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Instrument and Target Rules
As Specifications of Optimal Monetary Policy

Richard T. Froyen
Department of Economics
The University of North Carolina
Chapel Hill, NC 27599

Alfred V. Guender
University of Canterbury
Christchurch, New Zealand

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1. INTRODUCTION

For over a decade one issue in the literature on monetary policy in New Keynesian models has been the relative merits of instrument versus target rules. Svensson (1997), (1999), (2003), (2005), Svensson and Woodford (2004), and Woodford (2003), have argued for the superiority of target rules. McCallum (1999) and McCallum and Nelson (1999), (2004), (2005) have questioned the case for target rules arguing that instrument rules may be preferable. If one asks the question of whether in general a central bank would do better with an instrument rule or a target rule it is unlikely that one can answer the question conclusively. None of the just-cited papers proceeds at that level of generality but many are wide-ranging. In this paper we begin by focusing on *optimal* instrument and target rules within a series of widely-used models for monetary policy analysis. This focus enables us to exploit the equivalence between expressions for optimal policies as settings for an instrument or as a linear combination of currently observable variables. Instrument rules are obviously of the former class. Target rules are a limiting case of the latter class, for the case where the targets are observable.[1]

The optimal target and instrument rules will for the models we consider be equivalent. By this we mean that a central bank will take the same actions whichever rule they choose. McCallum and Nelson (2005) agree with Svensson (2003, p. 439), that "commitment to an optimal instrument rule has no advocates……" Still, comparisons of suboptimal instrument rules with the optimal one will clarify why they are suboptimal - - for example, "What is Wrong with Taylor Rules?" We will also argue that optimal instrument rules are relevant to current issues concerning the conduct of monetary policy.

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[1] More generally, in models where certainty equivalence holds optimal policies will remain unchanged when expected values replace actual values in target rules, as in Svensson and Woodford (2003).

Moreover even though optimal instrument and target rules result in identical policy actions there might be advantages to one or the other for a number of reasons. Robustness to model changes and transparency are two that feature prominently in the debate.

Most generally, we focus on the information set conditioning the policy setting. The equivalence of the optimal instrument setting and a policy setting as a linear combination of observables results from their being equivalent ways to respond to the same information set - - the same optimal conditional expectation of the target variables. Suboptimal policies will result from failure to fully exploit available information. Policy rules will change when information sets change. This implies that if optimal target rules and instrument rules are equivalent they will change in exactly the same circumstances.

Our examination of these issues proceeds as follows. Section 2 sets out the terminology adopted for the analysis. Section 3 considers target and instrument rules in two standard Keynesian models: the *IS-LM* model and the *AS-AD* model. A comparison of target versus instrument rules in these simple models provides useful background for the discussion of more modern models. Section 4 focuses on the canonical forward-looking New Keynesian model. Optimal instrument and target rules are derived and compared under different assumptions about policy conduct: discretion, simple commitment and global commitment (the "timeless perspective"). Section 5 examines issues including robustness and transparency which may lead to a preference of one type of rule over the other within variations of the New Keynesian model. Section 6 contains concluding comments.

2. TERMINOLOGY

For clarity we begin by defining terms. By a rule for monetary policy we mean, as do Svensson and other participants in this debate, a prescribed guide for the conduct of monetary policy. For each type of rule we consider there is a loss function that specifies the central bank's objectives - - the target variables. This specification of the objectives of monetary policy can be called a "targeting regime" [Svensson (2005, p. 622) Walsh (2003)], for example, inflation targeting, or just an objective function. We use the latter term. The term *target rule* is reserved for a condition to be fulfilled by the target variables (or forecasts thereof). Svensson calls this a *specific* target rule.

An instrument is defined as a variable administered by the central bank or as a financial market variable the central bank controls so closely that control error can be ignored. The U.S. federal funds rate is an example of the latter type of instrument. We consider explicit and implicit instrument rules. An explicit instrument rule will be defined as one that sets the instrument directly as a function of elements of a given information set. An implicit instrument rule will be one where the instrument setting is derived from an associated target rule. An optimal instrument rule will be one that minimizes the central bank's loss (objective) function. If it is an optimal explicit instrument rule then it must employ all available information. An optimal implicit instrument rule is simply the setting that implements the optimal target rule.

