ABSTRACT
This paper concerns optimal redistributive non-linear income taxation in an OLG model, where people care about their own consumption relative to (i) other people’s current consumption, (ii) own past consumption, and (iii) other people’s past consumption. We show that both (i) and (iii) affect the marginal income tax structure whereas (ii) does not. We also derive conditions under which atemporal and intertemporal consumption comparisons give rise to exactly the same tax policy responses. Based on available empirical estimates, comparisons with other people’s current and past consumption tend to substantially increase the optimal marginal labor income tax rates, while they may either increase or decrease the optimal marginal capital income tax rates.

Keywords: Optimal income taxation, asymmetric information, relative consumption, status, habit formation, positional goods.

JEL Classification: D62, H21, H23.
1. INTRODUCTION

A rapidly growing body of evidence suggests that people have positional preferences in the sense of deriving utility from their own consumption relative to that of others.\footnote{This includes evidence from happiness research (e.g., Easterlin, 2001; Blanchflower and Oswald, 2005; Ferrer-i-Carbonell, 2005; Luttmer, 2005; Clark and Senik, 2010) and questionnaire-based studies (e.g., Johansson-Stenman et al., 2002; Solnick and Hemenway, 2005; Carlsson et al., 2007). See also, e.g., Mamot (2004) and Daly and Wilson (2009) for evidence based on physiological studies, and Fliessbach et al. (2007) and Dohmen et al. (2011) for evidence based on brain science. Bowles and Park (2005) and Oh, Park and Bowles (2012) analyze variations in working hours between countries and over time, and find that social comparisons are important driving forces behind changes in work hours.} Alongside this development, a corresponding literature dealing with optimal policy responses to positional concerns has evolved, showing that such concerns may have a substantial effect on the incentive structure underlying public policy. Within the literature on optimal income taxation,\footnote{Other issues dealt with include public good provision (Ng, 1987; Aronsson and Johansson-Stenman, 2008; Wendner and Goulder, 2008; Wendner, forthcoming), social insurance (Abel, 2005), growth (Corneo and Jeanne, 1997, 2001; Wendner, 2010b, 2011), environmental externalities (Howarth, 1996, 2006; Brekke and Howarth, 2002; Wendner, 2005), stabilization policy (Ljungqvist and Uhlig, 2000) and tax evasion (Goerke, forthcoming). See also Frank (1999, 2005, 2007, 2008) for extensive and illuminating informal discussions of relative consumption concerns and how the society should deal with them.} it has for example been shown that social comparisons may motivate substantially larger and/or more redistributive income taxes than without such comparisons, see e.g. Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Tuomala (1990), Blomquist (1993), Ireland (2001), Aronsson and Johansson-Stenman (2008, 2010, forthcoming), Wendner and Goulder (2008) and Wendner (2010a).

Yet, almost all earlier studies on optimal policy responses to positional concerns that we are aware of assume that people only make “atemporal” consumption comparisons, by valuing their own current consumption relative to other people’s current consumption. A much more general approach has recently been presented by Rayo and Becker (2007). According to their evolutionary model, selfish genes would prefer that the humans they belong to are motivated by their own current consumption relative to (i) their own past consumption, (ii) other people’s current consumption, and (iii) other people’s past consumption. In the macroeconomic literature of dynamic consumption behavior, (i) corresponds to what is typically denoted habit formation (sometimes denoted internal habit formation), (ii)
corresponds to *keeping up with the Joneses*, while (iii) corresponds to *catching up with the Joneses* (sometimes denoted *external habit formation*). The present paper takes these three types of consumption comparisons as a point of departure in a study of optimal income taxation in a dynamic economy.

We develop and analyze an Overlapping Generations (OLG) model with endogenous labor supply and savings, where the consumers are concerned with their relative consumption, and where nonlinear taxes of labor income and capital income are used for purposes of correction and redistribution. A dynamic model allows us to explore intertemporal aspects of consumption comparisons, and provides a natural framework for studying capital income taxation. The latter is important not least due to the difficulties of explaining the widespread use of capital taxes with conventional public economics models. Earlier research shows that relative consumption concerns may motivate such taxes (Aronsson and Johansson-Stenman, 2010), and one might perhaps conjecture such concerns to be particularly important when the concept of relative consumption has more than one dimension, as we assume here.

The literature on optimal redistributive taxation under relative consumption concerns is scarce, and almost all earlier studies are based on static models. The only exception that we are aware of is Aronsson and Johansson-Stenman (2010), who analyze optimal nonlinear income taxation in a dynamic economy where each consumer compares his/her own current consumption with other people’s current consumption. Hence, their study neglects internal habit formation as well as the catching up with the Joneses type of comparison mentioned.

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3 The notion “keeping up with the Joneses” is unfortunately used with different meanings in the literature. It is either used to indicate social comparisons in the sense that my utility depends in part on my current consumption relative to your current consumption, as in our case, or it is used with more specific meanings, e.g., if you consume more now I will also consume more now. Similarly, the notion “catching up with the Joneses” may either, as here, simply mean that my utility today depends on my current consumption relative to your previous consumption, or it may reflect something more specific such that my consumption today increases with your previous consumption. No results in the present paper depend directly on the direction of people’s consumption and leisure adjustment in response to a change in the reference consumption.

4 The macroeconomics literature referred to above rarely analyzes the optimal policy responses to the externalities induced by relative consumption concerns. Ljungqvist and Uhlig (2000) and Gomez (2006) are two noteworthy exceptions.

above, and focuses solely on consumption comparisons based on keeping up with the Joneses preferences. The present paper, in contrast, addresses the implications of such atemporal comparisons for optimal income taxation simultaneously with the implications of relative consumption comparisons over time. Another study related to ours is Ljungqvist and Uhlig (2000), who consider optimal labor income taxation in a dynamic representative agent model where the consumer preference for relative consumption is driven by a catching up with the Joneses motive. We generalize their approach in several different ways by (1) considering a broader set of tax instruments, (2) analyzing redistribution policy alongside externality correction, and (3) allowing keeping up and catching up with the Joneses mechanisms to be operative simultaneously.

These extensions are important. In addition to the empirical evidence for between-people comparisons mentioned above, there is evidence suggesting that people also make comparisons with their own past consumption (e.g., Loewenstein and Sicherman, 1991; Frank and Hutchens, 1993); indeed, such comparisons were discussed already by Veblen (1899). It also makes intuitive sense that old people compare their own consumption with several different reference levels, including what they recall about their own and others’ consumption when they were young. Moreover, when growing up, most people are likely to receive information from parents and grandparents about the consumption (and other living conditions) characterizing earlier generations. The results from happiness studies have also documented that people’s happiness adapts to income changes, consistent with the idea that the reference income increases over time when actual income increases, see e.g. Stutzer (2004) and Di Tella et al. (2010). Specifically, Senik (2009) presents recent estimates regarding the importance of different kinds of comparisons over time, showing that subjective well-being is dependent on one’s own standard of living relative to both internal and external reference points. Such comparisons are also consistent with the empirical pattern of some financial puzzles, and (as mentioned) they are in line with recent research based on evolutionary models.

Section 2 presents the model and the outcome of private optimization. In Section 3, we present our optimal taxation findings in a first-best model, i.e., a model without asymmetric

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6 This includes various kinds of asset pricing puzzles, such as the equity premium puzzle; see, e.g., Abel (1990), Constantinides (1990), Campbell and Cochrane (1999), Chan and Kogan (2003), and Díaz et al. (2003).
information where the government can deal with its distributional objectives without any social costs. This provides a simple benchmark by which to compare our results with some earlier literature on optimal labor income taxation under positional concerns (which is based on such models). Our results show that relative comparisons with one’s own past consumption (internal habit formation) do not directly affect the policy rules for marginal income taxation (although they may, of course, influence the levels of marginal income tax rates). The intuition is that such comparisons are fully internalized at the individual level and do not generate any externalities. However, positional concerns governed by comparisons with other people’s current and past consumption give rise to externalities and will, therefore, also directly affect the incentive structure underlying marginal income taxation. We show that the appearance of positional concerns tends to increase the optimal marginal labor income tax rates, irrespective of whether individuals compare their own current consumption with other people’s current or past consumption. We also show how the marginal capital income tax rates are governed by differences in positional concerns over the individual lifecycle, where the relevant measure of reference consumption is again based on both the current and past consumption of others.

In general, positional concerns governed by other people’s past consumption give rise to much more complex policy responses than comparisons based on other people’s current consumption. This is so because consumption comparisons over time give rise to an intertemporal chain reaction with welfare effects in the entire future, whereas comparisons with other people’s current consumption only lead to “atemporal externalities.” We can nevertheless derive strong results for a natural benchmark case in which the concerns for relative consumption are constant over time, implying that relative consumption comparisons over time (based on the catching up with the Joneses preferences) give rise to exactly the same marginal tax rate responses as comparisons with other people’s current consumption (based on the keeping up with the Joneses preferences).

In practice, informational limitations are likely to prevent governments from using lump-sum taxation as a basis for redistribution. Therefore, in Section 4, we introduce asymmetric information between the government and the private sector with respect to individual ability (worker-productivity), where the public decision problem is described by a variant of the two-type optimal income tax model originally developed by Stern (1982) and Stiglitz (1982). Although simple, the two-type model provides a powerful framework for analyzing
externality correction and redistribution simultaneously. In such a second-best framework, tax distortions are the outcome of an optimal choice made by the government, subject to informational limitations, and not due to any arbitrary restrictions on the tax instruments (such as linearity) or the necessity to raise revenue per se. Therefore, our approach enables us to capture that the optimal income tax responses to positional concerns may involve purely corrective as well as redistributive elements.

In a second-best setting where ability is private information, there is also another policy incentive involved beyond positional externalities: the government may relax the incentive constraint by exploiting differences in positional concerns across ability-types. Our results show that while the externality-correcting mechanism unambiguously works to increase the marginal labor income tax rates, independently of whether individuals compare their own current consumption with other people’s current or past consumption (or use a combination of these two reference measures), the direction of the mechanism through the self-selection effect is ambiguous. We both present general optimal taxation results and derive sufficient conditions for when the overall net effect of positional concerns works to increase the marginal labor income tax rates. Section 5 illustrates with a particular Cobb-Douglas functional form and shows, based on parameter estimates from the literature, that positional preferences of both the keeping up with the Joneses and the catching up with the Joneses types substantially increase the optimal marginal labor income tax rates. Section 6 summarizes and concludes the paper; proofs are presented in the appendix.

2. CONSUMERS, FIRMS, AND MARKET EQUILIBRIUM

We start this section by describing the OLG framework and people’s preferences, followed by the definition of some useful measures of the extent to which people care about relative consumption. We then present the individual optimality conditions for labor supply and savings, followed by the corresponding profit maximization conditions for the firms and the condition for market equilibrium.

