frac{1}{2}\left(\overline{w} + \underline{w}\right)$ in both countries. The equilibrium number of people moving is $m^f \neq 0$ and $M^f = -m^f$ in periods 1 and 2 respectively. In both countries the consumption path satisfies the Euler equation:*

$$\frac{C^f_s}{c^f_s} = \overline{\delta}(1+r) \qquad \textit{for } s = \alpha, \beta \tag{41}$$

Intuitively, mobility is now useful, and the federal government has no reason to interfere with it. But efficiency enhancing mobility can be obtained by simply setting the same tax rate everywhere, because in this case workers will only move following the realization of the shock and therefore increasing aggregate production and average wage. Further mobility induced by different tax rates generates only costs and no aggregate benefits, as consumption vectors would still be equalized across countries. This also implies that when the

[16] The formulas for dM/dT and K remain unchanged as in the second period the shock disappears.

two countries differ in their time preferences ($\delta_A \neq \delta_B$), labor mobility introduces a trade-off. On one hand, both countries are forced to follow the same consumption path: under this regard, the federal policy maximizes the social welfare by implementing a consumption vector proportional to the average discount factor. On the other hand, labor mobility allows both countries to enjoy an efficiency gain: again, the federal fiscal policy plays a positive role, since in equilibrium people move to fully exploit the productivity shock. This process enables all workers to get a gross wage larger than the average wage they would get without any labour mobility. It is easy to see that both w^f and m^f are increasing in the size of the productivity shock ε.

When the two countries share the same time preferences ($\delta_A = \delta_B$), the constraint to follow the same consumption path over time is not binding, so the negative side of this trade-off vanishes. Therefore the federal fiscal policy is able to achieve a first best allocation in presence of labor mobility: the welfare of all consumers is higher than in absence of labor mobility, thanks to the gain in productivity.

Finally, notice that the allocation achieved under a strong federation cannot be replicated by a weak federation, because mobility would induce a violation in at least one of the budget constraints of the two countries. Intuitively, if $t_s^f > 0$, country B, that receives m immigrants in the first period, would not to able to finance the debt in the second period with T_s^f , when the immigrants go back home. In other words, the federal budget is necessary to achieve the allocation stated in Proposition 8. Intuitively, the federal government can impose the same (federal) tax/subsidy to workers whenever they decide to live; hence, it is not constrained by the national budget constraint and can implicitly introduce transfers across countries. Thus, in presence of asymmetric shocks the strong federation Pareto dominates the weak federation.

4.4 National fiscal policies

Consider finally the case where the fiscal policy is run at the national level. We can show that the two national fiscal policies lead to equilibria[17] which are necessarily different from that obtained by the federal government, and they lead to a lower social welfare. Quite interestingly, this happens for

[17] The discussion on existence and uniqueness of Nash equilibrium in Appendix 2 extends to the case of asymmetric shocks as well, just changing k and the formula for m.

any values of intertemporal preferences, in particular even for $\delta_A = \delta_B$. At the equilibrium with national fiscal policies, the number of people moving is necessarily different from that enabling the two countries to equate their marginal labour productivity (m^f), which is instead obtained with a federal fiscal policy; therefore a federal government is able to achieve a strict Pareto improvement over the national governments.

The national governments' problem is still defined by the objective function (**??**) together with the budget constraints (25) and (26). The FOCs are still given by equations (28), taking into account that – in equations (29) and (30) – the derivative $\frac{dm}{dt_{A\alpha}}$ is now given by (39) and k by (40).

Proposition 9 *With asymmetric productivity shocks, in a Nash equilibrium it is either $t^n_{As} \neq t^n_{Bs}$ or $T^n_{As} \neq T^n_{Bs}$ or both (for $s = \alpha, \beta$), and it is $m \neq m^f$. The equilibrium is not Pareto efficient.*

The reason why the two national governments cannot follow the same fiscal policy in both periods relies on their own budget constraints under mobility. To get the intuition behind this result, focus on the state $s = \alpha$. Suppose the two governments set the same subsidy rate $t > 0$ in the first period. Then people move in order to equate their wages across the two countries. In particular, people will move from A to B; then the subsidy will be given to a lower number of people in country A than in B. If the tax rate $T < 0$ were the same in the two countries, they would end up with the same number of people N in the second period (since wages are equated only if the supply of labour is the same across countries). But then the tax rate in period two should be lower in country A, since in the previous period a lower number of people have received the subsidy: this contradicts the starting point that the tax rates are the same across countries in both periods. Hence, even if countries have the same time preferences, any Nash equilibrium must be characterized by different tax rates across periods. But with different tax rate, mobility in the first period must be necessarily different from the one that maximizes production and wages. Hence, total production is necessarily below the optimal one that could be reached under a federal government with the ability to impose federal taxes. Hence, a strong federation introduce a Pareto improvement with respect to national policies. A weak federation could still improve on Nash equilibria, because it would solve a coordination problem. But it could not reach the optimum, because it would still be unduly constrained by the national budget constraints. Our conclusion therefore is that exactly when mobility is potentially useful, because it could introduce efficiency gains, that a strong federation and a federal budget is needed.

5 Costly mobility

How our conclusions would change if we introduced costly mobility? So far, we only considered the two extreme cases of infinitely costly, or not costly at all, labor mobility. But both cases are clearly unrealistic; in general, inside a federation, labor mobility is always possible, although costly for the people involved. It is then interesting and instructive to see how our results would change in this case. For simplicity, in this section we revert to a situation of symmetric shocks, as we already proved that federal solutions are superior when mobility is efficiency enhancing. In this section, then, labor mobility is again useless in efficiency terms and can only be generated by difference in the fiscal behavior of the two countries. In order to facilitate the analysis, we also simplify the model, by assuming that N is a very large number, so much that it can be considered a continuous variable, and normalizing this value to 1, so that population in each country consists now of a unit mass of people. As usual, we only discuss the case with negative symmetric shock in the first period, $s = \alpha$, as the results for this case can be immediately extended to the other case as well.

Let us then assume that now, when moving to a country different from the original one, a consumer pays a consumption cost $k \geq 0$. Suppose further that this cost is consumer specific, meaning that different consumers may have a different cost for moving. Thus, if, say, a consumer of mobility cost k, born in country i works in country j in period 1, his consumption is equal to $w_j + t_j - k$, where $i, j = A, B,\ i \neq j$. For reasons that will be clear below, we also assume that this cost is *permanent*; that is, if the same consumer still works in country j in period 2 (or moves from country i to country j in the second period), his consumption in the second period is still $W_j + T_j - k$. Let us finally assume that the parameter k is distributed across the population according to a uniform distribution with support $[0, \psi]$, $\psi > 0$. Notice that ψ can be interpreted here as a measure of mobility costs; if ψ increases (decreases), there will be less (more) people with low mobility costs. In what follows, we will then loosely talk of a case of "costly mobility" for $0 < \psi < \overline{\psi} < \infty$, of a case of "no mobility" for $\psi \to \infty$ and very loosely, of full mobility for $\psi \to 0$[18].

The introduction of mobility costs changes our condition of arbitrage in the labor market. Thus, for example, if in the first period, because of differences in the first period subsidy paid in the two countries, $w_j + t_j > w_i + t_i$, all the people in i with $w_j + t_j - k \geq w_i + t_i$ will move to country j. At the equilibrium, then, labor market clears at:

$$w_j + t_j - k^* = w_i + t_i \tag{42}$$

where the consumer with k^* mobility cost is just indifferent between living in i or moving to country j. Solving the equation above for $k^* = w_i + t_i - w_j - t_j$, and invoking the properties of the uniform distribution, we can easily establish that a share $m^* = \frac{k^*}{\psi}$ of the population living in i will move to j because of the difference in tax rates. Clearly, as shocks are symmetric, $t_j = t_i$ implies $m^* = 0$. Substituting for m^* in w_j and in w_i , and totally differentiating, we observe:

$$dk^*/dt_j = \frac{\psi}{\psi + \gamma\underline{\theta}(1-\gamma)\left[(1+m^*)^{\gamma-2} + (1-m^*)^{\gamma-2}\right]} = -dk^*/dt_i = \psi dm^*/dt_j = -\psi dm^*/dt_i > 0; \tag{43}$$

$$dk^*/d\psi = \frac{\gamma\underline{\theta}(1-\gamma)k^*\left[(1+m^*)^{\gamma-2} + (1-m^*)^{\gamma-2}\right]}{\psi(\psi + \gamma\underline{\theta}(1-\gamma)\left[(1+m^*)^{\gamma-2} + (1-m^*)^{\gamma-2}\right])} > 0 \tag{44}$$

$$dm^*/d\psi = \frac{1}{\psi}(dk^*/d\psi - m^*) = \frac{-m^*}{\psi + \gamma\underline{\theta}(1-\gamma)\left[(1+m^*)^{\gamma-2} + (1-m^*)^{\gamma-2}\right]} < 0 \tag{45}$$

These comparative statics results will be useful below.

