Efficient Subsidization of Human Capital Accumulation with Overlapping Generations and Endogenous Growth

by

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Abstract:

This paper studies second best policies for education, saving, and labour in an OLG model in which endogenous growth results from human capital accumulation. Government expenditures have to be financed by linear instruments so that growth equilibria are inefficient. The inefficiency is exacerbated if selfish individuals externalize the positive effect of education on descendents' productivity. It is shown to be second best to subsidize education even relative to the first best if the elasticity of the human capital investment function is strictly increasing.

Keywords: OLG model; endogenous growth; endogenous labour, education, and saving; intergenerational externalities; optimal taxation

JEL Classification: H21, I28, J24

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1. Introduction

According to data published by OECD (2009, Tables A8.2 and A8.4), various countries effectively subsidize education while other countries effectively tax education. Such a finding does not only raise the question of which policy is superior, it also raises the question of whether and how the effective subsidization of education can be justified in terms of efficiency. This paper studies this question in a framework of overlapping generations and endogenous growth. Two reasons of why education should be subsidized are highlighted. One of these is already known from the literature. It is the potential need to internalize the positive effect that human capital investments of selfish individuals have on the productivity of descendent generations. Efficient internalization requires subsidizing investments up to the first best. This paper stresses the second reason. This is the negative effect that distortionary taxation of labour has on education and growth. If the elasticity of the human capital investment function is strictly increasing, it is shown to be a second best policy to subsidize education even relative to the first best.

The traditional approach to optimal taxation follows Ramsey (1927) and takes the model of a representative taxpayer as a starting point. A critical feature of this literature is that the results characterizing optimal policy heavily depend on whether the representative taxpayer plans for finite or infinite periods. If the taxpayer's planning horizon is infinite, the rationale for employing distortionary linear taxes and subsidies turns out to be weak. This point was originally made by Chamley (1986) and Judd (1985) with respect to capital taxes. It extends, however, to the model with endogenous education, as has been demonstrated by Bull (1993), Jones, Manuelli, and Rossi (1993, 1997), and Atkeson, Chari, and Kehoe (1999). The question of whether human or nonhuman capital is accumulated is largely irrelevant. In the long run neither accumulation should be distorted.

The policy recommendations are less clear-cut if the taxpayer's planning horizon is finite. In the finite case it is primarily a matter of marginal rates of intertemporal substitution in consumption whether taxing saving is efficient or not. In particular, saving should be untaxed only if the taxpayer's utility is weakly separable between consumption and labour and homothetic in consumption (Atkinson and Stiglitz, 1972; Sandmo, 1974). By contrast, the design of efficient education policy is more a reflection of the specific properties of the earnings function. This function has to be weakly separable in qualified labour supply and education and the elasticity with respect to the latter has to be constant if it shall be second best not to distort the choice of education (Jacobs and Bovenberg, 2008; Bovenberg and

Jacobs, 2005). If weak separability holds and if the elasticity is strictly increasing, it is second best to subsidize education (Richter, 2009). If the planner trades off efficiency and equity and if education and qualified labour are complementary, it is equally second best to subsidize education (Jacobs et al., 2008).

It somewhat discredits the Ramsey approach that the suggested policy recommendations so critically depend on the taxpayer's planning horizon. That is why the present paper studies optimal taxation in a model with overlapping generations. Such a model stands between the static and dynamic Ramsey frameworks and it therefore promises less debatable policy recommendations. The broader objective of the present study is to characterize optimal policies for education, labour, and saving in a dynamic framework with overlapping generations. The narrower objective is to rationalize the effective subsidization of endogenous education. Such objectives may justify putting aside various shortcomings often turned against similar studies. In particular, we exclusively focus on efficiency and we stick to the representative taxpayer framework because one would not really be surprised to learn that subsidizing education can well be optimal when equity is traded off against efficiency. Furthermore, we rule out potential reasons of market failure because they may help to justify market intervention but certainly not the subsidization of education relative to the first best.

The model chosen is one with overlapping generations and endogenous growth. Individuals live for two periods. They decide on education, saving, and nonqualified labour in their youth. They supply qualified labour when old. The productivity of qualified labour increases in the stock of human capital inherited from preceding generations, and it also increases in own educational investments. Individuals either may be perfect altruists with respect to descendent generations or may behave selfishly. The implications of selfishness have been studied before by Wigger (2002, Sec. 3.4) and Docquier et al. (2007) for a framework in which the government is not constrained in the use of policy instruments. It is shown that decentralizing the first best requires subsidizing education up to the first best. The present paper goes beyond these earlier studies by endogenizing labour supply and by assuming that the government can only employ linear policy instruments. Most remarkably, major results characterizing efficient static policy extend to the dynamic framework. In particular, it is second best not to distort education if the human capital investment function is isoelastic in education. It is argued, however, that such constant elasticity has debatable implications in a dynamic framework. It implies that the human capital stock accumulated by preceding generations melts down to zero if just one generation stops investing. More appealing is the assumption that the elasticity of the investment function is increasing and that the human capital stock does not depreciate completely if just one generation fails to invest. If this is the case, it is second best at balanced growth to subsidize education even relative to the first best. This means that the marginal social cost of human capital should exceed the marginal social return in the long-run second-best optimum. This is a striking result. Not surprising is the need to subsidize education relative to *laissez faire*. This is so because the intergenerational externalities of human capital investments have to be internalized.[1] *A priori* it is not obvious, however, why investments should even exceed the first-best. Subsidizing education requires government revenue, which in the model has to be raised by distortionary taxes on labour and savings. With the intuition of Lipsey and Lancaster (1956/57) in mind, one might hypothesize that it is second best to provide insufficient incentives for education if labour has to be taxed and if the level of comparison is the first best. The contrary, however, is true. The key assumption is the strictly increasing elasticity of the human capital investment function with respect to education. The effect is that it is second best to subsidize education in static analysis, and this effect is shown to extend to the dynamic framework. At balanced growth the need to subsidize increases in the derivative of the investment function's elasticity and in two further factors. One factor is the Lagrange multiplier on the planner's implementability constraint, and the other is the gap between the marginal return to capital and the rate of balanced growth. In other words, the more binding the non-availability of lump-sum taxes is and the more deficient the growth is, the more should human capital accumulation overshoot the first best.

Assuming altruistic individuals changes some conclusions, but not all. Altruists internalize the positive effect that education has on descendents' productivity. Hence the need for government intervention is reduced. However, the second source of inefficiency modelled in this paper does not vanish. That second source is the need to employ distortionary taxes for financing government expenditures. The implications for second-best policy are shown to differ markedly between the first generation and all descendent generations. With respect to descendent generations the following results are obtained. The accumulation of human capital should not be distorted, and this result is obtained for arbitrary utility and human capital investment functions. Furthermore, qualified and nonqualified labour should be taxed uniformly across the life cycle when utility is homogeneous in consumption and multiplicative in the sub-utilities of consumption and non-leisure. Such results strongly contrast with those derived for the case of selfish individuals.

The results obtained for the first generation are less contrasting. In particular, it is second best not to distort the first generation's educational choice if the human capital investment function

[1] The need is highlighted by various earlier studies. An example is Del Rey and Racionero (2002).

is isoelastic in education. If, however, this function fails to be isoelastic, the optimal education policy for the first generation depends on initial values. On neutralizing the effect of initialization by assuming balanced growth and assuming a strictly increasing elasticity of the human capital investment function, it turns out to be second best to subsidize education. The reason is the same encountered when individuals are selfish. Strictly increasing elasticity is the reason why it is second best to subsidize education in static analysis. This effect extends to the dynamic framework. The need to subsidize is the stronger the larger the derivative of the investment function's elasticity is, the more binding the non-availability of lump-sum taxes is, and the more deficient growth is.

The unifying bottom line for selfish and altruistic individuals is as follows. Altruism well reduces the need to subsidize education relative to *laissez faire*, and altruism also implies that descendent generations should have non-distorted incentives to invest in human capital. The short-run policy recommendations for altruism, however, agree with the long-run recommendations for selfishness. Labour has to be taxed, and – given that the elasticity of the human capital investment function is strictly increasing – education should be subsidized relative to the first best.

The paper is structured as follows. Section 2 sets up the two-period overlapping-generations model with endogenous growth. The first-order conditions characterizing solutions of the planner's first-best maximization are derived. In Section 3 the utility functions are determined that are compatible with balanced growth in consumption and with constant use of labour and leisure. Section 4 studies the planner's problem when individuals behave selfishly and when no policy instruments but linear ones are available. Section 6 clarifies the relation between effective and efficient subsidization. Section 6 studies the planner's problem for individuals who are altruistic towards descendent generations. Section 7 summarizes.

