Personal Bankruptcy Law, Debt Portfolios and Entrepreneurship*

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Abstract

Every year 400,000 entrepreneurs fail and 60,000 file for bankruptcy. Thus the personal bankruptcy law has important implications for entrepreneurship. The option to declare bankruptcy encourages entrepreneurship through providing insurance since entrepreneurs may default on unsecured credit in bad times. However, perfectly competitive financial intermediaries take the possibility of default into account and they charge higher interest rates which reflect these default probabilities. Thus personal bankruptcy provides insurance at the cost of worsening credit conditions. Since the benefits depend on agents degree of risk-aversion but the costs depend on intertemporal preferences, we use Epstein-Zin preferences to investigate the robustness of our results. We develop a quantitative general equilibrium model of occupational choice that examines the effects of the US personal bankruptcy law on entrepreneurship. The model explicitly incorporates the US legislative framework and replicates empirical features of the US economy regarding entrepreneurship, wealth distribution and bankruptcy filings by entrepreneurs. Entrepreneurs in the model can obtain secured and unsecured credit. Secured credit must be repayed and therefore provides no insurance but is cheap. Our quantitative evaluations show: First, the current US bankruptcy law is too harsh. It does not provide enough insurance. According to our simulations, increasing the wealth exemption level to the optimal one would increase entrepreneurship, the median firm size, welfare and social mobility without increasing inequality. Second, and this is an important methodological contribution, the modeling of the credit market matters. Any analysis of unsecured credit and bankruptcy has to include secured credit as well. If agents had only access to unsecured credit (as is the case in most of the previous literature), the optimal bankruptcy law would be harsher. (JEL: M13, K10, O41, E20)

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1 Introduction

Entrepreneurs employ half of all workers in the US and they create three quarters of all new jobs. Over time, successful entrepreneurs, for example Bill Gates in 1978 or Larry Page and Sergey Brin in 1997, grow their small firms into big enterprises, for example Microsoft and Google today. Personal bankruptcy law is important for entrepreneurs because if an entrepreneur's firm is not incorporated he or she is personally liable for all the unsecured debts of this firm.[1] Many entrepreneurs fail each year, and around 60,000 file for bankruptcy.

This paper investigates quantitatively the effects of personal bankruptcy law on entrepreneurship. Bankruptcy introduces some contingency in a world of incomplete credit markets where only simple debt contracts are available. This contingency provides insurance against entrepreneurial failure at the cost of worsening credit conditions. If the bankruptcy law does not allow default under any circumstances, credit will be available at lower interest rates because borrowers will not default. This comes at the expense of borrowers having no insurance against business failure. If, however, the bankruptcy law makes default very easy, borrowers might be insured against bad outcomes. But in order to compensate for the default risk, banks have to charge higher interest rates or ration credit all together. In our model, as in the real world, entrepreneurs can also obtain secured credit. This modifies the trade-off between insurance and credit conditions by allowing agents, if they want to, to obtain cheap (secured) credit even in a world with a very generous bankruptcy law. We find that allowing entrepreneurs to obtain both, secured and unsecured credit, has quantitatively important effects on the model economy.

The trade-off between insurance and credit conditions is at the center of recent public discussions and policy changes in Europe and the US. In Europe, the bankruptcy law is much harsher than in the US. Many countries, for example Germany, the Netherlands and the UK, have made legislation more lenient with the explicit aim of fostering entrepreneurship.[2] The policy changes in the US went in the opposite direction. Following the huge increase in personal bankruptcy filings, US Congress in 2005 passed a law making personal bankruptcy less beneficial for filers. Even though the focus of this discussion has been on consumer bankruptcy, the effects on entrepreneurship are important because around 60,000 failed entrepreneurs file for bankruptcy each year. Our paper quantitatively assesses the relative strength of these two opposing forces: insurance versus credit conditions, on the number of entrepreneurs, on the access of poor agents to entrepreneurship, on firm size, and on welfare, inequality and social mobility.

We build an infinite horizon heterogeneous agent model, which has an occupational choice problem at its core. Agents differ with respect to their entrepreneurial and working productivity. During each period, they decide whether to become an entrepreneur or a worker. Cagetti and De Nardi (2006) also have this occupational choice at the center of their model, which is able to explain US wealth distribution, in particular its extremely skewed nature at the top. However, in their model, entrepreneurship is a risk-free activity because there is no uncertainty about current productivities. Thus

[1] Meh and Terajima (2008) report that unsecured debt accounts for around on ethird of all debt.

[2] In a companion paper, we are currently investigating the effects of introducing a US type of law in Europe.

there is no default in equilibrium and there is no insurance role for bankruptcy. We have default in our model because in the US 2.25% of all entrepreneurs file for bankruptcy.

Despite the importance of personal bankruptcy law for entrepreneurship, there is little quantitative literature on this topic. Starting with Athreya (2002), the literature so far has focused almost exclusively on consumer bankruptcy. For example, Livshits, MacGee, and Tertilt (2007) compare the US system under which future earnings are exempt after consumers have defaulted with a European type of system under which future earnings are garnished to repay creditors. They find that the welfare differences between the systems depend on the persistence and variance of the shocks. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) show that the recent tightening of the law in the US implies large welfare gains.[3] In this literature there are few papers that focus on secured and unsecured borrowing. Athreya (2006) finds that welfare is increasing in the wealth exemption level. Hintermaier and Koeniger (2008) examine the reasons for the increase in consumer bankruptcies in a model with durable and nondurable goods.

There are three closely related papers that analyze the effects of bankruptcy on entrepreneurship in a quantitative setting similar to our paper. Akyol and Athreya (2007) use an overlapping generations, partial equilibrium framework with heterogeneity in human capital. Their main results is that the current system is too generous. Meh and Terajima (2008) have a similar framework (partial equilibrium OLG model) in which they analyze bankruptcy decisions of both consumers and entrepreneurs. Mankart and Rodano (2007) have a model with temporary and permanent productivity shocks. The main result of all three papers is that the current system is too generous.[4]

Our model is able to replicate key macroeconomic variables of the US economy: the capital output ratio, the fraction of entrepreneurs in the population, the exit rate, the bankruptcy filings of entrepreneurs, the wealth of entrepreneurs compared to workers. Based on this model, we can conduct a policy experiment to assess whether the current exemption level (how much wealth a person can keep in case of a default) is optimal.

Our main result is that the current system is too harsh with respect to the exemption level. There are welfare gains from increasing the current exemption level to the optimal one. Entrepreneurship would increase from 7.2% of the population to 7.4% if the exemption level were increased because of the increased insurance effect. Moreover, eliminating bankruptcy exemptions would lead to a reduction of welfare and a reduction in entrepreneurship to 6.6% of the population.

Our results are strikingly different from other papers in the literature. Meh and Terajima (2008), Akyol and Athreya (2007) and Mankart and Rodano (2007) find that the current system is too generous[5]. The main difference is that all these paper do not allow entrepreneurs to obtain secured, in addition to unsecured, credit.

In a counterfactual experiment we find that if we exclude secured credit we get similar results as the previous literature: the current law appears to be too lenient. The reason is the following. When we exclude secured credit some agents are credit rationed because their incentive to default is too high. Therefore they become workers.

[3]Other papers in this growing literature are Athreya (2006), Athreya and Simpson (2006), Li and Sarte (2006), Mateos-Planas and Seccia (2006).

[4]Zha (2001) is a theoretical investigation of similar issues. However his model abstracts from occupational choice, which we show to be the crucial channel through which bankruptcy law affects entrepreneurship.

[5]This result is also common to most papers in the consumer bankruptcy literature.

Increasing the exemption level worsens this problem. If instead these agents can obtain secured credit (i.e. pledge collateral), they can run bigger firms and therefore find it profitable to become entrepreneurs. Excluding secured credit from the analysis overstates the role of credit rationing. Thus, the policy conclusion reached in the previous literature might be premature.

Our results, as those from Meh and Terajima (2008), Akyol and Athreya (2007) and Mankart and Rodano (2007) are consistent with the empirical finding of Berkowitz and White (2004) who show that in states with higher exemption levels, credit conditions are worse. But our paper is also consistent with the findings of Fan and White (2003) that show that entrepreneurship is higher in states with a more lenient bankruptcy law. This is not true in the work of Meh and Terajima (2008), Akyol and Athreya (2007) and Mankart and Rodano (2007).

Moreover, we use Epstein-Zin preferences. This allows us to distinguish between risk aversion and intertemporal elasticity of substitution. This is particularly interesting, given that the costs of a generous bankruptcy system, in terms of higher interest rates, depend mainly on the elasticity of intertemporal substitution, while the benefits, in terms of insurance, depend on risk aversion. Our choice of preferences allows us to examine these effects separately. We find that the optimal exemption level increases with the elasticity of intertemporal substitution. This result is quite intuitive since agents who are more willing to substitute consumption across time are less affected by the higher borrowing rates resulting from higher exemption levels. We also find that the optimal exemption level increases with risk aversion. The more risk averse agents are the more they value insurance.

The paper is organized as follows, Section 2 provides an overview of US bankruptcy law and presents data on entrepreneurial failure. In Section 3 we present our model and discuss the equilibrium condition. In Section 4 we discuss our calibration strategy and present the baseline results. Section 5 explains the main mechanism of the model. In Section 6, we conduct the main policy experiment. In Section 7 we present the effects of excluding secured credit and some robustness checks. Section 8 concludes.