2.1 The OLG Framework and Positional Preferences
Consider an OLG model where each individual lives for two periods and works during the first but not during the second. Since each individual only works during the first period of life, there is no evolution of productivity over time for a single individual, as in Kocherlakota (2005), although we allow for technical progress (discussed subsequently) that makes labor productivity increase over time. Individuals differ in ability, as measured by the before-tax wage rate. The number of individuals of ability-type $i$ in generation $t$, i.e., who were born at the beginning of period $t$, is denoted $n_i^t$. Each such individual derives utility from his/her consumption when young, $c_i^t$, consumption when old, $x_{i,t}^t$, and use of leisure when young, $z_i^t$, given by a time endowment, $H$, less hours of work, $l_i^t$ (when old, all available time is leisure). For further use, we define the average consumption in the economy as a whole in period $t$ as $\bar{c}_t = \left[ \sum_i n_i^t c_i^t + \sum_i n_i^t x_{i,t}^t \right] / \sum_i \left[ n_i^t + n_i^{t-1} \right]$.

People also care about their own consumption relative to that of others. In accordance with the bulk of earlier comparable literature, we focus on difference comparisons, where relative consumption is defined by the difference between the individual’s own consumption and a measure of reference consumption. The appropriate measure of reference consumption at the individual level is, of course, an empirical question; yet, as indicated above, there is very little information available. Our approach is to follow the recent contribution by Rayo and Becker (2007), who argue in the context of an evolutionary model of happiness that the reference point of an individual might be determined by three components: (i) other people’s current consumption, (ii) his/her own past consumption, and (iii) other people’s past consumption. In terms of our model, we interpret these three components such that people care about three different kinds of relative consumption: their own current consumption

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7 We follow earlier comparable literature in assuming that people do not care about their relative leisure; see Arrow and Dasgupta (2009) and Aronsson and Johansson-Stenman (2013) for analysis of the case where also relative leisure matters. See also Aronsson and Johansson-Stenman (in press) for a study of the case where leisure has a displaying role in making relative consumption more visible.

relative to \((i)\) the current average consumption such that \(c^i_t - \bar{c}_t\) and \(x^i_{t+1} - \bar{c}_{t+1}\) are the corresponding measures of relative consumption when young and when old, respectively; \((ii)\) their own consumption one period earlier, i.e., \(x^i_{t+1} - c^i_t\); and \((iii)\) the average consumption one period earlier such that \(c^i_t - \bar{c}_{t-1}\) and \(x^i_{t+1} - \bar{c}_{t+1}\) are the corresponding measures of relative consumption when young and when old, respectively.

The utility function of ability-type \(i\) born in the beginning of period \(t\) can then be written as

\[
U^i_t = V^i_t(c^i_t, z^i_t, x^i_{t+1}, c^i_t - \bar{c}_t, x^i_{t+1} - \bar{c}_{t+1}, x^i_{t+1} - c^i_t, c^i_t - \bar{c}_{t-1}, x^i_{t+1} - \bar{c}_{t+1})
\]

\[
= v^i_t(c^i_t, z^i_t, x^i_{t+1}, c^i_t - \bar{c}_t, x^i_{t+1} - \bar{c}_{t+1}, c^i_t - \bar{c}_{t-1}, x^i_{t+1} - \bar{c}_{t+1}).
\]

\[
= u^i_t(c^i_t, z^i_t, x^i_{t+1}, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}).
\]

The functions \(V^i_t(\cdot)\) and \(v^i_t(\cdot)\) are increasing in each argument, implying that \(u^i_t(\cdot)\) is decreasing in \(\bar{c}_{t-1}, \bar{c}_t, \) and \(\bar{c}_{t+1}\) and increasing in the other arguments; the three functions are also assumed to be twice continuously differentiable in their respective arguments and strictly concave.\(^9\) The first line of equation (1) is expressed in terms of the five consumption differences described above, as well as in terms of leisure and private consumption when young and when old, respectively. However, since \(c^i_t\) and \(x^i_{t+1}\) are decision variables of the individual, we can without loss of generality rewrite this utility formulation as the "reduced form" function on the second line, although the partial derivatives will now have a more complex interpretation than on the first line.\(^{10}\) For instance, the partial derivative of \(v^i_t(\cdot)\) with respect to \(c^i_t\) reflects both the direct utility effect of increased absolute consumption when young and the (presumably negative) utility effect due to lower relative consumption when old compared to when young.\(^{11}\) Therefore, all analytical results derived in a model where

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\(^9\) We do not attempt to explain why people care about relative consumption. While we share the view that signaling of some attractive characteristics constitutes a likely reason as to why people care about their own consumption relative to that of others (Ireland, 2001), we follow the considerably simpler modeling strategy of assuming that people’s preferences directly depend on relative consumption.

\(^{10}\) This approach is innocuous as long as the government commits to its future tax policy, as we will assume below. Without such commitment, internal habit formation may influence the optimal tax structure in a second best setting; see Guo and Krause (2011).

\(^{11}\) On the second line, the effect of \(x^i_{t+1} - c^i_t\) on utility is hence embedded in the effects of \(c^i_t\) and \(x^i_{t+1}\).
individuals do not compare their own current and past consumption will continue to hold also in the case where people make such comparisons. Intuitively, people will internalize such comparisons perfectly.

The third line contains the most general utility formulation and resembles a classic externality problem. Here, we do not specify anything regarding the structure of the social comparisons, beyond that others’ consumption levels cause negative externalities. As will be demonstrated, for some results we do not need any stronger assumptions regarding the preference structure. Yet, we need the more restrictive utility formulation based on the function \( v'_i(\cdot) \), where we specify that people care about additive comparisons, to establish a relationship between, on the one hand, the optimal tax policy and, on the other, the degree to which the utility gain from higher consumption is associated with increased relative consumption. The definition of such measures is the issue to which we turn next.

### 2.2 The Degree of Current versus Intertemporal Consumption Positionality

Since much of the subsequent analysis is focused on relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By using \( \Delta_i^c = c_i^t - \bar{c}_i^t \), \( \Delta_{t+1}^x = x_{t+1}^i - \bar{c}_{t+1}^i \), \( \Omega_i^c = c_i^t - \bar{c}_i^t \), and \( \Omega_{t+1}^x = x_{t+1}^i - \bar{c}_i^t \) as short notations for the measures of relative consumption in the function \( v'_i(\cdot) \) in equation (1), we can define the degree of current consumption positionality when young and when old, respectively, as

\[
\alpha_{i,t}^c = \frac{v'_{i,t,x}}{v'_{i,t,x} + v'_{i,t,c} + v'_{i,c}},
\]

\[
\alpha_{i,t+1}^x = \frac{v'_{i,t+1,x}}{v'_{i,t+1,x} + v'_{i,t+1,c} + v'_{i+1,c}},
\]

where the subindex indicates partial derivative, i.e., \( v'_{i,x} = \partial v'_i(\cdot) / \partial c_i^t \), and similarly for the other terms. The variables \( \alpha_{i,t}^c \) and \( \alpha_{i,t+1}^x \) are interpretable as the fraction of the overall utility increase from an additional dollar spent when young in period \( t \) and old in period \( t+1 \), respectively, that is due to the increased consumption relative to other people’s current
consumption (measured by the average consumption in the same period). By analogy, we can define the degree of intertemporal consumption positionality when young and when old, respectively, as

\[
\beta_{t}^{c,e} = \frac{\nu_{t,\Delta}^{c,e}}{\nu_{t,\Delta}^{c,e} + \nu_{t,\Delta}^{e,t} + \nu_{t,\epsilon}^{c,e}},
\]

\[
\beta_{t+1}^{c,x} = \frac{\nu_{t,\Delta}^{c,x}}{\nu_{t,\Delta}^{c,x} + \nu_{t,\Delta}^{e,t} + \nu_{t,x}^{c,x}}.
\]

Equations (3a) and (3b) have interpretations similar to equations (2a) and (2b); yet with the obvious modification of reflecting consumption comparisons over time: the variables \(\beta_{t}^{c,e}\) and \(\beta_{t+1}^{c,x}\) measure the fraction of the overall utility increase from an additional dollar spent in period \(t\) and \(t+1\) (i.e., when young and when old), respectively, that is due to the increased consumption relative to other people’s past consumption. We assume that \(0 < \alpha_{t}^{c,e}, \alpha_{t+1}^{c,x}, \beta_{t}^{c,e}, \beta_{t+1}^{c,x} < 1\) for all \(t\).

Let us next define the notions of the average degree of current consumption positionality and the average degree of intertemporal consumption positionality, which are given by

\[
\bar{\alpha}_{t} = \sum_{i} \alpha_{i}^{c,x} \frac{n_{t-1}^{i}}{N_{t}} + \sum_{i} \alpha_{i}^{c,e} \frac{n_{t}^{i}}{N_{t}} \in (0,1),
\]

\[
\bar{\beta}_{t} = \sum_{i} \beta_{i}^{c,x} \frac{n_{t-1}^{i}}{N_{t}} + \sum_{i} \beta_{i}^{c,e} \frac{n_{t}^{i}}{N_{t}} \in (0,1),
\]

respectively, where \(N_{t} = \sum_{i} [n_{t-1}^{i} + n_{t}^{i}]\). Note that both \(\bar{\alpha}_{t}\) and \(\bar{\beta}_{t}\) are measured among those alive in period \(t\).

2.3 The optimality conditions for individuals and firms and market equilibrium

The individual budget constraints are given by

\[
w_{t}^{t} - T_{t}(w_{t}^{t}) - s_{t}^{i} = c_{t}^{i},
\]
\[(5b) \quad s^i_t(1 + r_{t+1}) - \Phi_{t+1}(s^i_t r_{t+1}) = x^i_{t+1},\]

where \(w^i_t\) is the before-tax wage rate, implying that \(w^i_t l^i_t\) is the before-tax labor income, \(s^i_t\) is savings, \(r_{t+1}\) is the market interest rate, and \(T_i(\cdot)\) and \(\Phi_{t+1}(\cdot)\) denote the payments of labor income and capital income taxes, respectively. Thus, consumption levels when young are given by gross labor income net of labor income taxes and savings, whereas consumption levels when old are given by the sum of savings and capital income net of capital income taxes.