2. The model and the planner's first-best problem

Consider a sequence of overlapping generations with individuals living for two periods. The index t refers to the generation and to the period in which the representative individual of generation t is young and in her life period 0. Lifetime utility is given by $U_t \equiv U(C_{0t}, C_{1t}, L_{0t}, L_{1t})$ with the arguments $C_{0t}, C_{1t}, L_{0t},$ and L_{1t} denoting consumption and non-leisure in the life periods i=0,1. Utility is strictly increasing in consumption, is strictly decreasing in non-leisure, and is strictly concave. Additional restrictions on preferences required if the economy is to exhibit steady state growth are discussed in Section 3. Non-leisure in the second life period, L_{1t}, equals *qualified* labour supplied to the market in period t+1. By contrast, non-leisure in the first life period has to be divided between *nonqualified* labour supply $L_{0t}-E_t$ and education E_t. The effect of education is to increase human capital and labour productivity. H_{t-1} is the stock of human capital determining the productivity in period t. It is built up by generation t−1 and inherited by generation t. By spending time E_t on education, generation t determines the stock of human capital H_t effective in the second life period. The *human capital accumulation equation* is

$$G(E_t)H_{t-1}=H_t. \qquad (\mu_t\beta^t) \qquad (1)$$

$\mu_t\beta^t$ is a Lagrange multiplier associated with the planner's problem we are about to set up. The investment function $G_t \equiv G(E_t)$ is assumed to be non-negative and strictly monotone increasing with elasticity $\eta(E)\equiv EG'/G$ smaller than one. The case of constant elasticity η plays a prominent role in static models of endogenous education (Jacobs et al., 2008; Richter, 2009) and equally in what follows. A critical implication is G(0)=0 so that the stock of human capital built up by generation t−1 melts down to zero, H_t=0, if generation t does not spend positive time on education. If one assumes instead $G(E_t)\equiv \tilde{G}(E_t)+1-\delta_H$ with δ_H<1 and some function $\tilde{G}(E)$ of constant elasticity $\tilde{\eta}$, then $H_t=(1-\delta_H)H_{t-1}$ follows from E_t=0 so that some human capital is passed on to the next generation even if there are no new investments. In this case, the elasticity of the investment function, $\eta(E)=[1-\frac{1-\delta_H}{G(E)}]\tilde{\eta}$, is strictly increasing in E. To allow for both scenarios with constant and increasing elasticity of G(E) we assume $\eta'(E)\geq 0$ in what follows.

The functional specification (1) is standard in the endogenous growth literature. It can be traced back to Uzawa (1965), and it has been used since by Lucas (1988), Atkeson et al. (1999), and others. A key feature is that H_t is linear homogenous in H_{t-1}. A notable implication of (1) is that time spent on education (*learning*) is the only variable input in the production of human capital. In particular, learning cannot be substituted by physical inputs or services supplied by instructors. There is however some cost of instruction which accrues in fixed proportion with education. For simplicity's sake, it is modelled as a linear function of inherited human capital and time spent on education, $f E_t H_{t-1}$. It is suggestive to interpret the exogenous parameter f as *tuition fee*.

There is a second stock variable, K_t, to be interpreted as (nonhuman) capital built up by generation t in their first life period. It is not productive before the second life period, and it depreciates at the rate δ_K. Production F is linear homogenous in capital and effective labour. The *resource constraint* is

$$F_t + (1-\delta_K)K_{t-1} = C_{0t} + C_{1t-1} + f E_t H_{t-1} + K_t + A_t \qquad (\alpha_t \beta^t) \qquad (2)$$

with $F_t \equiv F(K_{t-1}, (L_{0t} - E_t)H_{t-1}, L_{1t-1}H_{t-1})$.

The variable A_t denotes exogenous government spending. Such spending may be of consumptive and/or productive use. As A_t is exogenous, we refrain from making it an explicit argument of the utility and/or production functions. When taking partial derivatives use is made of the following short forms:

$$F_{Kt} \equiv \frac{\partial F}{\partial K_{t-1}}, \qquad F_{L_0 t} \equiv \frac{\partial F}{\partial((L_{0t} - E_t)H_{t-1})}, \qquad F_{L_1 t} \equiv \frac{\partial F}{\partial(L_{1t-1}H_{t-1})}.$$

Qualified and nonqualified labour may be perfect substitutes in production, but they need not be. Human capital is obviously labour augmenting. Note that education incurs two kinds of cost. There is the *cost of forgone earnings*, $F_{L_0 t} E_t H_{t-1}$, and the *cost of tuition*, $f E_t H_{t-1}$.

The planner maximizes

$$\sum_{t=0}^{\infty} \beta^t U(C_{0t}, C_{1t}, L_{0t}, L_{1t}) \qquad (3)$$

in $C_{0t}, C_{1t}, L_{0t}, L_{1t}, E_t, H_t$, and K_t (t=0,1,...) subject to the human capital accumulation equation (1) and the resource constraint (2). The parameters $K_{-1}, H_{-1}, L_{1-1} = L_{1,t=-1}$ are exogenously given. 0<β<1 is a discount factor. Assume that this maximization – like all others still to follow – is well behaved and that it has an interior solution for which all choice variables are strictly positive. We abstain from stating all the assumptions needed to guarantee a well-behaved maximization with interior solutions. Identifying those assumptions must remain the object of independent research efforts. In the present paper we just state those assumptions explicitly needed to derive meaningful first-order conditions of second-best policies. We study neither second-order conditions nor questions of existence. As argued in Richter (2009) and as will become clearer below, a well-behaved maximization requires a specification of $U_t = U(C_{0t}, C_{1t}, L_{0t}, L_{1t})$ which is sufficiently concave to compensate for the lack of concavity of the human capital accumulation equation (1). The first-order conditions of the planner's maximization are as follows:

$$U_{C_0t} = \alpha_t\,, \quad U_{C_1t} = \alpha_{t+1}\beta\,, \quad F_{L_0t}\, H_{t-1}\, U_{C_0t} = -U_{L_0t}\,, \quad F_{L_1t+1} H_t\, U_{C_1t} = -U_{L_1t}\,, \tag{4}$$

$$F_{Kt+1} + 1 - \delta_K = U_{C_0t} / U_{C_1t} = U_{C_0t} / \beta\, U_{C_0t+1}\,, \tag{5}$$

$$\mu_t G_t^{'} = \alpha_t\,(f + F_{L_0t})\,, \tag{6}$$

$$\alpha_{t+1}\beta\,[F_{L_1t+1} L_{1t} + F_{L_0t+1} \cdot (L_{0t+1} - E_{t+1}) - f\, E_{t+1}] = \mu_t - \beta\, G_{t+1}\, \mu_{t+1}\,. \tag{7}$$

The conditions (4) characterize efficient consumption and labour choices. The condition (5) characterizes efficient saving and efficient capital. The condition (6) characterizes the efficient choice of E_t, and (7) is the condition characterizing the efficient choice of H_t. Solving (6) for μ_t and inserting into (7) yields, after some straightforward manipulations, the condition characterizing the *efficient accumulation of human capital*,

$$F_{L_1t+1} L_{1t} + F_{L_0t+1}\, L_{0t+1} - (F_{L_0t+1} + f\,)\, E_{t+1}$$

$$= [F_{Kt+1} + 1 - \delta_K\,] \frac{f + F_{L_0t}}{G_t^{'}} - G_{t+1} \frac{f + F_{L_0t+1}}{G_{t+1}^{'}}\,. \tag{8}$$

For the sake of brevity we also speak of *efficient education* if (8) holds. The first term on the left-hand side, $F_{L_1t+1} L_{1t}$, is the return to human capital accruing to generation *t* in the second life period, and the difference $F_{L_0t+1}\, L_{0t+1} - (F_{L_0t+1} + f\,)\, E_{t+1}$ is the return accruing to individuals of the next generation in their first life period. The factor

$$\frac{f + F_{L_0t}}{G_t^{'}} = (f + F_{L_0t}) H_{t-1} \frac{dE_t}{dH_t} \tag{9}$$

is the marginal cost of human capital in period *t*, and $\frac{f + F_{L_0t+1}}{G_{t+1}^{'}}$ is the marginal cost of human capital one period later. Hence the right-hand side of (8) captures the cost resulting from investing in period *t* instead of postponing the investment to the next period. By separating terms referring to generation *t* from terms referring to generation *t*+1, (8) can be written as

$$[F_{Kt+1}+1-\delta_K]\frac{f+F_{L_0t}}{G_t^{'}}-F_{L_1t+1}L_{1t}$$

$$=F_{L_0t+1}L_{0t+1}+(F_{L_0t+1}+f)E_{t+1}[\frac{1}{\eta_{t+1}}-1]\equiv MEB_{t,t+1}. \qquad (10)$$

Because $\eta_{t+1}<1$ by assumption, $MEB_{t,t+1}$ is positive. It is the *marginal external benefit* enjoyed by generation *t*+1 and generated by the human capital investment of generation *t*. This excess benefit has to be internalized by first-best policy when individuals are selfish. As a result of internalization, generation *t*'s cost, $[F_{Kt+1}+1-\delta_K]\frac{f+F_{L_0t}}{G_t^{'}}$, exceeds generation *t*'s return to human capital, $F_{L_1t+1}L_{1t}$.