5.1 National fiscal policy

Suppose $\delta_A \leq \delta_B$; country A then values future consumption weakly less than country B. In absence of any mobility, this would imply $t_{A\alpha} \geq t_{B\alpha}$, and we

[18] In the present formulation, m is not correctly defined for $\psi \to 0$, contrary to the case we studied above of perfect mobility.

proved above that this is true even with full mobility. Let us then maintain the hypothesis that $t_{A\alpha} \geq t_{B\alpha}$ even with costly mobility. We will show below that this is indeed the case. For simplicity, we will also work under the hypothesis that $dm = -dM$, that can be taken as a valid approximation for the case with limited mobility (see below). Under the assumed hypothesis, $t^*_{A\alpha}$, the optimal subsidy for country A in the first period, is chosen so as to maximize

$$F_A = \log\left(\gamma\underline{\theta}(1+m^*)^{\gamma-1} + t_{A\alpha}\right) + \delta_A \log(\gamma\theta - (1+r)(1+m^*)t_{A\alpha}) \quad (46)$$

where $m^* = \frac{w_{A\alpha}+t_{A\alpha}-w_{B\alpha}-t_{B\alpha}}{\psi} \geq 0$ for $t_{A\alpha} \geq t_{B\alpha}$. To interpret this equation, notice first that in the second period, as $M^* + m^* = 0$ by assumption, the size of people living in A is just 1, the size of the original population. This means that the people working in A in the second period will get a wage equal to $\gamma\theta(1+m^*+M^*)^{\gamma-1} = \gamma\theta$ and pay a lump sum tax equal to $(1+r)(1+m^*)t^*_{A\alpha}$. Notice further that since mobility costs are assumed to be permanent, the people working in A in the second period are indeed the original residents of A. The m^* people that moved in the first period from B to A are also the first in moving back to B in the second period (when the consumption in B becomes larger than in A) as for the B citizens living in A is costly, while this is not the case for the original A's resident. And as $m^* = -M^*$ only the B people who moved in A in the first period, return to B in the second. This in turn justifies the full weight given by A's benevolent policy maker to people living in A in the second period, and also the fact that the second period preferences are multiplied for δ_A, the (common) discount factor of the original residents in A. Notice finally that in the first period m^* people from country B lives in country A; however, under the national principle, their preferences do not matter for A's choices; B'residents in A only matter for A for the effect that their presence has on the wage paid to A original residents.

Solving, equation (46) the FOC for this problem are given by[19]:

$$\partial F_A/\partial t_{A\alpha} = \frac{1}{c^A}\frac{1}{1+s} - \delta^A(1+r)\frac{1}{C^A}(1+m^*(1+\sigma)) = 0 \quad (47)$$

where $\sigma = \frac{t^*_{A\alpha}}{m^*}(dm^*/dt_{A\alpha}) > 0$ is the elasticity of immigration with respect to $t_{A\alpha}$. In equation(47),$\frac{1}{1+s} = \partial c^A/\partial t_{A\alpha} = (\partial w_{A\alpha}/\partial m^*(dm^*/dt_{A\alpha}) + 1)$, and

[19] With costly mobility, the problem is certainly globally convex and the FOC indentifies the optimal solution for government.

$s = \frac{\gamma\underline{\theta}(1-\gamma)(1+m^*)^{\gamma-2}}{\psi+\gamma\underline{\theta}(1-\gamma)(1-m^*)^{\gamma-2}}$. Notice that $0 < s < 1$, so that $0 < \partial c^A/\partial t_{A\alpha} < 1$; intuitively, increasing the transfer by one unit in the first period does not provide a unit more of consumption to A's resident, because it attracts people from above and thus reduces $w_{A\alpha}$. Note also that at $m^* = 0$, $0 < s = \frac{\gamma\underline{\theta}(1-\gamma)}{\psi+\gamma\underline{\theta}(1-\gamma)} < 1$, $\partial s/\partial\psi < 0$, and $\lim_{\psi\to\infty} s = 0$, $\lim_{\psi\to 0} s = 1$. Thus, $\partial c^A/\partial t_{A\alpha}$ tends to 1 when mobility becomes infinitely costly and tends to $\frac{1}{2}$ when mobility costs tend to zero, exactly what we would have expected on the basis of the analysis of the previous sections.

Rewriting 47, we get:

$$\frac{C^A}{c^A} = \delta^A(1+r)(1+s)(1+m^*(1+\sigma)). \tag{48}$$

As $(1+s)(1+m^*(1+\sigma)) > 1$, equation (48) immediately implies by the second order condition that the level of transfer chosen by country A under costly mobility is lower than the one it would choose without labor mobility; that is, $t^*_{A\alpha} < \overset{\circ}{t}_{A\alpha}$, where $t^*_{A\alpha} = \arg\max F_A$ and $\overset{\circ}{t}_{A\alpha}$ is the optimal tax rate under no mobility. Intuitively, a unit of transfer more in the first period under labor mobility increases consumption less in the first period, and raises tax more in the second, than under no mobility, so pushing country A to choose a lower level of transfer in the first period.

Consider now country B. Under the assumed hypothesis that $t^*_{A\alpha} \geq t^*_{B\alpha}$, $t^*_{B\alpha}$ is the argument that maximizes the following welfare function:

$$F_B = (1-m^*)\log(\gamma\underline{\theta}(1-m^*)^{\gamma-1}+t_{B\alpha})+\frac{1}{\psi}\int_o^{k^*}\log(\gamma\underline{\theta}(1+m^*)^{\gamma-1}+t_{A\alpha}-k)dk+\delta_B\log(\gamma\theta-(1+r)(\tag{49}$$

The first and third term in F_B are obvious and can be explained along the same lines of the similar terms in F^A. The term under the integral sign captures the utility of the m^* people that from country B emigrate in the first period, gaining consumption c^A but paying a cost k in order to move and work in A. Solving, the FOC for this problem can be written as:

$$\partial F_B/\partial t_{B\alpha} = \frac{s}{1+s}\frac{(\log c^A - \log c^B)}{\psi}+(1-m^*)(\frac{1}{c^B}\frac{1}{1+p}-\delta^B(1+r)\frac{1}{C^B}(1+\frac{m^*}{1-m^*}\sigma\frac{t_{B\alpha}}{t_{A\alpha}})) \tag{50}$$

The first term on the RHS of equation (50) is positive and captures the increase in the utility of the B's people that emigrate and work in A in the first period, induced by an increase in $t_{B\alpha}$. Raising $t_{B\alpha}$ infact reduces m^* and as an effect increases the wage of the people who work in A $(\partial w_{A\alpha}/\partial m^*(dm^*/dt_{B\alpha}) = \frac{s}{1+s})$; in turn, this increases the utility of the B people who live in A by the factor $\frac{(\log c^A - \log c^B)}{\psi}$, the sum of the net utility of the emigrated people. In the second term, $p = \frac{\gamma\underline{\theta}(1-\gamma)(1-m^*)^{\gamma-2}}{\psi+\gamma\underline{\theta}(1-\gamma)(1+m^*)^{\gamma-2}}$, and p has the analogous interpretations and properties than s above; in particular, $0 < p < 1$, meaning that $0 < \partial c^A/\partial t_{B\alpha} < 1$.

Inspecting (50), we immediately see that there are two contrasting effects at work. On the one hand, and analogously to what happens to country A, labor mobility reduces the advantage of increasing transfer in the first period. As m^* would fall, an increase in $t_{B\alpha}$ would increase consumption in the first period only by $\frac{1}{1+p}$ and would decrease consumption in the second period, because the B nationals (all at home in the second period by assumption) would have to pay a higher tax to refund the debt. Forgetting the first term on the RHS of (50), this would certainly lead country B to select a suboptimal level of transfer with respect to the case with no labor mobility ($(1+p)(1+\frac{m^*}{1-m^*}\sigma\frac{t_{B\alpha}}{t_{A\alpha}}) > 1$). But there is also a contrasting effect, as increasing $t_{B\alpha}$ also rises the consumption of the B people who emigrate in A, and this mitigates the previous effect. Notice however that the first term on the RHS of (50), is very small for both high and low level of ψ, so that one would expect the second term to dominate at least for these cases, leading in general to under insurance for country B too under labor mobility. In all cases, the important conclusion is that the presence of this extra term leads to $|t^*_{A\alpha} - t^*_{B\alpha}| < \left|\overset{\circ}{t}_{A\alpha} - \overset{\circ}{t}_{B\alpha}\right|$, i.e. labor mobility reduces the absolute difference in tax rates between the two countries.

