# Aktienindex und Volatilität

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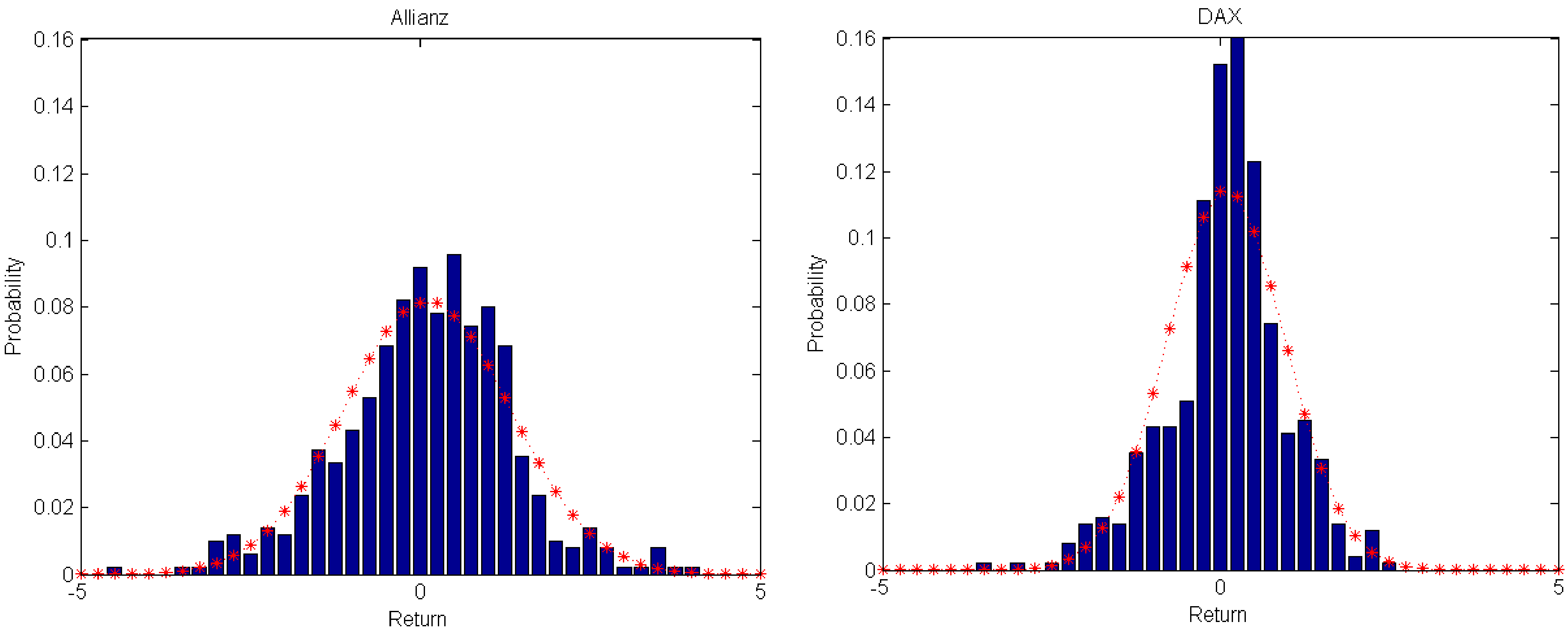
Dahlem Lectures on FACTS
an der Freie Universität Berlin
am 10. Januar 2007

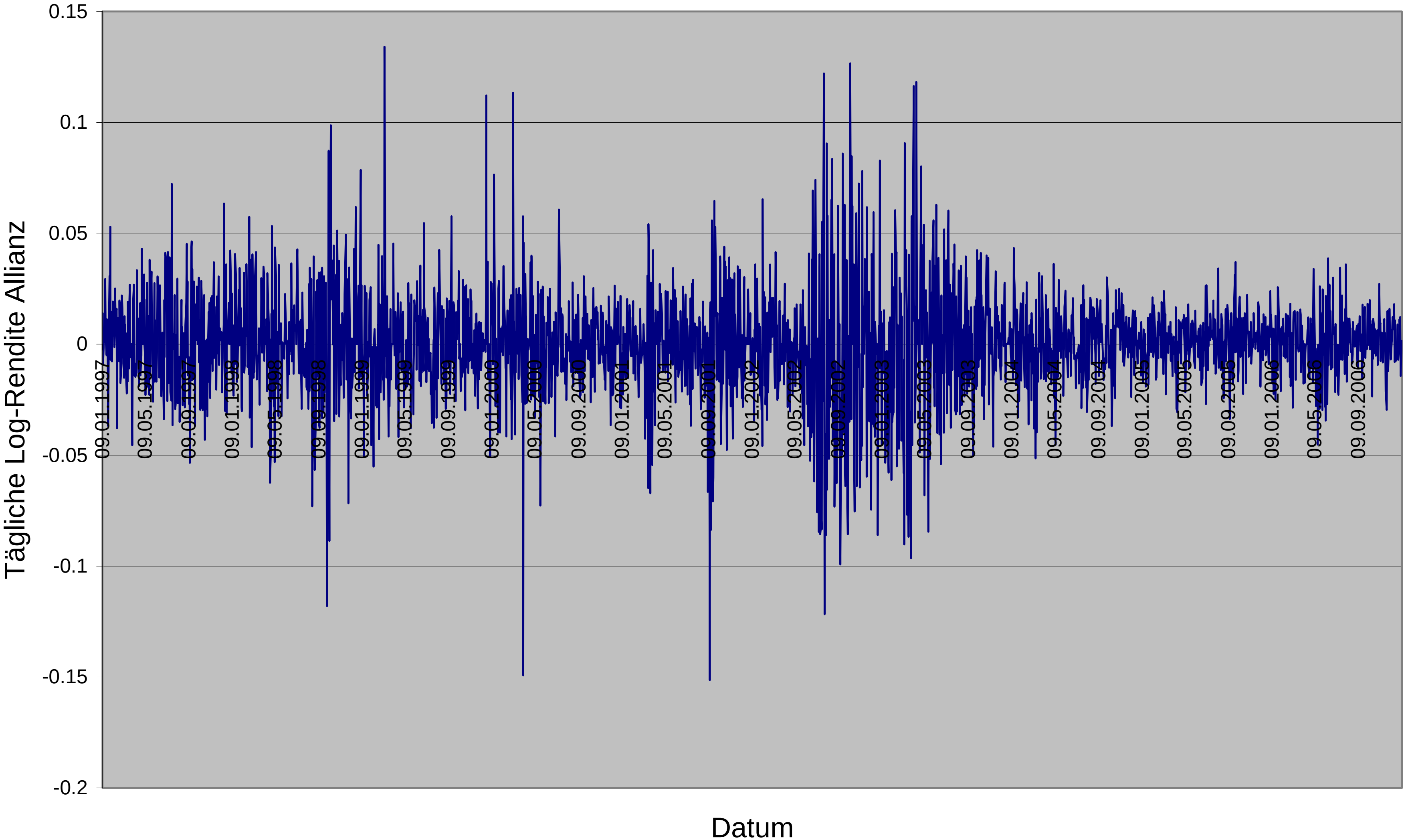
# Gliederung

1. Volatilität als Risikomaß
2. Typische Verlaufsmuster der Volatilität
3. Volatilität als Handelsobjekt
4. Höhe der Risikoprämie für Volatilität
5. Volatilität als Anlageobjekt
6. Ergebnisse