Although the measures of reference consumption are endogenous in our model, we assume that each individual treats them as exogenous, which is the conventional equilibrium assumption in models with externalities. To be more specific, and with reference to equation (1) above, this means that ability-type \(i\) of generation \(t\) treats \(\bar{c}_{t-1}, \bar{c}_{t}\), and \(\bar{c}_{t+1}\) as exogenous. The first-order conditions for the hours of work and savings can then be written as

\[(6) \quad u^i_{t,c} w^i_t \left[1 - T_i(w^i_t l^i_t)\right] - u^i_{t,z} = 0,\]
\[(7) \quad -u^i_{t,c} + u^i_{t,z} \left[1 + r_{t+1} \left(1 - \Phi_{t+1}(s^i_t r_{t+1})\right)\right] = 0,\]

where \(u^i_{t,c} = \partial u^i_t / \partial c^i_t\), \(u^i_{t,z} = \partial u^i_t / \partial z^i_t\), and \(u^i_{t,z} = \partial u^i_t / \partial x^i_t\), while \(T_i(w^i_t l^i_t)\) and \(\Phi_{t+1}(s^i_t r_{t+1})\) are the marginal labor income tax rate and marginal capital income tax rate, respectively.

The production sector consists of identical competitive firms producing a homogenous good with constant returns to scale; the number of firms is normalized to one for notational convenience. Following Aronsson and Johansson-Stenman (2010), the production function is given by

\[(8) \quad F(L^i_t, K^i_t; t) = g \left( \sum \theta^i L^i_t, K^i_t; t \right),\]

where \(L^i_t = n^i_t l^i_t\) is the total number of hours of work supplied by ability-type \(i\) in period \(t\), \(L^i_t = [L^i_1, L^i_2, \ldots]\) is a vector whose elements reflect the total number of work hours by each
ability-type, \( K_i \) is the capital stock in period \( t \), and \( \theta^i \) (for all \( i \)) is a positive constant. The direct time-dependency implies that we allow for exogenous technological change. Note that the functional form assumption implicit in \( g(\cdot) \) means that the relative wage rates, i.e., \( w_i^j / w_i^k = \theta^j / \theta^k \) for all \( j \) and \( k \), are fixed. This assumption simplifies the calculations in Section 4 below, where the public decision problem is affected by asymmetric information between the government and the private sector; it is not important for the policy incentives created by relative consumption concerns (which are the major concerns here).

The firm obeys the necessary optimality conditions

\[
F_L(L_i, K_i; t) = \frac{\partial g}{\partial (\sum_i \theta^i L_i)} \theta^i = w_i^j \quad \text{for all } I,
\]

\[
F_K(L_i, K_i; t) = \frac{\partial g}{\partial K_i} = r_i.
\]

### 3. FIRST-BEST TAXATION

In this section, we begin by specifying the social objective function, which is maximized subject to the overall resource constraint. Then we present the results in terms of optimal taxation, starting with the case where the average positionality degrees, population size, and the interest rate in the economy are all constant over time. This framework is technically convenient and allows us to characterize the basic policy incentives in an intuitive way. It also facilitates comparison with earlier studies. Finally, we present the optimal taxation results for the more general model where these assumptions are relaxed.

#### 3.1 The Social Decision Problem

We assume that the government faces a general social welfare function as follows:

\[
W = W(n_0^1 U_0^1, n_0^2 U_0^2, \ldots, n_1^1 U_1^1, n_1^2 U_1^2, \ldots),
\]
which is increasing in each argument. Since the optimum conditions are expressed for any such social welfare function, they are necessary optimum conditions for a Pareto efficient allocation.

Note that nonlinear taxation of labor and capital income allows the government to implement any desired combination of consumption, savings, and work hours for each individual. Therefore, we follow earlier literature on optimal nonlinear income taxation in dynamic economies by formulating the public decision problem as a direct decision problem in terms of private consumption, work hours, and the capital stock – an approach which will be particularly convenient in Section 4 below where we introduce asymmetric information between the government and the private sector. The marginal income tax rates implicit in the optimal resource allocation can then be derived by combining the social first-order conditions with the first-order conditions characterizing the private sector.

The resource constraint for the economy as a whole can be written as

\[ F(L_i, K_i; t) + K_i - \sum_j \left[ n_i^i c_j^i + n_i^i x_i^j \right] - K_{s+1} = 0, \]

such that production equals consumption plus investment at each instant. The Lagrangean associated with the public decision problem then becomes

\[ \mathcal{L} = W + \sum_i \gamma_i \left[ F(L_i, K_i; t) + K_i - \sum_j \left[ n_i^i c_j^i + n_i^i x_i^j \right] - K_{s+1} \right]. \]

Note finally that the government is assumed to treat the measures of reference consumption as endogenous, i.e., the government recognizes and incorporates into its decision problem how the measures of reference consumption change in response to public policy.

We are concerned with the optimal tax policy implemented for any ability-type \( i \) of any generation \( t \), which is based on the social first-order conditions for \( l_i^t, c_i^t, x_i^{t+1}, \) and \( K_{s+1} \). These social first-order conditions are given by (for all \( i \) and \( t \))

\[ \text{See, e.g., Brett (1997) and Aronsson and Johansson-Stenman (2010).} \]
where we have used \( w^j = F_k(L, K; t) \) and \( r_i = F_k(L, K; t) \) from the first-order conditions of the firm. For notational convenience, we have written equations (14b) and (14c) such that the right-hand side contains the derivative of the Lagrangean with respect to the appropriate measure of reference consumption, i.e., the measure of reference consumption that is affected by a change in \( c_i \) and \( x_{t+1} \), respectively. The derivative \( W_{t} = \partial\mathcal{L}/\partial c_i \) will be referred to as the positionality effect in period \( t \) and plays a crucial role in the subsequent analysis of optimal taxation.

### 3.2 Optimal First-Best Taxation with Constant Average Positionality Degrees

Let us start with some less general assumptions than those outlined above, and more specifically that:

i. The population size is constant over time, such that \( N_t = N \) for all \( t \).

ii. The average (current and intertemporal) degrees of positionality are constant over time, such that \( \bar{\alpha}_t = \bar{\alpha} \) and \( \bar{\beta}_t = \bar{\beta} \) for all \( t \).

iii. The interest rate is constant over time, such that \( r_t = r \) for all \( t \).

Although these assumptions of course reflect limitations, similar (or stronger) assumptions are typically made in the “catching up with the Joneses” literature, where one often assumes specific functional forms; see, e.g., Campbell and Cochrane (1999) and Díaz et al. (2003). It should also be noted that the model is still general enough to reflect different preferences between types, including different positionality degrees.
Assumption iii above implies from equation (14d) that \( \gamma_{t+k} = \gamma_t / [1 + r]^k \). This special case, which makes it easy to relate our results to findings in earlier literature, is either interpretable in terms of a steady state\(^{13}\) – provided that a steady state exists – or may follow as a consequence of adding additional assumptions about the preferences and technology (see Section 5 below).

Now, let us define \emph{the average degree of time-inclusive consumption positionality}, in present value terms, as follows:

\[
\bar{\rho} = \alpha + \frac{\beta}{1 + r}.
\]

Intuitively, \( \bar{\rho} \) reflects the overall social loss in a first-best world of consuming an additional dollar today. The first term, \( \alpha \), reflects the part of this loss that will occur through current consumption positionality, whereas the second term, \( \beta / [1 + r] \), reflects the loss due to intertemporal consumption positionality. The reason why the latter loss is discounted is, of course, that it will occur in the next period. We can then derive

\[
W_t = \frac{\partial \mathcal{L}}{\partial c_t} = -N \gamma_t \frac{\alpha + \beta / [1 + r]}{1 - \alpha - \beta / [1 + r]} = -N \gamma_t \frac{\rho}{1 - \rho}.
\]

Using equations (14a)-(14d) and (15) we are ready to present the following benchmark results:

**PROPOSITION 1.** Under assumptions i-iii, the first-best marginal labor and capital income tax rates, respectively, can be written as (for \( i=1,2 \))

\[
(16) \quad T'_i(w^i_l^i) = \bar{\rho} > 0 \]

\[
(17) \quad \Phi'_{i,t1}(s'_i r_{i+1}) = 0.
\]

\(^{13}\) This requires that the preferences and technology do not change over time, and that the economy approaches a stationary equilibrium in which \( l'_i, c'_i, x'_i \) (for all \( i \)), and \( K'_i \) all remain constant over time.
To interpret Proposition 1, note first that since the government may reach its distributional objectives by using lump-sum taxes/subsidies, there is no distributional reason for using distortionary taxation. As such, the non-zero marginal labor income tax rates are solely due to the externalities that positional concerns give rise to. By analogy to earlier comparable literature (see the introduction), we find that positional concerns motivate positive marginal labor income tax rates. The novelty here is that the marginal labor income tax rate is given by the average degree of time-inclusive positionality; there are no additional effects associated with the two separate components $\alpha$ and $\beta/(1+r)$. This has a strong implication: current positionality (reflecting the keeping up with the Joneses motive) and intertemporal positionality (reflecting the catching up with the Joneses motive) affect the marginal labor income tax rates in exactly the same way. Note also that equation (16) nests corresponding results derived by, e.g., Dupor and Liu (2003) (where $\bar{p} = \alpha$ and $T_i'(w_i' l_i') = \alpha$) and Ljungqvist and Uhlig (2000) (where $\bar{p} = \beta/(1+r)$ and $T_i'(w_i' l_i') = \beta/(1+r)$).

Second, capital income taxation plays no corrective role here. The intuition is that the positionality effects in periods $t$ and $t+1$ cancel out. Therefore, as long as assumptions i-iii hold, the marginal capital income tax rates are zero in the first best, irrespective of whether the consumers have keeping up or catching up with the Joneses preferences (or a mix of them).

### 3.3 Optimal First-Best Taxation with Time-Varying Degrees of Positionality

As indicated above, the optimal tax policy depends on the consumption externalities, which in turn depend on the positionality effect. As long as we made a couple of simplifying assumptions (i-iii), both the positionality effect and the optimal marginal income tax formulas became simple and easily interpretable. Not surprisingly, when relaxing these assumptions, things become more complex. The positionality effect is now given by:

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14 Ljungqvist and Uhlig use a model with infinite time horizons to describe the consumer behavior, and assume that the reference consumption relevant today is a geometric average of the consumption in earlier periods. The result discussed above refers to their special case where the reference consumption is given by the average consumption in the previous period. The study by Dupor and Liu is based on a static model.
Equation (18) comprises three distinct (negative) effects. The first term on the right-hand side, \(-N_i \gamma_i \tilde{\alpha}_i / (1 - \tilde{\alpha}_i) < 0\), measures the direct welfare loss in period \(t\) of an increase in \(\tilde{c}_i\); the intuition is that an increase in \(\tilde{c}_i\), ceteris paribus, leads to lower utility for all consumers via the argument \(c_i' - \tilde{c}_i\) in the function \(v_i'()\) in equation (1). This effect depends on the average degree of current positionality. The analogous second term, \(-N_{i+1} \gamma_{i+1} \tilde{\beta}_{i+1} \tilde{\alpha}_{i+1} / (1 - \tilde{\alpha}_{i+1}) < 0\), is interpretable as the direct welfare loss in period \(t+1\) of an increase in \(\tilde{c}_{i+1}\), and the underlying mechanism here is that \(\tilde{c}_i\) affects individual utility negatively via the argument \(\chi_{i+1}' - \tilde{c}_i\) in the function \(v_i'()\). This effect captures the intertemporal consumption externality and depends on the average degree of intertemporal positionality.