A final characterization of policy we will consider is as a linear combination of currently observable variables. A prominent early example of this type of policy setting is Poole's combination policy in the *IS-LM* model. LeRoy (1975) and LeRoy and Waud (1977) clarify the relationship of Poole's combination policy and instrument rules that

exploit currently observable financial market information. Target rules are examples of this type of policy setting for the case where the current values of the target variable are observable.

3. OPTIMAL POLICY SPECIFICATIONS IN KEYNESIAN MODELS

As background to the later discussion of optimal policy rules in the New Keynesian model we consider two versions of the earlier Keynesian model.

A. The *IS-LM* Model

A stochastic version of the *IS-LM* model, as in Poole (1970), is given by

1. $y = a_0 - a_1 i + v$

2. $m = b_0 + b_1 y - b_2 i + \mathbf{\eta}$

with $E(v) = E(\mathbf{\eta}) = 0$

$$E(v^2) = \sigma_v^2 \quad E(\eta^2) = \sigma_\eta^2 \quad E(v\eta) = 0$$

Where we ignore time subscripts and where: y = output; i = the interest rate; m = money supply.

The policymaker's objective function is

3. $\text{Min } E[L_t] = E[(y_t - E[y_t])^2] = \sigma_y^2$

The Optimal Instrument Rule

Substitution of (1) into (2) yields

2. $m = b_0 + a_0 b_1 - (b_2 + a_1 b_1)i + (\mathbf{\eta} + b_1 v)$

The assumption in Poole (1970) was that the financial market variables (m and i) were currently observable but the target variable (y) was not.[2] With this assumption the central bank can observe $(\mathbf{O} + b_1 v)$.

Using this information, the policymaker can form the optimal conditional expectation of the target variable (y) and construct the optimal instrument rule.

If we assume that the interest rate is chosen as the instrument, the optimal rule will be

4. $\qquad i* = -a_1^{-1}\left[y* - a_0 - E_t v\right]$

where the subscript t is used on the expectations operator to indicate the expectation conditional on available current information. Using the standard formula for conditional expectations this term is

5. $\qquad E_t\left(v|\eta + b_1 v\right) = \dfrac{b_1 \sigma_v^2 \left(\eta + b_1 v\right)}{b_1^2 \sigma_v^2 + \sigma_\eta^2}$

<u>Optimal Policy as a Linear Combination of Observable Variables</u>

An alternative expression for the optimal policy is as a linear combination of observable variables. If we continue to assume that the target variable (y) is unobservable, while m and i are observed, this results in Poole's (1970) combination policy which is of the form

6. $\qquad m = \mathbf{8}_0 + \mathbf{8}i$

The optimal combination policy can be derived by substituting equation (6) into equation $(2')$ for m and proceeding to calculate the values of $\mathbf{8}_0$ and $\mathbf{8}$ that minimize the

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[2] Papers such as Friedman (1975) made the more realistic assumption that a reserve aggregate was currently observable but the money supply was not.

loss function (3). Alternatively, the optimal combination policy can be expressed directly from the optimal instrument rule (4). Using (5) in (4) we solve for $(O + b_1 v)$ and substitute the result into (2'). With some rearrangement this yields

$$7. \qquad m = b_0 + y*b_1 + \frac{\sigma_\eta^2}{b_1 \sigma_v^2}(y* - a_0) + \frac{a_1 \sigma_\eta^2 - b_2 b_1 \sigma_v^2}{b_1 \sigma_v^2} i$$

Equation (7) and the optimal combination policy derived directly from equation (6) are identical. This is the result in LeRoy (1975) and LeRoy and Waud (1977). Both specifications exploit the information content of observable variables to learn about the realizations of the error terms affecting the target variable.

A Target Rule

In the fixed price *IS-LM* model a target rule would simply be

$$8. \qquad y = y*$$

To be operational, contrary to the assumption in the Poole framework, output must be assumed to be observed. There is no substantive policy problem. The target rule would be equivalent to the optimal instrument rule (4) with the actual value of v replacing its expectation.