2 Entrepreneurial failure and personal bankruptcy in the US

Personal bankruptcy procedures in the US consist of two different procedures: Chapter 7 and Chapter 13. Under Chapter 7, all unsecured debt is discharged immediately, while a secured creditor can fully seize the assets pledged as collateral. Future earnings cannot be garnished. This is why Chapter 7 is known as providing a "fresh start". At the same time, a person filing for bankruptcy has to surrender all wealth in excess of an exemption level. The exemption level varies across US states, ranging from $11,000 in Maryland to unlimited for housing wealth in some states, for example Florida. Therefore, we calculate the population-weighted median across states. The resulting average exemption level is $47,800 in 1993.[6]

Under Chapter 13 agents can keep their wealth, debt is not discharged immediately and future earnings are garnished. Entrepreneurs are better off under Chapter 7 for

[6]The wealth exemption level does not change much over time. We choose 1993 because it is in the middle of the sample years for our data on entrepreneurship wealth distribution and bankruptcies.

three reasons: they have no non-exempt wealth, their debt is discharged immediately and they can start a new business straight away, since their income will not be subject to garnishment (see White, 2007). 70% of total bankruptcy cases involving entrepreneurs are under Chapter 7. Therefore we will focus on Chapter 7 only.

Persons can file for bankruptcy only once every six years. The bankruptcy filing remains public information for ten years. Therefore, agents have difficulties obtaining unsecured credit for some time after having defaulted. Secured credit, credit that is collateralized, is always available.

The US. Small Business Administration reports an exit rate of on average 9.7% per annum for small firms in the period from 1990-2005.[7] Out of these failing firms 9.3% file for bankruptcy, according to the official data from the Administrative Office of the Courts.[8] Unfortunately, the official data on personal bankruptcy caused by a business failure seem to be severely downward biased. Lawless and Warren (2005) estimate that the true number could be three to four times as big. Their own study is based on an in-depth analysis of bankruptcy filers in five different judicial districts. Their explanation of this discrepancy is the emergence of automated classification of personal bankruptcy cases. Almost all software used in this area has "consumer case" as the default option. Thus reporting a personal bankruptcy case as a "business related" case requires some - even though small - effort while being completely inconsequential for the court proceedings. In addition to their own study they report data from Dun & Bradstreet according to which business bankruptcies are at least twice the official number.[9]

In the calibration of our model we set the baseline exemption level equal to $47,800. The baseline exclusion period is set to two year.[10] We calibrate the model such that the default rate of entrepreneurs is 2.25%.

3 The model

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents face idiosyncratic uncertainty, but there is no aggregate uncertainty. At the

[7]The U.S. Small Business Adminstration splits small firms into employer and non-employer firms. Employer firms have at least one employee working in the firm. There are roughly five million employer and 15 million non-employer firms in the U.S. Since the focus of our paper is on entrepreneurs who own and manage the firm we use only the data for employer firms since non-employer firms have in many cases the owner not working in the firm. To ensure consistency across our three databases, when we use data from the Survey of Consumer Finance (SCF) and the Panel Study of Income Dynamics (PSID) we define entrepreneurs as business owners who manage a firm with at least one employee.

[8]While one can obtain exit rates from the PSID data (Quadrini, 2000), it is impossible to obtain reliable bankruptcy data from the PSID. There is only one wave in which respondents were asked about past bankruptcies.

[9]Dun & Bradstreet (D&B) is a credit-reporting and business information firm. D&B compiles its own independent business failure database. Until the emergence of automated software for law firms and courts in the mid 1980s, the official business bankruptcy data and the index compiled by D&B have a positive and significant correlation of 0.73. From 1986-1998 this correlation coefficient becomes negative and insignificant. Extrapolating from the historic relationship between the D&B index and personal bankruptcy cases caused by business failures leads to the conclusion that the official data under report business bankruptcy cases at least by a factor of two.

[10]We choose a short exclusion period because there is evidence that entrepreneurs obtain unsecured credit even after defaulting. However as a robustness check, we set the exclusion period to six years and the results do not change much.

beginning of every period, agents decide whether to become workers or entrepreneurs. An entrepreneur must decide how much to invest, how much to borrow secured and, if he is allowed to, how much to borrow unsecured. An entrepreneur who has defaulted on unsecured credit is excluded from unsecured credit for two year but is allowed to obtain secured credit. Since we focus on the implications of personal bankruptcy for entrepreneurs, workers are not allowed to borrow. Agents productivities evolve over time and agents are subject to uninsurable production risk. After the shocks are realized, production takes place. At the end of the period unsecured borrowers decide whether to repay or whether to default and how much to consume and how much to save. If they default, they will be borrowing constrained in the next period. Anticipating this behavior, banks who give unsecured credit vary the interest rate charged for each loan taking into account the individual borrower's default probability. The remainder of this section presents the details of the model.

3.1 Credit and bankruptcy law

Agents can get two types of credit: secured and unsecured. Both types of credit are subject to a limited commitment problem.[11] After getting credit, all borrowers have two options: take all liquid assets, their own wealth plus the amount borrowed, and run or start the entrepreneurial activity. If they run the agents can keep a fraction λ of the liquid assets. If the agents start the entrepreneurial activity then the only difference is that secured credit must be repaid (and it has priority in the bankruptcy proceedings), while unsecured credit is subject to Chapter 7 bankruptcy procedure, if the agent exercises his default option.

In the event of a default the agent still must repay her secured debt. Unsecured debt, however, is discharged. Any assets remaining after repaying the secured debt which is in excess of an exemption level X are liquidated.

An agent who has defaulted in the past is excluded from the market for unsecured credit for a certain period of time. During this period he still can obtained secured credit and can become an entrepreneur. We call this agent *borrowing constrained* and we denote his credit status as BC. It is important to note that this agent is not fully excluded from the credit market. He can still obtain *secured* credit. However he cannot obtain *unsecured* credit. We assume that every *borrowing constrained* agent, whether worker or entrepreneur, faces a credit status shock at the end of the period. With probability $(1 - \varrho)$ the agent remains *borrowing constrained*. With probability ϱ the agent regain access to *unsecured* credit. He becomes an *unconstrained* agent with credit status UN.[12] This probability ϱ captures the duration of exclusion period from the market of unsecured borrowing. It is calibrated such that the average exclusion period is two year.

[11] We introduce this limited commitment problem to obtain reasonable leverage ratios. As pointed out by Heaton and Lucas (2002) models without information asymmetries yield counterfactually large leverage ratios.

[12] The length of the exclusion period is transformed into a probability in order to avoid an additional state variable that keeps track of the numbers of years left before the solvency status is returned to UN. This procedure is standard in the literature, see Athreya (2002) and Chatterjee et al. (2007).

3.2 Households

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents differ with respect to their level of assets a, their entrepreneurial productivity θ, their working productivity φ, and their credit market status $S \in \{UN, BC\}$.

3.2.1 Preferences

For simplicity we abstract from labor-leisure choice. All agents supply their unit of labor inelastically either as workers or as entrepreneurs. In order to disentangle the effects of risk aversion from that of the elasticity of inetertemporal substitution we assume that agents have Epstein-Zin preferences. A stochastic consumption stream $\{c_t\}_{t=0}^{\infty}$ generates an utility $\{u_t\}_{t=0}^{\infty}$ according to

$$u_t = U(c_t) + \beta U\left(\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right]\right)$$

where β is the discount rate and $\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right] \equiv \Gamma^{-1}\left[\mathbb{E}_t\Gamma(u_{t+1})\right]$ is the consumption equivalent of u_{t+1} given information at period t. The utility function $U(c) = c^{1-\frac{1}{\psi}}/\left(1-\frac{1}{\psi}\right)$ aggregates consumption across dates and ψ is the intertemporal elasticity of substitution. The utility function $\Gamma(c) = c^{1-\gamma}/(1-\gamma)$ aggregates consumption across states and γ is the coefficient of relative risk aversion.

3.2.2 Productivities

Each agent is endowed with a couple of stochastic productivity levels which are known at the beginning of the period: one as an entrepreneur θ, and one as a worker φ. We make the simplifying assumption that the working and entrepreneurial ability processes are uncorrelated.

The workers' ability process Following the literature, we assume that labor productivity follows the following AR(1) process

$$\log\varphi_t = (1-\rho)\,\mu + \rho\log\varphi_{t-1} + \varepsilon_t$$

where ε_t is *iid* and $\varepsilon \sim N(0, \sigma_\varepsilon)$. If the agent becomes a worker his labor income during current period is given by $w\varphi$.

The entrepreneurs' ability process In contrast to the case of working ability, there are no reliable estimates of the functional form for the case of entrepreneurial ability. Therefore, following Cagetti and De Nardi (2006), we will assume a parsimonious specification where entrepreneurial productivity follows a 2-state Markov process with $\theta^L = 0$ and $\theta^H > 0$ and transition matrix

$$P_\theta = \begin{bmatrix} p^{LL} & 1-p^{LL} \\ 1-p^{HH} & p^{HH} \end{bmatrix}$$

We calibrate the 3 parameters (θ^H, p^{HH} and p^{LL}) to match some observed features of entrepreneurial activity in the US economy.

3.3 Technology

Entrepreneurial sector Every agent in the economy has access to a productive technology that, depending on her entrepreneurial productivity θ, produces output according to the production function

$$\begin{aligned} Y &= \theta k^{\nu} \\ k &= \chi I \end{aligned}$$

where θ is the agent's persistent entrepreneurial productivity described above.