Finally, the third term on the right-hand side of equation (18) reflects an intertemporal chain reaction. The intuition is that the intertemporal aspect of the consumption comparisons, i.e., that other people’s past consumption affects utility, means that the welfare effects of changes in the reference consumption are not time-separable (as they would be without intertemporal comparisons). This is so because a change in the reference consumption today means behavioral adjustments in the future, which in turn influence the reference consumption relevant for future generations. In the absence of relative comparisons over time, i.e., if \(\tilde{\beta}_i = 0\) for all \(t\), the right-hand side of equation (18) collapses to \(-N_i \gamma_i \tilde{\alpha}_i / (1 - \tilde{\alpha}_i)\).

Before presenting the optimal marginal income tax rates, define the marginal rate of substitution between leisure and private consumption in period \(t\), \(MRS_{u_i,c_i} = u_{i,c_i}' / u_{i,l} '\), and the marginal rate of substitution between consumption in periods \(t\) and \(t+1\), \(MRS_{c_{t+1},c_t} = u_{t,c_t}' / u_{t+1,c_t}'\), for ability-type \(i\) of generation \(t\), and let \(1 + \eta_{t+1} = N_{t+1} / N_t\) denote the population growth factor. The optimal tax structure can then be characterized as follows:
PROPOSITION 2. The first-best marginal income tax rates faced by ability-type $i$ can be written as

\begin{equation}
T_i'(w_i' l_i') = - \frac{MRS_{z,c}^{i,t}}{\gamma_i w_i' N_i} W_i > 0
\end{equation}

\begin{equation}
\Phi_{t+1}(s_i^t r_{t+1}) = \frac{1}{\gamma_{t+1} r_{t+1} N_i} \left[ W_{t+1} - \frac{MRS_{z,c}^{i,t}}{1 + \eta_{t+1}} W_{t+1} \right].
\end{equation}

Again, since the government may reach its distributional objectives by using lump-sum taxes/subsidies, there is no distributional reason for using distortionary taxation, and the non-zero marginal labor income tax rates are therefore solely due to the negative externalities that positional concerns give rise to. Therefore, the marginal labor income tax rates are positive for all types. Yet, since the positionality effects become very complex in the case where the average positionality degrees vary over time, it is not possible to express the optimal marginal labor income tax rates in a simple way in terms of the degrees of positionality.

To interpret equation (20), note that the marginal capital income tax rates reflect a desired tradeoff for society between present and future consumption. As a consequence, the right-hand side of equation (20) is decomposable into two parts (in subsection 3.2, these two parts cancelled out due to assumptions $i$-$iii$). The basic intuition is that each individual generates positional externalities both when young and when old. Therefore, whether positional concerns lead to a higher or lower marginal capital income tax rate in period $t+1$ depends on the difference between the positionality effect in period $t$ and the discounted positionality effect in period $t+1$. Again, this result holds irrespective of whether the preferences for relative consumption are governed by a keeping up or catching up with the Joneses motive, or by a mix of them.

4. OPTIMAL SECOND-BEST TAXATION

In reality, governments are not likely to be able to redistribute on a lump-sum basis, due to that individual ability is private information. This may, in turn, have important implications for how the tax system should be used in response to positional concerns. In this section, we
will generalize the findings obtained in the previous section to the case where asymmetric information prevents redistribution through lump-sum taxes.

As we indicated above, the analysis in this section is based on a two-type model, which simplifies the calculations considerably compared to the continuum-type framework, while still being able to convey the main insights as to how and why externality correction interacts with redistribution policy under asymmetric information. Yet, it is rather straightforward to generalize our model to a multi-type formulation, but with little value added in terms of economic insights. In addition, the two-type model also enables us to compare our results with those presented in Aronsson and Johansson-Stenman (2010), which – to our knowledge – is the only earlier study on optimal second-best taxation in an intertemporal model where the consumers have positional preferences (yet of the keeping up with the Joneses type).

4.1 The Social Decision Problem

In each generation, therefore, there are two types of individuals, where the low-ability type (type 1) is less productive than the high-ability type (type 2). Following the convention in earlier literature on optimal nonlinear taxation, we assume that the government is able to observe income, that ability is private information, and that the government wants to redistribute from the high-ability to the low-ability type. To prevent the high-ability type from becoming a mimicker, we impose the following self-selection constraint:

\[
U_1^2 = u^2(c_1^2, c_2^2, x_{t+1}, c_t, c_{t+1}) \geq u^2(c_1^1, H - \phi l_1^1, x_{t+1}, c_{t-1}, c_t, c_{t+1}) = \hat{U}_1^2,
\]

which means that the high-ability type weakly prefers the allocation intended for him/her over the allocation intended for the low-ability type. The variable \( \phi = w_1^1 / w_2^1 = \theta^1 / \theta^2 \) denotes the wage ratio, which is a constant by the assumptions about the technology made earlier. The left-hand side of the weak inequality in (21) measures the utility faced by the high-ability type if revealing his/her true ability, while the right-hand side represents the utility of the high-ability mimicker, i.e., a high-ability type who chooses the same income-consumption points as the low-ability type. Although the mimicker enjoys the same consumption as the
low-ability type in each period, he/she reaches this consumption with less work-effort (due to the difference in productivity).\textsuperscript{15}

The social welfare function, the production technology and the resource constraint are the same as in Section 3. Therefore, the Lagrangean can be written as

\begin{equation}
\mathcal{L} = W + \sum_t \lambda_t \left[ U_t^2 - \hat{U}_t^2 \right] + \sum_t \gamma_t \left[ F(L_t, K_t; t) + K_t - \sum_{i=1}^2 [n_i^t c_i^t + n_i^t x_i^t] - K_{t+1} \right]
\end{equation}

where $\lambda_t$ is the Lagrange multiplier associated with the self-selection constraint faced by generation $t$. As above, the government treats the measures of reference consumption as endogenous.

Following the bulk of earlier literature on optimal nonlinear income taxation in dynamic economies, we assume that the government commits to its future tax policy.\textsuperscript{16} Let

$\hat{u}_t^2 = u_t^2(c_t^1, H - \phi l_t^1, x_{t+1}^1, \bar{c}_t, \bar{c}_{t+1})$ denote the utility of the mimicker based on the third utility formulation in equation (1). The direct decision-variables relevant for generation $t$ are $l_t^1, c_t^1, x_{t+1}^1, l_t^2, c_t^2, x_{t+1}^2$, and $K_{t+1}$, and the social first-order conditions are given by

\begin{align}
\frac{\partial W}{\partial (n_t^1 L_t^1)} n_t^1 c_t^1 + \phi \lambda_t \hat{u}_t^2 + \gamma_t n_t^1 W_t^1 &= 0, \\
\frac{\partial W}{\partial (n_t^1 L_t^1)} n_t^1 x_{t+1}^1 - \lambda_t \hat{u}_t^2 - \gamma_t n_t^1 W_{t+1} &= 0,
\end{align}

\textsuperscript{15} Given the set of available policy instruments assumed here, it is possible for the government to control the present and future consumption as well as the hours of work of each ability-type. As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen by the low-ability type on each tax function (both the labor income tax and the capital income tax), and thus consume the same amount as the low-ability type in both periods.

\textsuperscript{16} See Brett (1997), Aronsson et al. (2009), and Aronsson and Johansson-Stenman (2010). For recent studies of time-consistent optimal nonlinear income taxation without commitment, see Brett and Weymark (2008) and Guo and Krause (2011). The latter study shows that the policy incentives created by (internal) habit formation depend on whether or not the government commits to its future tax policy. See also Acemoglu et al. (2011) for a political economy model with taxation of capital and labor when self-interested politicians cannot commit to future policies.
\[
(23c) \quad \frac{\partial W}{\partial (n_t U_1^t)} n_t^1 u_{1,t}^1 - \lambda_t \hat{u}_{1,t}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} W_{t+1} = 0 ,
\]
\[
(23d) \quad - \left[ \frac{\partial W}{\partial (n_t U_2^t)} n_t^2 + \lambda_t \right] u_{1,t}^2 + \gamma_t n_t^2 w_t^2 = 0 ,
\]
\[
(23e) \quad \left[ \frac{\partial W}{\partial (n_t U_1^t)} n_t^2 + \lambda_t \right] u_{1,t}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} W_t = 0 ,
\]
\[
(23f) \quad \left[ \frac{\partial W}{\partial (n_t U_2^t)} n_t^2 + \lambda_t \right] u_{1,t}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_{t+1}} W_{t+1} = 0 ,
\]
\[
(23g) \quad \gamma_{t+1} [1 + r_{t+1}] - \gamma_t = 0 ,
\]

where (as before) \( u_{1,c}^t = \partial u_t^c / \partial c_t^1 \), \( u_{1,z}^t = \partial u_t^z / \partial c_t^1 \) and \( u_{1,x}^t = \partial u_t^x / \partial x_{t+1} \), while \( W_t = \partial \mathcal{L} / \partial \bar{v}_t \) denotes the positionality effect in period \( t \), i.e., the partial welfare effect of an increase in \( \bar{v}_t \).