Analyzing both FOCs simultaneously allows us to reach two other important conclusions. First, assume that $\delta_A = \delta_B$; then $t^*_{A\alpha} = t^*_{B\alpha} = t^*_\alpha$ is clearly a solution for two FOCs as can be seen by substituting for $t^*_{A\alpha} = t^*_{B\alpha} = t^*_\alpha$ in the formulas above (recall that at $t^*_{A\alpha} = t^*_{B\alpha} = t^*_\alpha, m^* = 0$, $c^A = c^B$ and $p = s$). But this common tax rate would now solve $\frac{C}{c} = \delta(1+r)(1+p)$ that still involves suboptimal transfers in the first period, as $s > 0$ for costly but imperfect mobility. *Hence, differently to the case with perfect mobility, even with identical preferences, costly labor mobility induces a distortion in the optimal transfers of the two countries, leading to suboptimal insurance under the negative shock and to over insurance for positive ones.* The reason lies in an externality effect; under costly mobility, the level of consumption in the two countries is not longer the same in each period, so that if say country A reduces his tax in the first period and as an effect lose people to B in this period, these A people living in B would not get the same utility as the B original residents. This distortion leads the A country to count less the increase in B utility that would do under full mobility, leading to inefficient choices.

The second observation is that, analogously to the full mobility case, $\delta_A \neq \delta_B$ implies $t^*_{A\alpha} \neq t^*_{B\alpha}$. As can be easily checked, substituting for any $t^*_{A\alpha} = t^*_{B\alpha} = t^*_\alpha$ in eqs. (48) and (50) above, would necessarily lead to one of the two equations not to be satisfied with equality, contradicting the argument that t^*_α is the optimal choice for both countries. Finally, this also allows us to conclude that indeed $\delta_A \leq \delta_B$ implies $t_{A\alpha} \geq t_{B\alpha}$. To see this, assume that $t_{A\alpha} = t_{B\alpha} = t_\alpha$ and consider a small variation in the two transfers, with one of the two countries choosing alternatively a higher and a lower level of transfers with respect to t_α. Invoking (48) and (50) , at $t_{A\alpha} = t_{B\alpha} = t_\alpha$, the change in utility for each country i would be given by

$\Delta F_i = (\partial F_i(t_{A\alpha} = t_{B\alpha} = t_\alpha)/\partial t_i)dt_i + (\partial F_i(t_{A\alpha} = t_{B\alpha} = t_\alpha)/\partial t_i)dt_j$, for $i, j = A, B$, $i \neq j$ and $dt_i > (<)0, dt_j < (>)0$

Substituting, $dt_{A\alpha} > 0$ generates a higher increase or lower decrease in welfare to country A than country B for any level of t_α; hence, A would have a larger incentive than B to deviate and raise transfer in the first period. We summarize the results of this section in the following:

Proposition 10 *Assume $dm^* + dM^* = 0$ for any first period choices of the tax/transfers rates by the two countries. Then under costly mobility, i) the Nash equilibrium in tax/transfers under national fiscal policies is always Pareto inefficient. ii)For $\delta_A = \delta_B$, $t^*_{As} = t^*_{Bs} = t^*_s$, and t^*_s involves under insurance (overinsurance) under $s = \alpha$ $(s = \beta)$ for both countries. iii) $\delta_i \leq \delta_j$ entails $t^*_{i\alpha} \geq t^*_{j\alpha}$ $i, j = A, B$, $i \neq j$. IV) $t^*_{i\alpha}, t^*_{j\alpha}$ induce under insurance (over insurance) for the country that sets the higher transfer (tax) under $s = \alpha$ $(s = \beta)$. V) Finally, $|t^*_{As} - t^*_{Bs}| < \left|t^\circ_{As} - t^\circ_{Bs}\right|$, where t°_{is} is the optimal tax/transfer rate of country i in the case of no mobility, with $i = A, B$ and $s = \alpha, \beta$.*

5.2 Federal fiscal policy

Again, we distinguish here between two possible forms of federal intervention, a weak (coordination) and a strong federation (federal budget). The distinction is even more relevant in the contest of costly mobility because as we argued above costly mobility also induces an externality effect across countries, an issue that is not present under perfect mobility (and the national principle). Thus, federal coordination surely Pareto dominates the national fiscal choices, and it is interesting to see which characteristics the federal coordination solution presents, so as to better understand the source of inefficiency under national policies.

5.2.1 Federal coordination

Let us then suppose that the federal government maximizes, by choices of $t_{A\alpha}, t_{B\alpha}$ the aggregate welfare function:

$$\Gamma = F_A(t_{A\alpha}, t_{B\alpha}) + F_B(t_{A\alpha}, t_{B\alpha}) \tag{51}$$

under the constraints that $(1+r)(1+m)t_{A\alpha} + T_{A\alpha} = 0$ and $(1+r)(1-m)t_{B\alpha} + T_{B\alpha} = 0$. Substituting for the two constraints, and differentiating, we get under the maintained assumption $t_{A\alpha} \geq t_{B\alpha}$, the following first order equations:

$$\partial\Gamma/\partial t_{A\alpha} = \partial F_A/\partial t_{A\alpha} + \frac{1}{1+s}\frac{(\log c^A - \log c^B)}{\psi} + \frac{1}{c^B}(1-m)\frac{p}{1+p} + \delta^B(1+r)\frac{1}{C^B}\frac{t_{B\alpha}}{t_{A\alpha}}m\sigma; \tag{52}$$

$$\partial\Gamma/\partial t_{B\alpha} = \partial F_B/\partial t_{B\alpha} + \frac{1}{c^A}\frac{s}{1+s} + \delta^A(1+r)\frac{1}{C^A}m\sigma; \tag{53}$$

where $\partial F_A/\partial t_{A\alpha}$ and $\partial F_B/\partial t_{B\alpha}$ are as in eqs. (47) and (**??**) above and the remaining terms in (52) and (53) captures the cross derivatives, $\partial F_B/\partial t_{A\alpha}$ and $\partial F_A/\partial t_{B\alpha}$. Notice that both are not negative; $\partial F_B/\partial t_{A\alpha} \geq 0$ and $\partial F_A/\partial t_{B\alpha} \geq 0$. Intuitively, a larger transfer in the first period in each country increases the welfare of the other country. If country B increases $t_{B\alpha}$,

m falls, and this increases A's consumption in both periods. If country A increases $t_{A\alpha}$, not only the consumption of the B people living in country B increases in both periods as m rises, but also the utility of the B people living in country A increases. These positive external effects are however not taken into account by the national decision makers and this leads to suboptimal policies. Indeed, as we have shown already, at least for one country, transfers in the first period are below the first best level.