We assume that investment is subject to an *iid* idiosyncratic shock. Each unit of the *numeraire* good which is invested in the entrepreneurial activity is transformed in χ units of capital with $log\chi \sim N(0, \sigma_{\chi})$ This *iid* shock represents the possibility that an inherently talented entrepreneur (i.e. an agent with high and persistent θ) might choose the wrong project or could be hit by an adverse demand shock. Quadrini (2000) shows that the entry rate of workers with some entrepreneurial experience in the past, is much higher than the entry rate of those workers without any experience. Therefore it seems that entrepreneurs come mostly from a small subset of total population. If their firms fail, they are very likely to start a new firm within a few years. The *iid* shock χ helps us to capture this difference in the entry rates.

Corporate sector Many firms are both incorporated and big enough not to be subject to personal bankruptcy law. Therefore we follow Quadrini (2000) and Cagetti and De Nardi (2006) and assume a perfectly competitive corporate sector which is modeled as a Cobb-Douglas production function

$$F(K_c, L_c) = AK_c^{\xi} L_c^{1-\xi}$$

where K_c and L_c are capital and labor employed in this sector. Given perfect competition and constant returns to scale the corporate sector does not distribute any dividend. Capital depreciates at rate δ in both sectors.

3.4 Credit market

We assume that there is perfect competition (free entry) in the credit market. Therefore banks must make zero expected profit on any contract. The opportunity cost of lending to entrepreneurs is the rate of return on capital in the corporate sector. This is also equal to the deposit rate.[13] Agents can get two types of credit: *secured* credit and *unsecured* credit. *Secured* credit represents collateralized borrowing. Thus, it is available at the risk free rate plus a small transaction cost ($r^s = r^s + \tau^s$). *Unsecured* credit requires higher transaction costs ($\tau^u > \tau^s$) that reflect the higher information costs which are present in the real world and from which we abstract in the model.

Both types of contracts are subject to a limited commitment constraint. Instead of investing the money in the entrepreneurial firm the agent can take the money and run

[13] In our model, banks are isomorphic to a bond market in which each agent has the possibility to issue debt.

away with a fraction λ of the credit plus assets. Anticipating this behavior, banks will never lend any amount such that the agent prefers to run.[14]

There are no information asymmetries in the credit market. Banks know the agent's assets, the amount he borrowed secured s and his productivities. For any given value of (a, s, θ, φ) and for any amount lent unsecured b, by anticipating the behavior of the entrepreneur, banks are able to calculate the probability of default and the recovery rate in case of default. Perfect competition implies that they set the interest rate, $r(a, s, \theta, \varphi, b, X)$, such that they expect to break even. This interest rate depends on the exemption level X because it affects the incentives to default and the amount the bank recovers in this event. Therefore banks offer a menu of one period debt contracts which consist of an amount lent b and a corresponding interest rate $r(a, s, \theta, \varphi, b, X)$ to each agent (a, s, θ, φ).

3.5 Timing

Figure 1 shows the timing of the model. Given the focus of the paper we choose the timing such that workers can never default. Entrepreneurs' borrowing and default decisions are taken within the period. At the beginning of the period all agents face an occupational choice: they choose whether they become entrepreneurs or workers. Agents know their current productivities (φ, θ).

Workers deposit all their wealth at the banks, receiving a rate of return r^d. After production has taken place, they choose consumption and savings. At the end of the period the *borrowing constrained* worker receives the credit status shock. With probability ϱ he remains *borrowing constrained* next period (i.e. $S' = BC$). With probability $(1 - \varrho)$ he becomes *unconstrained* next period (i.e. $S' = UN$).

The *borrowing constrained* entrepreneur chooses how much secured credit s to obtain or whether to save. After having obtained secured credit s, the *borrowing constrained* entrepreneur decides whether to take s and his own wealth a and run (with a fraction λ of it). In this case the bank receives nothing. Anticipating this, the bank will never lend an amount s with which the agent would run. The entrepreneur decides how much to invest before the *iid* shock χ is realized. After χ is realized and production has taken place, he chooses consumption and savings. At the end of the period he receives the credit status shock.

The *unconstrained entrepreneur* can obtain both: secured credit s and unsecured credit b. Before knowing χ, he chooses his capital stock by deciding how much to borrow (or invest at rate r^d). He obtains secured credit s at the interest rate r^s. Unsecured borrowing is done by picking from the menu $\{b, r(a, \theta, \varphi, s, b, X)\}$ offered by the the banks. As for the *borrowing constrained*, the *unconstrained constrained* can take $a+b+s$ and run. And as before, the bank will never lend in a way that induces the agent to run. After χ is realized and production has taken place, the entrepreneur must repay his secured debt. Then he can decide whether to repay his unsecured debt as well and be *unconstrained* next period (i.e. $S' = UN$) or whether to declare bankruptcy and be *borrowing constrained* next period(i.e. $S' = BC$). After that he chooses consumption and savings.

[14]This means that running with the money is an out of equilibrium behavior. We introduce it to limit the leverage ratio to empirically plausible levels.

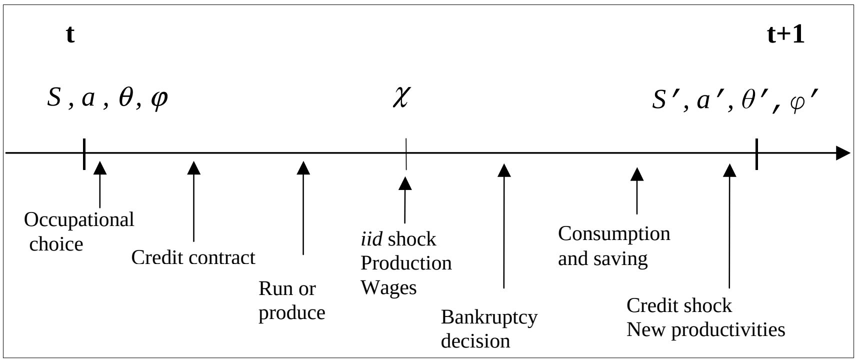

Figure 1: Timing of the model

Since the credit status S consists only of the two states BC and UN, we define the individual state variable as (a, θ, φ), and we solve for two value functions $V^{UN}(a, \theta, \varphi)$ and $V^{BC}(a, \theta, \varphi)$ one for each credit status.

3.6 The problem of the *borrowing constrained* agent

The *borrowing constrained* agent can only obtain secured credit. Therefore he can either save or borrow at a rate r^d subject to the limited commitment constraint. At the beginning of the period he can choose whether to become an entrepreneur, which gives utility $N^{BC}(a, \theta, \varphi)$ or a worker which yields utility $W^{BC}(a, \theta, \varphi)$. Therefore the value of being a *borrowing constrained* agent with state (a, θ, φ) is

$$V^{BC}(a, \theta, \varphi) = \max\left\{N^{BC}(a, \theta, \varphi), W^{BC}(a, \theta, \varphi)\right\}$$

where the "max" operator reflects the occupational choice.

Worker At the beginning of the period the *borrowing constrained* worker deposits all his wealth at the bank and he receives labor income $w\varphi$. At the end of the period, he chooses consumption and saving, taking into account that he will receive a credit status shock in addition to productivity shocks. With probability ϱ he will be still *borrowing constrained* next period which yields utility $V^{BC}(a', \theta, \varphi)$, while with probability $(1 - \varrho)$ he will become *unconstrained* which yields utility $V^{UN}(a', \theta, \varphi)$. His saving problem is the following

$$\begin{aligned} W^{BC}(a, \theta, \varphi) &= \max_{c, a'}\left\{U(c) + \beta U\left(\mathbb{CE}_t\left[\varrho V^{BC}(a', \theta', \varphi') + (1-\varrho) V^{UN}(a', \theta', \varphi')\right]\right)\right\} \\ s.t.\ c + a' &= w\varphi + \left(1 + r^d\right) a \\ a' &\geq 0 \end{aligned}$$

Entrepreneur At the beginning of the period the *borrowing constrained* entrepreneur decides how much to invest in his firm $I = a + s$ by choosing how much secured credit $(s > 0)$ or save, at rate r^d $(s < 0)$. Each unit of investment is transformed in χ units of capital, $(k = \chi I)$. After he has got credit he could take the money and run away with a fraction λ. If he does so his utility is given by

$$\begin{aligned}\Upsilon\left[a+s,\theta,\varphi\right] &= \max_{c,a'}\left\{U\left(c\right)+\beta U\left[\mathbb{CE}_{t}V^{BC}\left(a',\theta',\varphi'\right)\right]\right\}\\ s.t.\ c+a' &= \lambda\left(a+s\right)\\ a' &\geq 0\end{aligned}$$

After the shock χ is realized he will decide how to allocate the resources $(\chi I)^{\nu}\theta+(1-\delta)\chi I-\left(1+r^{d}\right)s$ among consumption and savings. His saving problem, after uncertainty is resolved,[15] is

$$\begin{aligned}\tilde{N}^{BC}\left(a,\theta,\varphi,\chi,s\right) &= \max_{a',c}\left\{U\left(c\right)+\beta U\left(\mathbb{CE}_{t}\left[\varrho V^{BC}\left(a',\theta',\varphi'\right)+\left(1-\varrho\right)V^{UN}\left(a',\theta',\varphi'\right)\right]\right)\right\}\\ s.t.\ c+a' &= \left[\chi\left(a+s\right)\right]^{\nu}\theta+\left(1-\delta\right)\chi\left(a+s\right)-\left(1+r^{d}\right)s\\ a' &\geq 0\end{aligned}$$

Therefore the optimal investment decisions of the agent at the beginning of the period is

$$\begin{aligned}N^{BC}\left(a,\theta,\varphi\right) &= \max_{s}U\left(\mathbb{CE}_{t}\left\{\tilde{N}^{BC}\left(a,\theta',\varphi',\chi,s\right)\right\}\right)\\ s.t.\ N^{BC}\left(a,\theta,\varphi\right) &> \Upsilon\left[a+s,\theta,\varphi\right]\end{aligned}$$

3.7 The problem of the *unconstrained* agent

At the beginning of the period the *unconstrained* agent faces the following occupational choice

$$V^{UN}\left(a,\theta,\varphi\right)=\max\left\{W^{UN}\left(a,\theta,\varphi\right),N^{UN}\left(a,\theta,\varphi\right)\right\}$$

where $W^{UN}\left(a,\theta,\varphi\right)$ is the utility of becoming a worker and $N^{UN}\left(a,\theta,\varphi\right)$ of becoming an entrepreneur.