# Volatilität als Risikomaß: tägl. Log-Renditen 2005-2006



# Clustering: Rendite Allianz-Aktie 1997-2006

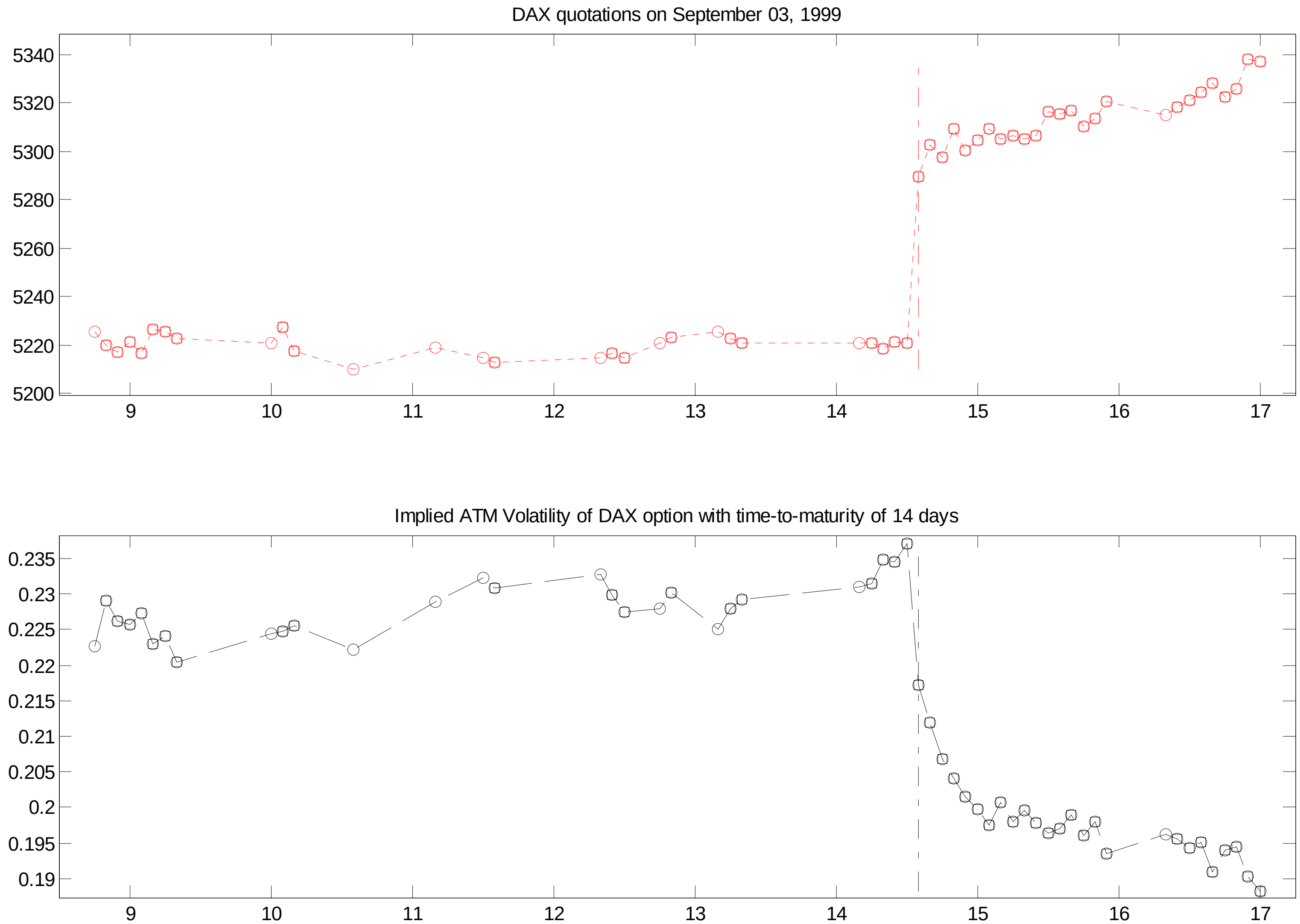


# DAX und VDAX am 24. Juli 2006

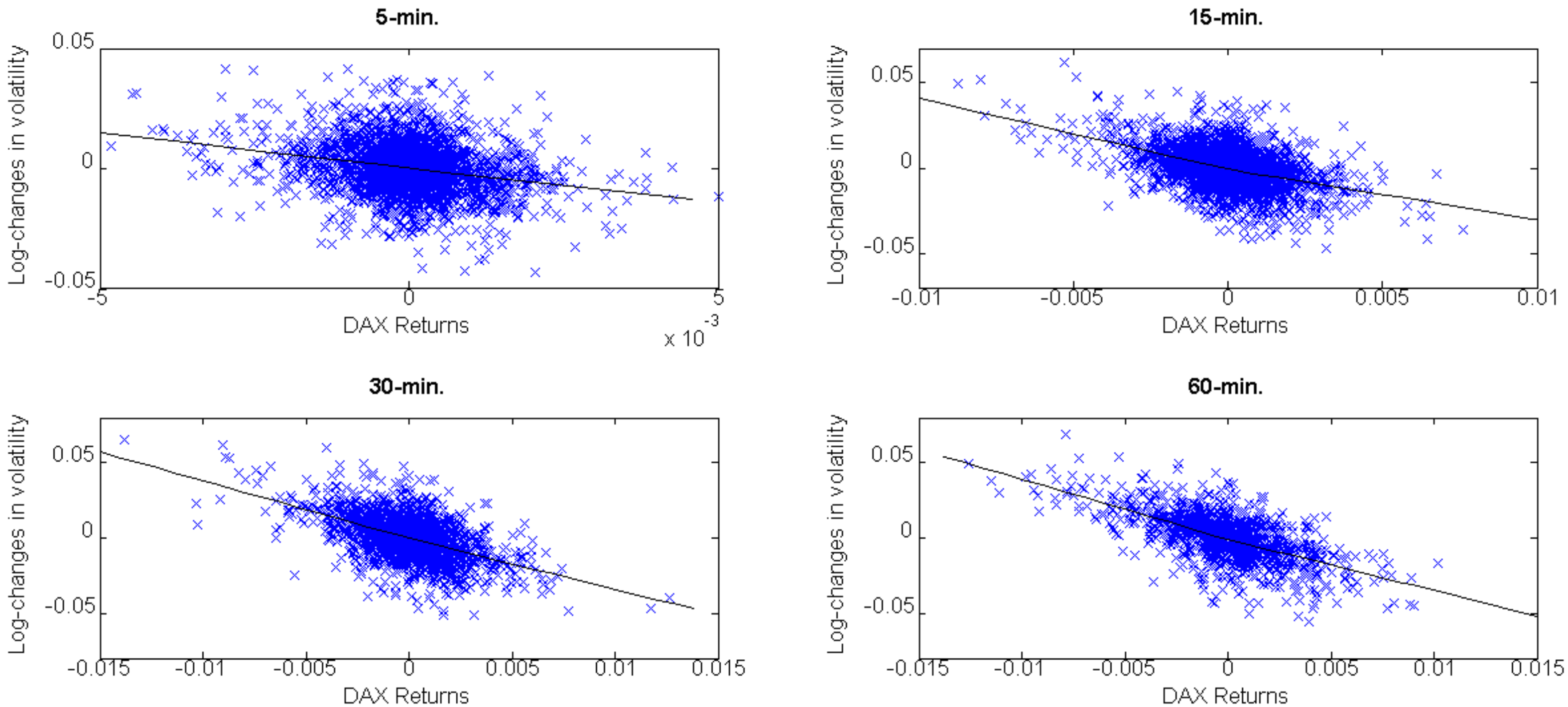
**DAX**

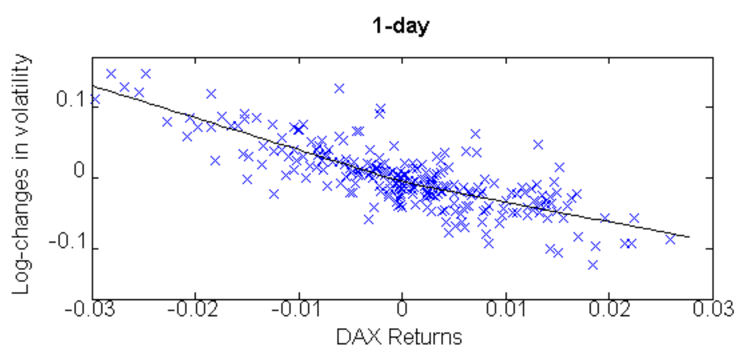

**VDAX**

# Kurs- und Volatilitätssprünge: 3. September 1999



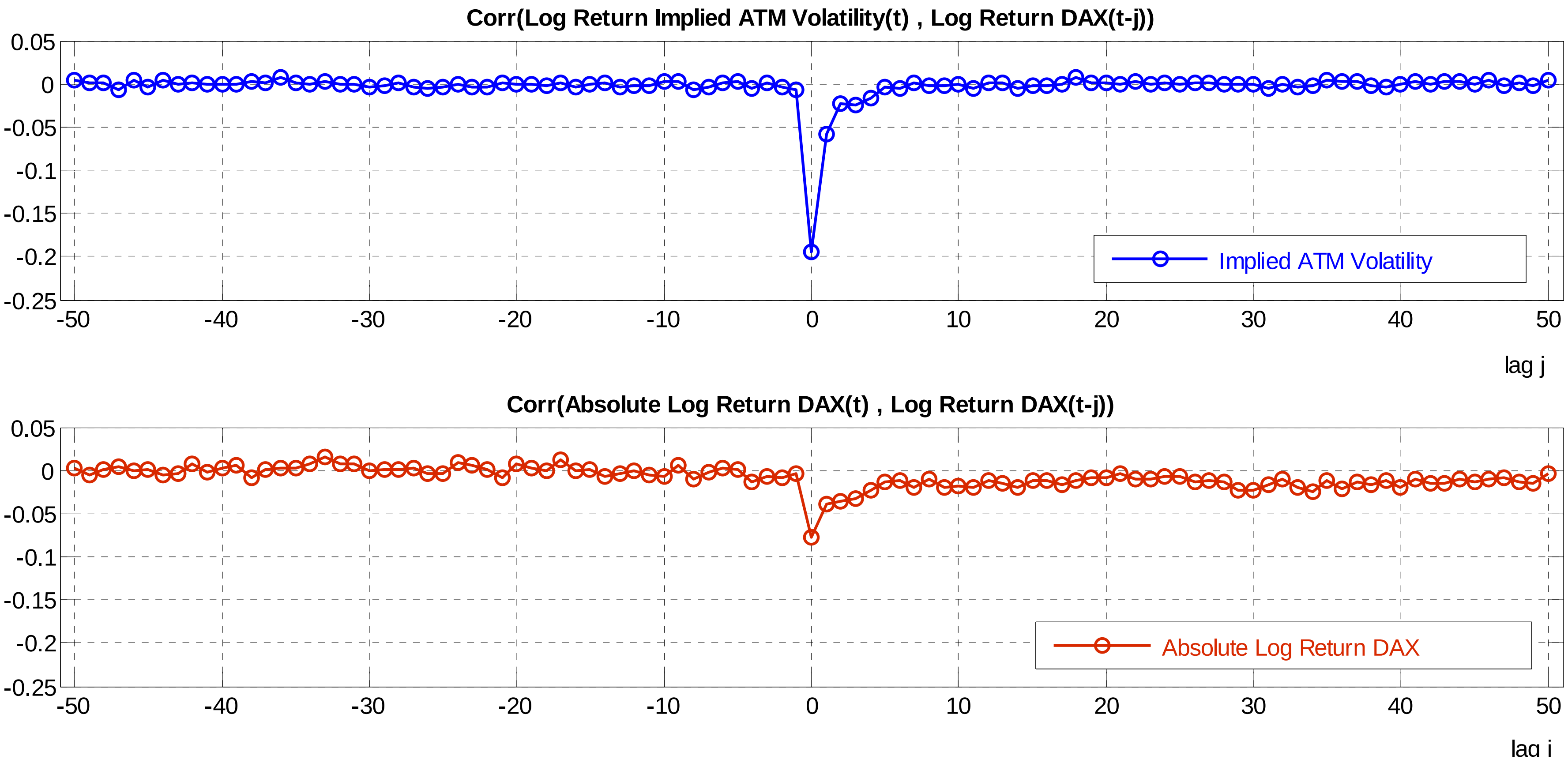
# Contemporaneous correlations 2004 for different sampling frequencies




| | | |
|---|---|---|
| 5-min. | : | -0.2626 |
| 15-min. | : | -0.4576 |
| 30-min. | : | -0.5335 |
| 60-min. | : | -0.6345 |
| 1-day | : | -0.7999 |

# 5-minute cross-correlations 1995-2004



# Gliederung

# Volatility as an asset class

UBS Investment Bank

## Volatility Certificate on DJ Euro STOXX 50®

**Up to 1'500'000 Certificates**

Performance

### *Final Terms*

**Product Description**

This product is principally aimed at professional portfolio managers as a portion of a strategic portfolio because volatility is generally highly inversely correlated with stock prices. Adding inversely correlated assets to a balanced portfolio generally increases diversification and may increase risk adjusted return.