To get further insights, let us solve for the optimal policies, setting both $\partial\Gamma/\partial t_{A\alpha}$ and $\partial\Gamma/\partial t_{B\alpha}$ equal to zero. Summing over the two resulting equations, we get that at the optimum:

$$\sum_{i=A,B} \frac{1}{C^{iF}}\left(\frac{C^{iF}}{c^{iF}}-\delta^{i}(1+r)\right)-m^{F}\left[\frac{\delta^{A}(1+r)}{C^{AF}}+\frac{1}{C^{BF}}\left(\frac{C^{BF}}{c^{BF}}-\delta^{B}(1+r)\right)\right]+\frac{(\log c^{AF}-\log c^{BF})}{\psi}=0 \tag{54}$$

where the suffix F indicates that (**??**) is evaluated at the optimal choices under federal coordination. Equation (**??**) allows for a ready interpretation; the two terms in the summation measure the distortions in the fiscal choices with respect to the first best with no labor mobility; the second two, measure the further cost and benefit of labor mobility. In fact, letting $t^{F}_{A\alpha} > t^{F}_{B\alpha}$induce a positive m^{F}; this has the cost that m^{F} people in B do not get the consumption in country B in the two periods and that the A people pay a higher tax in the second (evaluated at the respective marginal utility); but it has the positive effect that m^{F} B people now move to A, thus earning a higher utility, whose total sum is $\frac{(\log c^{AF}-\log c^{BF})}{\psi}$. At the optimum, the federal government trade off the distortions in the consumption across periods in the two countries with these further net costs of mobility. (**??**) allows us to get a first conclusion:

Proposition 11 *Suppose* $\delta_A = \delta_B$; *then* $t^{F}_{A\alpha} = t^{F}_{B\alpha} = t^{F}_{\alpha} = t^{\circ}_{\alpha}$, *where* t^{F}_{α} *solves* $\frac{C^{F}}{c^{F}} = \delta(1+r)$

Thus, coordination with identical preferences it is enough to reach the first best solution. Notice that the federal solution is in this case not only Pareto superior to national fiscal policies but it is also a strict Pareto improvement, as both countries are strictly better off under coordination than they are under fiscal policy. It is also clear from the equation above that for $\delta_A \neq \delta_B$,

$t^F_{A\alpha} = t^F_{B\alpha}$ cannot be the optimal federal solution, as for $t^F_{A\alpha} = t^F_{B\alpha}$ the last two terms in (**??**) are zero, while for $\delta_A \neq \delta_B$ at least one of the first two terms is different from zero. Putting it differently, and *differently from the full mobility case, the federal government would then not choose to equalize the tax rates of the two countries with different preferences.* Costly mobility allows consumption to differ across countries and across periods, and the federal government exploits this advantage, trading it off with the costs that a choice of different first period transfer induce in terms of positive and costly mobility.

To see these effects more clearly, let us use a first order Taylor approximation to write $\frac{(\log c^{AF} - \log c^{BF})}{\psi} = \frac{1}{c^{AF}} m^F + R$; where R represents the higher terms of the expansion. Note that by concavity of the log function, $R > 0$. Substituting, the optimal federal policy solves:

$$(1+m^F)\frac{1}{C^{AF}}(\frac{C^{AF}}{c^{AF}} - \delta^A(1+r)) + (1-m^F)\frac{1}{C^{BF}}(\frac{C^{BF}}{c^{BF}} - \delta^B(1+r)) + R = 0 \quad (55)$$

Inspecting (55), some results are evident. First, under costly mobility, the federal government would not choose the optimal first best solutions $t^\circ_{A\alpha}, t^\circ_{B\alpha}$. Doing so, in fact, could not possibly satisfy (55), making equal to zero the first two terms, but leaving $R > 0 = 0$, a contradiction. The first best solutions could only be chosen if $\psi \to \infty$, that is, for no mobility, as in this case $m^F \to 0$ and $R \to 0$. Second, (55) also implies that the optimal solutions under federal coordination must involve either suboptimal transfers in the first period for both countries (that is, $\frac{C^{iF}}{c^{iF}} < \delta^i(1+r)$ for both $i = A, B$), or suboptimal transfer for one country and sovraoptimal transfer for the other. Indeed, it is reasonable to argue that in order to avoid costly mobility while allowing some differences in consumption across periods for the two countries, so as to respect their time preferences, the federal government would reduce the difference in the two transfers by setting $t^\circ_{A\alpha} > t^F_{A\alpha} > t^F_{B_\alpha} > t^\circ_{B\alpha}$. Summing up,

Proposition 12 *Let $\delta_A > \delta_B$; then, under costly mobility, i) $t^F_{As} \neq t^F_{Bs}$ and ii) $t^F_{is} \neq t^\circ_{is}$, $i = A, B$. iii)The optimal federal solution involves either underinsurance (resp. over insurance) for both countries under $s = \alpha$ (resp. $s = \beta$), or over insurance for a country and under insurance for the other. IV) Finally,$\left|t^F_{As} - t^F_{Bs}\right| < \left|t^\circ_{As} - t^\circ_{Bs}\right|$.*

5.2.2 Federal budget

Let us suppose now that the federal government not only coordinates national policies, but it can enforce a federal budget. The problem of the federal government could then be written as:

$$\Lambda = F_A(t_{A\alpha}, t_{B\alpha}, T^A) + F_B(t_{A\alpha}, t_{B\alpha}, T^B) - \lambda(((1+m)t_{A\alpha} + (1-m)t_{B\alpha})(1+r) - (T^A + T^B)) \tag{56}$$

Notice that we now distinguish in the welfare of the two countries the effect of first and second period tax/ transfers; indeed with a federal budget an increase in the transfer today by a country does not automatically lead to an increase in tax payment tomorrow; this depends on federal choices on the allocation of the extra burden across the two countries.

Solving the federal problem, and writing the first order conditions in full we get:

$$t_{A\alpha} : \frac{1}{c^A}\frac{1}{1+s} + \frac{1}{1+s}\frac{(\log c^A - \log c^B)}{\psi} + \frac{1}{c^B}(1-m)\frac{p}{1+p} = \lambda(1+r)((1+m) + t_{A\alpha}\partial m/\partial t_{A\alpha}); \tag{57}$$

$$t_{B\alpha} : \frac{1}{c^A}\frac{s}{1+s} + \frac{s}{1+s}\frac{(\log c^A - \log c^B)}{\psi} + (1-m)\frac{1}{c^B}(\frac{1}{1+p}) = \lambda(1+r)((1-m) + t_{B\alpha}\partial m/\partial t_{A\alpha}); \tag{58}$$

$$T^A : \frac{\delta^A}{C^A} = \lambda; \tag{59}$$

$$T^B : \frac{\delta^B}{C^B} \tag{60}$$

$$\lambda : (1+r)\ ((1+m)t_{A\alpha} + (1-m)t_{B\alpha}) = (T^A + T^B) \tag{61}$$

Inspection of the first order conditions immediately implies that with different time preferences, the federal policy maker would not equalize consumption across periods; indeed, equations (59) and (60) imply $\frac{C^A}{C^B} = \frac{\delta^A}{\delta^B}$, and simple inspection of (57) and (58) shows that $t_{A\alpha} = t_{B\alpha}$ could not possible satisfy these two equations. To get further insights, let us sum together

(57) and (58), substitute inside for (59) and (60), and use again our previous approximation for $\frac{(\log c^A - \log c^B)}{\psi}$. Collecting terms, this gives

$$(1+m^{F\prime})\frac{1}{C^{AF\prime}}(\frac{C^{AF\prime}}{c^{AF\prime}}-\delta^A(1+r))+(1-m^{F\prime})\frac{1}{C^{BF\prime}}(\frac{C^{BF\prime}}{c^{BF\prime}}-\delta^B(1+r))+R'-m^{F\prime}\sigma'\lambda(1+r)(1+\frac{t'_{B\alpha}}{t'_{A\alpha}})=0 \tag{62}$$

Where the prime upon the variables serves as a remind to the reader that equation (62) is evaluated at the optimal choices with a *federal* budget. Comparing ((62) with (55) we see that the equations have similar terms, except for an extra term in the former, $-m^{F\prime}\sigma'\lambda(1+r)(1+\frac{t'_{B\alpha}}{t'_{A\alpha}})<0$. (62) clearly implies $t_{is}^{F\prime} \neq t_{is}^{\circ}$, $i = A, B$. Furthermore, the presence of the extra term in (62) suggests that the two first terms in (62) are smaller in absolute values than the corresponding terms in (55), meaning that $\frac{C^{iF\prime}}{c^{iF\prime}}$is closer to $\delta^i(1+r)$ than $\frac{C^{iF}}{c^{iF}}$. In other words, the structure of the solution under the federal budget is similar to the solution under coordination only; that is, for $\delta_A \neq \delta_B, t_{As}^{F\prime} \neq t_{Bs}^{F\prime}$ and $t_{is}^{F\prime} \neq t_{is}^{\circ}$, $i = A, B$. But the federal government use the extra room provided by the federal budget to ease the distortions in the optimal consumption plans of the two countries, making $t_{is}^{F\prime}$ closer to t_{is}° than t_{is}^{F}.