Worker The problem of the *unconstrained* worker is identical to the *borrowing constrained* one except that the agent will be *unconstrained* in the future for sure. His saving problem is the following

$$\begin{aligned}W^{UN}\left(a,\theta,\varphi\right) &= \max_{c,a'}U\left(c\right)+\beta U\left(\mathbb{CE}_{t}\left[V^{UN}\left(a',\theta',\varphi'\right)\right]\right)\\ s.t.\ c+a' &= w\varphi+\left(1+r^{d}\right)a\\ a' &\geq 0\end{aligned}$$

Entrepreneur The *unconstrained entrepreneur* decides how much to invest in his firm $I=a+b+s$ by choosing how much to borrow from secured credit ($s>0$) from unsecured credit ($b>0$) or save at rate r^{d} ($s<0$). If he borrows unsecured credit he can choose from the menu $\{b,r\left(a,\theta,\varphi,b,s,X\right)\}$ offered by competitive banks. After the

[15] We denote with a "~" all the value functions, *after* uncertainty (about χ) is resolved. The value functions without "~" are *before* uncertainty is resolved.

shock χ is realized he can choose whether to declare bankruptcy (default) or whether to repay and how much to consume and save. He solves the problem backwards.

If he repays his unsecured debt, he has to choose how to allocate his resources, $\theta\left[(a+b+s)\chi\right]^{\nu} + (1-\delta)(a+b+s)\chi - b\left[1+r(a,\theta,\varphi,b,s,X)\right] - (1+r^{d})s$, between consumption and savings. Given that the decision of repaying is done when current productivities (θ,φ) and the shock χ are known, his utility from repaying is given by

$$\begin{aligned}\tilde{N}^{pay}(a,b,s,\theta,\varphi,\chi) &= \max_{c,a'}\left\{U(c)+\beta U\left(\mathbb{CE}_t\left[V^{UN}(a',\theta',\varphi')\right]\right)\right\}\\ s.t.\ a'+c &= \theta\left[(a+b+s)\chi\right]^{\nu}+(1-\delta)(a+b+s)\chi-\cdots\\ &\quad -b\left[1+r(a,\theta,\varphi,b,s,X)\right]-(1+r^{d})s\\ a' &\geq 0\end{aligned}$$

If he defaults, his unsecured debt is discharged. But he must repay any secured debt he had and he loses all assets in excess of the exemption level X. Thus, the resources to allocate between consumption and savings are $\min\left\{\theta\left[(a+b+s)\chi\right]^{\nu}+(1-\delta)(a+b+s)\chi-(1+r^{d})s, X\right\}$. Moreover if he defaults he will be *borrowing constrained* next period. Therefore by declaring bankruptcy he gets

$$\begin{aligned}\tilde{N}^{bankr}(a,b,s,\theta,\varphi,\chi) &= \max_{c,a'}\left\{U(c)+\beta U\left(\mathbb{CE}_t\left[V^{BC}(a',\theta',\varphi')\right]\right)\right\}\\ s.t.\ a'+c &= \min\left\{\theta\left[(a+b+s)\chi\right]^{\nu}+(1-\delta)(a+b+s)\chi-(1+r^{d})s, X\right\}\\ a' &\geq 0\end{aligned}$$

He will declare bankruptcy if $N^{bankr}(a,b,s,\theta,\varphi\,\chi) > N^{pay}(a,b,s,\theta,\varphi,\chi)$ and vice versa. Thus, at the beginning of the period the agent choose the optimal amount of b from the menu $\{b, r(a,\theta,\varphi,b,X)\}$ and the optimal s anticipating his future behavior. Therefore his utility is given by

$$\begin{aligned}N^{UN}(a,\theta,\varphi) &= \max_{\{b,r(\cdot)\},s}\mathbb{CE}_t\left[\max\left\{\tilde{N}^{pay}(a,b,s,\theta,\varphi,\chi), \tilde{N}^{bankr}(a,b,s,\theta,\varphi,\chi)\right\}\right]\\ s.t.\ N^{UN}(a,\theta,\varphi) &\geq \Upsilon^{UN}\left[a+s+b,\theta,\varphi\right]\end{aligned}$$

where the "max" operator inside the square brackets reflects the bankruptcy decision, and the "max" operator outside the square brackets reflects the borrowing decision. The last equation represents the limited commitment constraint where

$$\begin{aligned}\Upsilon\left[a+s+b,\theta,\varphi\right] &= \max_{c,a'}\left\{U(c)+\beta U\left[\mathbb{CE}_t V^{BC}(a',\theta',\varphi')\right]\right\}\\ s.t.\ c+a' &= \lambda(a+s+b)\\ a' &\geq 0\end{aligned}$$

3.8 The zero profit condition of the banks

Banks observe the state variables (a,θ,φ) at the moment of offering the contract.There is perfect competition (free entry) in the credit market therefore banks make zero profit on each secured and unsecured loan contract. Therefore the bank is indifferent between issuing secured and unsecured loans. For each unit of secured credit the bank know that the agent will repay for sure: free entry will push the interest rate on secured credit to the risk free rate plus the transaction cost τ^s. For any given state (a, θ, φ) and for any given amount of secured borrowing the agent is doing (s) and for any unsecured loan (b), banks know in which states of the world the agent will file for bankruptcy. Therefore, they are able to calculate the probability that a certain agent with characteristics (a, θ, φ), and secured loan s, will default for any given amount b. This default probability, $\pi^{bankr}(a, \theta, \varphi, b, s, X)$, depends on the exemption level X because X affects the incentive to default directly.

If the agent repays banks receive $[1 + r(a, \theta, \varphi, b, s, X)]\, b$. If the agent defaults banks sells the firm's un-depreciated capital. Therefore they receive: nothing if $\theta\left[(a + b + s)\,\chi\right]^{\nu} + (1 - \delta)\,(a + b + s)\,\chi - \left(1 + r^d\right) s < X$, while banks receive $\theta\left[(a + b + s)\,\chi\right]^{\nu} + (1 - \delta)\,(a + b + s)\,\chi - \left(1 + r^d\right) s - X$ otherwise.

The zero profit condition of the banks is given by

$$\begin{pmatrix} \left[1 - \pi^{bankr}(a, \theta, \varphi, b, s, X)\right] [1 + r(a, \theta, \varphi, b, s, X)]\, b + \\ + \pi^{bankr}(a, \theta, \varphi, b, s, X) \\ \max\left\{\theta\left[\chi I\right]^{\nu} + (1 - \delta)\,\chi I - \left(1 + r^d\right) s - X, 0\right\} \end{pmatrix} = (1 + r^d)(1 + \tau^u) b,$$

where $I = a + b + s$

3.9 Equilibrium

Let $\eta = (a, \theta, \varphi, S)$ be a state vector for an individual, where a denotes assets, θ entrepreneurial productivity, φ working productivity and S the credit status. From the optimal policy functions (savings, capital demand, default decisions), from the exogenous Markov process for productivity and from the credit status shocks, we can derive a transition function, that, for any distribution $\mu(\eta)$ over the state provides the next period distribution $\mu'(\eta)$. A stationary equilibrium is given by

- a deposit rate of return r^d and a wage rate w
- an interest rate function
- a set of policy functions $g(\eta)$ (consumption and saving, secured and unsecured borrowing, capital demand, bankruptcy decisions and occupational choice)
- a constant distribution over the state η, $\mu^*(\eta)$

such that, given r^d and w and a bankruptcy regime X and ϱ:

- $g(\eta)$ solves the maximization problem of the agents;
- the corporate sector representative firm is optimizing;
- capital, labor and goods market clear:
  - capital demand comes from both, entrepreneurs and the corporate sector, while supply comes from the saving decisions of the agents;

- labor demand comes from the corporate sector, while labor supply comes from the occupational choice of the agents;

- the interest rate function reflects the zero profit condition of the banks
- The distribution $\mu^* (\eta)$ is the invariant distribution associated with the transition function generated by the optimal policy function $g (\eta)$ and the exogenous shocks.

The model has no analytical solution and must be solved numerically. The algorithm used to solve the model and other details are presented in the appendix.

4 Results

4.1 Parametrization

4.1.1 Fixed parameters

Following standard practice in the literature we try to minimize the number of parameters of the model used to match the data. We therefore select some parameters which have already been estimated in the literature. We choose $\rho = 0.95$ for the autoregressive coefficient of the earnings process.[16] The variance of the earnings process is chosen to match the Gini index of labor income as observed in the PSID, where it is 0.38.[17] The process is approximated using a 4-state Markov chain, using the Tauchen (1986) method as suggested by Adda and Cooper (2003).[18] Total factor productivity is normalized to 1, while the share of capital in the Cobb-Douglas technology for the corporate sector is set to $\xi = 0.36$. The depreciation rate is set $\delta = 0.08$. These parameters are summarized in table 1.