The volatility certificate is for those who believe that at the money implied volatility on the strike setting date will be higher than that which they can buy in the forward market.

**Product Details**

| | |
|---|---|
| **Underlying Index** | **DJ EuroSTOXX50 Index®**<br>(Bloomberg SX5E; Reuters .STOXX50E) |

# Varianzswap

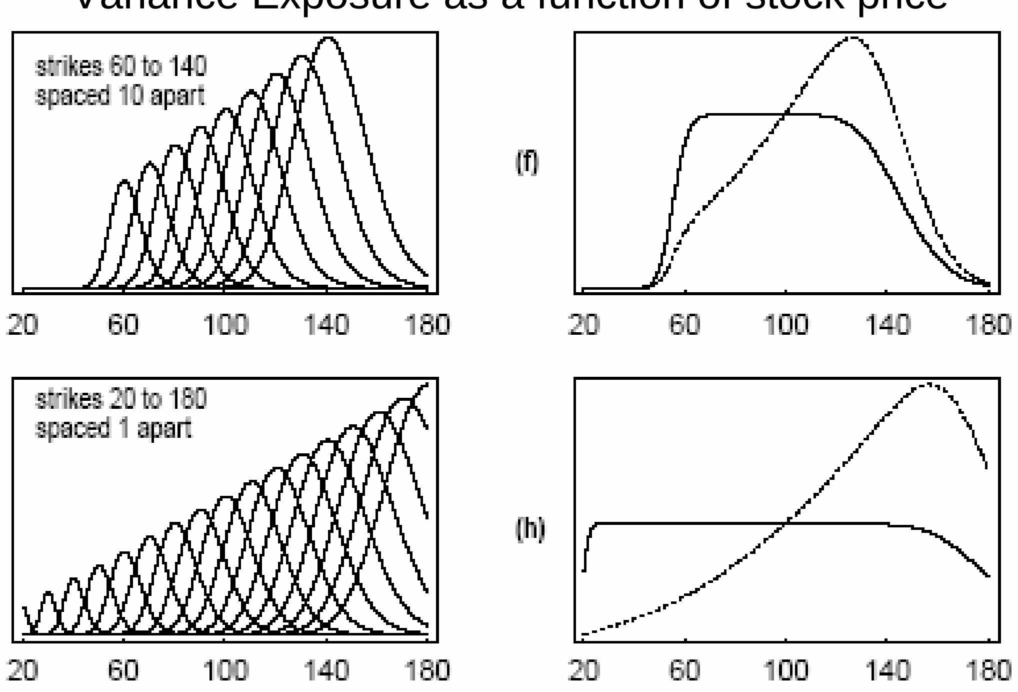
Termingeschäft, bei dem die Vertragsparteien den Austausch folgender Zahlungen vereinbaren:

- **Käufer:** bezahlt den bei Vertragsabschluss vereinbarten Preis
- **Verkäufer:** bezahlt einen Betrag in Höhe der während der Vertragslaufzeit realisierten annualisierten Varianz der Log-Renditen des Basispapiers (hier: DAX)

Bei Vertragsabschluss wird der Preis so festgelegt, dass der faire Wert des Geschäfts Null beträgt.

# Variance swaps: replication

Variance Exposure as a function of stock price




Source:
Demeterfi/Derman/Kamal/Zhou

$$K_{VARS} = \frac{2}{T} e^{rT} \int_0^{F_0(T)} \frac{1}{K^2} P_{BS}(K, T, \sigma_0(K, T))\,dK$$

$$+ \frac{2}{T} e^{rT} \int_{F_0(T)}^{\infty} \frac{1}{K^2} C_{BS}(K, T, \sigma_0(K, T))\,dK$$

# Previous papers

On variance swaps:

Neuberger, A. (1994), The log contract, *Journal of Portfolio Management* (Winter), pp. 74–80.