3. Balanced growth

We speak of *balanced growth* if the non-leisure choices $L_{0t}=L_0, L_{1t}=L_1$, and $E_t=E$ are constant across time while consumption, output, and both types of capital all grow at the common gross rate *G*=*G*(*E*), so that we have $H_{t-1}=G^tH_{-1}, K_{t-1}=G^tK_{-1}$, $C_{it}=G^tC_{i0}\equiv G^tC_i$. At balanced growth, $F_{Kt+1}=F_K$ is constant in *t*. If an efficient allocation is to be compatible with balanced growth, then conditions (4) and (5) require the rates of substitution

$$\frac{U_{C_0}(G^tC_0,G^tC_1,L_0,L_1)}{U_{C_0}(G^{t+1}C_0,G^{t+1}C_1,L_0,L_1)},\ \frac{U_{C_0}(G^tC_0,G^tC_1,L_0,L_1)}{U_{C_1}(G^tC_0,G^tC_1,L_0,L_1)}$$

to be both constant in *t*. Taking total derivatives with respect to *t* and setting the total derivatives equal to zero implies constancy of

$$[G^tC_0\cdot U_{C_0C_0}(G^tC_0,..)+G^tC_1\cdot U_{C_1C_0}(G^tC_0,..)]/U_{C_0}(G^tC_0,..)$$

$$=[G^tC_0\cdot U_{C_0C_1}(G^tC_0,..)+G^tC_1\cdot U_{C_1C_1}(G^tC_0,..)]/U_{C_1}(G^tC_0,..)\equiv d-1 \qquad (11)$$

in *t*. Upon substituting $\tilde{C}_i$ for G^tC_i and integrating in $\tilde{C}_i$ one obtains

$$C_0U_{C_0}+C_1U_{C_1}=dU+cX \qquad (12)$$

where *d*, *c* are constants and where *X* is a function of L_0, L_1. The following two types of utility specifications satisfy this condition:

(i) $U(C_0, C_1, L_0, L_1) = V(C_0, C_1) \cdot \Lambda(L_0, L_1) - D(L_0, L_1)$ (13)

where $V(C_0, C_1)$ is homogeneous of degree $d \neq 0$;

(ii) $U(C_0, C_1, L_0, L_1) = [a_0 \ln C_0 + a_1 \ln C_1]\Lambda(L_0, L_1) - D(L_0, L_1)$. (14)

Utility functions of type (13) satisfy condition (12) when setting $c \equiv d \neq 0$, $X \equiv D$ and utility functions of type (14) satisfy condition (12) when setting $c \equiv a_0 + a_1$, $d \equiv 0$, $X \equiv \Lambda$. In the latter case homogeneity in consumption does clearly not hold in the strict sense, but (11) still holds with d=0. For the sake of brevity we choose to speak of homogeneity in both cases (13) and (14). In what follows homogeneity in consumption is assumed whenever second best policies are evaluated at balanced growth.

An earlier characterization of utility functions compatible with growth in consumption and constancy in leisure is due to King, Plosser, and Rebelo (1988, 2002). These authors however restrict their study to dynamic equilibria and government policies in a Ramsey-type framework with exogenous growth. Furthermore, they work with utility functions $U(C, L)$ which have only two arguments. (13) and (14) extend their findings.

Assuming balanced growth and utility to be homogeneous of degree d in consumption, we obtain $U_{C_0t} = G^{(d-1)t} U_{C_0 0}$. Hence $F_K + 1 - \delta_K = G^{1-d} / \beta$ by (5). Furthermore, the *condition of transversality*, $\beta^t U_{C_0t} K_t \to 0$ for $t \to \infty$, implies $(\beta G^{d-1})^t U_{C_0 0} G^t K_{-1} \to 0$ for $t \to \infty$, i.e., $\beta G^d < 1$. As a result, the return to capital exceeds the growth rate:

$$F_K + 1 - \delta_K = G^{1-d} / \beta > G. \tag{15}$$

The following analysis studies second-best policy with regard to education, to saving, and also to labour. The focal question, however, is whether it is second best to provide or not to provide efficient incentives for education. As we shall see, much depends on the elasticity of the elasticity of the investment function $G(E)$ and on whether individuals are perfect altruists towards their children or not. In the altruistic model – also called the dynasty model – individuals are assumed to maximize (3). In the other case the representative individual is assumed to maximize own lifetime utility

$$U(C_{0t}, C_{1t}, L_{0t}, L_{1t}) \tag{16}$$

subject to the own lifetime budget constraint. We study both scenarios, and we start by analyzing efficient taxation in the standard OLG framework with selfish individuals. The approach taken is called the primal approach in optimal taxation.

4. Optimal taxation in the standard OLG model with selfish individuals

The selfish individual representing generation t is assumed to maximize (16) in the five variables $C_{0t}, C_{1t}, L_{0t}, L_{1t}, E_t$, and savings S_t subject to the life-period budget constraints

$$\omega_{0t}(L_{0t} - E_t)H_{t-1} = C_{0t} + \varphi_t E_t H_{t-1} + S_t \qquad (\lambda_{0t}) \qquad (17a)$$

$$\omega_{1t}L_{1t}G(E_t)H_{t-1} + R_{t+1}S_t = C_{1t}. \qquad (\lambda_{1t}) \qquad (17b)$$

In this optimization H_{t-1} is treated as an exogenous parameter. By assumption, any excess supply of savings, $S_t - K_t$, is invested in government bonds. ω_{0t} is the wage rate of nonqualified labour, ω_{1t} is the wage rate of qualified labour, φ_t is the tuition fee, and R_{t+1} is the return earned on savings. All these prices and costs are after tax and subsidy. For each t there are six first-order conditions

$$U_{C_0t} = \lambda_{0t}, \quad U_{C_1t} = \lambda_{1t}, \qquad (18)$$

$$\omega_{0t}\, H_{t-1}\, U_{C_0t} = -U_{L_0t}, \quad \omega_{1t}\, G(E_t)H_{t-1}\, U_{C_1t} = -U_{L_1t}, \qquad (19)$$

$$\omega_{1t}L_{1t}\, G_t^{'}\, U_{C_1t} = (\varphi_t + \omega_{0t})\, U_{C_0t}, \quad R_{t+1} = \lambda_{0t} / \lambda_{1t}. \qquad (20)$$

They are constraints in the planner's optimal taxation problem we are about to set up. In the primal approach to optimal taxation these conditions are used to substitute for the four relative prices $\omega_{0t}, \omega_{1t}, \varphi_t, R_{t+1}$, and the two Lagrange multipliers $\lambda_{0t}, \lambda_{1t}$. After substituting, the lifetime budget constraint derived from (17a,b) can be written as

$$\sum_{i=0}^{1}[C_{it}U_{C_it} + L_{it}U_{L_it}] = \eta_t\, L_{1t}U_{L_1t}. \qquad (\tilde{\lambda}_t\beta^t) \qquad (21)$$

The condition (21) assumes the role of an *implementability constraint* in the planner's second-best problem. Because

$$-\eta_t \frac{L_{1t}U_{L_1t}}{U_{C_0t}} \underset{(19),(20)}{=} (\varphi_t + \omega_{0t})\, E_t H_{t-1}, \qquad (22)$$

the right-hand side of (21) can be interpreted as the private cost of education. As it turns out, the marginal increase in H_t is of particular significance when characterizing second-best policies. Let us call the marginal increase the *private marginal cost of human capital*. The formal definition is

$$PMC_t^{HC} \equiv -\frac{d}{dH_t}[\ \eta_t\ \frac{L_{1t}U_{L_1t}}{U_{C_0t}}] = -\frac{L_{1t}U_{L_1t}}{U_{C_0t}}\frac{dE_t}{dH_t}\frac{d}{dE_t}\eta(E_t)$$

$$= -\frac{L_{1t}U_{L_1t}}{U_{C_0t}}\frac{\eta_t^{'}}{G_t^{'}H_{t-1}} = -\frac{L_{1t}U_{L_1t}}{U_{C_0t}}\frac{1}{H_t}\frac{E_t}{\eta_t}\frac{d\eta_t}{dE_t}\ . \tag{23}$$

The private marginal cost is obviously increasing in the elasticity of the elasticity of $G(E_t)$. If the elasticity $\eta_t = \eta(E_t)$ is constant, $PMC_t^{HC}=0$ results. If the elasticity is however strictly increasing, PMC_t^{HC} is positive.

The planner maximizes the sum of discounted lifetime utilities (3) in $C_{0t}, C_{1t}, L_{0t}, L_{1t}, E_t, H_t$, and K_t (t=0,1,...) subject to the implementability constraint (21), the human capital accumulation equation (1), and the resource constraint (2). In a fully-fledged description of the planner's maximization one would have to include the first-order conditions of profit maximization. However, these conditions can be used to substitute for the endogenous factor prices before taxes and subsidies. Hence, they are not constraining the planner. The solutions are second best in the sense that they have to fulfil the implementability constraint in addition to the first-best constraints (1) and (2). If lump-sum taxes were available, the planner could ignore (21). Inclusion of (21) in the set of constraints implies that the planner is restricted in the choice of policy instruments. The restriction is however not an arbitrary one. Quite to the contrary, implicit in the derivation of (21) is the assumption that the planner is not constrained in setting consumer prices $\omega_{0t}, \omega_{1t}, \varphi_t$, and R_{t+1}. This means in particular that labour income can be taxed at different rates over an individual's life cycle. If such differentiation is ruled out by assumption, the planner has to respect an additional constraint, which may have strong implications for the design of optimal taxation. See Erosa and Gervais (2002) for a discussion of this point in an OLG model without endogenous education.