4.1.2 Preference parameters

The option to default provides agents with an insurance against bad outcomes. The value of this insurance depends crucially on the agents attitudes towards risk. As described above, the price of this insurance are worsened credit conditions. Agents who still borrow face higher interest rates. Thus, the value of the costs of the insurance depends mainly on the agents elasticity of intertemporal substitution. Therefore, we separate these two parameters and conduct our main policy experiment for different values of these parameters. In the baseline model, we set the coefficient of relative risk aversion $\sigma = 3$ and the elasticity of intertemporal substitution $\psi = 1.1$. Later on we investigate values for σ ranging from 1.5 to 4.5 and ψ ranging from 0.5 to 1.5. Table 2 summarizes preferences.

[16] In a life cycle setting, Storesletten, Telmer, and Yaron (2004) and Storesletten, Telmer, and Yaron (2001) find ρ in the range between 0.95 and 0.98. We choose $\rho = 0.95$ to take into account that the agents in our model are infinitely lived and that the intergenerational auto-regressive coefficient is lower. Solon (1992) estimates it around 0.4.

[17] The exact value of the variance is $\sigma_\varepsilon^2 = .08125$. This is higher than the estimate of Storesletten et al. (2004) of about 0.02. We abstract from many important factors that are empirically relevant for the earnings distribution, e.g. human capital, life-cycle savings. Therefore, in order to generate the

Table 1: The fixed parameters

Parameter	Symbol	Baseline
TFP	A	1 (normalization)
Share of capital	ξ	0.36
Transaction cost secured credit	τ^s	0.01
Transaction cost unsecured credit	τ^u	0.05
Depreciation rate	δ	0.08
Working productivities	φ	$\begin{bmatrix} \varphi_1 = 0.316, \varphi_2 = 0.745 \\ \varphi_3 = 1.342, \varphi_4 = 3.163 \end{bmatrix}$
Transition matrix	P_φ	$\begin{bmatrix} 0.8393 & 0.1579 & 0.0028 & 0.0000 \\ 0.1579 & 0.6428 & 0.1965 & 0.0028 \\ 0.0028 & 0.1965 & 0.6428 & 0.1579 \\ 0.0000 & 0.0028 & 0.1579 & 0.8393 \end{bmatrix}$

Table 2: Preference parameters

Parameter	Symbol	Value
CRRA	σ	3
IES	ψ	1.1

4.1.3 Bankruptcy policy parameters

The two policy parameters are the exemption level X and the probability ϱ of being able to obtain unsecured credit again. The law does not state any formal period of exclusion from unsecured credit after a bankruptcy filing. For our baseline specification, we set $\varrho = 0.5$ which corresponds to an average exclusion period from credit of two years. This is lower than most values in the consumer bankruptcy literature.[19] We think that this is warranted since there is evidence the entrepreneurs have access to unsecured credits relatively fast after having defaulted, see for example Lawless and Warren (2005). However, we conduct a robustness check and also investigate a considerably longer exclusion period of six years. The exemption level differs across US sates. Using US state-level data for 1993 we calculate the median across states of the total exemption[20] ("homestead" plus "personal property" exemption). The resulting median exemption level is $47,800, taking an average household labor income of $48,600 corresponds to a value of **0.98** for the exemption/wage ratio.[21] Table 3 summarizes the bankruptcy

observed inequality, we need a higher variance of the earnings process.

[18] Floden (2008) shows that for highly correlated processes the method of Adda and Cooper (2003) achieves a higher accuracy than the original methods of Tauchen (1986) and Tauchen and Hussey (1991).

[19] Athreya (2002) sets the exclusion period to 4 years, Li and Sarte (2006) to 5 years, Chatterjee et al. (2007) to 10 years.

[20] We took the data from Berkowitz and White (2004) and top-coded the unlimited homestead exemption to the maximum state exemption.

[21] As a further robustness check, we increase the exemption level by 50% and the results do not change

parameters.

Table 3: the bankruptcy parameters

Parameter	Symbol	Value
Exemption/wage	X/w	0.98
Unsecured credit exlsuion (expressed as probability)	ϱ	0.5

4.1.4 Calibrated parameters

We are left with the following 7 parameters to be calibrated: high entrepreneurial productivity (θ^H), entrepreneurial productivity transition matrix (p^{HH}, p^{LL}), concavity of entrepreneurial production function (ν), fraction of cash on hand with which an agent can run (λ), discount factor (β) and the variance of the transitory shock (σ_χ).

We choose these 7 parameters such that the model matches the following 7 moments of the US economy. First we want the model to match the *capital-output ratio* (K/Y) in the US economy. In the literature we find values ranging from 2.8 to 3.1. We target it to be 3.0. We target the *fraction of defaults*. Given the discussion in Section 2 we set this equal to 2.25%. The *fraction of entrepreneurs in the total population* is 7.3% in the Survey of Consumers Finances.[22] Based on PSID data the *exit rate* of entrepreneurs is equal to 15%. The median leverage ratio of entrepreneurs [23] in the SCF is around 15%.

Since the benefits of bankruptcy depend crucially on the wealth of an agent we match some features of the wealth distribution. The US wealth distribution is extremely skewed with the top 40% of richest households holding around 94% of total assets. As a last target we choose to match the *ratio of the median wealth of entrepreneurs to the median wealth in the whole population.* This target captures features of both the wealth distribution and entrepreneurial productivity and technology. We set the target to 6.3 as found in the SCF. The targets are summarized in the second column of Table 5.

4.2 The baseline calibration results

We first present the baseline version of the model. Table 4 reports the value of the calibrated parameters in the baseline specification.

Table 5 reports the value of the targets and the actual results achieved in the baseline specification.

The marginal product of capital in the corporate sector (r^d) is 2.9%. Less than one percent (0.79%) of the total population is in the constrained state. Our model does replicate the ratio of medians and the share of the wealth held by the richest 40% fairly well. It captures the main features that entrepreneurs are several times richer than workers and that most of the wealth is held by the richest. The Gini coefficient of wealth is 0.83 in the model, slightly higher than the data (0.8). For the purpose of our policy experiments it is important that the model replicates the middle and lower part of the wealth distribution since bankruptcy law affects almost exclusively these agents.

[22]See Mankart and Rodano (2007, appendix B) for data sources, definitions and further details.
[23]Leverage is defined as the ratio of debt to the sum of debt and equity.

Table 4: the calibrated parameters

Parameter	Symbol	Benchmark Value
High entrepreneurial productivity	θ^H	0.662
Entrepreneurial productivity transition	p^{HH}, p^{LL}	0.890 , 0.989
Concavity of entrepreneurial technology	ν	0.876
Fraction with which agent can run	λ	0.963
Discount factor	β	0.895
Variance of transitory shock	σ_χ	0.346

Table 5: the baseline calibration targets

Moment	Target	Model
Fraction of Entrepreneurs (in %)	7.3	7.3
Ratio of medians (in %)	6.3	6.1
Share of net-worth of top 40%	94.0	94.1
K/Y	3.0	3.0
Exit Rate (in %)	15.0	15.0
Bankruptcy Rate (in %)	2.25	2.25
Median leverage (in %)	15.0	15.0

Another feature that we do not target but that our model captures fairly well is the difference in the entry rate between workers with previous business experience and those without previous business experience. Based on PSID data[24], those who had some experience within the past three years are 13 times as likely to enter entrepreneurship than the average worker. In the model this ratio is 10.

Quadrini (2000) reports that around 35-40% of total capital is invested in the entrepreneurial sector. In our baseline specification this fraction is slightly lower, around 31.3%.

4.3 Investigating the model's mechanisms

4.3.1 Occupational choice

The key ingredient of the model is occupational choice. Figure 2 represents the occupational choice of an *unconstrained* agent with high entrepreneurial productivity and low working productivity.

The dotted line shows the value function of becoming a worker, whereas the solid line shows the value function of becoming an entrepreneur.

The first result is that, otherwise identical agents choose differently according to their wealth: poor agents become workers while rich agents become entrepreneurs. This result is standard in the occupational choice under credit market imperfections

[24] See Mankart and Rodano (2007, appendix B)

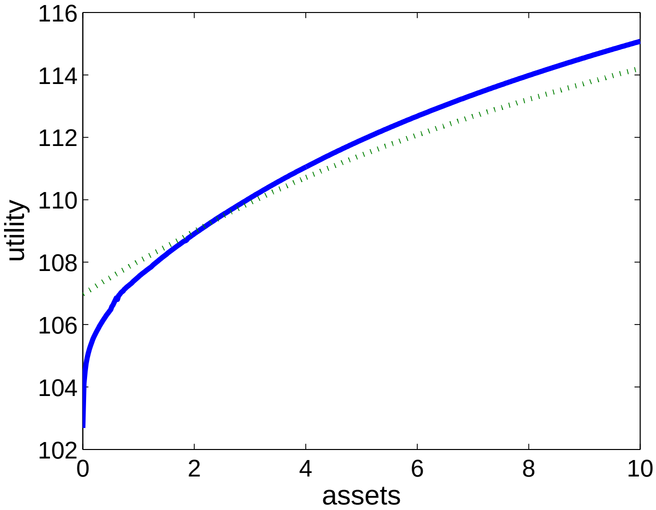

Figure 2: Occupational choice ($S = UN, \theta = \theta^H, \varphi = \varphi_3$)

literature (see e.g. Banerjee and Newman, 1993). The main reasons are that poor agents have smaller firms and face higher interest rates. They have smaller firms because, being poor, they need to borrow more but they face higher rates on the loans. The cost of financing is higher for the poor for two reasons. First, they have a higher incentive to default. Defaulting rich agents have to give up all their wealth above the exemption level. Second, in the event of default the bank gets less when the agent is poor. Thus, to break even, the bank has to charge a higher interest rate. That is, in this model, wealth acts as collateral.