Demeterfi, K./Derman, E./Kamal, M./Zou, J. (1999), A guide to volatility and variance swaps, *Journal of Derivatives* **6**(4), pp. 9–35.

On the variance risk premium:

Bondarenko, O. (2004), Market price of variance risk and performance of hedge funds, *Working paper*, University of Illinois.

Doran, J./Ronn, E. (2004b), *On the Market Price of Volatility Risk*, Working Paper Florida State University.

Driessen, J./Maenhout, P. (2003), *The World Price of Jump and Volatility Risk*, Working Paper University of Amsterdam.

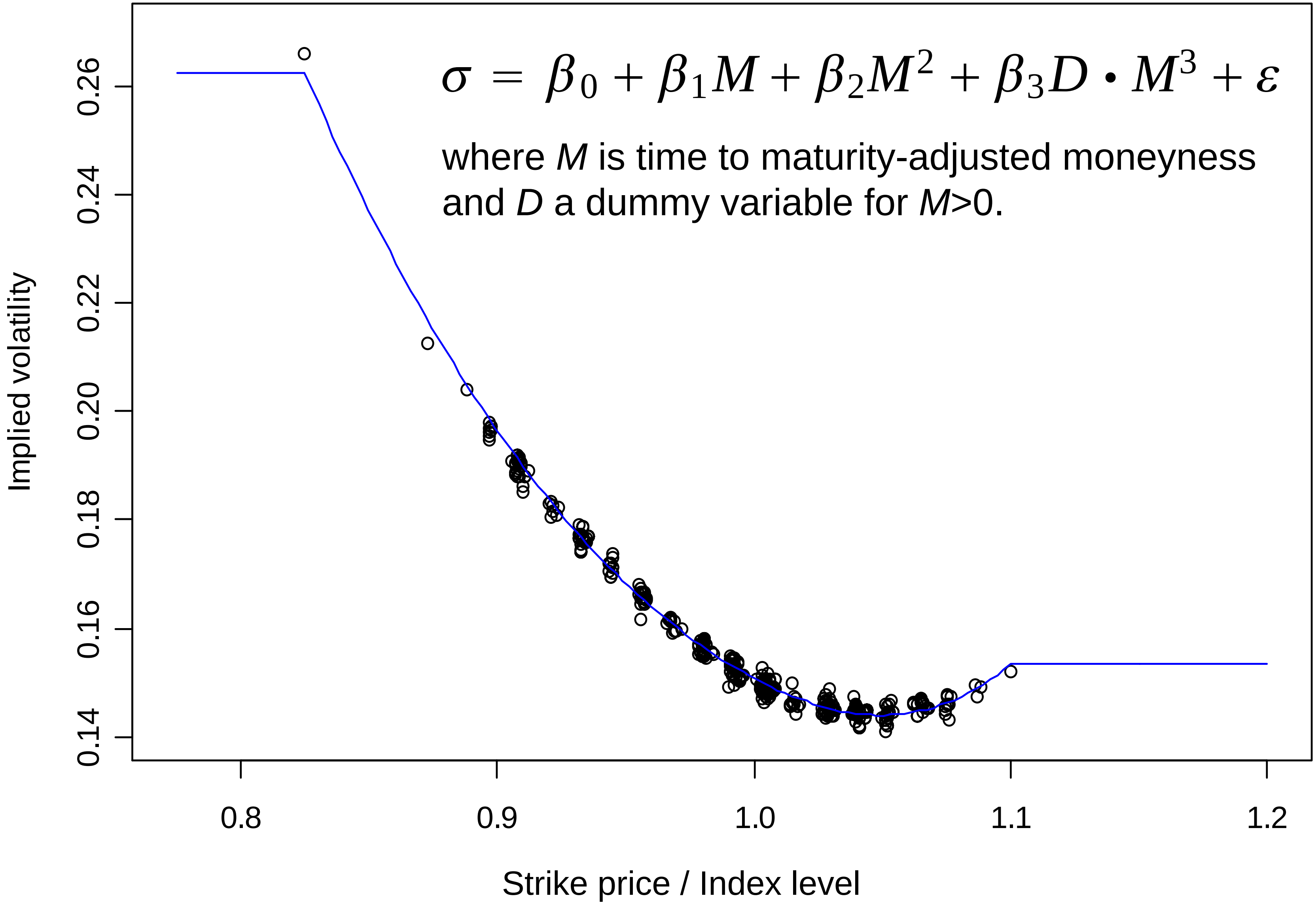
Carr, P./Wu, L. (2004), Variance risk premia, *Working paper*, Courant Institute.

# Set-up of the empirical study

1. Estimate the strike price structure of option prices (smile).
2. Calculate the fair value of variance swaps.
3. Measure realized (ex post) variance during the time to maturity.
4. Calculate the profit or loss for the buyer of a variance swap as the difference between realized variance and $K_{VARS}$.
5. Initiate such a variance swap trade on each trading day over the 10 year period from January 1995 to December 2004.

# Strike price structure of option prices

DAX option on Dec. 10, 2004 with time to maturity of 42 days



# Effect of the volatility skew on variance swap rates

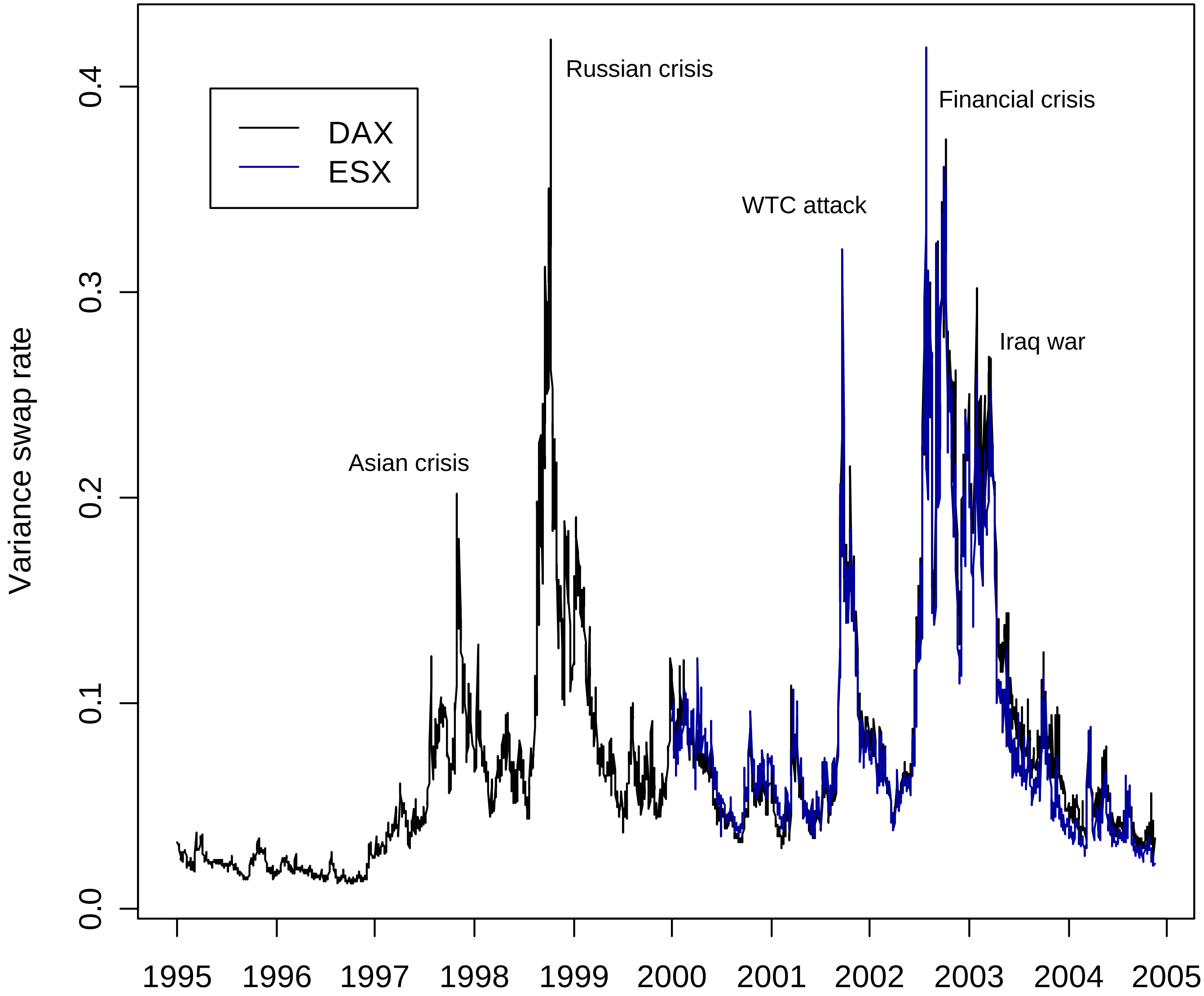
$$\sigma_{imp} = b_0 + b_1 M + b_2 M^2 + \varepsilon$$