To solve the planner's problem set

$$W_t \equiv U_t + \tilde{\lambda}_t\ \{\sum_{i=0}^{1}[C_{it}U_{C_it} + L_{it}U_{L_it}] - \eta_t\ L_{1t}U_{L_1t}\}\ . \tag{24}$$

The first-order conditions are as follows:

$$\frac{\partial}{\partial C_{0t}},\ \frac{\partial}{\partial L_{0t}}:\quad W_{C_0t} = \alpha_t = -\frac{W_{L_0t}}{F_{L_0t} H_{t-1}}, \tag{25}$$

$$\frac{\partial}{\partial C_{1t}},\ \frac{\partial}{\partial L_{1t}}:\quad W_{C_1t} = \alpha_{t+1}\,\beta = -\frac{W_{L_1t}}{F_{L_1t+1} H_t}, \tag{26}$$

$$\frac{\partial}{\partial K_t}:\quad \alpha_{t+1}\beta\,[\,F_{Kt+1} + 1 - \delta_K\,] = \alpha_t\,, \tag{27}$$

$$\frac{\partial}{\partial E_t}:\quad \mu_t\, G_t^{'}\, H_{t-1} = \tilde{\lambda}_t\, \eta_t^{'}\, L_{1t} U_{L_1t} + \alpha_t\,(\,f + F_{L_0t}\,)\, H_{t-1} \;\Rightarrow$$

$$\frac{\mu_t}{\alpha_t} \underset{(23)}{=} \frac{f + F_{L_0t}}{G_t^{'}} - \frac{\tilde{\lambda}_t}{\alpha_t}\, U_{C_0t}\, PMC_t^{HC}\,, \tag{28}$$

$$\frac{\partial}{\partial H_t}:\quad \alpha_{t+1}\,\beta\,[\,F_{L_1t+1} L_{1t} + F_{L_0t+1}\cdot(\,L_{0t+1} - E_{t+1}\,) - \; f\,E_{t+1}\,] + \mu_{t+1}\,\beta\, G_{t+1} = \mu_t\,. \tag{29}$$

We wish to derive characterizations of second-best policy with regard to saving, education, and labour. We start with saving. As has been shown by Atkinson and Stiglitz (1972), Sandmo (1974), Atkeson, Chari and Kehoe (1999), and others, it is efficient not to distort saving if utility is weakly separable between consumption and non-leisure and is homothetic in consumption, $U = U(V(C_0, C_1), L_0, L_1)$ with a linear homogeneous function *V*. The utility functions defined in (13) and (14) are examples of weakly separable and homothetic functions. Weak separability and homotheticity implies

$$\frac{W_{C_i}}{U_{C_i}} = 1 + \tilde{\lambda}\,\{1 + \sum_{j=0}^{1}[C_j \frac{U_{C_jC_i}}{U_{C_i}} + L_j \frac{U_{L_jC_i}}{U_{C_i}}] - \eta\, L_1 \frac{U_{L_1C_i}}{U_{C_i}}\}$$

$$= 1 + \tilde{\lambda}\,\{1 + V\frac{U_{VV}}{U_V} + \sum_{j=0}^{1} L_j \frac{U_{VL_j}}{U_V} - \eta\, L_1 \frac{U_{VL_1}}{U_V}\} \;=\; \text{constant in } i=0,1. \tag{30}$$

Relying on (25) – (27) and (30) this implies

$$F_{Kt+1} + 1 - \delta_K = \frac{\alpha_t}{\alpha_{t+1}\beta} = \frac{W_{C_0t}}{W_{C_1t}} = \frac{U_{C_0t}}{U_{C_1t}}\,. \tag{31}$$

This has to be interpreted as saying that it is optimal from the planner's perspective to equate the marginal rate of return to capital with the private marginal rate of substitution in consumption.

Proposition 1: If behaviour is selfish and if utility is weakly separable between consumption and non-leisure and homothetic in consumption, it is second best not to distort saving.

We turn next to education. We first prove that it is efficient not to distort human capital accumulation if the investment function *G* is isoelastic. We do so by relying on (27)–(29), which are the first-order conditions with respect to K_t, E_t, and H_t. By making use of (27) and (28), (29) can be written as

$$[F_{L_1t+1}L_{1t} + F_{L_0t+1}\cdot(L_{0t+1} - E_{t+1}) - f\,E_{t+1}] + [\frac{f + F_{L_0t+1}}{G'_{t+1}} - \frac{\tilde{\lambda}_{t+1}}{\alpha_{t+1}} U_{C_0t+1}\, PMC^{HC}_{t+1}]G_{t+1}$$

$$= [\frac{f + F_{L_0t}}{G'_t} - \frac{\tilde{\lambda}_t}{\alpha_t} U_{C_0t}\, PMC^{HC}_t][F_{Kt+1} + 1 - \delta_K]. \tag{32}$$

Obviously, (32) equals (8) whenever $PMC^{HC}_{t+1} = PMC^{HC}_t = 0$ which is the case if $\eta(E_t)$ is constant.

Proposition 2: Assume selfish behaviour. It is second best not to distort education if the human capital investment function *G*(*E*) is isoelastic.

Proposition 2 is a dynamic version of the *education efficiency proposition*, well known from static tax analysis (Jacobs and Bovenberg, 2008; Bovenberg and Jacobs, 2005). An intuitive explanation is the following. The planner cares about two objectives. One objective is to minimize the efficiency loss resulting from distorted choices of consumption and leisure. The other objective is to minimize losses in the rent income generated by education. In general, these two minimizations are not separable, so that the planner has to trade off. Separability is only ensured if the human capital investment function is isoelastic. If this is the case and if the set of policy instruments is sufficiently rich, it is efficient not to distort education and to minimize the efficiency loss resulting from distorted choices of consumption and leisure. According to Proposition 2 this result extends to the dynamic framework and it does not explicitly rely on the utility specifications (13) and (14). Things are different if the private marginal cost of human capital is positive.

To study this case set

$$\Delta_t \equiv \frac{\tilde{\lambda}_t}{\alpha_t} U_{C_0 t}\; PMC_t^{HC} \cdot (F_{Kt+1} + 1 - \delta_K) - \frac{\tilde{\lambda}_{t+1}}{\alpha_{t+1}} U_{C_0 t+1}\; PMC_{t+1}^{HC} \cdot G_{t+1}. \tag{33}$$

With this definition (32) can be written as

$$\Delta_t = \frac{f + F_{L_0 t}}{G_t^{'}}\; (F_{Kt+1} + 1 - \delta_K) - \frac{f + F_{L_0 t+1}}{G_{t+1}^{'}}\, G_{t+1}$$

$$- F_{L_1 t+1} L_{1t} - [\, F_{L_0 t+1} \cdot (L_{0t+1} - E_{t+1}) - f\, E_{t+1}]. \tag{34}$$

Comparison of (34) and (8) reveals that Δ_t is the efficient wedge between the social cost and the social benefit of investing in human capital in period *t* instead of postponing the investment by one period. A positive wedge stands for subsidizing relative to the first best. *A priori* the sign of Δ_t is indeterminate. This is different if (33) is evaluated at a balanced growth path. By definition, balanced growth means that the non-leisure choices $L_{0t} = L_0, L_{1t} = L_1$, and $E_t = E$ are constant in *t* while consumption, output, and both types of capital all grow at the common gross rate *G*=*G*(*E*), so that we have $H_{t-1} = G^t H_{-1}$, $K_{t-1} = G^t K_{-1}$, $C_{it} = G^t C_{i0} \equiv G^t C_i$. At balanced growth $F_{Kt+1} = F_K$, $G_{t+1} = G$ in *t*. Because the utility functions are as specified in (13) and (14), the other variables entering (33) take on the following values:

(i) $U_{C_0 t} = G^{(d-1)t}\, U_{C_0 0} \equiv G^{(d-1)t}\, U_{C_0}$.

(ii) $$PMC_t^{HC} \underset{(23)}{=} -\frac{L_{1t} U_{L_1 t}}{U_{C_0 t}} \frac{1}{G'(E_t) H_{t-1}} \eta'(E_t) = -\frac{L_1 U_{L_1 0} G^{dt}}{U_{C_0 0} G^{(d-1)t}} \frac{1}{G'(E) H_{-1} G^t} \eta'(E)$$

$$= -\frac{L_1 U_{L_1}}{U_{C_0}} \frac{1}{G' H_{-1}} \eta' = PMC_0^{HC} \equiv PMC^{HC}.$$

Because *U* is homogeneous of degree *d* in consumption, *W* is likewise homogeneous of degree *d* in consumption. As a result, the growth factor G^t cancels out in equation (25): $W_{C_0 t} = -\dfrac{W_{L_0 t}}{F_{L_0 t} H_{t-1}}$. After cancelling out, the only variable carrying an index *t* in this equation is the Lagrange multiplier $\tilde{\lambda}_t$. Hence

(iii) $\tilde{\lambda}_t = \tilde{\lambda}$, and *a fortiori*

(iv) $\alpha_t = W_{C_0 t} = G^{(d-1)t}\, W_{C_0 0} \equiv G^{(d-1)t}\, W_{C_0}$ and $\dfrac{U_{C_0 t}}{\alpha_t} = \dfrac{G^{(d-1)t} U_{C_0}}{G^{(d-1)t} W_{C_0}} = \dfrac{U_{C_0}}{W_{C_0}}$.

Eventually, setting $R \equiv F_K + 1 - \delta_K$, (33) can be written as

$$\Delta = \tilde{\lambda} \frac{U_{C_0}}{W_{C_0}} \cdot PMC^{HC} \cdot (R - G). \tag{35}$$

Interpret $\tilde{\lambda} U_{C_0} / W_{C_0}$ as the social cost associated with the implementability constraint. This factor is positive if the implementability constraint is binding, $\tilde{\lambda}$>0, which is the case if the non-availability of lump-sum taxes is a binding constraint.[2] In this sense the factor measures the cost resulting from the non-availability of lump-sum taxes. PMC^{HC} is the private marginal cost of human capital, which is positive by assumption and increasing in η'. Finally, $R - G$ is the growth gap, which by (15) must be positive as well. Hence Δ is the product of three positive factors.