4.3.2 The behavior of the unconstrained agents

The second important ingredient is the decision of the *unconstrained* entrepreneurs. The solution of the entrepreneurs' problem is represented in Figure 3.

The top panel shows demand for unsecured debt (b). The second panel shows demand for secured debt (s). The third panel shows the corresponding price of unsecured credit[25] The bottom panel shows the resulting firm size ($(a + b + s)$). Poorer agents (e.g. agents with assets $a < 0.8$) become workers while all the others become entrepreneurs ($a > 0.8$). The very rich entrepreneurs ($a > 2.4$) will never find it profitable to default. Their wealth is so high that defaulting is too costly for them. Therefore they borrow only secured since secured credit is cheaper than unsecured.[26] The "middle class" entrepreneurs (e.g. $a = 2$) will instead default if the shock is sufficiently bad, since the cost of bankruptcy is lower for them. In order to break even, the bank charges a higher interest rate, i.e. the unsecured credit is more expansive. The interest rate depends (negatively) on the assets of the entrepreneur, because in the event of default the bank will be able to seize the difference between the assets of the entrepreneur and the exemption level. Capital demand for the "middle-class" entrepreneurs is increasing because of the cost of borrowing is declining. The spikes in the demand for unsecured credit reflect the discretization of the investment shock.

[25] For readability, we show the price of credit instead of the interest rate.

[26] The transaction cost for secured credit is lower than for unsecured credit.

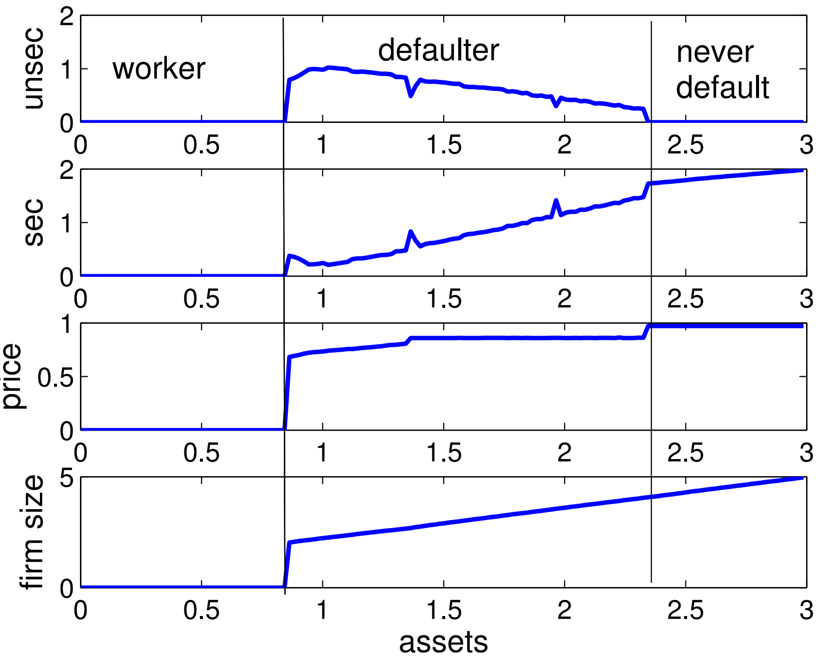

FIGURE 3: INTEREST RATE AND FIRM SIZE ($\theta = \theta^H$, $\varphi = \varphi_2$)

4.3.3 A first look at the effects of bankruptcy

Bankruptcy affects the problem of the unconstrained agents, because it changes credit conditions and the amount of insurance available. We examine these effects with the following experiment. We compare the behavior of the unconstrained agents in two different situations: one in which bankruptcy is allowed and one in which bankruptcy is absent. Figure 4 shows the policy functions in these situations.

The effects of allowing bankruptcy depend on the wealth of the agent. First, the default behavior of the rich (e.g. $a > 2.4$) is not affected. They are entrepreneurs and they repay their debt even in the bad states. As explained above, even if bankruptcy is available, it is too costly for them. They demand a little bit more secured credit due to a general equilibrium effect. Second, allowing bankruptcy affects the behavior of the less rich agents (e.g. $a = 1.5$). They are entrepreneurs in both situations. But when bankruptcy is allowed they borrow more unsecured because they are better insured at cost of more expansive credit. We call this increase in the firm size the *intensive margin*. Third, the occupational choice of even less rich agents (e.g. $a = 1$) is affected. When bankruptcy is not allowed they are not insured against bad outcomes. Therefore they do not want to borrow, even though they could borrow at rate r^s. They become workers. When bankruptcy is allowed they are insured against bad outcomes. Therefore they borrow, even though they have to pay a high interest rate. This increases the rewards of entrepreneurship enough to change their occupational choice. We call this increase in the number of entrepreneurs the *extensive margin*. Fourth, the occupational choice of the very poor agents (e.g. $a < 0.7$) is not affected, they are workers in both situations.

In this particular experiment abolishing bankruptcy reduces entrepreneurship and firm size, the intensive and the extensive margins are negative. The negative effect

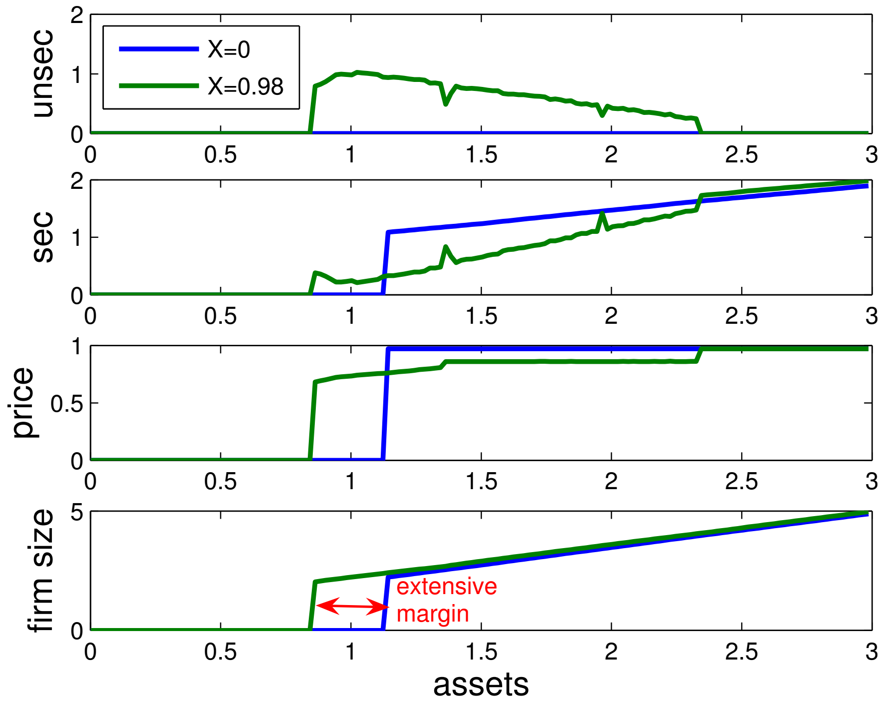

FIGURE 4: FIRM SIZE AND INTEREST RATE $(S = UN, \theta = \theta^H, \varphi = \varphi_2)$

of lowering the amount of insurance available dominates the positive effect of better credit conditions.

4.4 Changing the exemption level

Our main policy experiment is to analyze the effects of changing the exemption level.

Figure 5 shows the effects of changing the exemption level on welfare, entrepreneurship, exit rates and defaults. Table 6 reports the variables of interest for 3 values of X/w. Column 2 reports results when bankruptcy is very harsh $(X/w = 0)$. Column 3 reports results for the baseline calibration $(X/w = 0.98)$ and column 4 for the optimal exemption level $(X/w = 7.3)$.

Welfare Increasing the exemption level from zero increases welfare. The insurance effect is dominating the worsening credit market effect. More agents become entrepreneurs (see also Table 6) and welfare increases. However, increasing the exemption level beyond the optimal level worsens credit market conditions so much that agents borrow less, and therefore fewer agents find it profitable to become entrepreneurs. The current exemption level in the US, $X/w = 0.98$, is too low. The bankruptcy law is too harsh. The welfare gains in increasing the exemption level are substantial. The change in consumption equivalent (see row 10 in 6) is 2.2% of annual consumption. The rich and the poor both gain from increasing the exemption level.