$$M = \frac{X/F - 1}{\sqrt{T}}$$

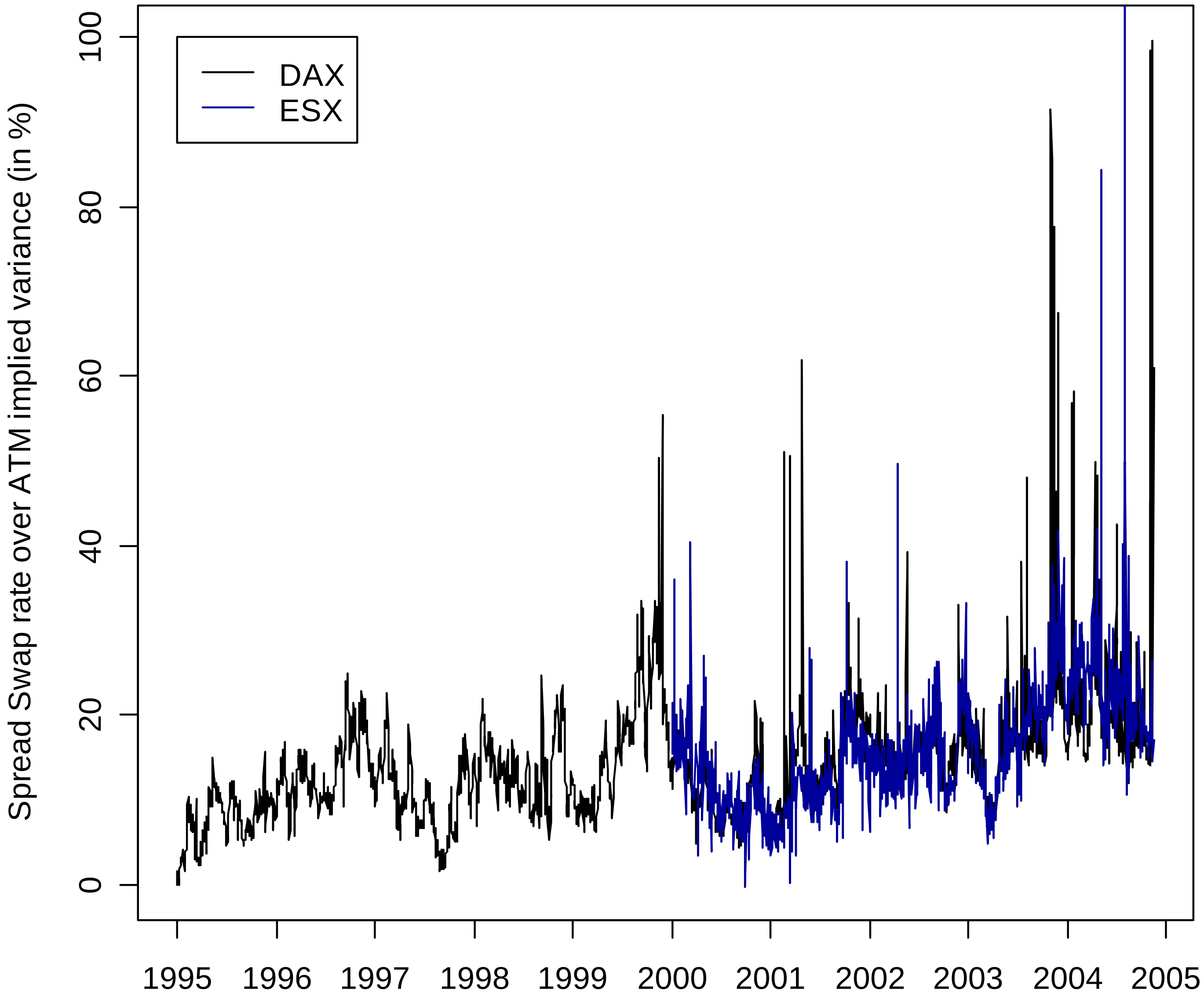
$$K_{VARS} \approx \Sigma^2 + 3b_1^2\Sigma^2 + 2b_2\Sigma^3$$