For further use, we define the following measures of differences in the degree of current and intertemporal positionality, respectively, between the mimicker and the low-ability type in period \( t \):

\[
(24a) \quad \alpha_t^d = \frac{\lambda_t \hat{u}_{1,t}^2}{\gamma_t N_t} \left[ \hat{\alpha}_{t,c}^2 - \alpha_{t,c}^1 \right] + \frac{\lambda_t \hat{u}_{1,z}^2}{\gamma_t N_t} \left[ \hat{\alpha}_{t,z}^2 - \alpha_{t,z}^1 \right]
\]
\[
(24b) \quad \beta_t^d = \frac{\lambda_t \hat{u}_{1,x}^2}{\gamma_t N_t} \left[ \hat{\beta}_{t,x}^2 - \beta_{t,x}^1 \right] + \frac{\lambda_t \hat{u}_{1,z}^2}{\gamma_t N_t} \left[ \hat{\beta}_{t,z}^2 - \beta_{t,z}^1 \right],
\]

where the symbol “\(^d\)” denotes “mimicker” (as before), while the superindex “\(^d\)” stands for “difference.” Note that \( \alpha_t^d \) and \( \beta_t^d \) reflect positionality differences between the young mimicker and the young low-ability type and between the old mimicker and the old low-ability type, respectively. Note also that the variables \( \alpha_t^d \) and \( \beta_t^d \) are related to the self-selection constraint, since each component in equation (24a) and (24b), respectively, is proportional to the Lagrange multiplier of the self-selection constraint (either the constraint facing generation \( t-1 \) or the constraint facing generation \( t \)). As such, \( \alpha_t^d \) and \( \beta_t^d \) are fundamentally related to the second-best framework with asymmetric information set out here (they would vanish in a first-best economy, where \( \lambda_t = 0 \) for all \( t \)).
4.2 Optimal Second-Best Taxation and Time-Invariant Positionality Degrees

Consider first the second-best analogue to the stationary regime addressed in subsection 3.2, where the degrees of positionality are constant over time. This special case facilitates comparison with earlier literature on optimal second-best taxation under relative consumption as well as provides straightforward (second-best) analogues to equations (16) and (17). Therefore, and by analogy to subsection 3.2, let us again make assumptions i, ii, and iii, such that, for all $t$, $N_t = N$, $\bar{\alpha}_t = \bar{\alpha}$, $\bar{\beta}_t = \bar{\beta}$, and $r_t = r$, and in addition make the following assumption:

iv. The indicators of positionality differences between the mimicker and the low-ability type are constant over time in the sense that $\alpha^d_t = \alpha^d$, and $\beta^d_t = \beta^d$ for all $t$.

As before, this case is either interpretable in terms of a steady state or may follow as a consequence of adding additional assumptions about the preferences and technology. By analogy to the average degree of time-inclusive consumption positionality defined in subsection 3.2, i.e., $\bar{\rho} = \bar{\alpha} + \bar{\beta} / (1 + r)$, we define the difference in the time-inclusive degree of consumption positionality between the mimicker and the low-ability type (also in present value terms) as

$$\rho^d = \alpha^d + \frac{\beta^d}{1 + r}.$$

We can then express the positionality effect, which characterizes the partial welfare effect of an increase in $\bar{c}_t$, as

$$W_{\tau_t} = -N \gamma \frac{\bar{\rho} - \rho^d}{1 - \bar{\rho}}.$$

The positionality effect presented in equation (25) contains two parts. The first, $-N \gamma / (1 - \bar{\rho}) < 0$, is equivalent to the right-hand side of equation (15) and is interpretable as the direct welfare loss of an increase in $\bar{c}_t$. This effect arises because a higher $\bar{c}_t$ leads to lower utility for all consumers through a reduction in relative consumption, ceteris paribus.
Therefore, this component is a pure externality and depends only on the average degree of time-inclusive consumption positionality. The second part, \( N\gamma \rho^d / (1 - \bar{\rho}) \), reflects the difference in the degree of time-inclusive consumption positionality between the mimicker and the low-ability type (when young and when old). As we explained above, the intuition is that an increase in \( \bar{\xi} \) may either relax (\( \rho^d > 0 \)) or tighten (\( \rho^d < 0 \)) the self-selection constraint. Also, since \( \bar{\rho} \) and \( \rho^d \) are based on the time-inclusive positionality concept, it follows that equation (25) accurately captures the welfare effects of consumption comparisons driven by both the keeping up with the Joneses mechanism (through \( \bar{\alpha} \) and \( \alpha^d \)) and the catching up with the Joneses mechanism (through \( \bar{\beta} / (1 + r) \) and \( \beta^d / (1 + r) \)).

With equation (25) at our disposal, we can relate the marginal income tax rates to the average degree of time-inclusive consumption positionality and to differences in this measure of positionality between the mimicker and the low-ability type. Starting with the marginal labor income tax rates, we combine equations (6), (23a), (23b), and (25) to derive the marginal labor income tax rate faced by the low-ability type, and equations (6), (23d), (23e), and (25) to derive the marginal labor income tax rate faced by the high-ability type. To simplify the notations, we use \( \tau_1^i \) and \( \tau_2^i \) to denote the optimal marginal labor income tax rates in the original Stiglitz (1982) model, i.e., the rates that would follow in the absence of relative consumption concerns,

\[
\tau^1_i = \frac{\lambda^2 \lambda^2}{w' \hat{H}_i} \left[ MRS^{1,1}_e^{1,1}_e - \phi \hat{MRS}^{2,2}_e^{2,2}_e \right] \quad \text{and} \quad \tau^2_i = 0,
\]

where \( \lambda^2 = \lambda / \gamma \).

We can then characterize the marginal labor income tax rates as follows:

**PROPOSITION 3.** Under assumptions i-iv, the second-best marginal labor income tax rate can, for each ability type, be written in the following additive form (for \( i=1, 2 \)):

\[
(26) \quad T^i_i (w/l_i) = \tau^i_i + [1 - \tau^i_i] \bar{\rho} - [1 - \tau^i_i] [1 - \bar{\rho}] \frac{\rho^d}{1 - \rho^d}.
\]
The calculations behind equation (26) are presented in the Appendix. The first term on the right-hand side is the expression for marginal labor income taxation that would follow in the standard optimal income tax model without any positional concern. For the low-ability type, this component is typically positive (at least if the consumers share a common utility function), while it is zero for the high-ability type due to that the relative wage rate is constant. The intuition is that the government may relax the self-selection constraint by taxing low-ability labor, since the mimicker attaches a higher marginal value to leisure than the low-ability type, whereas no such option exists for the high-ability type. These effects are well understood from earlier research (Stiglitz, 1982).

The second term measures the marginal external cost of consumption as reflected by the average degree of time-inclusive consumption positionality, although its contribution to the marginal labor income tax rates is modified compared to the first-best formula given by equation (16). The intuition is that the fraction of marginal income that is already taxed away does not give rise to positional externalities. Therefore, if \( \tau^*_1 > 0 \), this “second-best modification” tends to reduce the externality-correcting component in the formula for the low-ability type.

The third term on the right-hand side of equation (26) reflects self-selection effects of the positional concerns. To provide intuition, suppose first that \( \rho^d > 0 \), meaning that the mimicker has a higher degree of time-inclusive positionality than the low-ability type. In this case, increased reference consumption gives rise to a larger utility loss for the mimicker than for the low-ability type, and the government may relax the self-selection constraint through a tax policy that leads to increased reference consumption. In turn, this provides an incentive for the government to implement a lower marginal labor income tax rate. Consequently, if \( \rho^d \) is positive and sufficiently large, relative consumption concerns may actually contribute to reduce the marginal labor income tax rates (let be that this scenario seems unlikely). If \( \rho^d < 0 \), on the other hand, then increased reference consumption tightens the self-selection constraint, meaning that the third term on the right-hand side contributes to increase the marginal labor income tax rate. In this case, therefore, the positionality effect as a whole leads to increased marginal labor income taxation.
An important message of the analysis carried out in subsection 3.2 was that the keeping up and catching up with the Joneses motives for relative consumption comparisons affect the first-best tax policy in the same way. This insight carries over to the second-best policy examined here. Equation (26) means that the effects of positional concerns on the marginal labor income tax rates are fully captured by two variables: the average degree of time-inclusive positionality, \( \overline{\rho} \), and differences in the degree of time-inclusive positionality between the mimicker and the low-ability type, \( \rho^d \). As a consequence, the average degree of current positionality, \( \overline{\alpha} \), and the present value of the average degree of intertemporal positionality, \( \overline{\beta} / (1 + r) \), affect each marginal labor income tax rate in exactly the same way, and the measure of differences in the degree of current positionality between the mimicker and the low-ability type, \( \alpha^d \), affects each marginal labor income tax rate in exactly the same way as the corresponding measure based on intertemporal positionality, \( \beta^d / (1 + r) \). Special cases of equation (26) are derived by Aronsson and Johansson-Stenman (2010), where the preferences for relative consumption are based solely on the keeping up with the Joneses motive (\( \overline{\rho} = \overline{\alpha} \) and \( \rho^d = \alpha^d \)), and by Ljungqvist and Uhlig (2000), where the consumption comparisons are based on the catching up with the Joneses motive and there is no asymmetric information (\( \overline{\rho} = \overline{\beta} / (1 + r) \) and \( \rho^d = 0 \)).

Turning to the capital income tax structure, we can derive the marginal capital income tax rate implemented for the low-ability type by combining equations (7), (23b), (23c), (23g), and (25), and the marginal capital income tax rate implemented for the high-ability type by combining equations (7), (23e), (23f), (23g), and (25). To simplify the presentation of the results, we use the short notation \( \delta_i^t \) for the optimal marginal capital income tax rate for ability-type \( i \) that would follow in the absence of relative consumption concerns, where

\[
\delta_i^t = \frac{\hat{\lambda}_t \hat{u}_{t,s}^2}{\gamma_{t,s} \eta_{t+1} r_{t+1}} \left[ MRS^{1,t}_{s,x} - MRS^{2,t}_{s,x} \right] \quad \text{and} \quad \delta_i^2 = 0.
\]

We can then derive the following result:

**PROPOSITION 4.** Under assumptions i-iv, the second-best marginal capital income tax rate can, for each ability-type, be written as (for \( i=1, 2 \)): 

\[
\Phi_{t+1}(s_{t+1}) = \frac{1 - \bar{\rho}}{1 - \rho^d} \delta_i^1.
\]

The variable \(\delta_i^1\) on the right-hand side of equation (27), which does not directly depend on positional concerns, is due to the self-selection constraint and is well understood and explained in earlier research (e.g., Brett, 1997). It appears here because the government may relax the self-selection constraint by exploiting that the mimicker and the low-ability type differ with respect to the marginal rate of substitution between present and future consumption (in which case \(\delta_i^1 \neq 0\)), whereas no such option exists with regard to the marginal capital income tax rate of the high-ability type (\(\delta_i^2 = 0\)).