Entrepreneurs Increasing the exemption level increases the fraction of entrepreneurs by 0.2 percentage points. Thus, there is a positive extensive margin. In particular,

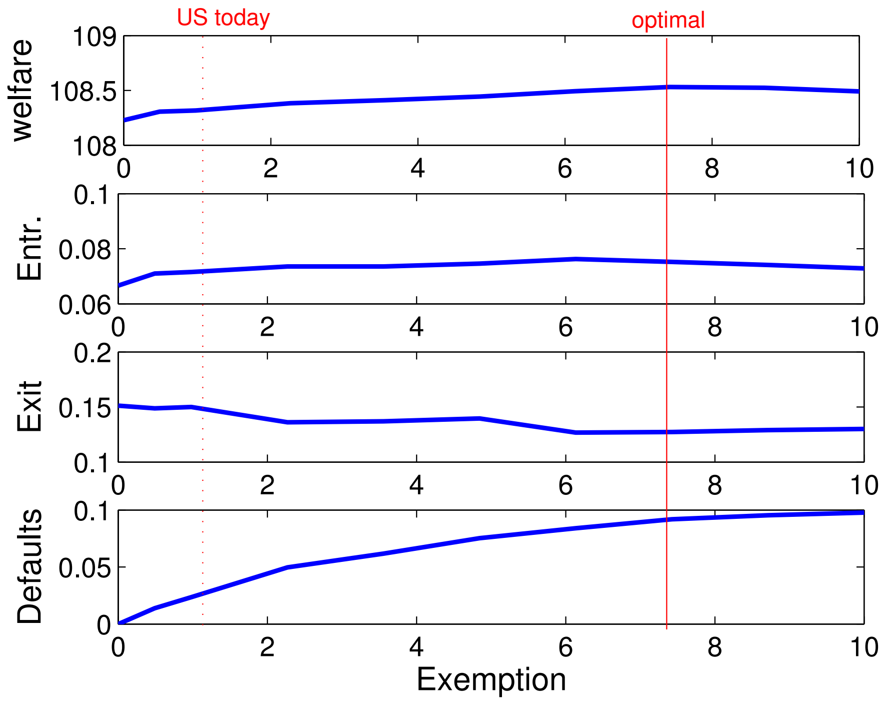

FIGURE 5: CHANGES IN THE EXEMPTION LEVELS

the optimal exemption level allows entrepreneurs who have defaulted to remain entrepreneurs because they can keep more assets in the default case. However, as can be seen in figure 5, the entrepreneurship rate peaks earlier than welfare. This implies that the intensive margin, i.e. bigger firms, is important in explaining the welfare results. As expected the default rate is increasing in the exemption level. The exit rate however is declining in the exemption level. The reason for this is that entrepreneurs who have defaulted keep enough assets to remain entrepreneurs despite being excluded from unsecured credit.

Access to entrepreneurship of the poor Next we turn to how bankruptcy law affects the determinants of entry into entrepreneurship. There is allocative inefficiency in our model because insurance markets are missing. Part of this inefficiency is reflected in some poor highly productive agents not becoming entrepreneurs, either because they receive too little insurance or because the conditions at which credit is available are too bad. Table 7 reports the effects of different exemption levels on the minimum assets needed for the highly productive ($\theta_{-1} = \theta^H$) agent to become an entrepreneur.

The rows show these values for the levels of working productivity (φ). The attractiveness of becoming a worker is increasing in working productivity, i.e. the outside option of entrepreneurs is increasing in working productivity. Thus in order to enter entrepreneurship, the expected profits must be higher for an agent with high working productivity. Since richer agents need to borrow relatively less and since they receive better credit conditions, their expected profits are higher. This implies that, to become an entrepreneur, an agent with high working productivity must be richer than an agent with low working productivity to enter entrepreneurship.

Increasing the exemption level to the optimal induces agents with high levels of

Table 6: the effects of changes in the exemption level

X/w	0	0.98	7.3
Exit rate (in %)	15.1	15.0	12.9
Fraction of Entrepreneurs (in %)	6.7	7.2	7.4
Bankruptcy/Exit (in %)	0	15.0	73.8
Capital/Output	3.02	3.02	3.02
Median assets of Entr/ Median assets	7.2	6.3	7.3
Share of Capital in entr. sector (in %)	30.9	31.4	33.2
Gini of Assets	0.84	0.84	0.83
Share of assets in top 40% of pop (in %)	94.6	94.6	94.5
Median output in entrepreneurial sector	9.7	8.9	11.4
Welfare in CE	-0.5	0.0	2.2
Welfare of rich in CE	-0.9	0.0	2.46
Welfare of poor in CE	0.1	0.0	2.02

labor productivity to enter entrepreneurship earlier. Poorer agents however will enter only when they are richer. The reason for this is that the credit market conditions worsen so much that they can obtain only secured credit and therefore lose the insurance coming from unsecured credit.

Table 7: minimum wealth for entrepreneurship

minimum wealth for entrepreneurship			
X/w	0	0.98	7.3
$\varphi = 0.316$	0.32	0.28	0.32
$\varphi = 0.745$	1.14	0.86	1.08
$\varphi = 1.342$	2.34	2.24	2.20
$\varphi = 3.163$	6.87	6.83	6.75

4.5 Modeling of credit markets matter

Almost all paper in the bankruptcy literature allow unsecured borrowing only.[27] Notable exceptions are Athreya (2006) and Hintermaier and Koeniger (2008) in the consumption literature. The reason is that the computational burden of allowing for secured credit as well is considerable. However, according to data from Sullivan, Warren, and Westbrook (1989), secured borrowing is as important as unsecured borrowing.[28].

Not only is secured credit empirically relevant, but also, as we show in this section, it is crucial for the results. To show this, we set up a model identical to the one discussed so far except that there is no secured credit available, neither for the *borrowing constrained* nor for the *unconstrained* entrepreneur. This implies that the former cannot borrow at

[27]See for example Akyol and Athreya (2007), Meh and Terajima (2008), Athreya (2002), Livshits et al. (2007), Chatterjee et al. (2007), Athreya and Simpson (2006), Li and Sarte (2006), Mateos-Planas and Seccia (2006).

[28]Mean secured debt over mean total debt is about 55%

all and must finance his projects with his own wealth. We first recalibrate the model and then conduct the same policy experiment as before. The results in figure 6 are striking. The optimal bankruptcy law now would be to abolish bankruptcy completely. This would increase welfare and lead to a higher number of entrepreneurs.

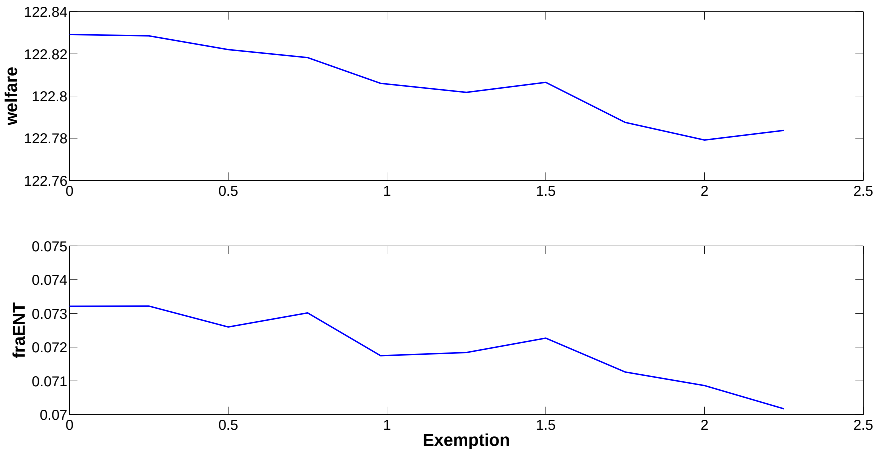

FIGURE 6: WELFARE EFFECTS OF CHANGES IN X IF ONLY SECURED CREDIT AVAILABLE

TABLE 8: CALIBRATION UNSECURED CREDIT ONLY

Moment	**Target**	**Unsec credit only**	**Sec and Unsec**
Entrepreneurs (in %)	7.3	7.17	7.44
Exit Rate (in %)	15.0	13.55	12.76

Table 8 shows what happens if we use the calibrated parameters of the model without secured borrowing and now allow secured borrowing. Since the financial market is now relatively more complete, we see that there are more entrepreneurs and fewer exits.

The reason for this can be seen in figure 7. All agents is region (2) are not able to obtain unsecured credit because their default incentive is too high. If secured credit is not available, these agents become workers. However, if secured credit is available, these agents can borrow secured and so become entrepreneurs. Agents in region (3) use secured credit to run bigger firms.

This mechanism explains why the optimal exemption level in a model with secured and unsecured credit is much higher than the optimal exemption level in a model with only unsecured credit. Absent secured credit, an increase in the exemption level prices out many more agents. It would expand regions (1) and (2). Thus, the agents become workers because they are credit rationed. The availability of secured credit dampens this negative effect.

Another way of looking at this is the following. The optimal policy is a very harsh bankruptcy law. This implies that the agents do not value the insurance that is provided by the bankruptcy law. They would like to have less insurance but therefore have better credit market conditions. This means essentially that the agents want a

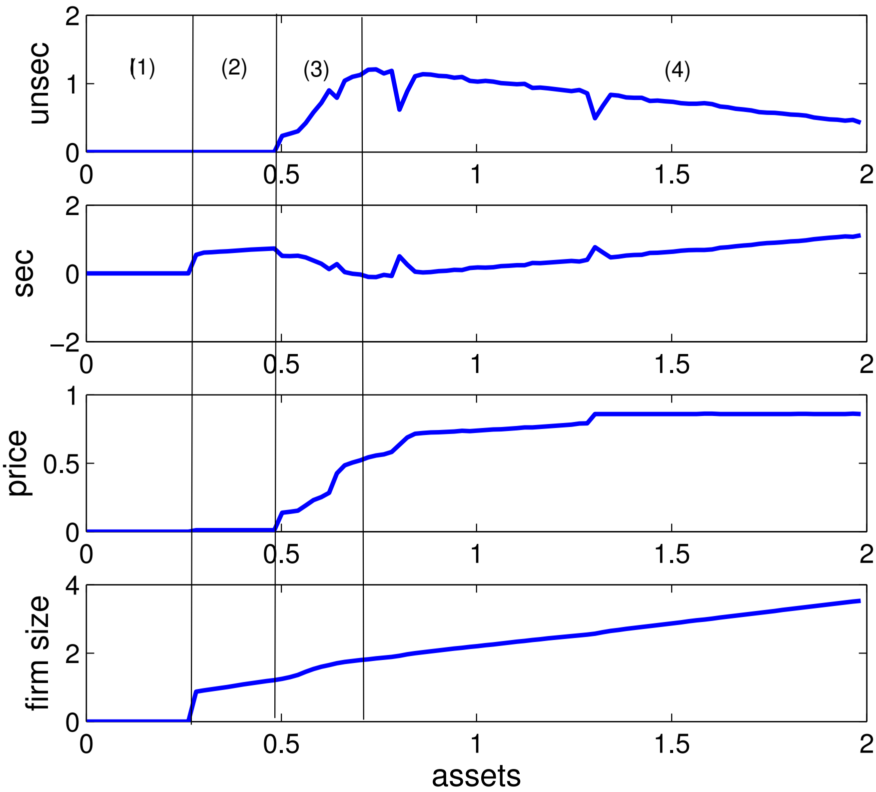

FIGURE 7: POLICY FUNCTIONS WITH SECURED CREDIT

commitment device that takes away the default option. One way to achieve his is to make the law harsher. Another way, however, is to use secured credit. Secured credit is the commitment device that the agents want.