6 Concluding remarks

In this paper we reexamine the well known proposition that an optimal currency area requires perfect labor mobility in order to work smoothly. Since first formulated by Robert Mundell, this proposition has become part of the common wisdom on optimal currency areas, and has been used, for instance, to criticize the current functioning of the EMU. We enrich the usual framework by considering an intertemporal setting and the possibility of efficiency enhancing fiscal policy, due to imperfections in the working of capital markets. We get quite different results from the conventional view. Even when labor mobility is perfect and it is potentially efficiency enhancing because the member countries of the currency union are hit by temporary asymmetric shocks, national fiscal policies alone cannot reach the first best equilibrium. The reason is that at the equilibrium countries are induced to compete in order to attract the mobile workers and their tax bases and this leads to inefficient level of mobility. We also show that a federal government with the ability of imposing federal labor taxes could solve the problem, generating a strict Pareto improvement and allowing only for that regional mobility that maximizes aggregate production and welfare. The reason is that the federation has the advantage of being able to tax individuals whenever they decide to live and therefore to implicitly introduce transfers across countries, a possibility that is not open to national countries or regions. Hence, contrary to the conventional vision, a federal budget appears to be essential to the good working of a currency area, in particular when labor mobility is efficiency enhancing. Labor mobility and federal budget are not substitute but complement each other in a optimal currency union.

The model we discussed is of course a far cry from realism, but certainly our conclusions do not support the view that, for instance, lack of labor mobility is the main problem of the EMU. Even in our narrow perspective, lack of a federal government and of a federal budget would seem to be at least as problematic.

Our point has been made in an extremely simple model, although we also show that its basic insights extend at least to the case with costly labor mobility. Of course, governments are not always welfare maximizers, public expenditure and debt are not only used to smooth consumptions across periods, private citizens and not only governments can to some extent gain access to capital markets, governments do bankrupt sometimes and markets are not perfectly competitive. But while discussing all these possible extensions could be interesting, we believe that our basic point is sound and would survive all these extensions. Labor mobility (and more generally factor mobility) may interfere with national (or regional) dynamic fiscal policy, because by moving people (and factors) might reduce future tax bases. This may make unsustainable a dynamic fiscal path that would otherwise be beneficial for citizens. A federation with tax powers and the possibility of introducing transfers across countries or regions can ease this problem.

7 APPENDIX 1

Proof of Proposition 1

Period 2. By applying the implicit function theorem to equation (18), we get:

$$\frac{dM}{dT_{A\alpha}} = \frac{1}{\Theta\gamma(\gamma-1)[(N-m-M)^{\gamma-2}+(N+m+M)^{\gamma-2}]} < 0 \quad (63)$$

$$\text{and } \frac{dM}{T_{B\alpha}} = -\frac{dM}{T_{A\alpha}} > 0 \quad (64)$$

From equations (16) and (17) we can derive:

$$\frac{dW_{A\alpha}}{dT_{A\alpha}} = \frac{dW_{A\alpha}}{dM}\frac{dM}{dT_{A\alpha}} = -\frac{1}{1+K} < 0 \text{ and } \frac{dW_{B\alpha}}{dT_{B\alpha}} = \frac{dW_{B\alpha}}{dM}\frac{dM}{dT_{B\alpha}} = -\frac{K}{1+K} < 0 \quad (65)$$

where

$$K \equiv \left(\frac{N+m+M}{N-m-M}\right)^{\gamma-2} > 0 \quad (66)$$

Period 1. By applying the implicit function theorem to equation (24), we get:

$$\frac{dm}{dt_{A\alpha}} = \frac{1}{\underline{\theta}\gamma(\gamma-1)[(N-m)^{\gamma-2}+(N+m)^{\gamma-2}]} < 0 \text{ and } \frac{dm}{t_{B\alpha}} = -\frac{dm}{t_{A\alpha}} > 0 \quad (67)$$

From equations (22) and (23) we can derive:

$$\frac{dw_{A\alpha}}{dt_{A\alpha}} = \frac{dw_{A\alpha}}{dm}\frac{dm}{dt_{A\alpha}} = -\frac{1}{1+k} < 0 \text{ and } \frac{dw_{B\alpha}}{dt_{B\alpha}} = \frac{dw_{B\alpha}}{dm}\frac{dm}{dt_{B\alpha}} = -\frac{k}{1+k} < 0 \quad (68)$$

where

$$k \equiv \left(\frac{N+m}{N-m}\right)^{\gamma-2} > 0 \quad (69)$$

Proof of Proposition 2.

Let be $\delta \equiv \delta_A = \delta_B$.

Consider the state $s = \alpha$. Let be $m = M = 0$. From equations (16) and (17) we get: $W_{i\alpha} = W^*$ for $i = A, B$. From equations (22) and (23) we get $w_{i\alpha} = \underline{w}$ for $i = A, B$. The budget constraints (25) and (26) boil down to $t_{i\alpha}(1+r) + T_{i\alpha} = 0$ for $i = A, B$; hence $\frac{dT_{i\alpha}}{dt_{i\alpha}} = -(1+r)$ for $i = A, B$. It is also $K = k = 1$. Therefore the two FOCs (28) boil down to the standard

Euler equation (10) with $\delta_i = \delta$ for $i = A, B$, from which the equilibrium tax/subsidy rates are $t_{i\alpha} = t^*_\alpha$, $T_{i\alpha} = -t^*_\alpha(1+r)$ for $i = A, B$.

To show that this is a Nash equilibrium, consider a deviation by one country, say A, from t^*_α to $t^*_\alpha + dt_{A\alpha}$ and suppose that this deviation increases the intertemporal utility of consumers of country A. However, as for $\delta_i = \delta$ the two countries have the same preferences, and by perfect labor mobility consumption is equalized in each country in each period, this means that consumers of country B would also gain from the change. But this would mean that the change from t^*_α to $t^*_\alpha + dt_{A\alpha}$ is a Pareto improving move, which would contradict the fact that $t_{i\alpha} = t^*_\alpha$ is Pareto efficient. This means that there cannot exist a unilateral deviation from t^*_α that benefits a single country. Hence, $t_{i\alpha} = t^*_\alpha$ is a Nash equilibrium. The same reasoning applies to the state $s = \beta$. QED

Proof of Proposition 3

Consider the state $s = \alpha$. Let $m = M = 0$. From equations (16) and (17) we get: $W_{i\alpha} = W^*$ for $i = A, B$. From equations (22) and (23) we get $w_{i\alpha} = \underline{w}$ for $i = A, B$. From equations (18) and (24), it follows that $t_{A\alpha} = t_{B\alpha}$ and $T_{A\alpha} = T_{B\alpha}$. Hence $C_{A\alpha} = C_{B\alpha}$ and $c_{A\alpha} = c_{B\alpha}$, and $\frac{C_{A\alpha}}{c_{A\alpha}} = \frac{C_{B\alpha}}{c_{B\alpha}}$. But $m = M = 0$, $t_{A\alpha} = t_{B\alpha}$ and $T_{A\alpha} = T_{B\alpha}$ also imply that $\frac{dT_{A\alpha}}{dt_{A\alpha}} = \frac{dT_{B\alpha}}{dt_{B\alpha}}$ (see equations 29 - 30), and from the FOCs (28) we get: $\frac{C^n_{A\alpha}}{c^n_{A\alpha}} < \frac{C^n_{B\alpha}}{c^n_{B\alpha}}$. (since $\delta_A < \delta_B$). Therefore at least one country has an incentive to deviate. So in equilibrium it cannot be $m = M = 0$.

Now let $t_{A\alpha} = t_{B\alpha}$ and $T_{A\alpha} = T_{B\alpha}$. From equations (18) and (24), it follows that $m = M = 0$, but this cannot be the case in equilibrium, as we have seen above. So in equilibrium it cannot be $t_{A\alpha} = t_{B\alpha}$ and $T_{A\alpha} = T_{B\alpha}$.

Finally, the equilibrium conditions in the labor market (equations 15 and 21) imply that it must be $\frac{C^n_{A\alpha}}{c^n_{A\alpha}} = \frac{C^n_{B\alpha}}{c^n_{B\alpha}}$. But the Euler equation (10) imply that $\frac{C^*_{A\alpha}}{c^*_{A\alpha}} < \frac{C^*_{B\alpha}}{c^*_{B\alpha}}$ (with $\delta_A < \delta_B$). Therefore it cannot be $\frac{C^n_{A\alpha}}{c^n_{A\alpha}} = \frac{C^*_{A\alpha}}{c^*_{A\alpha}}$ and $\frac{C^n_{B\alpha}}{c^n_{B\alpha}} = \frac{C^*_{B\alpha}}{c^*_{B\alpha}}$.