Proposition 3: Assume selfish behaviour, and *U* to satisfy (13) or (14). At balanced growth it is second best to subsidize education relative to the first best if the private marginal cost of human capital, PMC^{HC}, is positive. The strength of positive distortion increases in (i) the private marginal cost of human capital, (ii) the growth gap, and (iii) the cost resulting from the non-availability of lump-sum taxes.

This is a remarkable result, for reasons explained before. It is rather evident, and has been noted before, that the *laissez-faire* level of education is inefficient from the first-best perspective. Without government intervention, selfish individuals externalize the positive effect of own education on descendent generations' welfare. Not so evident is the result that human capital accumulation should be distorted along balanced growth while capital accumulation should not be distorted, subject to appropriately chosen utility functions. The

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[2] We abstain from proving in detail that the Lagrange multiplier is positive. Jones et al. (1997, p. 109) do this for a maximization which comes close to the present one. The intuition is the following. Paying generation *t* some positive lump-sum income would show up on the right-hand side of (21). The Lagrange multiplier must be positive if increasing such a lump-sum income can be shown to have a negative effect on the planner's objective function. The effect is indeed negative, because such a lump-sum transfer must be paid at the expense of government funds, which are generated by distortive taxes. Although the government budget constraint is not modelled explicitly, it has to be respected. This follows from Walras's law. In summary, the non-availability of lump-sum taxes is the reason why $\tilde{\lambda}$ is positive.

sign of the efficient distortion is even less obvious. Note that any revenue needed to subsidize the cost of tuition has to be raised by distortionary labour taxes. With the intuition of Lipsey and Lancaster (1956/57) in mind, one could have hypothesized that it is second best to give negative incentives for human capital accumulation relative to the first best if labour has to be taxed. The contrary, however, is true. The key assumption is the strictly increasing elasticity of the human capital investment function with respect to education. If the elasticity is strictly increasing, the private marginal cost of human capital is positive. With a positive private marginal cost of human capital it is second best to subsidize education. This has been shown before by Richter (2009) to hold in static analysis, and it is shown here to extend to the dynamic framework. The need to subsidize increases in the factors listed in Proposition 3. In particular, it increases in the elasticity of the human capital investment function's elasticity.

We finally turn to the study of labour taxation. Of particular interest is the efficient taxation of nonqualified labour relative to qualified labour. As the definition of W_t in (24) is structurally asymmetric in L_{0t} and L_{1t}, one may easily conjecture that qualified and nonqualified labour should be taxed differently. To make a clear case for differentiated taxation and to obtain clear-cut results, we focus on balanced growth and specific utility functions. Thus we assume

$$U \equiv V(C_0, C_1) - \sum_{i=0}^{1} D_i(L_i) \tag{36}$$

where V which is homogenous. In this particular case the first-order condition (25) implies:

$$W_{L_0} + F_{L_0} H_{-1} W_{C_0} = 0. \Leftrightarrow$$

$$(1+\tilde{\lambda} d)[U_{L_0} + F_{L_0} H_{-1} U_{C_0}] = \tilde{\lambda}[L_0 D_0''(L_0) + (1-d) D_0'(L_0)] \,. \tag{37}$$

Similarly, (26) implies

$$W_{L_1} + F_{L_1} G H_{-1} W_{C_1} = 0 \Leftrightarrow$$

$$(1+\tilde{\lambda} d)[U_{L_1} + F_{L_1} G H_{-1} U_{C_1}] = \tilde{\lambda}[(1-\eta) L_1 D_1''(L_1) + (1-d-\eta) D_1'(L_1)] \,. \tag{38}$$

Denote by $\nu_i \equiv L_i U_{L_i L_i} / U_{L_i}$ >0 the elasticity of marginal utility of leisure in life-period i, and define tax rates τ_i by setting $(1-\tau_0) F_{L_0} H_{-1} = -U_{L_0} / U_{C_0}$, $(1-\tau_1) F_{L_1} G H_{-1} \equiv -U_{L_1} / U_{C_1}$ $\Leftrightarrow$ $(1-\tau_i) F_{L_i} = \omega_i$. Dividing (38) through by (37) gives us

$$\frac{\tau_1/(1-\tau_1)}{\tau_0/(1-\tau_0)} = \frac{(1-\eta)\nu_1 - \eta + 1 - d}{\nu_0 + 1 - d} \,. \tag{39}$$

For $\eta = 0$ and d=1, (39) is the familiar (*inverse*) *elasticity rule*. According to this rule, wage taxes τ_i should increase in ν_i. If utility were quasi-linear, the ν_i would be the inverse of the wage elasticity of labour supply in life-period *i*. Hence taxes would have to vary inversely with the wage elasticities rendering the rule its name. The rule is extended by (39) to allow for endogenous education. The effect of education is to reduce the tax on qualified labour relative to the tax on nonqualified labour. The deviation from the elasticity rule increases in the elasticity of the human capital investment function, η. See Richter (2009), who derives (39) with d=1 for the static framework. It has to be noted that the given interpretation of (39) assumes a positive numerator. Such positivity is only ensured if the convexity of D_1 (as measured by ν_1) and/or the concavity of *V* (as measured by $1-d$) is sufficiently strong to compensate for the lack of concavity of the human capital accumulation equation (1). This lack of concavity is measured by η, and positivity of the numerator requires η to be less than $(\nu_1 + 1 - d)/(\nu_1 + 1)$.

Proposition 4: Assume selfish behaviour, and *U* to satisfy (36). On a balanced growth path it is then second best to tax labour according to the elasticity rule (39). The effect of endogenous education is to reduce the tax on qualified labour relative to the tax on nonqualified labour.

5. Efficient and effective subsidization of education

As mentioned in the introduction, OECD data suggest that various countries effectively subsidize education while others effectively tax education. Before substantiating such a statement one has to clarify the underlying notion of effective subsidization and its relation to efficient subsidization.

In the recent publication of 2009 the OECD reports estimates of the private and public net present values for individuals obtaining tertiary education as part of initial education in 2005. In present notation the private net present value is

$$NPV_{priv} \equiv \omega_1 L_1 G H_{-1} U_{C_1} / U_{C_0} - (\varphi + \omega_0) E H_{-1}$$

$$\underset{(20)}{=} (\varphi+\omega_0)[\frac{G}{EG'}-1]EH_{-1} = \frac{1-\eta}{\eta}(\varphi+\omega_0)EH_{-1}.$$

For the sake of brevity, the time index t is dropped. The public net present value is the difference between the social and the private net present values where the social value

$$NPV_{soc} \equiv F_{L_1}L_1GH_{-1}/[F_K+1-\delta_K] - (f+F_{L_0})EH_{-1}$$

captures only the return to education accruing to the investing generation. Denote by

$$PRR \equiv \frac{NPV_{priv}}{(\varphi+\omega_0)EH_{-1}} = \frac{1-\eta}{\eta},$$

$$SRR \equiv \frac{NPV_{soc}}{(f+F_{L_0})EH_{-1}} = \frac{F_{L_1}L_1G}{[F_K+1-\delta_K](f+F_{L_0})E} - 1$$

the *private rate of return* and the *social rate of return*, respectively. Our suggestion is to speak of effective subsidization only to the extent that the private rate exceeds the social rate. Hence denote by

$$s \equiv \frac{PRR-SRR}{PRR} \tag{40}$$

the *effective rate of subsidization*. The efficient value s_{eff} of this rate is determined by

$$(1-\eta)s_{eff} \underset{def}{=} 1-\frac{\eta F_{L_1}L_1G}{[F_K+1-\delta_K](f+F_{L_0})E} = \frac{[F_K+1-\delta_K](f+F_{L_0})-G'F_{L_1}L_1}{[F_K+1-\delta_K](f+F_{L_0})}$$

$$\underset{(10),(34)}{=} \frac{\Delta+MEB}{(F_K+1-\delta_K)(f+F_{L_0})/G'} \tag{41}$$

where $(f+F_{L_0})/G'$ is the social marginal cost of human capital and $MEB = F_{L_0}L_0 + (F_{L_0}+f)E(1-\eta)/\eta$ the marginal external benefit as specified by (9) and (10). With Δ and MEB, s_{eff} is positive as well. Equation (41) confirms the view that there are two reasons for effective subsidization of education. One is the need to internalize the intergenerational externality and the other is the need to compensate for distortionary labour taxation. Just for the sake of illustration we report the empirical values of s for men as they can be computed by means of the data published by OECD (2009, tables A8.2 and A8.4). Positive values for s are obtained in case of TUR (.47), POL (.34), ESP (.22), POR (.20), AUT (.19), CAN (.18), NOR (.10), ITA (.09), and HUN (.04). Negative values are obtained for SWE (-.03), KOR (-.05), DEN (-.05), FIN (-.06), CZE (-.14), USA (-.16), NZL (-.20), GER (-.20), IRL (-.20), FRA (-

.32), BEL (-.32), and AUS (-.40). Such extreme differences in effective rates and even more the opposing signs clearly raise questions. A deeper analysis however has to remain the object of future research. The numbers are only reported to illustrate the empirical relevance of the theoretical investigation undertaken in this paper.