B. Aggregate Supply – Aggregate Demand Framework

For two decades a workhorse model for optimal monetary policy was the aggregate demand and supply framework. A version of the model consists of the following three equations:

$$9. \qquad y_t = c_0 + c_1 \left(p_t - p_{t,t-1}^e \right) + u_t$$

$$10. \qquad y_t = a_0 - a_1 \left(i_t - \left(p_{t+1,t-1}^e - p_{t,t-1}^e \right) \right) + v_t$$

$$11. \qquad m_t = p_t + b_0 + b_1 y_t - b_2 i_t + {}_t$$

where new variables and error terms are:

$$p_t = \text{aggregate price level}$$

$$p^e_{t+j,t-1} = \text{rational expectation of p for } t + j; j = 0,1 \text{ taken at t-1.}$$

$$u_t = \text{supply shock with mean zero variance } \sigma^2_u \text{ and zero covariances}$$
$$\text{with other disturbances in the model.}$$

Equations (10) and (11) are modified *IS* and *LM* equations from the previous sub section. The real interest rate replaces the nominal rate in the *IS* equation (10). The price level now appears in the *LM* equation. The new equation (9) is a Lucas-type supply specification.

The loss function to be minimized is

12. $\quad E[L_t] = V(y_t) + V(p_t)$

We consider the same three types of optimal policy specifications that we did for the simple *IS-LM* model. We do this in a simplified set-up of the model:

13. $\quad p_t = ky_t + u_t$

14. $\quad y_t = -a_1 i_t + v_t$

15. $\quad m_t = p_t + b_1 y_t - b_2 i_t + \epsilon_t$

In (13-15) each variable is replaced by the deviation from its expected value.[3] The supply specification (13) is rearranged with the price level (in deviation form) on the left-hand side. This rearrangement facilitates comparisons with the New Keynesian model in Section 4.

Policy as an Optimal Instrument Rule

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[3] We use the same symbols for variables in deviation form with hopefully no confusion.

As with the *IS-LM* model we assume that the financial market variables m and i are observed. Output (y) and the price level (p) are assumed to be unobserved. Substitution of (13) and (14) into (15) yields

16. $m_t = (b_2 - a_1(k + b_1))i_t + (k + b_1)v_t + u_t + \eta_t$

and thus an observation on a linear combination of the model's three structural disturbances.

With the interest rate as the chosen instrument, the optimal instrument rule will then be of the form

17. $i_t = \gamma_1 E_t v_t + \gamma_2 E_t u_t$

where the expectation of each of the disturbances relevant to the loss function (12) are conditional on the linear combination of the error terms in (16).

Combining the instrument rule (17) with the *IS* equation (14) and substituting for the conditional expectations yields the reduced form equation for output.

18. $y = \dfrac{1}{D}\big((W + Z)v - a_1 X(\eta + u)\big)$

From the (13) reduced form for the price level is:

19. $p = \dfrac{k}{D}\big((W + Z)v - a_1 X\eta)\big) + \dfrac{1}{D}(D - ka_1 X)u$

where $W = (k + b_1)^2 \sigma_v^2 (1 - a_1\gamma_1) \quad Z = (1 - a_1\gamma_2(k + b_1))\sigma_u^2 + \sigma_\eta^2$

$X = \gamma_1(k + b_1)\sigma_v^2 + \gamma_2\sigma_u^2 \quad D = (k + b_1)^2 \sigma_v^2 + \sigma_u^2 + \sigma_\eta^2$

From (18) and (19) we calculate the variances: of y and p under an instrument rule.

20. $V(y) = \dfrac{1}{D^2}\big[(W + Z)^2\sigma_v^2 + (a_1 X)^2(\sigma_\eta^2 + \sigma_u^2)\big]$

21. $\quad V(p) = \dfrac{1}{D^2} \left[k^2 (W + Z)^2 \sigma_v^2 + (ka_1 X)^2 \sigma_\eta^2 + (D - ka_1 X)^2 \sigma_u^2) \right]$

Then, minimizing the loss function (12) with respect to the parameters of the rule yields:

$$\gamma_1^* = \frac{1}{a_1} \qquad \gamma_2^* = \frac{\mu k}{a_1 (1 + \mu k^2)}$$

Policy as a Linear Combination of Observable Variables

As in the previous sub section, we assume that the financial market variables m and i are observable while output and the price level are not. In this case the optimal combination policy is still of the form given by (6), now with time subscripts.

(6) $M_t = {}_0 + \ i_t$

The model is then composed of equations (13) – (15) and (6).