The same reasoning applies to the state $s = \beta$. QED

Proof of Proposition 4

Recall that a Nash equilibrium, $\frac{C^n_{A\alpha}}{c^n_{A\alpha}} = \frac{C^n_{B\alpha}}{c^n_{B\alpha}} = \frac{C^n}{c^n}$. Using the notation of Appendix 2 for convenience, this implies that the two country FOC at the equilibrium can be written as:

$\frac{1}{c^n}\frac{k}{1+k} = -\frac{\delta_A}{C^n} dC_{Aa}/dt_{A\alpha}$

$\frac{1}{c^n}\frac{1}{1+k} = -\frac{\delta_B}{C^n} dC_{Ba}/dt_{B\alpha}$

Hence
$\left(\frac{N-m}{N+m}\right)^{2-\gamma} = k = \frac{\delta_A dC_{Aa}/dt_{A\alpha}}{\delta_B dC_{Ba}/dt_{B\alpha}}$ or $S \equiv k\frac{dC_{Ba}/dt_{B\alpha}}{dC_{Aa}/dt_{A\alpha}} = \frac{\delta_A}{\delta_B}$

Assume $\delta_B > \delta_A$. Note first that at $t_{B\alpha} = t_{A\alpha}$, $S = 1$ that would contradict the equality. Hence, $t_{B\alpha} \neq t_{A\alpha}$. To determine whether $t_{B\alpha} > t_{A\alpha}$ or viceversa, note first that the two components of S move in opposite directions with respect to the sign of the difference between the two tax rates. For instance, if $t_{B\alpha} > t_{A\alpha}$, $m > 0$ and $k < 1$. But then$\frac{dC_{Ba}/dt_{B\alpha}}{dC_{Aa}/dt_{A\alpha}} > 1$ and it is unclear which of the two effects dominates the other. To make further progress consider a small variation in one of two taxes, say $t_{B\alpha}$, to $t_{B\alpha}+dt_{B\alpha}$, starting from the situation where $t_{B\alpha} = t_{A\alpha}$. This would mean to compute:

$dS = (\frac{dk}{dt_{B\alpha}}\frac{dC_{Ba}/dt_{B\alpha}}{dC_{Aa}/dt_{A\alpha}} + k\frac{d}{dt_{B\alpha}}(\frac{dC_{Ba}/dt_{B\alpha}}{dC_{Aa}/dt_{A\alpha}}))dt_{B\alpha}$

and evaluated it at $t_{B\alpha} = t_{A\alpha}$ i.e. at $m = 0$. That is:

$dS|_{t_{B\alpha}=t_{A\alpha}} = (-\frac{(2-\gamma)N^{1-\gamma}}{\underline{\theta}\gamma(1-\gamma)} - \frac{2(d^2C_{Ba}/dt_{B\alpha}^2 - d^2C_{Aa}/dt_{A\alpha}dt_{Ba})}{1+r})dt_{B\alpha}$

The first term in round brackets is certainly negative; the second (evaluated at $m = 0$) is certainly positive. However, inspection of equation (**) in Appendix 2 would suggest that at least for γ close to $\frac{1}{2}$, the second term dominates the first, because the function $C_{i\alpha}$ is strongly concave at $m = 0$. If this is the case, then $\frac{dS}{dt_{B_\alpha}}|_{t_{B\alpha}=t_{A\alpha}} > 0$. But then increasing $t_{B\alpha}$ would make S even larger than 1. It follows that instead $t_{A\alpha}$ should raise. Invoking continuity, this then implies that $\delta_B \geq \delta_A$, requires $t_{A\alpha} \geq t_{B\alpha}$ at the equilibrium.

Proof of Proposition 5

Consider the state $s = \alpha$. The cross-country equality of net wages (see equations (15 and 21)) implies $\frac{C^f_{A\alpha}}{c^f_{A\alpha}} = \frac{C^f_{B\alpha}}{c^f_{B\alpha}}$, which in turn implies (by equating the RHS of the FOCs 33 - 34) the following condition:

$$\frac{(N-m)(1+r) + \frac{dm}{dt_A}\left[(1+r)(t_{B\alpha} - t_{A\alpha}) + T_{B\alpha} - T_{A\alpha}\right]}{(N-m-M) + \frac{dM}{dT_{A\alpha}}(T_{B\alpha} - T_{A\alpha})}\frac{K}{k} =$$

$$= \frac{(N+m)(1+r) + \frac{dm}{dt_A}\left[(1+r)(t_{A\alpha} - t_{B\alpha}) + T_{A\alpha} - T_{B\alpha}\right]}{(N+m+M) + \frac{dM}{dT_{A\alpha}}(T_{A\alpha} - T_{B\alpha})}$$

and a solution to this equation is: $t_{i\alpha} = t_\alpha$, $T_{i\alpha} = T_\alpha$ for $i = A, B$ and $m = M = 0$. Hence both the FOCs boil down to the Euler equation (35).

The equilibrium conditions in the labor market give: $W_{i\alpha} = W^*$ and $w_{i\alpha} = \underline{w}$, for $i = A, B$.

The same reasoning applies to the state $s = \beta$.

Proof of Proposition 8

Consider the state $s = \alpha$. The cross-country equality of net wages (see equations (15 and 21)) implies $\frac{C^f_{A\alpha}}{c^f_{A\alpha}} = \frac{C^f_{B\alpha}}{c^f_{B\alpha}}$, which in turn implies (by equating the RHS of the FOCs 33 - 34) the following condition:

$$\frac{(N-m)(1+r) + \frac{dm}{dt_A}\left[(1+r)(t_{B\alpha} - t_{A\alpha}) + T_{B\alpha} - T_{A\alpha}\right]}{(N-m-M) + \frac{dM}{dT_{A\alpha}}(T_{B\alpha} - T_{A\alpha})} \frac{K}{k} =$$

$$= \frac{(N+m)(1+r) + \frac{dm}{dt_A}\left[(1+r)(t_{A\alpha} - t_{B\alpha}) + T_{A\alpha} - T_{B\alpha}\right]}{(N+m+M) + \frac{dM}{dT_{A\alpha}}(T_{A\alpha} - T_{B\alpha})}$$

and a solution to this equation is: $t^f_{i\alpha} = t^f_\alpha$, $T^f_{i\alpha} = T^f_\alpha$ for $i = A, B$, and $M^f = -m^f$ with m^f given by:

$$m^f = N \frac{(1+\varepsilon)^{\frac{1}{1-\gamma}} - (1-\varepsilon)^{\frac{1}{1-\gamma}}}{\Omega} > 0 \tag{70}$$

where $\Omega \equiv (1+\varepsilon)^{\frac{1}{1-\gamma}} + (1-\varepsilon)^{\frac{1}{1-\gamma}}$. Hence both the FOCs boil down to the Euler equation (41). The equilibrium conditions in the labor market give: $W_{i\alpha} = W^*$ (defined by equation 4) and $w_{i\alpha} = w^f$ for $i = A, B$, where:

$$w^f = W^* \left(\frac{\Omega}{2}\right)^{1-\gamma} > W^* \tag{71}$$

and $\Omega > 2$ is obtained by applying the Jensen's inequality to the function $y = x^{\frac{1}{1-\gamma}}$. Since $W^* = \frac{1}{2}\left(\overline{w} + \underline{w}\right)$, it is $w^f > \frac{1}{2}\left(\overline{w} + \underline{w}\right)$.

The same reasoning applies to the state $s = \beta$, with $m^f < 0$.

Proof of Proposition 9

Part A: it is either $t^n_{As} \neq t^n_{Bs}$ or $T^n_{As} \neq T^n_{Bs}$ or both. Consider the state $s = \alpha$, and let be $t_{i\alpha} = t_\alpha$ and $T_{i\alpha} = T_\alpha$ for $i = A, B$. Equation (18) and $T_{A\alpha} = T_{B\alpha}$ imply $M = -m$. Equation (**??**) and $t_{A\alpha} = t_{B\alpha}$ imply $m = m^f$

(defined by 70). By inserting these values of m and M into the national budget constraints (25) and (26), we get:

$$\begin{aligned} T_{A\alpha} &= -\frac{(N-m^f)t_\alpha(1+r)}{N} \\ T_{B\alpha} &= -\frac{(N+m^f)t_\alpha(1+r)}{N} \end{aligned}$$

which in turn imply that it must be $T_{A\alpha} \neq T_{B\alpha}$. Hence assuming that $t_{i\alpha} = t_\alpha$ and $T_{i\alpha} = T_\alpha$ for $i = A, B$ leads to a contradiction.