Note that there is no direct effect of relative consumption concerns in equation (27). As explained in subsection 3.2, the intuition is that the positionality effects in periods \(t\) and \(t+1\) largely cancel out when the degrees of positionality are constant over time. Yet, there is a second best adjustment of the marginal capital income tax rate faced by the low-ability type due to positional concerns, which was not present in subsection 3.2. To interpret this adjustment, consider the situation where \(MRS_{c,x}^{1,t} > MRS_{c,x}^{2,t}\), in which the mimicker values an additional dollar today in terms of consumption tomorrow less than does the low-ability type, implying that \(\delta_i^1 > 0\) and \(\Phi_{t+1}(s_{t+1}) > 0\). The term \(1 - \bar{\rho}\) serves to reduce the effect that \(\delta_i^1\) would otherwise have on the marginal capital income tax rate. The intuition is, of course, that capital income taxation leads to an increase in \(\bar{c}_i\) and, therefore, to an increase in the externality that \(\bar{c}_i\) gives rise to. As such, positional externalities affect the socially optimal allocation of consumption over the lifecycle for the low-ability type, even if the degrees of positionality are constant over time. Note also that the policy incentive captured by \(1 - \bar{\rho}\) in equation (27) is, in turn, either counteracted or further strengthened by the component \(1 - \rho^d\) in the denominator. This adjustment follows because increased reference consumption may either relax (\(\rho^d > 0\)) or tighten (\(\rho^d < 0\)) the self-selection constraint, depending on whether a mimicking high-ability type is more or less positional than the low-ability type, where the comparison is based on the time-inclusive measure of positionality. Analogous results and interpretations hold for the case where \(MRS_{c,x}^{1,t} < MRS_{c,x}^{2,t}\).
It is worth emphasizing once again that $\bar{\rho}$ reflects the average degree of time-inclusive consumption positionality in present value terms, i.e., $\bar{\rho} = \bar{\alpha} + \bar{\beta}/(1+r)$, and $\rho^d$ is an indicator of differences in the degrees of positionality between the mimicker and the low-ability type based on the time-inclusive concept, so $\rho^d = \alpha^d + \beta^d/(1+r)$. Therefore, all qualitative results in this subsection hold in the special case without consumption comparisons over time, i.e., where $\bar{\rho} = \bar{\alpha}$ and $\rho^d = \alpha^d$, which is the case addressed by Aronsson and Johansson-Stenman (2010). Similarly, all qualitative results hold in the other extreme situation where $\bar{\rho} = \bar{\beta}/(1+r)$ and $\rho^d = \beta^d/(1+r)$, in which there are no comparisons with other people’s current consumption.

4.2.1 When the Optimal Capital Income Taxes Vanish

An important issue in the literature on capital income taxation is to determine conditions for when it is optimal not to use such taxes. In this brief subsection we add some further assumptions that together are sufficient for an optimal tax structure without capital income taxes. In doing so, we will relate to a classical result by Ordover and Phelps (1979), regarding when it is optimal not to use capital taxation at all. From Proposition 4, it is straightforward to derive such conditions also in our model. The following result is an immediate consequence of Proposition 4:

**COROLLARY 1.** Under assumptions i-iv, and if leisure is weakly separable from private consumption in the sense that $U_t = q_t(f_t(c_t, x_{t+1}^c, \Delta_t^{x,c}, \Delta_{t+1}^{x,c}, \Omega_t^{x,c}, \Omega_{t+1}^{x,c}), z_t)$ describes the utility function, both optimal marginal capital income tax rates are zero.

This result follows from acknowledging that the mimicker and the low-ability type differ only with respect to preferences and use of leisure. Given the separability assumption and that the consumers share a common sub-utility function $f_t(\cdot)$, it follows that $MRS_{c,x}^{1,t} = M\tilde{RS}_{c,x}^{2,t}$, and hence that the possibility of relaxing the self-selection constraint through capital income taxation vanishes. The quite remarkable consequence of Corollary 1 is that the separability-result by Ordover and Phelps continues to apply under a fairly general formulation of the relative consumption concerns, which allows for both the keeping up and catching up with the Joneses mechanisms.
4.3 Optimal Second-Best Taxation with Time-Varying Degrees of Positionality

Let us finally – as we also did in subsection 3.3 – relax the stationarity assumptions (assumptions i-iv). While the policy rules will be more complex, we are still able to show some important findings for how the relative consumption concerns affect the optimal marginal income tax rates. A general characterization of the optimal marginal income tax rates in the second-best is given as (for $i=1,2$)

\begin{equation}
T_i(w_i^l I_i^l) = \tau_i^l - \frac{MRS_i^{l,t}}{1 - \gamma_i W_i^l N_i^t},
\end{equation}

\begin{equation}
\Phi_i^{s,t}(s_i^l r_i^l) = \delta_i^l + \frac{1}{1 - \gamma_i r_i^l N_i^t} \left[ W_i^l - \frac{MRS_i^{l,t}}{1 + \eta_i r_i^l W_i^l} \right],
\end{equation}

where we have used the short notations $\tau_i^l$ and $\delta_i^l$ for the marginal labor income tax rate and marginal capital income tax rate, respectively, implemented for ability-type $i$ in the standard two-type OLG model without positional concerns. Equations (28) and (29) are straightforward generalizations of the corresponding first-best formulas given by equations (19) and (20).

To be able to say more about the relationship between the relative consumption concerns and the marginal income tax rates, we must explore the positionality effect in more detail. By using the expressions for the differences in the degree of current ($\alpha_i^d$) and intertemporal ($\beta_i^d$) positionality between the mimicker and the low-ability type in period $t$, as given by equations (23a) and (23b), together with the short notations

\begin{align*}
A_{t+k} &= \frac{N_{t+k} \gamma_{t+k} [\alpha_{t+k} - \bar{\alpha}_{t+k}]}{1 - \bar{\alpha}_{t+k}}, \\
B_{t+k} &= \frac{N_{t+k+1} \gamma_{t+k+1} [\beta_{t+k+1} - \bar{\beta}_{t+k+1}]}{1 - \bar{\alpha}_{t+k}},
\end{align*}

we obtain the following second-best analogue to equation (18):
Equation (30) extends equation (25) to the case where neither the degrees of positionality, nor the population and interest rate are (necessarily) constant over time. We can see that equation (30) has the same structure as equation (18) above, with the exception that the terms $-\tilde{\alpha}_{t+k}$ and $-\tilde{\beta}_{t+k+1}$ in equation (18) are here replaced with $\alpha^d_{t+k} - \tilde{\alpha}_{t+k}$ and $\beta^d_{t+k+1} - \tilde{\beta}_{t+k+1}$, respectively. The only important difference between equations (25) and (30) is that we can no longer make use of the (analytically convenient) time-inclusive rates of positionality in the same way as before. As a consequence, although comparisons with other people’s past consumption (the catching up motive) give rise to the same qualitative policy implications as those associated with comparisons with other people’s current consumption (the keeping up motive), the welfare consequences of intertemporal consumption comparisons are, in general, much more complex than those associated with atemporal consumption comparisons. This is seen by recognizing that in the special case without intertemporal consumption comparisons, i.e., when $\tilde{\beta}_t = \beta^d_t = 0$ for all $t$, equation (25) reduces to read

$$W_{t'} = -N_{\gamma_t} \frac{\tilde{\alpha}_t - \alpha^d_t}{1-\tilde{\alpha}_t}$$

for all $t$, which takes exactly the same form as equation (25) above.

From equation (30), we can derive the following result regarding the conditions for when the sign of the positionality effect is unambiguously negative:

**LEMMA 1.** If, from period $t$ and onwards, the low-ability type is at least as positional as the mimicker on average, or if the positionality differences are sufficiently small, in any of the following senses:

(i) $$\frac{N_{\gamma_t} \alpha^d_t + N_{\gamma_t} \alpha^d_t \beta^d_{t+1}}{1-\tilde{\alpha}_t} + \sum_{k=1}^{\infty} N_{\gamma_t} \alpha^d_{t+k} \frac{\beta^d_{t+k+1}}{1-\tilde{\alpha}_{t+k}} \prod_{j=1}^{k} \frac{\tilde{\beta}_{t+j}}{1-\tilde{\alpha}_{t+j-1}} \leq 0,$$

(ii) $\alpha^d_{t+k} < \tilde{\alpha}_{t+k}$ and $\beta^d_{t+k+1} < \tilde{\beta}_{t+k+1}$ for all $k \geq 0$,

(iii) $\alpha^d_{t+k} \leq 0$ and $\beta^d_{t+k+1} \leq 0$ for all $k \geq 0$,

then increased reference consumption in period $t$ reduces the welfare.
Given that the individual degrees of positionality (both in the current and intertemporal dimensions) are always between zero and one, (i) gives a sufficient condition for when increased reference consumption in period \( t \) leads to lower welfare. Yet, condition (i) is not necessary, since the terms in equation (30) that solely reflect the average degrees of positionality (i.e., the pure externality terms) contribute to lower welfare as well. Condition (ii) is not necessary either, since \( \bar{W}_k \) can clearly be negative even if (ii) does not hold for some \( k \). Note finally that condition (iii), which we refer to due to its straightforward interpretation, is actually redundant since it implies condition (ii).

By combining Lemma 1 with equation (28), we obtain the following result:

**PROPOSITION 5.** *If any of the conditions in Lemma 1 hold, so that increased reference consumption leads to lower welfare, ceteris paribus, then the positionality effect in period \( t \) contributes to increase the marginal labor income tax rates for both ability-types in period \( t \).*

The interpretation of Proposition 5 is straightforward. If the low-ability type is at least as positional as the mimicker on average, or if loosely speaking the positionality differences are sufficiently small, and given that the individual degrees of positionality are always between zero and one, then we obtain from equation (27) that \( \bar{W}_k < 0 \). As such, Proposition 5 also extends the result of Oswald (1983) – that “marginal tax rates are higher than in the conventional model when the population is predominantly jealous” (page 82) – to a model with consumption comparisons over time.\(^\text{17}\)

Similarly, by combining Lemma 1 with equation (29), we can derive the following result for how positional concerns contribute to the marginal capital income tax rates:

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\(^{17}\) By “jealousy,” Oswald meant that an increase in the reference consumption leads to decreased utility for the individual consumer (as compared to “altruism,” which has the opposite effect). He analyzed nonlinear taxation of commodities instead of income, and the result referred to above is based on a utility function that is separable in the reference measure; as assumption that limits the influence of the incentive constraint on the marginal tax rates.
PROPOSITION 6. Suppose that any of the conditions in Lemma 1 hold, so that increased reference consumption leads to lower welfare, ceteris paribus. Then, if the preferences become less (more) positional over time in the sense that

\[
\left| W_r \right| > (<) \frac{MRS_{c,x}^{t+1}}{1 + \eta_{t+1}} \left| W_r \right|
\]

i.e., the positionality effect in period \( t \) dominates (is dominated by) the positionality effect in period \( t+1 \), then the joint contribution of the positionality effects in periods \( t \) and \( t+1 \) is to decrease (increase) the marginal capital income tax rate for ability-type \( i \) in period \( t+1 \).

If Lemma 1 applies, such that the positionality effect is negative in all periods, the interpretation of the first-best policy rule for marginal capital income taxation in equation (20) carries over with some modifications to the second-best framework addressed here. Therefore, if the positionality effect in period \( t \) dominates the corresponding effect in period \( t+1 \), there is an incentive for the government to discourage the consumption in period \( t \) relative to the consumption in period \( t+1 \). The difference by comparison with the first-best policy analyzed in subsection 3.3 is, of course, that the policy incentives analyzed here are due to both externality correction and redistribution effects of positional concerns through the self-selection constraint. This has been discussed at some length above. The analogous policy incentive to encourage the consumption in period \( t \) relative to the consumption in period \( t+1 \) arises if the positionality effect in period \( t+1 \) dominates. Again, these insights follow irrespective of whether the positional concerns are driven by the keeping up or catching up mechanism or a mix between them.