As already mentioned, most previous papers do not include secured credit in their models and most of them find that the current bankruptcy law is too lenient.[29] Our results imply that these results might not be robust towards including secured borrowing.

4.6 Robustness

In this section, we show the effects of changing the agent's preferences. We separate the elasticity of intertemporal substitution from the coefficient of relative risk aversion because they have different effects. With a standard utility function, one is the inverse of the other. In this case, an increase in risk aversion as for example examined in Athreya (2006) conflates two effects. On the one hand, since agents are more risk averse, they value insurance more so the optimal exemption level is likely to be higher. On the other hand, with standard preferences, an increase in risk aversion simultaneously lowers the elasticity of intertemporal substitution. Thus, agents are less willing to transfer consumption across time. But a higher exemption level will increase the interest rate agents face because banks have to charge higher interest rates in order to break

[29]The two other papers (Akyol and Athreya (2007),Meh and Terajima (2008)) in the entrepreneurial bankruptcy literature find significant welfare gains from making the law harsher. The papers in the consumer bankruptcy literature reach similar conclusions.

even. Thus, a decrease in the elasticity of intertemporal substitution is likely to lead to a lower optimal exemption level. By not separating the two, one examines only their net effect. It is possible that each of these two effects is big but that they cancel each other so that the net effect is small.

4.6.1 Changing EIS

In this subsection we investigate the robustness of the results towards different values of elasticity of intertemporal substitution. The costs of a lenient bankruptcy law are higher interest rates which make substitution across time more costly. If agents' willingness to substitute consumption across period is low (i.e. eis is small), higher interest rates will be particularly costly. Therefore the optimal exemption level should be an increasing function of the elasticity of intertemporal substitution. We recalibrate the model once with a low elasticity of intertemporal substitution, ($\psi = 0.6$) and once with a high elasticity of intertemporal substitution ($\psi = 1.4$). We keep the coefficient of risk aversion constant. The results are shown in table 9. The optimal exemption level is increasing in the elasticity of intertemporal substitution as expected. While the magnitude of the effects is not huge, they are quantitatively significant.

Table 9: Optimal exemption level for different EIS

CRRA	**Optimal X**
0.6	6.7
1.1	7.3
1.4	7.9

4.6.2 Changing RRA

In this subsection we investigate the robustness of the results towards different degrees of risk aversion. The possibility to default provides insurance against bad outcomes. The value agents attach to this insurance depends on their risk aversion. We recalibrate the model once with a low coefficient of risk aversion, ($\sigma = 1.5$) and once with a high coefficient of relative risk aversion ($\sigma = 4.5$). We keep the elasticity of intertemporal substitution constant since we want to isolate the importance of risk attitudes.

The optimal exemption level, the amount of insurance, is increasing in σ. This result is qualitatively not surprising. However it is also quantitatively important. If agents were less risk averse, the optimal exemption level would be 13% lower. However, the effects are rather small in welfare terms. Welfare never changes by more than a fraction of a percent. This is due to the fact that all exemption levels are pretty high.

Table 10: Optimal exemption level for different CRRA values

CRRA	Optimal X
1.5	6.3
3.0	7.3
4.5	8.7

5 Conclusion

This is the first paper to explore quantitatively the effects of personal bankruptcy law on entrepreneurship in a general equilibrium setting with heterogeneous agents and secured and unsecured credit. First, we developed a dynamic general equilibrium model with occupational choice which explicitly incorporates the US bankruptcy law. Our model endogenously generates interest rates that reflect the different default probabilities of the agents. Our model accounts for the main facts on entrepreneurial bankruptcy, entrepreneurship, wealth distribution and macroeconomic aggregates in the US.

Then, we used the model to quantitatively evaluate the effects of changing the US bankruptcy law. The simulation results show that increasing the exemption level would increase the fraction of entrepreneurs and welfare. These effects are significant: increasing the exemption level to the optimal one has positive welfare effects in the order of 2.2% of average consumption. All households, rich and poor, would be better off.

The most important contribution of our paper is to show that the modeling of the credit market matters. Investigating the optimal exemption level in a model without secured credit gives misleading results because it overstates credit rationing.

We are currently extending our research program along two dimensions. First, we are incorporating the transition to the new steady state. So far, our results are based on a comparison of steady-states. Transitional effects might be important to evaluate welfare. In addition it might explain why the current law is too lenient. It could be that some groups lose during the transition and therefore oppose changes.

Second, we are expanding our model to incorporate explicitly a European type of bankruptcy law. The laws in European countries are much harsher than the law in the US. For example in Italy, debt is never discharged. A defaulter is liable forever. We are analyzing the effects of introducing a US type of law on the Italian economy.

Appendices

A Computational strategy

The state vector for an individual is given by $\eta = (a, \theta, \varphi, S)$. The aggregate state is a density $\mu_t (a, \theta, \varphi, S)$ over the individual state variables. We assume that a takes on value on a grid G_a of dimension n_a. Therefore the dimension of the individual state space is $n = n_a \times n_\theta \times n_\varphi \times 2$ where $n_\theta = 2$ is the number of states for the entrepreneurial productivity and $n_\varphi = 4$ is the number of states for the working productivity.

In order to solve the model we use the following:

Algorithm 1 *Our solution algorithm is:*

1. *Assign all parameters values*
2. *Guess a value for the endogenous variable r.*
3. *Given r the FOC of the corporate sector uniquely pin down the wage rate w. The representative competitive firm in the corporate sector will choose K_c and L_c such as*

$$r^d = \xi A K_c^{\xi-1} L_c^{1-\xi} = \xi A \left(\frac{K_c^d}{L_c^d}\right)^{\xi-1} \tag{A-1}$$

$$w = (1-\xi) A K_c^{\xi} L_c^{-\xi} = (1-\xi) A \left(\frac{K_c^d}{L_c^d}\right)^{\xi} \tag{A-2}$$

   Therefore r uniquely pins down $\left(\frac{K_c}{L_c}\right)$ and in turn uniquely pins down w.

4. *Given (r, w) we solve for the optimal value functions and corresponding policy functions by value function iteration. Within the period we solve backwards in time.*
   (a) *We guess a value function $\mathsf{V}(\eta)$*
   (b) *We solve the consumption-savings problem of the constrained and unconstrained agent for a grid of cash on hand.*
   (c) *We approximate the resulting continuation value functions.*
   (d) *Since the worker faces no uncertainty within the period, these value functions give us the values for the workers.*
   (e) *Given the continuation value, we solve the problem of the unconstrained entrepreneur:*
      - *We set up a grid for secured credit.*
      - *For each value of secured credit, we set up a grid for unsecured credit.*
      - *For each value of unsecured credit, we price the credit according to the zero profit condition.*
      - *We identify the optimal grid point and then bisect around that optimal point to get a more accurate choice of unsecured credit.*

- *We calculate the value for each combination of secured.*

(f) *The problem of the constrained entrepreneur is solved similarly.*

(g) *Occupational choice gives us the updated value functions* $\hat{\mathsf{V}}(\eta)$.

(h) *We iterate until convergence.*

(i) *As a byproduct we obtain the policy functions.*

5. *The policy functions, the exogenous transition matrix for the shocks (both for* θ *and for* φ*), the* iid *investment shock and the credit status shock* ϱ *induce a transition matrix* P_η *over the state* η.

6. *The transition matrix* P_η *maps the any current distribution*[30] μ_η *into a next period distribution* μ'_η *by simply*

$$\mu_{\eta,t+1} = P_\eta^{'} \times \mu_{\eta,t}$$

We calculate the steady state distribution over the state μ_η^* *by solving for a*

$$\mu_\eta^* = P_\eta^{'} \times \mu_\eta^*$$

7. *From the policy functions and the steady state distribution, we derive the market clearing conditions*

8. *Labor market clearing implies that labor supply* $L^s(r)$ *is equal to labor demand (that comes from corporate*L_c^d*). Plugging this into the FOC (A-1) of the corporate sector we get capital demand from corporate sector:*

$$K_c^d(r) = \left(\frac{r}{\xi A}\right)^{\frac{1}{\xi-1}} L^S(r)$$

9. *Now we look at capital market clearing:*

$$K_{ENTR}^d(r) + K_c^d(r) = SA(r)$$

10. *If there is not equilibrium at point 9 we adjust interest rate, we go back to point 3 and we iterate until market clears*[31].

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[30] Note that in our framework the distribution of household over the state μ_η, is vector of dimension n whose elements sum up to 1.

[31] In practice we first run a grid search over different values for r and then bisect until we get market clearing.

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