6. Optimal taxation in the OLG model with altruistic individuals

The perfectly altruistic individual is assumed to maximize $\tilde{U}_t \equiv U(C_{0t}, C_{1t}, L_{0t}, L_{1t}) + \beta \tilde{U}_{t+1}$, which by recursive substitution amounts to maximizing the sum of discounted lifetime utilities (3) in $C_{0t}, C_{1t}, L_{0t}, L_{1t}, E_t, H_t$, and K_t (t=0,1, ...). This objective is maximized subject to the human capital accumulation constraint (1) and the dynasty's budget constraint,

$$\sum_{t=0}^{\infty}[\pi_{t+1}\omega_{1t}L_{1t}H_t + \pi_t\omega_{0t}(L_{0t} - E_t)H_{t-1}]$$

$$= \sum_{t=0}^{\infty}[\pi_t C_{0t} + \pi_{t+1}C_{1t} + \pi_t\varphi_t E_t H_{t-1} + (\pi_t - R_{t+1}\pi_{t+1})K_t] \qquad (\lambda). \qquad (42)$$

The price and cost variables have the same meaning as before. The first-order conditions are (t=0,1, ...)

$$\beta^t U_{C_0t} = \lambda\pi_t, \;\; \beta^t\, U_{C_1t} = \lambda\pi_{t+1}, \;\; \omega_{0t}\, H_{t-1}\, U_{C_0t} = -U_{L_0t}, \;\; \omega_{1t}H_t\, U_{C_1t} = -U_{L_1t}, \qquad (43)$$

$$\mu_t G_t' = (\varphi_t + \omega_{0t})\, U_{C_0t}, \quad R_{t+1} = \pi_t / \pi_{t+1}, \qquad (44)$$

$$\lambda\pi_{t+1}[\,\omega_{1t}L_{1t} + \omega_{0t+1}(L_{0t+1} - E_{t+1}) - \varphi_{t+1}\, E_{t+1}\,] = \beta^t\, \mu_t - \beta^{t+1}\, G_{t+1}\, \mu_{t+1}. \qquad (45)$$

The last condition implies

$$\lambda\sum_{t=0}^{\infty}\pi_{t+1}[\omega_{1t}L_{1t} + \omega_{0t+1}\,(L_{0t+1} - E_{t+1}) - \varphi_{t+1}E_{t+1}]H_t \underset{(44)}{=} \sum_{t=0}^{\infty}[\beta^t\mu_t H_t - \beta^{t+1}\mu_{t+1}H_{t+1}]$$

$$= \mu_0 H_0 \underset{(44)}{=} \frac{\varphi_0 + \omega_{00}}{G_0'} U_{C_0 0} H_0. \qquad (46)$$

Multiplying the budget constraint (42) through by λ and using (43), (44), and (46) to substitute for $\lambda\pi_t, \lambda\pi_{t+1}, \omega_{0t}, \omega_{1t}$, and R_{t+1} in (42) yields the implementability constraint

$$\sum_{t=0}^{\infty}\beta^t\sum_{i=0}^{1}C_{it}U_{C_it} = B \qquad (\tilde{\lambda}) \qquad (47)$$

with

$$B \equiv \{[\omega_{00}(L_{00}-E_0)-\varphi_0 E_0]H_{-1}+\frac{\varphi_0+\omega_{00}}{G_0^{'}}H_0\}U_{C_0 0} \ .$$

Similarly, (43) and (44) can be used to substitute for $\lambda\pi_{t+1}, \omega_{0t+1}, \omega_{1t}$, and μ_t in (45), which leaves us with (t=0,1, ...)

$$-L_{1t}U_{L_1 t}-\beta\,[(L_{0t+1}-E_{t+1})U_{L_0 t+1}+\varphi_{t+1}\,E_{t+1}\,U_{C_0 t+1}\,H_t\,]$$

$$\underset{(45)}{=}\ \{\mu_t-\beta\ G_{t+1}\ \mu_{t+1}\}\,H_t\ =\ \mu_t\ H_t-\beta\ \mu_{t+1}\ H_{t+1}$$

$$\underset{(44)}{=}\ [\varphi_t\ U_{C_0 t}-U_{L_0 t}\ \frac{1}{H_{t-1}}]\frac{H_t}{G_t^{'}}-\beta\,[\varphi_{t+1}\ U_{C_0 t+1}-U_{L_0 t+1}\ \frac{1}{H_t}]\frac{H_{t+1}}{G_{t+1}^{'}}\ . \qquad (\gamma_t\beta^t)\ (48)$$

The planner maximizes the sum of discounted lifetime utilities (3) in $C_{0t}, C_{1t}L_{0t}, L_{1t}, E_t, H_t, K_t$, and φ_t (t=0,1, ...) subject to the resource constraint (2), the accumulation constraint (1), and the behavioural constraints (47) and (48). It is important to note that the cost of tuition φ_{t+1} (t=0,1,...) only appears explicitly in the condition (48). By contrast, the planner's objective function and the constraints (1), (2), and (47) are independent of φ_{t+1}. The condition (48) can therefore be treated as a relationship by which the "free" policy variable φ_{t+1} can be determined. This solution procedure is feasible because the coefficient of φ_{t+1} in (48) does not vanish. The coefficient equals $\beta U_{C_0 t+1}[E_{t+1}H_t-\frac{H_{t+1}}{G_{t+1}^{'}}]\ =\ \beta U_{C_0 t+1}E_{t+1}H_t(1-\frac{1}{\eta_{t+1}})\ <0$. Hence the planner's problem is equivalent to the simplified version in which (3) is maximized in $C_{0t}, C_{1t}L_{0t}, L_{1t}, E_t, H_t, K_t$ (t=0,1, ...), and φ_0 subject to (1), (2), and (47). The same kind of solution procedure has been applied by Atkeson, Chari, and Kehoe (1999) and others before.

We first study those first-order conditions of the simplified planner's problem which are associated with variables which do not enter the implementability constraint (47) or which drop out when making particular assumptions. The optimization with respect to those variables is not affected by (47) and should therefore remain undistorted.

Proposition 5: Assume altruistic behaviour. Then it is second best not to distort education for all generations except the first.

Proposition 6: Assume altruistic behaviour and the utility function to be weakly separable between consumption and non-leisure and homothetic in consumption. Then it is second best not to distort the accumulation of capital for all generations except the first.

Proposition 7: Assume altruistic behaviour and $U = V(C_0, C_1) \cdot \Lambda(L_0, L_1)$ with homogeneous *V*. Then it is second best to tax qualified and nonqualified labour uniformly. This holds for all generations except the first.

The proof of Proposition 5 is rather straightforward. Just note that the variables E_t, H_t, K_t (*t*>0) do not enter the implementability constraint. Taking partial derivatives of the Lagrange function with respect to these variables and substituting for the Lagrange multipliers μ_t, α_t yields the efficiency condition (8) for *t*>0. The proof of Proposition 6 parallels the one of Proposition 1 and is therefore skipped. The proof of Proposition 7 is as follows. Set

$$W_t \equiv U_t + \tilde{\lambda} \sum_{i=0}^{1} C_{it} U_{C_i t} \quad .$$

If *V* is homogeneous of degree $d \neq 0$, then $W_{L_i t} = (1 + \tilde{\lambda} d) U_{L_i t}$ (*i*=0,1). Hence the social and the private marginal rates of intertemporal substitution in non-leisure are equal,

$$\frac{W_{L_1 t}}{W_{L_0 t}} = \frac{U_{L_1 t}}{U_{L_0 t}} \underset{(43)}{=} \frac{\omega_{1t} H_t U_{C_1 t}}{\omega_{0t} H_{t-1} U_{C_0 t}} . \tag{49}$$

The equation (49) is equally obtained if *V* is homogeneous of degree zero in the sense of (14) with $D \equiv 0$. Taking partial derivatives of the Lagrange function with respect to K_t, L_{0t}, L_{1t}, yields (27) and $W_{L_0 t} = -\alpha_t F_{L_0 t} H_{t-1}$, $W_{L_1 t} = -\alpha_{t+1} \beta F_{L_1 t+1} H_t$ (*t*>0). Therefore, (49) $\Leftrightarrow$

$$\frac{\alpha_{t+1} \beta F_{L_1 t+1} H_t}{\alpha_t F_{L_0 t} H_{t-1}} = \frac{\omega_{1t} H_t U_{C_1 t}}{\omega_{0t} H_{t-1} U_{C_0 t}} \Leftrightarrow \frac{F_{L_1 t+1}}{F_{L_0 t}} \underset{(27)}{=} [F_{Kt+1} + 1 - \delta_K] \frac{\omega_{1t}}{\omega_{0t}} \frac{U_{C_1 t}}{U_{C_0 t}} .$$

Define tax rates τ_{it} by setting $1 - \tau_{1t} \equiv \omega_{1t} / F_{L_1 t+1}$, $1 - \tau_{0t} \equiv \omega_{0t} / F_{L_0 t}$. Hence, (49) $\Leftrightarrow$

$$\frac{1 - \tau_{0t}}{1 - \tau_{1t}} = [F_{Kt+1} + 1 - \delta_K] \frac{U_{C_1 t}}{U_{C_0 t}} . \tag{50}$$

The utility functions assumed to hold for Proposition 7 are weakly separable between consumption and non-leisure and homothetic in consumption. Hence Proposition 6 applies and it is second best not to distort saving. As a result, the right-hand of (50) equals one and labour tax rates are independent of age.