An interesting implication of the proposition is that it would be optimal with increasing marginal capital income taxation over time in an economy where the preferences become more positional over time (i.e., if we tend to attach a higher value to increased relative consumption than to increased absolute consumption as time passes). Such a pattern is actually broadly consistent with some empirical evidence: Clark et al. (2008) analyze the impact of relative income on happiness and conclude that the concern for relative income tends to increase as the average income in a country increases. Note also that we can interpret the component \( \frac{MRS_{c,x}^{t+1}}{1 + \eta_{t+1}} \) as the effective discount factor for ability-type \( i \), which is used to discount the positionality effect in period \( t+1 \) to period \( t \).
5. RESULTS BASED ON A COBB- DOUGLAS UTILITY FUNCTION

In order to more clearly illustrate some implications of the relative consumption comparisons for optimal income taxation, let us use the same assumptions as in Section 5 with respect to a constant population size and interest rate, but in addition consider the following Cobb- Douglas utility function:

\[ U'_i = k^i(\zeta^i)^k(c'_{net,t})^k(x'_{net,t+1})^k, \]

where \( k^i, k^i, k_z, k_x > 0 \) are constants and \( k_z + k_c + k_x < 1; \) \( c'_{net,t} \) and \( x'_{net,t+1} \) reflect what we may think of as consumption net of relative consumption concerns, when young and when old, for an individual of ability-type \( i \) born in period \( t \), as defined below:\(^{18}\)

\[(32a) \quad c'_{net,t} = \left[1 - a - a'\right]c'_t + a\left[c'_t - \bar{c}_t\right] + a'\left[c'_t - \bar{c}_{t-1}\right] = c'_t - a\bar{c}_t - a'\bar{c}_{t-1}, \]

\[(32b) \quad x'_{net,t+1} = \left[1 - b - b'\right]x'_{t+1} + b\left[x'_{t+1} - \bar{c}_{t+1}\right] + b'\left[x'_{t+1} - \bar{c}_t\right] = x'_{t+1} - b\bar{c}_{t+1} - b'\bar{c}_t. \]

By substituting equations (32a) and (32b) into equation (31), we obtain:

\[(33) \quad U'_i = k^i(\zeta^i)^k\left[c'_t - a\bar{c}_t - a'\bar{c}_{t-1}\right]^{k} \left[x'_{t+1} - b\bar{c}_{t+1} - b'\bar{c}_t\right]^{k}. \]

Although the utility functions are allowed to differ between the ability-types, through the parameters \( k^i \) and \( k^i \), the individual degrees of current and intertemporal consumption positionality are clearly the same between types, and are also constant over time. The degrees of current consumption positionality when young and when old for each type are equal to \( a \) and \( b \), respectively, whereas the corresponding degrees of intertemporal consumption positionality are given by \( a' \) and \( b' \).

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\(^{18}\) For analytical simplicity, we ignore the possibility that each individual also compares his/her own current consumption with his/her own previous consumption. As mentioned before, allowing for such comparisons would not directly affect the optimal tax formulas, since the individuals would internalize these effects themselves.
From equation (26), we then have that the optimal marginal labor income tax rate for type $i$ is given by

\begin{equation}
T_i'(w_i^l) = \sigma_i^l + [1-\sigma_i^l] \left[ \frac{a+b}{2} + \frac{a'+b'}{2(1+r)} \right],
\end{equation}

where the first expression in brackets thus represents the average current degree of consumption positionality and the second the (one period discounted) average intertemporal degree of consumption positionality. As before, $\sigma_i^l$ reflects the optimal marginal labor income tax rate for ability-type $i$ without relative consumption concerns.

Regarding optimal capital income taxation, it is easy to see that the separability assumption in Corollary 1 above is fulfilled by the utility function in Equation (34). Therefore, we know that the optimal marginal capital income tax rate is zero for each ability-type and in all time periods, irrespective of the parameter values of the utility function.

5.1 Orders of Magnitude

Let us now briefly discuss possible orders of magnitude of the optimal marginal income taxes. A couple of studies have attempted to measure the average degree of current consumption positionality, corresponding to $(a+b)/2$ in equation (34). According to the survey-experimental evidence of Solnick and Hemenway (1998), Johansson-Stenman et al. (2002), Alpizar et al. (2005), and Carlsson et al. (2007), the average degree appears to be in the order of magnitude of 0.5. Wendner and Goulder (2008) argue, based on the existing empirical evidence, for a value between 0.2 and 0.4, whereas evidence from happiness studies such as Luttmer (2005) suggests a much larger value in the order of magnitude of 0.8.

There is less direct evidence regarding the average intertemporal degree of consumption positionality, corresponding to $(a'+b')/2$ in equation (37). Alvarez-Cuadrado et al. (2004) refer to a benchmark value used by Carrol et al. (1997), with a value of a parameter that can
be interpreted as an intertemporal degree of consumption positionality equal to 0.5.\textsuperscript{19} As a sensitivity analysis, they use a value of 0.8, based on Fuhrer (2000).

As an illustrative example, consider the case where the optimal marginal labor income tax rate in the absence of relative consumption concerns equals 0.3 for ability-type $i$, and where both the average degree of current consumption positionality and the average degree of intertemporal consumption positionality are also 0.3, i.e., $\sigma'_i = (a+b)/2 = (a'+b')/2 = 0.3$. Then, if the real interest rate between the periods is given by $r = 1$,\textsuperscript{20} it follows that the optimal marginal labor income tax rate is equal to $T_i'(w_i'/l_i') = 0.3 + 0.7[0.3 + 0.3/2] = 0.615$. In other words, the optimal marginal labor income tax rate would be above 60% rather than 30% as in the absence of relative consumption comparisons. While the underlying estimates of the current and intertemporal degrees of positionality presented above are highly uncertain, and can hardly be interpreted as completely independent of each other,\textsuperscript{21} it nevertheless seems as if their joint effect on the marginal labor income tax rates may be substantial.

\section*{6. CONCLUSION}

The present paper simultaneously recognizes three mechanisms behind relative consumption concerns: comparisons with (i) other people’s current consumption (keeping up with the Joneses), (ii) own past consumption (habit formation), and (iii) other people’s past consumption (catching up with the Joneses). We are not aware of any previous normative economic analysis in such a setting.

\textsuperscript{19} In Carrol et al. (1997), the reference consumption is not others’ average consumption one period earlier (since their study is not based on an OLG model), but instead a weighted average of others’ average consumption where the weight is larger the closer to the present the consumption takes place.

\textsuperscript{20} This corresponds to an annual real interest rate of slightly less than 2 percent if we assume 40 years between the periods.

\textsuperscript{21} We are not aware of any study that simultaneously attempts to estimate the average degree of current and intertemporal consumption positionality.
We start by deriving a first-best tax policy to correct for the positional externalities in the case where the government is able to redistribute through lump-sum instruments. We show that comparisons with one’s own past consumption do not affect the optimal policy rules, since such comparisons are internalized by each individual (although internal habit formation may, of course, affect the levels of marginal income tax rates), whereas comparisons with other people’s current and past consumption generate positional externalities. In a stationary regime where the degrees of positionality are time-invariant, the optimal tax policy is derived in terms of the average degree of time-inclusive consumption positionality, which is essentially the sum of the average degree of current consumption positionality and the average degree of intertemporal consumption positionality. Results derived in earlier literature such as Ljungqvist and Uhlig (2000) and Dupor and Liu (2003) follow as special cases of our first-best model. We also show that the optimal marginal labor income tax rates become larger the more positional people are on average, in terms of the average degree of time-inclusive consumption positionality. It is also demonstrated that this modifying effect can be substantial.

The second-best analysis carried out in Section 4 is based on the two-type optimal income tax model with asymmetric information between the government and the private sector. In this case, the net effects of relative consumption concerns also depend on whether the low-ability type is more or less positional (broadly speaking) than the mimicker. The reason is that this determines whether an increase in the reference consumption works to relax or tighten the self-selection constraint. If the degrees of positionality are constant over time, there are no direct effects of relative consumption concerns on the marginal capital income tax rates; in a second-best setting, such concerns will, nevertheless, affect the marginal capital income tax structure through the self-selection constraint. We are also able to reproduce the well-known result of Ordover and Phelps (1979) for when there should be no capital income taxes, in a model where people compare their own current consumption with several different measures of reference consumption.

When we generalize the model to allow for time variation also with regard to the positionality degrees, the population size, and the interest rate, then the optimal policy responses become considerably more complex and the optimal policy rules are no longer possible to express in a simple way in terms of time-inclusive positionality degrees. This applies both to the first-best and the second-best model. Yet, we were able to obtain important findings regarding the
qualitative effects of positional concerns on the optimal marginal income tax rates, and in particular, when such concerns unambiguously work to increase or decrease these rates.

Finally we illustrate with a Cobb-Douglas functional form and show, based on parameter estimates from the literature, that positional preferences of both the keeping up with the Joneses and catching up with the Joneses types substantially increase the optimal marginal labor income tax rates for both types. Since the leisure separability conditions are fulfilled for this form, the optimal marginal capital income tax rates are consequently zero for both types.

We believe that the research area consisting of normative economic analysis when relative consumption matters is still underexplored. Examples of important issues that remain to be analyzed include a multi-country setting, public provision of private (non-positional) goods, public good provision in a dynamic economy, and long-term social discounting.

APPENDIX

To save space, we have chosen to derive the more general results first, such that the results following from more restrictive formulations of the model appear as special cases.

Labor Income Taxation: derivation of equations (19) and (28)

Consider the tax formula for the low-ability type. By combining equations (23a) and (23b), we obtain

\[
\frac{u_{t,z}}{u_{t,x}} \left[ \lambda_i h_{t,x}^2 + \gamma_i n_i - \frac{n_i^t}{N_i} \frac{\partial L}{\partial C_i} \right] = \lambda_i \phi h_{t,z}^2 + \gamma_i n_i w_i^t.
\]

By substituting \( T^r_i(w_i^l l_i w_i^l)w_i^l = w_i^l - u_{t,z} / u_{t,x} \) from equation (6) into equation (A1) and rearranging, we obtain equation (28) for the low-ability type. The corresponding formula for the high-ability type can be derived in the same general way by combining equations (6), (23d), and (23e). Equation (19) follows as the special case where \( \lambda_i = 0 \).
Capital income taxation: derivation of equations (20) and (29)

Consider first the formula for the low-ability type. By combining equations (23b) and (23c), we obtain

(A2) \[ MRS_{t,i}^{L,t} = \lambda_i \hat{u}_{t,i}^2 + n_i^1 \frac{\partial L}{\partial c_{t+1}} = \lambda_i \hat{u}_{t,i}^2 + \gamma_i n_i^1 - \frac{n_i^1}{N_i} \frac{\partial L}{\partial c_{t+1}}. \]

We can then use equations (7) and (23g) to derive \( MRS_{t,i}^{L,t} = 1 + r_{i+1} - r_{i+1} \Phi_{i+1}(s_{i+1}^i, r_{i+1}) \) and \( \gamma_i = \gamma_{i+1}[1 + r_{i+1}] \), respectively. Substituting into equation (A2) and rearranging, we obtain equation (29) for the low-ability type. We can derive the corresponding expression for the high-ability type in the same general way by combining equations (7), (23e), (23f), and (23g). Equation (20) follows as the special case where \( \lambda_{i+1} = \lambda_i = 0 \).