$\Sigma^2$ : Variance swap price when smile is flat at level $b_0$

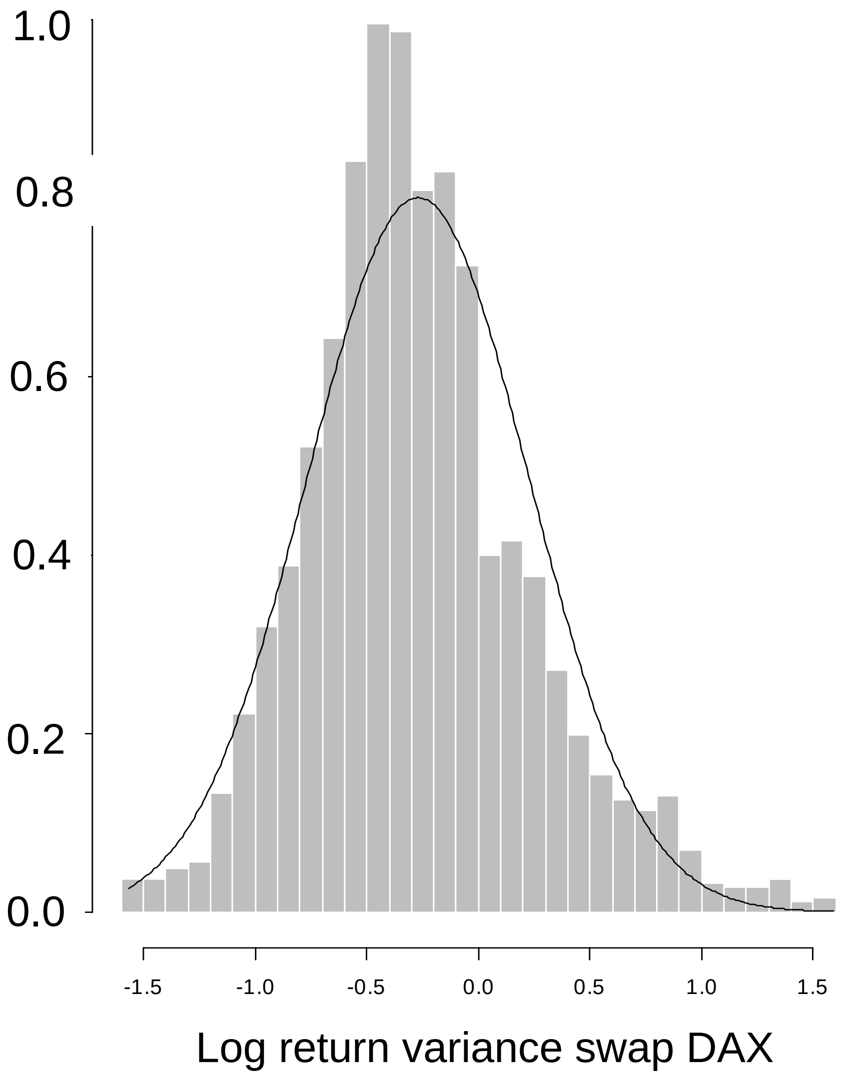
# Variance swap rates over time

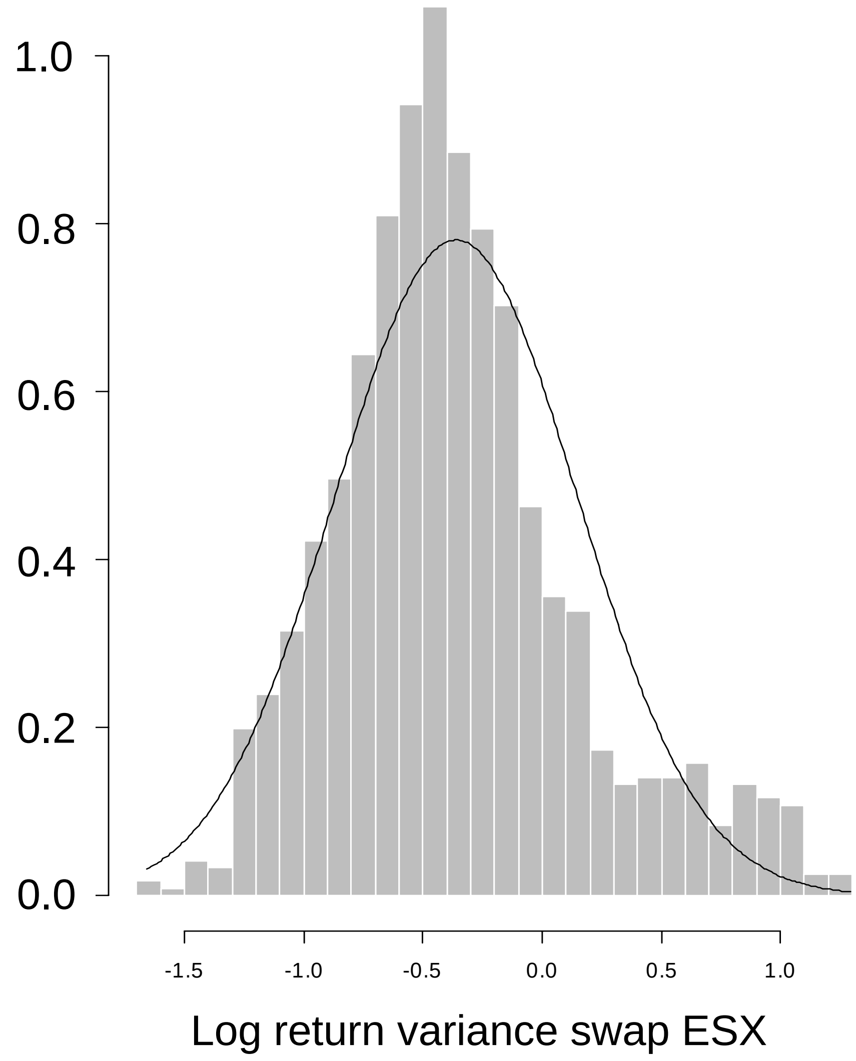


# Spread swap rate over ATM implied variance

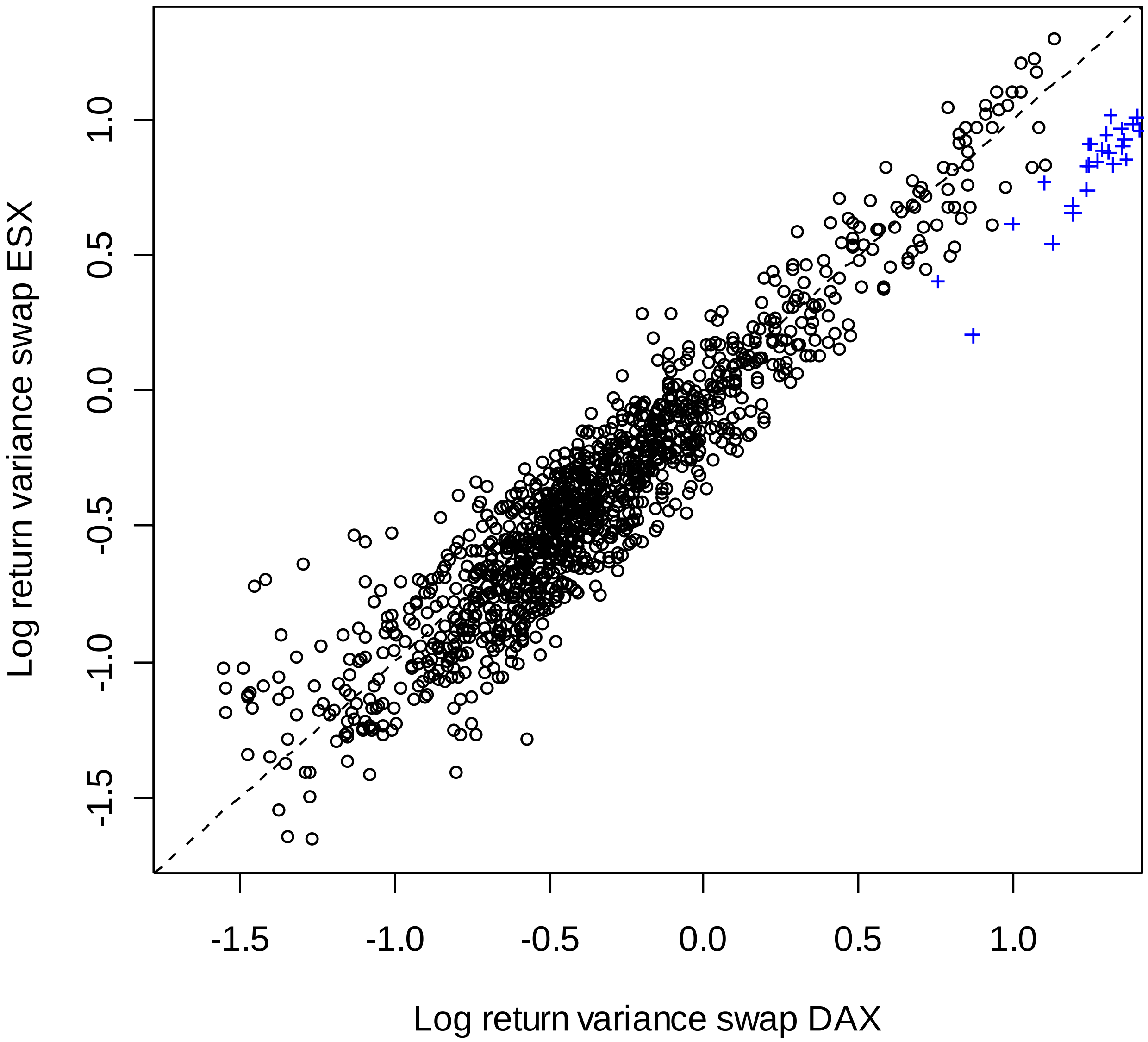


# Histograms of log returns variance swap DAX and ESX

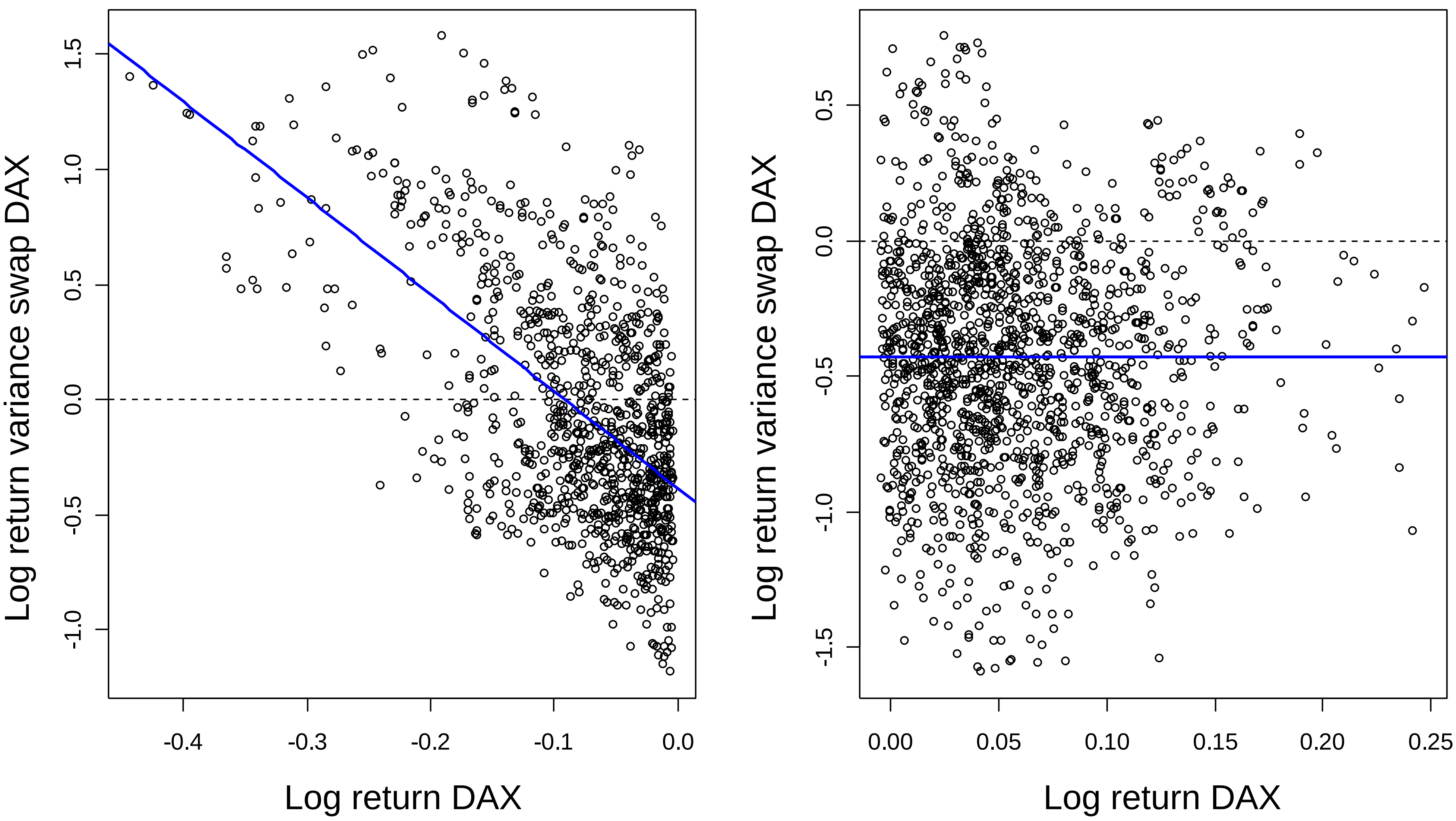


# Log returns var swap DAX versus ESX 2000 to 2004



# Relationship with index returns



# Equilibrium analysis of variance swap returns

Leland modification of CAPM: $$\alpha_{L,i} = \mathbb{E}(R_i^e) - \mathbb{E}(R_M^e)\beta_{L,i}$$

$$\beta_{L,i} = \frac{\text{cov}[R_i, -(1+R_M)^{-\theta}]}{\text{cov}[R_M, -(1+R_M)^{-\theta}]}$$

| | Single period CAPM | | | Leland-modification | | Continuous time CAPM | | |
|---|---|---|---|---|---|---|---|---|