Proposition 6 is just what one would expect in view of the literature. Proposition 7 is less obvious, and it even allows us to qualify the main result of Erosa and Gervais (2002) stating that it is generally optimal to differentiate labour taxes across the individual life cycle. The intuitive explanation for this result is that labour supplied in the second life period differs from labour supplied in the first period. While Proposition 4 confirms the result of Erosa and Gervais on assuming selfish individuals, Proposition 7 does not. Obviously, in the present framework altruism removes the need to employ age-dependent labour taxes for descendent generations. Age-dependent labour taxes would then be used only as a correcting device if it were second best to distort saving. This becomes clearer when considering utility functions which are additive separable between consumption and non-leisure, $U = V(C_0, C_1) + \Lambda(L_0, L_1)$. In this case (50) would equally hold but the right-hand side of (50) would only equal one in the optimum if *V* were homothetic. This is a noteworthy qualification of Erosa et al. (2002). Above, it is derived from the equality of the social and private marginal rates of intertemporal substitution in non-leisure, (49). For this equality to hold we have to assume not only altruism, but also a sufficiently rich set of policy instruments. In particular, the planner must be able to choose ω_{it} independently of φ_t. In other words, the planner must be able to optimize the taxation of labour separately from the subsidization of education.

Finally, Proposition 5 is interesting in that it is much stronger than the results derived in the Chamley-Judd literature. It holds for arbitrary utility functions, and it does not assume balanced growth. That is, Proposition 5 is logically stronger than Propositions 6 and 7. And it is also much stronger than Proposition 2, which assumes the human capital investment function to be isoelastic. By contrast, Proposition 5 even holds for functions *G* which fail to be isoelastic. All this strongly reminds one of the Production Efficiency Theorem of Diamond and Mirrlees (1971). According to this theorem the allocation of intermediate goods should not be distorted in second best if no lump-sum income accrues to the private sector. This is just what holds in the present model. Investment in human capital is modelled as an intermediate good in the sense that it does not affect the implementability constraint (47) for t>0. Furthermore, the only lump-sum income modelled is income earned by the parent generation living in period 0. On setting $\pi_0 = 1$, this income equals $F_{K0}K_{-1} + F_{L_1 0}L_{1-1}H_{-1}$ +

$(1-\delta_K)K_{-1}$. It does not show up in the dynasty's budget constraint (42). It must therefore be income accruing to the government budget. The Production Efficiency Theorem is applicable, and Proposition 5 can be considered to be a corollary.

The recommendation not to distort education is not easily translated into explicit tax and subsidy rates. The reason is that private incentives are affected by a whole set of tax and subsidy rates, which all must be optimally set. Just inspect the altruist's first-order condition (45) determining the optimal amount of human capital. After substituting for the Lagrange multipliers one obtains

$$\omega_{1t}L_{1t}+\omega_{0t+1}\,L_{0t+1}-(\,\omega_{0t+1}+\varphi_{t+1}\,)\,E_{t+1}\;=\;R_{t+1}\,\frac{\varphi_t+\omega_{0t}}{G_t^{'}}\;-G_{t+1}\,\frac{\varphi_{t+1}+\omega_{0t+1}}{G_{t+1}^{'}}\,. \qquad (51)$$

This condition reveals that the altruist's incentive to invest in human capital is not only affected by taxes on own labour income and the subsidy paid to the own cost of tuition. It is additionally affected by the tax on savings, by the next generation's tax on nonqualified labour, and finally by the subsidy paid to the next generation's cost of tuition. More can be said only after making specific assumptions. Just for the sake of illustration, assume $U=V(C_0,C_1)\cdot\Lambda(L_0,L_1)$ with homogeneous *V*. Hence Propositions 6 and 7 apply, and it is optimal not to tax saving, $R_{t+1}=F_{Kt+1}+1-\delta_K$, and to tax labour independently of age, $1-\tau_t\equiv\omega_{1t}/F_{L_1t+1}=\omega_{0t}/F_{L_0t}$ (*t*>0). Only if optimal wage taxes do neither differentiate across generations, $\tau_t=\tau$, can one infer that it is compatible with efficiency for the cost of tuition to be subsidized at the same rate as labour income is taxed, $\varphi=(1-\tau)f$. This follows immediately from comparing (51) with (8). If the mentioned assumptions do not hold, it is difficult to make definite statements about the efficient structural relationship between labour tax rates and education subsidy rates.

The government has to finance the exogenous cash flow of government expenditures A_t (*t*>0). If the amount of pure profit earned by the government is insufficient, distortionary taxes have to be employed to balance the budget. In this case, the implementability constraint (47) is binding, and it cannot be ruled out that it is efficient to distort the choice of education of generation 0. This raises the question of how to design optimal human capital policy for generation 0. As we are going to learn, the answer comes close to what has been shown to be efficient in the world of selfish individuals. More precisely, generation 0's education should not be distorted if the human capital investment function is isoelastic. If however the private marginal cost of human capital is positive, education should be positively distorted relative to the first best. To show this we maximize (3) subject to (1), (2), (47), and (48). Taking partial derivatives of the Lagrange function yields the following results after some tedious but straightforward manipulations have been made:

$$\frac{\partial}{\partial \varphi_0}: \quad \gamma_0 = -\tilde{\lambda}(1-\eta_0), \tag{52}$$

$$\frac{\partial}{\partial \varphi_1}: \quad \gamma_1 = \gamma_0(1-\eta_1), \tag{53}$$

$$\frac{\partial}{\partial E_0}: \quad \frac{\mu_0}{\alpha_0} \underset{(52)}{=} \frac{f+F_{L_0 0}}{G_0^{'}} - \frac{\tilde{\lambda}}{\alpha_0} U_{C_0 0} \frac{\varphi_0+\omega_{00}}{G_0^{'}}[1+\frac{E_0 G_0^{''}}{G_0^{'}}], \tag{54}$$

$$\frac{\partial}{\partial E_1}: \quad \frac{\mu_1}{\alpha_1} \underset{(52),(53)}{=} \frac{f+F_{L_0 1}}{G_1^{'}} - \frac{\tilde{\lambda}}{\alpha_1} U_{C_0 1} \frac{\varphi_1+\omega_{01}}{G_1^{'}}[1-\frac{E_0 G_0^{'}}{G_0}][1+\frac{E_1 G_1^{''}}{G_1^{'}}]. \tag{55}$$

The first-order condition with respect to K_0 is the same as (27) for t=0. By making use of (52)–(55) and (27) for t=0 we end up with

$$\frac{\partial}{\partial H_0}: \quad \Delta_0 = \frac{f+F_{L_0 0}}{G_0^{'}}(F_{K1}+1-\delta_K) - \frac{f+F_{L_0 1}}{G_1^{'}} G_1 - F_{L_1 1} L_{10} - [F_{L_0 1}\cdot(L_{01}-E_1) - f E_1], \tag{56}$$

where

$$\Delta_0 \equiv \frac{\tilde{\lambda}}{\alpha_0} U_{C_0 0}\cdot PMC_0^{HC}\cdot(F_{K1}+1-\delta_K) - \frac{\tilde{\lambda}(1-\eta_0)}{\alpha_1} U_{C_0 1}\cdot PMC_1^{HC}\cdot G_1 \tag{57}$$

and (t=0,1)

$$PMC_t^{HC} \equiv -\frac{L_{1t} U_{L_1 t}}{U_{C_0 t}} \frac{\eta_t^{'}}{G_t^{'} H_{t-1}} \underset{(22)}{=} \frac{\varphi_t+\omega_{0t}}{G_t^{'}} \frac{E_t \eta_t^{'}}{\eta_t} = \frac{\varphi_t+\omega_{0t}}{G_t^{'}}[1-\frac{E_t G_t^{'}}{G_t}+\frac{E_t G_t^{''}}{G_t^{'}}]. \tag{58}$$

The variables Δ_0 and PMC_t^{HC} are defined so that the parallels with (33) and (23) show up. As PMC_t^{HC} vanishes for isoelastic $G(E_t)$, we obtain

Proposition 8: Assume altruistic behaviour and the human capital investment function G to be isoelastic. Then it is second best not to distort the first generation's educational choice.

Proposition 8 is just the altruistic analogue to Proposition 2. It is a result that one could easily conjecture. Altruism goes beyond selfishness in internalizing efficiency effects. If it is second best not to distort education given that *G* is isoelastic and behaviour selfish, then it should all the more be second best not to distort education given that *G* is isoelastic and behaviour altruistic.

Things are less straightforward if the private marginal cost of human capital is positive. Without making further assumptions, it is difficult to sign Δ_0. However, we are able to derive a direct analogue to Proposition 3. More precisely, Δ_0 can be shown to be positive if the growth path is balanced and if utility is homogeneous in consumption. The assumption of balanced growth has the effect of neutralizing the impact of initialization.