Part B: $m \neq m^f$. Let be $\delta_A = \delta_B$. Equation (**??**) and $m = m^f$ (defined in 70) imply that $t_{A\alpha} = t_{B\alpha}$. Then condition (**??**) becomes

$$\frac{(N-m^f)}{(N-m^f-M) - T_{A\alpha}\frac{dM}{dT_{A\alpha}}}\frac{K}{k} = \frac{(N+m^f)}{(N+m^f+M) - T_{B\alpha}\frac{dM}{dT_{A\alpha}}} \tag{72}$$

since $\frac{dm}{dt_{A\alpha}} = 0$ for $t_{A\alpha} = t_{B\alpha}$. The solution to this equation is $M = -m^f$ and $T_{A\alpha} = T_{B\alpha}$. Hence $m = m^f$ implies that $t_{A\alpha} = t_{B\alpha}$ and $T_{A\alpha} = T_{B\alpha}$, but this cannot be the case, as shown in Part A.

The same reasoning (for both Parts A and B) applies to the state $s = \beta$.

8 Appendix 2

We focus on country A and state of the world α; the results for country B and state β are analogous. We begin with the case of symmetric shocks. From the intertemporal budget constraint in this case:

$$T_{A\alpha} = -\frac{(N-m)t_{A\alpha}(1+r)}{(N-m-M)}; T_{B\alpha} = -\frac{(N+m)t_{B\alpha}(1+r)}{(N+m+M)}$$

Substituting in the labor market equilibrium conditions for the second period,

$$\Theta\gamma[(N-m-M)^{\gamma-1} - (N+m+M)^{\gamma-1}] + T_{A\alpha} - T_{B\alpha} = 0$$

and totally differentiating for $t_{A\alpha}$, we get the relationship between first period and second period mobility:

$$\left\{\Theta\gamma(1-\gamma)[(N-m-M)^{\gamma-2} + (N+m+M)^{\gamma-2}] - \frac{(N-m)t_{A\alpha}(1+r)}{(N-m-M)^2} - \frac{(N+m)t_{B\alpha}(1+r)}{(N+m+M)^2}\right\} dM/dt_{A\alpha}$$
$$-\left\{\Theta\gamma(1-\gamma)[(N-m-M)^{\gamma-2} + (N+m+M)^{\gamma-2}] - \frac{Mt_{A\alpha}(1+r)}{(N-m-M)^2} + \frac{Mt_{B\alpha}(1+r)}{(N+m+M)^2}\right\} dm/dt_{A\alpha}+$$

$+\frac{(N-m)(1+r)}{(N-m-M)}$;

Manipulating the formulas:

(*) $dM/dt_{A\alpha} = -dm/dt_{A\alpha} - A^{-1}(\frac{t_{A\alpha}\,(1+r)}{(N-m-M)} + \frac{t_{B\alpha}\,(1+r)}{(N+m+M)})dm/dt_{A\alpha} + A^{-1}\frac{(N-m)(1+r)}{(N-m-M)}$

where $A = \frac{\{C_{Aa}-\gamma W_{A\alpha}\}}{(N-m-M)} + \frac{\{C_{Ba}-\gamma W_{B\alpha}\}}{(N+m+M)} > 0$ is the coefficient of $dM/dt_{A\alpha}$ in the equation above. Eq. (*) shows that $dM/dt_{A\alpha} > 0$, and it is larger in absolute terms than $dm/dt_{A\alpha}$.

Consider now consumption in the second period in country A:

$$C_{Aa} = W_{A\alpha} + T_{A\alpha} = \Theta\gamma(N-m-M)^{\gamma-1} - \frac{(N-m)t_{A\alpha}\,(1+r)}{(N-m-M)}$$

totally differentiating, we get:

$$dC_{Aa}/dt_{A\alpha} = \left\{\Theta\gamma(1-\gamma)(N-m-M)^{\gamma-2} - \frac{(N-m)t_{A\alpha}\,(1+r)}{(N-m-M)^2}\right\} dM/dt_{A\alpha} +$$

$$\left\{\Theta\gamma(1-\gamma)[(N-m-M)^{\gamma-2} - \frac{Mt_{A\alpha}\,(1+r)}{(N-m-M)^2}\right\} dm/dt_{A\alpha} - \frac{(N-m)(1+r)}{(N-m-M)}$$

which can be rewritten, by manipulating the formulas, as

$$dC_{Aa}/dt_{A\alpha} = BdM/dt_{A\alpha} + \left\{B + \frac{t_{A\alpha}\,(1+r)}{(N-m-M)}\right\} dm/dt_{A\alpha} - \frac{(N-m)(1+r)}{(N-m-M)}$$

where $B = \frac{\{C_{Aa}-\gamma W_{A\alpha}\}}{(N-m-M)}$. Substituting for $dM/dt_{A\alpha}$ from (*), we finally get

(**)

$$dC_{Aa}/dt_{A\alpha} = (1+r)\left\{\frac{t_{A\alpha}}{(N-m-M)}(1-BA^{-1}) - BA^{-1}\frac{t_{B\alpha}}{(N+m+M)})\right\} dm/dt_{A\alpha} - \frac{(N-m)(1+r)}{(N-m-M)}(1-BA^{-1})$$

Note that $BA^{-1} < 1$. The second term is then certainly negative while the sign of the first depends on the difference between the two tax rates. At $t_{A\alpha} = t_{B\alpha}$ (that implies $m = M = 0$), $BA^{-1} = 1/2$ and the first term is zero. At $t_{A\alpha} > (<) t_{B\alpha}$ the first term is negative (positive). By continuity, this term is however small and dominated by the second for m small, that is, as long as the two tax rates are not too different. Hence, $dC_{Aa}/dt_{A\alpha} < 0$ for all $t_{A\alpha} \geq t_{B\alpha}$ and it is still negative for $t_{A\alpha} < t_{B\alpha}$ provided that $t_{B\alpha}$ is not too larger than $t_{A\alpha}$. Intuitively, if $t_{A\alpha} \geq t_{B\alpha}$,increasingly further $t_{A\alpha}$ implies a higher cost in terms of second period consumption for country A. If $t_{A\alpha} < t_{B\alpha}$ this cost is lower, and possibly, if $t_{B\alpha}$ is much higher than

$t_{A\alpha}$, the cost might even become negative. Finally, note that at $t_{A\alpha} = t_{B\alpha}$, $dC_{Aa}/dt_{A\alpha} = \frac{-(1+r)}{2}$.

Recalling the definition of $\frac{dm}{dt_{A\alpha}}$ from Proposition 1, it can be proved that $\frac{d^2m}{dt^2_{A\alpha}} = 0$ for $m = 0$ and $\frac{d^2m}{dt^2_{A\alpha}} < (>)0$ for $m < (>)0$. Using this fact, and differentiating again (**), one gets that $d^2C_{Aa}/dt^2_{A\alpha} < 0$ for $t_{A\alpha} \geq t_{B\alpha}$; for $t_{A\alpha} < t_{B\alpha}$ the sign is uncertain, but it is negative by continuity for $t_{B\alpha}$ not too larger than $t_{A\alpha}$, for the same reasons already discussed above. Finally, $d^2C_{Aa}/dt_{A\alpha}dt_{B\alpha} > 0$ under the same conditions.

Convexity of government problem under perfect mobility.

Consider again government $A's$ problem in state α in the case of symmetric shocks. Government max $F^A = \ln(c_{Aa}) + \delta_A \ln(C_{Aa})$, where all mobility and budget constraints are directly substituted in the objective function. The FOC for this problem, invoking Proposition 1 and the above, can be written as:

$$dF^A/dt_{A\alpha} = \frac{1}{c_{Aa}}\frac{k}{1+k} + \frac{\delta_A}{C_{Aa}} dC_{Aa}/dt_{A\alpha} = 0$$

where $k \equiv \left(\frac{N+m}{N-m}\right)^{\gamma-2}$ and $dC_{Aa}/dt_{A\alpha}$ is given by the equation (**) above. Clearly, $dC_{Aa}/dt_{A\alpha}$ must be negative for this equation to hold. If $dC_{Aa}/dt_{A\alpha} > 0$, that as we showed above can however only happen for $t_{B\alpha}$ much larger than $t_{A\alpha}$, the best solution for government A would be to keep increasing $t_{A\alpha}$ up to the point in which $dC_{Aa}/dt_{A\alpha}$ becomes negative and equal the first term. Hence, the equation above certainly characterizes government behavior. The equation defines a (local) maximum if the SOC are also satisfied. Taking the second derivative:

$$d^2F^A/dt^2_{A\alpha} = -\frac{1}{(c_{Aa})^2}\left(\frac{k}{1+k}\right)^2 - \frac{\delta_A}{(C_{Aa})^2}(dC_{Aa}/dt_{A\alpha})^2 + \frac{\delta_A}{C_{Aa}} d^2C_{Aa}/dt^2_{A\alpha} - \frac{1}{c_{Aa}}\frac{k}{(1+k)^2}\frac{2N(2-\gamma)}{(N+m)(N-m)} dm/dt_{A\alpha}$$

the first two terms are certainly negative, the third is negative under the conditions stated above, and the fourth is certainly positive as $dm/dt_{A\alpha} < 0$. The sign is therefore uncertain. Notice however that for m close to zero, the fourth term is certainly dominated by the first, so that $d^2F^A/dt^2_{A\alpha} < 0$. By continuity, then the SOC are certainly satisfied if m is not too large, that means that the two tax rates cannot be two far one from the others and that the parameter determining the elasticity of m to tax rate differentials, γ, must be close to 1/2.