| | $\widehat{\alpha}$ | $\widehat{\beta}$ | $R^2$ | $\widehat{\alpha}_L$ | $\widehat{\beta}_L$ | $\widehat{\alpha}_c$ | $\widehat{\beta}_c$ | $R^2$ |
| DAX 95-04 | −0.1180<br>(−2.54) | −3.29<br>(−4.25) | 0.183 | −0.1166<br>(−2.60) | −3.68 | −0.2728<br>(−6.64) | −2.93<br>(−5.75) | 0.177 |
| DAX 95-99 | −0.0340<br>(−0.58) | −3.26<br>(−4.03) | 0.229 | −0.0264<br>(−0.55) | −3.56 | −0.1619<br>(−2.97) | −3.32<br>(−5.31) | 0.207 |
| DAX 00-04 | −0.2192<br>(−3.54) | −4.12<br>(−3.43) | 0.203 | −0.2282<br>(−3.14) | −4.60 | −0.3880<br>(−6.54) | −3.51<br>(−4.71) | 0.230 |
| ESX 00-04 | −0.2641<br>(−5.05) | −3.56<br>(−3.48) | 0.225 | −0.2721<br>(−4.59) | −3.98 | −0.4369<br>(−7.45) | −3.36<br>(−4.53) | 0.222 |

**Table 6:** Equilibrium analysis of variance swap returns. MSCI world index serves as market proxy.

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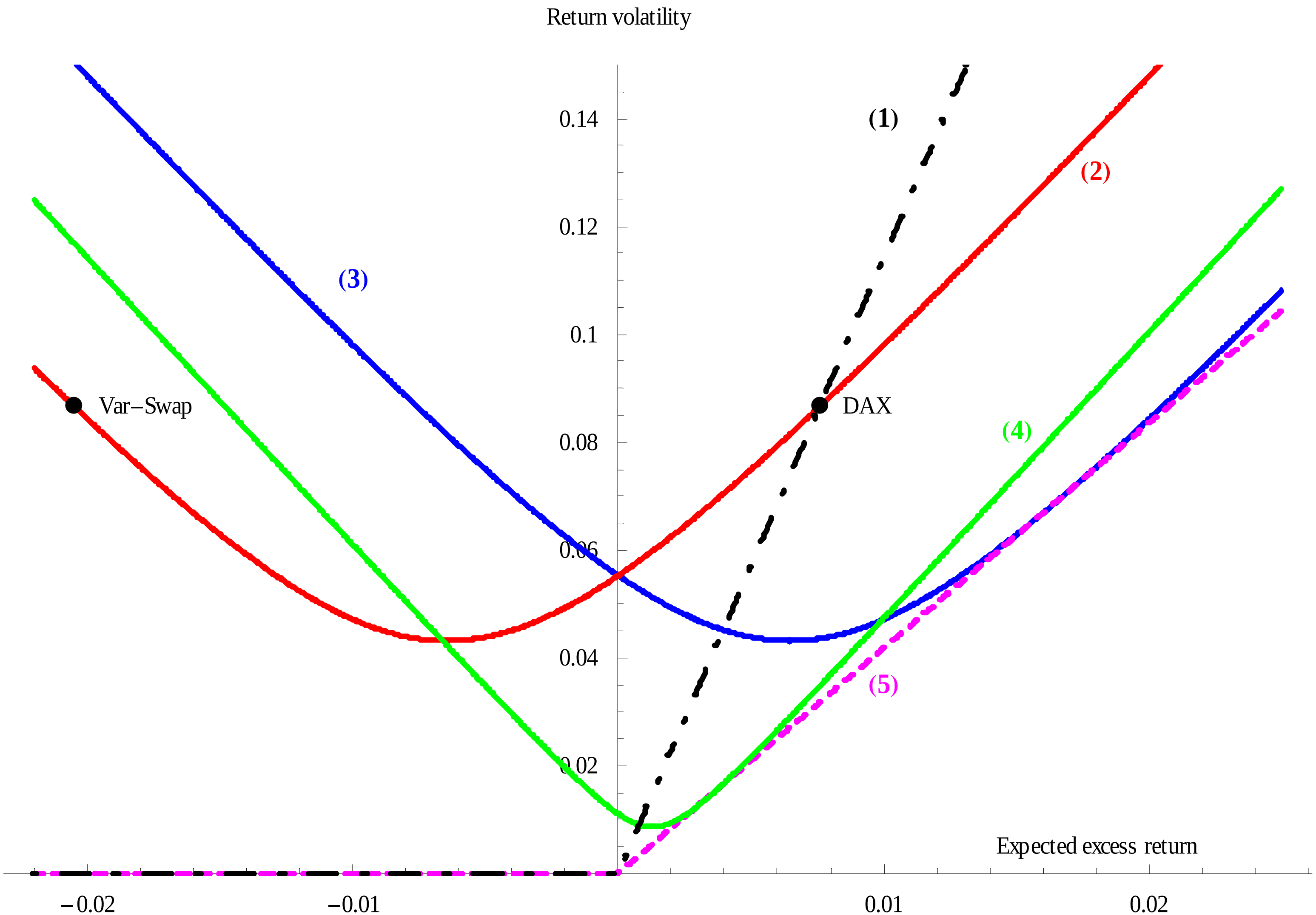
# Mean variance analysis (DAX 1995-2004)