Derivation of equations (18) and (30)

Consider first equation (30). By using equation (22), we can derive

(A3) \[ \frac{\partial L}{\partial c_t} = \sum_{i=1}^2 \frac{\partial W}{\partial (n_i^1 U_{t-1}^i)} n_i^{1i} u_{t-1,i} \] \[ + \lambda_{i+1} n_i^1 \left[ u_{t-1,i}^2 - u_{t-1,i}^2 \right] + \lambda_i n_i^1 \left[ u_{t+1,i}^2 - u_{t+1,i}^2 \right] \]

From equation (1), we have

\[ u_{t,ce} = v_{t,ce} + v_{t,ce} + v_{t,ce} = \frac{v_{t,ce}}{\alpha_{t,ce}} = \frac{v_{t,ce}}{\beta_{t,ce}}, \]

\[ u_{t,cs} = v_{t,cs} + v_{t,cs} + v_{t,cs} = \frac{v_{t,cs}}{\alpha_{t,cs}} = \frac{v_{t,cs}}{\beta_{t,cs}}, \]

\[ u_{t,ca} = -v_{t,ca} - v_{t,ca}, \]

\[ u_{t,ca} = -v_{t,ca}, \]

\[ u_{t,ca} = -v_{t,ca}, \]

so

(A4) \[ u_{t,ce} = -\alpha_{t,ce} u_{t,ce} - \beta_{t,ce} u_{t,ce}, \]

(A5) \[ u_{t,cs} = -\beta_{t,cs} u_{t,ce}, \]

(A6) \[ u_{t,ca} = -\alpha_{t,ca} u_{t,ce}, \]
which substituted into equation (A3) imply
\[
\frac{\partial \mathcal{L}}{\partial \tilde{c}_t} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{i-1}U_{i-1})} n_{i-1}^i \tilde{c}_t^{i-x} u_{t-1,x}^i - \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{i-1}U_{i-1})} n_{i-1}^i \left[ \alpha_{t}^{i,x} u_{t,x}^i + \beta_{t}^{i,x} u_{t,x}^i \right]
\]
\[
-2 \frac{\partial W}{\partial (n_{i}U_{i+1})} n_{i}^i \beta_{t+1}^{i,x} u_{t+1,x}^i + \lambda_{t-1} \left[ -\alpha_{t}^{2,x} u_{t-1,x}^2 + \alpha_{t}^{2,x} u_{t-1,x}^2 \right] + \lambda_{t} \left[ -\beta_{t+1}^{2,c} u_{t+1,c} + \beta_{t+1}^{2,c} u_{t+1,c} \right]
\]
\[
\hat{c}_t \left[ -\alpha_{t}^{2,c} u_{t,c}^2 - \beta_{t}^{2,c} u_{t,c}^2 + \hat{c}_t \alpha_{t}^{2,c} u_{t,c}^2 + \hat{c}_t \beta_{t+1}^{2,c} u_{t+1,c} \right] + \lambda_{t+1} \left[ -\beta_{t+1}^{2,c} u_{t+1,c} + \beta_{t+1}^{2,c} u_{t+1,c} \right]
\]
From equations (23b), (23c), (23e), and (23f), we have
\[
\frac{\partial W}{\partial (n_{i}U_{i})} n_{i}^i u_{t,c}^i = \lambda_{t} u_{t,c}^i - N_{i}^t \frac{n_i^t}{\partial \tilde{c}_t},
\]
\[
\frac{\partial W}{\partial (n_{i}U_{i+1})} n_{i}^i u_{t-1,x}^i = -\lambda_{t} u_{t-1,x}^i + N_{i}^t \frac{n_i^t}{\partial \tilde{c}_t},
\]
\[
\frac{\partial W}{\partial (n_{i-1}U_{i})} n_{i-1}^i u_{t-1,x}^i = \lambda_{t} u_{t-1,x}^i - N_{i}^t \frac{n_i^t}{\partial \tilde{c}_t},
\]
\[
\frac{\partial W}{\partial (n_{i}U_{i+1})} n_{i}^i u_{t-1,x}^i = -\lambda_{t} u_{t-1,x}^i + N_{i}^t \frac{n_i^t}{\partial \tilde{c}_t}.
\]
By substituting equations (A8)-(A11) into equation (A7), and collecting terms, we obtain
\[
\frac{\partial \mathcal{L}}{\partial \tilde{c}_t} = \frac{\partial \mathcal{L}}{\partial \tilde{c}_t} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} - N_{t} \frac{\bar{\alpha}_t}{1-\bar{\alpha}_t} - N_{t+1} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t}
\]
\[
\frac{\lambda_{t} \hat{c}_t}{1-\bar{\alpha}_t} \left[ \alpha_{t}^{2,x} - \alpha_{t}^{2,x} \right] + \frac{\lambda_{t} \hat{c}_t}{1-\bar{\alpha}_t} \left[ \alpha_{t}^{2,c} - \alpha_{t}^{2,c} \right] + \frac{\lambda_{t} \hat{c}_t}{1-\bar{\alpha}_t} \left[ \beta_{t+1}^{2,x} - \beta_{t+1}^{2,x} \right] + \frac{\lambda_{t+1} \hat{c}_t}{1-\bar{\alpha}_t} \left[ \beta_{t+1}^{2,c} - \beta_{t+1}^{2,c} \right]
\]
\[
= \frac{1}{1-\bar{\alpha}_t} \left[ \frac{\bar{\beta}_{t+1}}{\partial \tilde{c}_t} \frac{\partial \mathcal{L}}{\partial \tilde{c}_t} + N_{t} \gamma_{t} \left[ \alpha_{t}^{d} - \bar{\alpha}_t \right] + N_{t+1} \gamma_{t+1} \left[ \beta_{t+1}^{d} - \bar{\beta}_{t+1} \right] \right]
\]
where we have used the short notations \( \alpha_{t}^{d} \) and \( \beta_{t}^{d} \) as defined earlier. Using the short notations
\[
A_t = \frac{N_{t} \gamma_{t} \left[ \alpha_{t}^{d} - \bar{\alpha}_t \right]}{1-\bar{\alpha}_t},
\]
\[
B_t = \frac{N_{t+1} \gamma_{t+1} \left[ \beta_{t+1}^{d} - \bar{\beta}_{t+1} \right]}{1-\bar{\alpha}_t},
\]
\[
\phi_t = \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t},
\]
the recursive equation (A12) can more conveniently be rewritten and expanded as
\[
\frac{\partial L}{\partial c_i} = A_i + B_i + \varphi_i \frac{\partial L}{\partial c_{t+1}} = A_i + B_i + \varphi_i \left[ A_{i+1} + B_{i+1} + \varphi_{t+1} \frac{\partial L}{\partial c_{t+2}} \right]
\]
\[
= A_i + B_i + \varphi_i \left[ A_{i+1} + B_{i+1} + \varphi_{t+1} \left[ A_{i+2} + B_{i+2} + \varphi_{t+2} \frac{\partial L}{\partial c_{t+3}} \right] \right].
\]
\[
= A_i + B_i + \sum_{i=1}^{\infty} \left[ A_{i+1} + B_{i+1} \right] \varphi_{i+1} \left[ A_{i+2} + B_{i+2} + \varphi_{i+2} \frac{\partial L}{\partial c_{i+3}} \right].
\]

(A13)

Substituting back \( \varphi_i = \frac{\bar{\beta}_{i+1}}{\left(1 - \bar{\alpha}_i\right)} \) into equation (A13) implies equation (30). Equation (18) follows as the special case where \( \lambda_i = 0 \) for all \( t \).

Finally, by adding assumptions i-iv (i.e. i. \( N_t = N \); ii. \( \bar{\alpha}_t = \bar{\alpha} \) and \( \bar{\beta}_t = \bar{\beta} \); iii. \( r = r \); and iv. \( \bar{\alpha}_t = \alpha^d \) and \( \bar{\beta}_t = \beta^d \), for all \( t \)), equation (30) reduces to equation (25), and equation (18) reduces to equation (15).

**Derivation of equations (16) and (26)**

Consider first equation (26) for the low-ability type. By combining equations (25) and (28), we obtain

\[
(T'_i (w^l_i) = \frac{\lambda^*_i}{w^l_i n^l_t} \left[ MRS_{z,c}^{1,i} - \phi \hat{MRS}_{z,c}^{2,i} \right] \frac{MRS_{z,c}^{1,i}}{w^l_t} \rho^d - \bar{\rho}}{1 - \bar{\rho}}.
\]

(A14)

Then, by using \( MRS_{z,c}^{1,i} / w^l_t = 1 - T'_i (w^l_i) \) and rearranging, we obtain equation (26) for the low-ability type. The corresponding marginal income tax rate for the high-ability type is derived in a similar way. Equation (16) follows as the special case where \( \lambda_i = 0 \).

**Derivation of equations (17) and (27)**

Consider first equation (27) for the low-ability type. Substituting equation (25), for period \( t \) and period \( t+1 \), into equation (29), we obtain

\[
(\Phi'_{t+1}(s^*_i r^*_t) = \delta^*_i + \frac{1}{r} \frac{\rho^d - \bar{\rho}}{1 - \bar{\rho}} \left[ \frac{\gamma_t - MRS_{c,s}^{1,i}}{\gamma_t} \right].
\]

(A15)
for \(i=1,2\), where we have used the short notations \(\delta^{i}_1\) and \(\delta^{i}_2\) as defined earlier. Using \(\text{MRS}^{i}_{r,s} = 1 + r \Phi^{i}_{r,s}(s^{i}_{r,s})\) together with \(\gamma_i / \gamma_{i+1} = 1 + r\) in equation (A15) and rearranging, we obtain equation (27) for the low-ability type. Equation (27) for the high-ability type can be derived in the same general way. Equation (17) follows as the special case where \(\lambda_{i-1} = \lambda_i = 0\).

REFERENCES