Nash equilibria

Assuming the SOC is satisfied, the FOC above identifies the optimal choice of $t_{A\alpha}$, $t^*_{A\alpha}$. In particular, we have the identity:

$dF^A/dt_{A\alpha}(t^*_{A\alpha}; t_{B\alpha}) \equiv 0$, where $t_{B\alpha}$ is the tax choice by country B. Totally differentiating:

$dt^*_{A\alpha}/dt_{B\alpha} = -\frac{d^2F^A/dt_{A\alpha}dt_{B\alpha}}{d^2F^A/dt^2_{A\alpha}}$

thus $sign(dt^*_{A\alpha}/dt_{B\alpha}) = sign(d^2F^A/dt_{A\alpha}dt_{B\alpha})$, as $d^2F^A/dt^2_{A\alpha} < 0$. Computing:

$d^2F^A/dt_{A\alpha}dt_{B\alpha} =$

$-\frac{1}{(c_{Aa})^2}(\frac{k}{1+k})^2 - \frac{\delta_A}{(C_{Aa})^2}(dC_{Aa}/dt_{A\alpha})^2 + \frac{\delta_A}{C_{Aa}}d^2C_{Aa}/dt_{A\alpha}dt_{Ba} - \frac{1}{c_{Aa}}\frac{k}{(1+k)^2}\frac{2N(2-\gamma)}{(N+m)(N-m)}dm/dt_{B\alpha}$

The first two elements derive by perfect mobility (see proposition 1); an increase in $t_{B\alpha}$ have the same effect on c_{Aa} than an increase in $t_{A\alpha}$; similarly for C_{Aa}. The third element is positive, under the conditions stated above, the fourth is negative (as $dm/dt_{B\alpha} > 0$). Hence, unless the third element is very large, the reaction function is negatively sloped; that is, a higher $t_{B\alpha}$ would reduce $t^*_{A\alpha}$.

Notice further that as long as the problem of the govern is well behaved, the reaction functions are continuous and map a compact set in itself. Hence a Nash equilibrium in the first period tax rates surely exists. Uniqueness would also follow if each reaction function were a contraction in the tax rates. That is, provided that

$$dt^*_{A\alpha}/dt_{B\alpha} = \left| -\frac{d^2F^A/dt_{A\alpha}dt_{B\alpha}}{d^2F^A/dt^2_{A\alpha}} \right| < 1.$$

Computing, this condition can be written as

$$\frac{\delta_A}{C_{Aa}}d^2C_{Aa}/dt_{A\alpha}dt_{Ba} - \frac{\delta_A}{C_{Aa}}d^2C_{Aa}/dt^2_{A\alpha} > -\frac{2}{c_{Aa}}\frac{k}{(1+k)^2}\frac{2N(2-\gamma)}{(N+m)(N-m)}dm/dt_{A\alpha}$$

The terms on both the RHS and the LHS are positive under the conditions stated above; but the complexity of the formulas do not allow us to determine if this condition is satisfied or not. Hence we cannot rule out the existence of multiple Nash equilibrium.

Consider now the case with asymmetric shocks. Here, mobility in the first period also depend on the realization of the shock; workers in the first period now move even if $t_{A\alpha} = t_{B\alpha}$. Let $m = m^*$ and the corresponding M^* be workers' mobility with identical taxation. But what matters for the governments' problem to be well behaved and for the existence of the equilibrium is the *additional mobility* generated by the difference in the tax rates. This still depends by the functions we discussed above, just substituting in the formula for m^* and M^* when $t_{A\alpha} = t_{B\alpha}$ and reinterpreting m as variation with respect to m^*. With this sostitution, and reintepreting k as discussed in the text for the case of asymmetric shock, our analysis in this Appendix still holds.

9 References

Belke A, D. Gros, (2009a),"Is a unified macroeconomic policy necessarily better for a common currency area?", *European Journal of Political Economy*, 25: 98-101.

Belke A, D. Gros, (2009b),"On the benefits of fiscal policy coordination in a currency union: a note", *Empirica*, 36: 45-49.

Bofinger P., E. Mayer (2007), "Monetary and fiscal policy interaction in the Euro area with different assumptions on the Phillips curve", *Open Economy Review*, 18: 291-305

Canzoneri M., V. Grilli, P. Masson (1991), *Establishing a Central Bank: Issues in Europe and Lessons from the US*, Cambridge, Cambridge University Press.

Commission of the European Communities (1977), *Report of the Study Group on the Role of Public Finance in European Integration*, vol. 1: General Report, Brussels, April 1977.

Delors J. (1989), "Regional implications of economic and monetary integration", in Committee for the Study of Economic and Monetary Union, *Report on Economic and Monetary Union in the European Community*, Luxembourg, Office for Official Publications of the EC.

Evers M.P. (2006), "Federal fiscal transfers in monetary unions: a NOEM approach", *International Tax and Public Finance*, 13: 463-488.

Fair D., C. de Boissieux, eds (1992), *Fiscal Policy, Taxes, and the Financial System in an Increasingly Integrated Europe*, Deventer, Kluwer.

Hairault J.O., P.Y. Henin, F. Portier, eds. (1997), *Should We Rebuild Built-in Stabilizers*, Dodrecht, Kluwer.

Hughes-Hallet A., M. Hutchison, S. Jensen, eds. (1999), *Fiscal Aspects of European Monetary Integration*, Cambridge, Cambridge University Press.

Jonung L., E.Drea (2009), "The euro: it can't happen. It's a bad idea. It won't last. US economists on the EMU, 1989-2002", *European Economy*, Economic Papers 395/ December 2009.

Kenen P. (1969), "The theory of optimum currency areas: an eclectic view", in Mundell, Swoboda, eds. (1969).

Kletzer K. (1999), "Monetary union, asymmetric productivity shocks and fiscal insurance: an analytical discussion of welfare issues", in Hughes-Hallet, Hutchison, Jensen, eds. (1999).

Kletzer K., W. Buiter (1997), "Monetary union and the role of automatic stabilizers", in Hairault, Henin, Portier, eds. (1997).

Kletzer K. J. von Hagen (2001), "Monetary union and fiscal federalism", in Wyplosz, ed. (2001).

Krugman P. (2011), "Can Europe be saved?", *The New York Times*, January, 12.

Maćkowiak B., F.P. Mongelli, G. Noblet, F. Smets (2008), *The Euro at Ten - Lesson and Challenges*, Frankfurt am mein, ECB.

Mundell R. (1961), "A theory of optimum currency areas", *American Economic Review*, 51: 657-665.

Mundell R., A. Swoboda, eds. (1969), *Monetary Problems of the International Economy*, University of Chicago Press.

Sala-i-Martin X., J. Sachs (1991), "Fiscal federalism and optimum currency areas: evidence for Europe from the United States", in Canzoneri, Grilli, Masson, eds. (1991).

Sapir A. (2004), *An Agenda for a Growing Europe*, New York, Oxford University Press.

Sapir (2008), "The OCA theory", in Maćkowiak, Mongelli, Noblet, Smets (2008), 264-267.

Tobin J. (1998), "Currency unions: Europe versus the United States", *Policy Options*, May: 31-33.

von Hagen J. (1992), "Fiscal arrangements in a monetary union: some evidence from the US", in Fair, de Boissieux, eds. (1992).

von Hagen J., C. Wyplosz (2008), "EMU's decentralized system of fiscal policy", *European Economy*, Economic Papers 306/February 2008.

Wyplosz C., ed. (2001), *The Impact of EMU on europe and the Developing Countries*, Oxford, Oxford University Press.