The proof is only sketched. First note that $\omega_{0t} = \omega_0$ follows from (43). In a second step G^{dt} is shown to be a factor that cancels out of the constraint (48), so that φ_t and φ_{t+1} are the only remaining variables in (48) carrying an index *t*. The equation can then be used to solve for $\varphi_t = \varphi_{t+1} \equiv \varphi$. This is a feasible procedure, as the coefficient of φ does not vanish. Just note that after dividing through by G^{dt} the coefficient equals $\beta G^d U_{C_0 0}[EH - \frac{GH}{G'}] + U_{C_0 0}\frac{GH}{G'}$ $= U_{C_0 0}\, EH[\beta G^d + \frac{1}{\eta}(1-\beta G^d)]$. The condition of transversality, $\beta G^d < 1$, implies that the coefficient is positive. Plugging φ into (58) yields $PMC_t^{HC} = PMC^{HC}$. Assume $PMC^{HC} > 0$ and prove $\Delta_0 = \Delta > 0$ by inspecting (57) and by noting

$$\frac{\tilde{\lambda}}{\alpha_0} U_{C_0 0} \cdot (F_{K1} + 1 - \delta_K) > \frac{\tilde{\lambda}(1-\eta)}{\alpha_1} U_{C_0 1} \cdot G \underset{(27)}{\Leftrightarrow} U_{C_0 0} > \beta(1-\eta)\, G^{d-1}\, U_{C_0 0} \cdot G$$

$$\Leftrightarrow\ 1 > (1-\eta) \cdot \beta\, G^d .$$

The last inequality follows from $\eta < 1$ and, once more, from the condition of transversality.

Proposition 9: Assume altruistic behaviour, and *U* to be homogeneous in consumption. At balanced growth it is second best to subsidize the first generation's educational choice relative to the first best if the private marginal cost of human capital, PMC^{HC}, is positive.

It would be nice if one could similarly characterize second-best policy with regard to the first generation's choice of labour and saving. However, analogues to Propositions 1 and 4 seem not to hold. In particular, it seems that the first generation's saving decision is systematically distorted. The reason is the factor $U_{C_0 0}$ entering the right-hand side of (47). This factor implies a lack of symmetry when taking partial derivatives of *B* with respect to C_{i0} (i=0,1). As a result it is second best to distort saving.

The parallelism between Propositions 9 and 3 allows us to tell a unifying story for selfish and altruistic individuals. Altruism well reduces the need to subsidize education relative to *laissez-faire*. Altruism also implies that the second-best tax policy for descendent generations is more like the first-best policy. The accumulation of human capital should remain undistorted, and – if utility functions are well selected – labour taxes need not be differentiated across the individual life cycle. The short-run policy recommendations for altruism, however, parallel the long-run recommendations for selfishness. Labour has to be taxed, and – given that the elasticity of the human capital investment function is strictly increasing – education should be subsidized relative to the first best. Whether saving should be taxed is not a matter of selfishness or altruism. With regard to descendent generations it primarily depends on assumptions made with regard to the marginal rate of intertemporal substitution in consumption.

7. Summary

The accumulation of human capital may suffer from all sorts of potential inefficiencies. Most of them have simply been assumed away in the present study. Such a procedure is, no doubt, debatable. Critical is the ignoring of possible causes of capital market or policy failure. Even more critical is the ignoring of individual heterogeneity and informational asymmetry. Still, the procedure is defended with the objective of studying efficient taxation in Ramsey's tradition. More precisely, this paper aims at bridging the gap that separates the two strands of Ramsey tax analyses which exist for the finite and the infinite planning horizon. Our knowledge of efficient human capital policy in Ramsey's tradition is largely shaped by incompatible results derived for the different horizons. The results derived for the infinite horizon suggest that education should not be distorted in the long run, just as saving should not be distorted in the long run. Hence it seems as if efficient policy does not differentiate between human and nonhuman capital. By way of contrast, the results in finite horizon strongly suggest differentiated policies. Whether education should be distorted or not appears to depend primarily on how education affects the individual's earning potential. More precisely, only if the earnings function is weakly separable in qualified labour supply and education and if the elasticity with respect to the latter is constant, should the choice of education be not distorted by second-best policy (Jacobs and Bovenberg, 2008). By way of contrast, the question of whether saving should be distorted or not primarily has to be answered with regard to the taxpayer's preferences. More precisely, saving should not be taxed if the taxpayer's utility is weakly separable between consumption and labour/non-leisure and homothetic in consumption (Atkinson and Stiglitz, 1972).

The model filling the gap between finite and infinite Ramsey tax analyses is one with overlapping generations. The present paper studies second-best policy for education, saving, and labour in such an overlapping-generations model with endogenous growth. There have been earlier attempts to do the same. In view of the present study, two attempts deserve to be cited more than others. These are by Atkeson, Chari, and Kehoe (1999) on one side and by Wigger (2002, Sec. 3.4) and Docquier et al. (2007) on the other side. The most conspicuous differences to the present study are the following ones. The focus of the present study is on human capital accumulation, while the focus of Atkeson et al. is on nonhuman capital. Their paper contains extensions to both endogenous education and overlapping generations, but it fails to integrate the two. The work of Wigger and Docquier et al. does integrate them. However, it does not allow for endogenous labour supply and second-best taxation. The authors assume the availability of non-distortionary tax instruments, which the present study does not. In a sense, the present paper starts where Atkeson et al. and where Wigger and Docquier et al. stop. It goes beyond Atkeson et al. by integrating endogenous education and overlapping generations, and it goes beyond Wigger and Docquier et al. by endogenizing labour supply and by doing second-best tax analysis.

The present paper studies two possible reasons for allocational inefficiency. One is the non-availability of non-distortionary tax instruments. The other is individual selfishness. Taxpayers are assumed to externalize the positive effect that their human capital investments have on the productivity of descendent generations. As stressed by Wigger and by Docquier et al., selfishness is the source of an intergenerational externality. It gives reason to subsidize education relative to *laissez-faire*. Such subsidization, however, requires government revenues. In the framework studied by Wigger and by Docquier et al. it is efficient to subsidize education up to the first-best level where marginal social costs equal marginal social returns. The result assumes the availability of non-distortionary tax instruments. The key assumption of the present study, however, is that no tax instruments are available that would allow the government to raise the revenue needed to subsidize education without creating distortions. As it turns out, it is still second best not to distort education if only the human capital investment function is isoelastic. This result can be considered to be the dynamic version of the education efficiency proposition known from static Ramsey analysis.

It is, however, argued that an isoelastic investment function has the unappealing implication that all human capital accumulated by past generations melts down to zero if only one generation stops investing. If, by way of contrast, human capital depreciates just by some fraction and if the investment function's elasticity is strictly increasing, then investment incentives should overshoot the first best at balanced growth. In other words, it is efficient in the long run to combine positive tax wedges in the labour market with an effective subsidy wedge for education. The need to subsidize is shown to increase in (i) the private marginal cost of human capital, (ii) the cost resulting from the non-availability of lump-sum taxes, and (iii) the growth gap. Furthermore, it turns out to be efficient to tax labour such that qualified labour is less distorted than nonqualified labour.

If taxpayers are altruists with respect to descendent generations, one clear reason for government intervention does not apply. The effect that education has on descendent generations' productivity is internalized by altruists. The only remaining inefficiency modelled in this paper is caused by the need to employ distortionary taxes for financing government expenditures. As it turns out, all generations except the first one should still be given non-distorted incentives for accumulating human and nonhuman capital. Furthermore, labour should be taxed uniformly across the individual life cycle when utility is homogeneous in consumption and multiplicative in the sub-utilities of consumption and non-leisure. This result allows us to qualify the main result of Erosa and Gervais (2002), who stress the need to employ age-dependent labour taxes in second best. In the present framework, however, altruism has the effect of implying equality of the social and private marginal rates of intertemporal substitution in non-leisure. The optimality of uniform labour taxation is an immediate though intriguing corollary to this equality. In view of the Chamley-Judd literature, results suggesting non-distortionary taxation may not be too surprising.

Striking, however, is the strength of the result concerning human capital accumulation. While the other results on non-distortionary taxation require specific utility functions, the result on human capital accumulation holds without any comparable qualification. One only has to assume that no lump-sum income accrues to the private sector. It is argued that this result on efficient education policy is best interpreted as a corollary to the Production Efficiency Theorem of Diamond and Mirrlees (1971).

The results on non-distortionary taxation do not require removing all distortions. On the contrary, the labour supply of descendent generations will be distorted if the government has to finance exogenous government expenditures by relying on distortionary instruments. Nor do the results on non-distortionary taxation extend to the dynasty's first generation, indexed by zero in the present paper. A more precise characterization of optimal policy for generation 0 is difficult, as the specific features not only depend on the shape of the human capital investment function but also on initial values of key variables. As in the case with selfish individuals, it is efficient not to distort education if the investment function is isoelastic in education. If, however, the elasticity is strictly increasing and if the impact of initialization is suppressed by assuming balanced growth, it is second best to subsidize education relative to the first best. The reason is the same as the one given before in the scenario with selfish individuals. A strictly increasing elasticity of the investment function has the effect that it is second best to subsidize education in static analysis, and this effect extends to the dynamic framework. At balanced growth the need to subsidize increases in the derivative of the investment function's elasticity, and it is the stronger, the more binding the non-availability of lump-sum taxes is and the more deficient growth is.

The unifying bottom line for selfish and altruistic individuals is as follows. Altruism well reduces the need to subsidize education relative to *laissez faire*, and altruism also implies that descendent generations should be given non-distorted incentives for accumulating human capital. The short-run policy recommendations for altruism, however, agree with the long-run recommendations for selfishness. Labour has to be taxed, and – given that the elasticity of the human capital investment function is strictly increasing – education should be subsidized relative to the first best. Whether saving should be taxed is not a matter of selfishness or altruism. It primarily depends on assumptions made with regard to the marginal rate of intertemporal substitution in consumption.

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