# Characteristics of mean-variance efficient portfolios

| | $SR_0$ | Case 1: Base case | | | | | |
|---|---|---|---|---|---|---|---|
| | | $SR$ | $x_{VARS}$ | $x_S$ | $x_{rf}$ | $LL: \frac{x_{VARS}}{x_S}$ | $HL: \frac{x_{VARS}}{x_S}$ |
| DAX 95-04 | 0.0876 | 0.2394 | $< 0$ | $< 0$ | $> 1$ | 6.1140 | 0.9655 |
| DAX 95-99 | 0.3632 | 0.3743 | $< 0$ | $> 0$ | $< 1$ | −0.3394 | −0.0566 |
| DAX 00-04 | −0.1336 | 0.3728 | $< 0$ | $< 0$ | $> 1$ | 1.1667 | 0.1709 |
| ESX 00-04 | −0.1740 | 0.5924 | $< 0$ | $< 0$ | $> 1$ | 1.2438 | 0.1816 |

# Optimal portfolio weights under power utility

| | DAX 1995 - 2004 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Base case | | | +2 STD | | | -2 STD | | |
| $\alpha$ | $x_{VARS}$ | $x_S$ | $x_{rf}$ | $x_{VARS}$ | $x_S$ | $x_{rf}$ | $x_{VARS}$ | $x_S$ | $x_{rf}$ |
| 1 | -1.56 | -0.31 | 2.87 | -0.10 | 0.89 | 0.21 | -1.73 | -0.27 | 3.00 |
| 1.5 | -1.22 | -0.25 | 2.47 | -0.08 | 0.60 | 0.48 | -1.58 | -0.42 | 3.00 |
| 2 | -0.98 | -0.20 | 2.18 | -0.06 | 0.45 | 0.61 | -1.44 | -0.56 | 3.00 |
| 5 | -0.44 | -0.09 | 1.53 | -0.03 | 0.18 | 0.85 | -0.73 | -0.33 | 2.06 |
| 10 | -0.22 | -0.04 | 1.26 | -0.01 | 0.09 | 0.92 | -0.39 | -0.17 | 1.56 |

# Polynomial Goal Programming

Method to explicitly consider higher moments in portfolio optimization.

Definition of sharpe ratio (SR), skewness (SK) and kurtosis (KT):

$$SR = \frac{E[R]}{\sigma};\ SK = \frac{E\left[(R-E[R])^3\right]}{\sigma^3};\ KT = \frac{E\left[(R-E[R])^4\right]}{\sigma^4}$$

# Polynomial Goal Programming

Maximiere

$$Z(x_{VARS}, x_S) = (1 + d_{SR}(x_{VARS}, x_S))^{\alpha} + (1 + d_{SK}(x_{VARS}, x_S))^{\beta} + (1 + d_{KT}(x_{VARS}, x_S))^{\gamma}$$

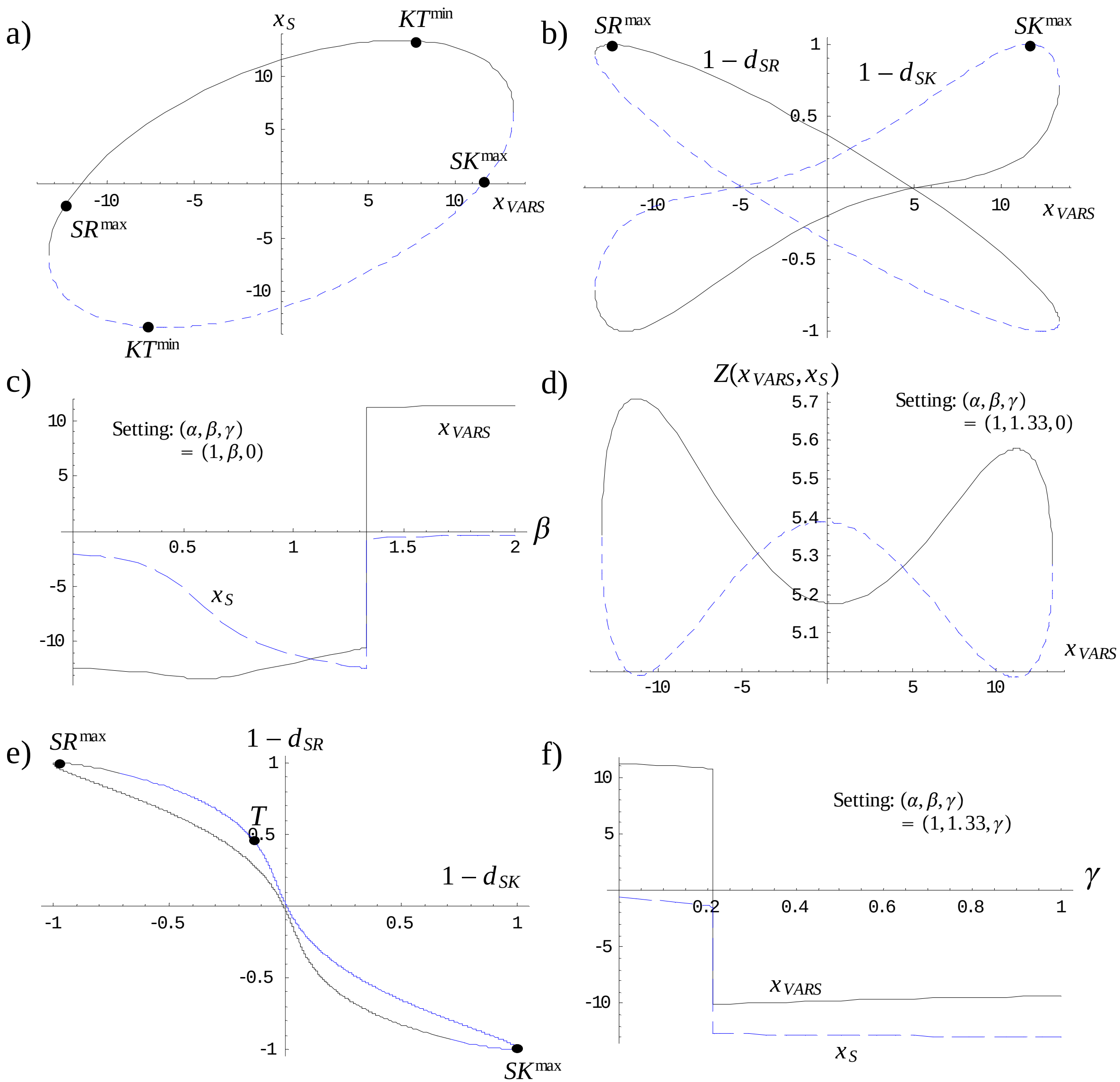
unter den Nebenbedingungen

$$d_{SR}(x_{VARS}, x_S) = \frac{SR^{\max} - SR(x_{VARS}, x_S)}{SR^{\max}},$$

$$d_{SK}(x_{VARS}, x_S) = \frac{SK^{\max} - SK(x_{VARS}, x_S)}{SK^{\max}},$$

$$d_{KT}(x_{VARS}, x_S) = \frac{KT(x_{VARS}, x_S) - KT^{\min}}{KT^{\min}}.$$

# Ergebnis der Optimierung



# Conclusion

- The profile of log variance swap returns against log index returns on average resembles the payoff of a long put position. → Crash protection.
- Due to the option-like profile of returns it is crucial to account for the non-normality of returns in measuring the performance of variance swap investments.
- The volatility risk premium at the German as well as the European stock market is strongly negative.
- Its magnitude is not compatible with standard equilibrium pricing models.
- Our backtests result in significant short volatility positions in optimal portfolios during the sample period. Typically, the stock index weight is also negative, since the diversification gain exceeds the loss in expected return.

# Conclusion

- The objectives of a high Sharpe ratio, high skewness and low kurtosis are highly conflicting. In particular, the short positions required to profit from negative variance swap returns inevitably lead to an undesired negative skewness.
- A balanced tradeoff between Sharpe ratio and skewness often does not exist. Investors tend to the extreme portfolios (Sharpe ratio driven, skewness driven or kurtosis driven) and avoid to be stuck in the middle.
- This ‘all-or-nothing' characteristic is reflected in jumps of asset weights when certain thresholds of preference parameters are crossed.