We solve for y_t and p_t, take their variances and substitute the result into the loss function (12). Minimization with respect to   yield the optimal value of the combination parameter:

22. $\quad \lambda^* = \dfrac{a_1 (1 + \mu k^2)\sigma_\eta^2 + (a_1 - (a_1 b_1 + b_2)k\mu)\sigma_u^2 - b_2 (b_1 + k)(1 + \mu k^2)\sigma_v^2}{\mu k \sigma_u^2 + (b_1 + k)(1 + \mu k^2)\sigma_v^2}$

The optimal combination parameter is a function of all the parameters of the model and the variances of the model's structural disturbances. It is further the case that at this optimal value of , the variances of p_t and y_t are equal to those at the optimal values of ${}_1$ and ${}_2$ for the instrument rule. The two policies are equivalent.

<u>Policy as a Target Rule</u>

Policy as an optimal target rule is of more interest in the *AS-AD* framework than in the *IS-LM* model. In the *AS-AD* model there is a substantive policy choice even if price and output are observed; there is a price-output tradeoff.

In line with the literature on New Keynesian models, a target rule would be of the form:

23. $y_t + p_t = 0$

The Optimal Target Rule if p_t and y_t are Observed:

An optimal target rule can be viewed as an optimal policy conditioned on a linear combination of observable variables where the target variables themselves are observable. The observability of p_t and y_t is required to enable the policymaker to achieve a non random linear combination of two endogenous variables subject to stochastic shocks.

If p_t and y_t are observable and the interest rate is the policy instrument, the model comprises (13), (14), (23) and the loss function (12). Substituting (13) and (14) into the target rule and solving for the interest rate yields the instrument setting

24. $i_t = \dfrac{1}{a_1} v_t + \dfrac{1}{(\theta + k)a_1} u_t$

Substituting (24) back into the *IS* equation (14) and solving for y_t and p_t yields

25. $y_t = -\dfrac{u_t}{\theta + k}$

26. $p_t = \dfrac{\theta u_t}{\theta + k}$

If we then compute the variances of the two target variables, substitute them into the loss function (12) and minimize with respect to , we find the optimal parameter of the target rule

27. $\theta^* = \dfrac{1}{\mu k}$

This result is analogous to the optimal policy setting in New Keynesian models. Notice that the only parameter that matters (apart from the preference parameter μ) is k. Neither *IS* nor *LM* parameters affect the optimal policy setting. The policymaker moves the output gap, via movements in the policy instrument, to a degree that depends on his ability to affect the price level and the relative weight he attaches to the variance of the price level in the expected loss function. The relative size of the variance of the stochastic disturbances also play no role in the optimal target rule.

The Optimal Target Rule if p_t and y_t are Not Observed:

If p_t and y_t are not observed then the target rule (23) cannot be implemented. Such a target rule is not a monetary policy rule in accord with our definition in Section 2 (or Svensson's) as a prescribed guide for the conduct of monetary policy.

An implementable target rule is

28. $E_t y_t + E_t p_t = 0$

We assume, as in previous sections, that while p_t and y_t are not observable, the financial market variables i_t and m_t are observed.[4]

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[4] As in the earlier sections, it would be more realistic to distinguish between a bank reserve measure and a money aggregate and assume the former to be observed.

Combining the *IS* equation (14) with the *LM* and aggregate supply equations [(13) and (15)] yields a relationship between the observable variables m_t and i_t and a linear combination of the error terms.

29. $\quad m_t = -((k + b_1) a_1 + b_2)i_t + (k + b_1)v_t + \eta_t + u_t$

From (29) we observe $((k + b_1)v_t + \eta_t + u_t)$.

Forming conditional expectations of the disturbances yields:

30. $\quad E_t v_t = \dfrac{(k + b_1)\sigma_v^2}{(k + b_1)^2 \sigma_v^2 + \sigma_\eta^2 + \sigma_u^2} ((k + b_1)v_t + \eta_t + u_t)$

31. $\quad E_t u_t = \dfrac{\sigma_u^2}{(k + b_1)^2 \sigma_v^2 + \sigma_\eta^2 + \sigma_u^2} ((k + b_1)v_t + \eta_t + u_t)$

Supposing that the interest rate is still the instrument we now have the reaction function

32. $\quad i = \dfrac{1}{a_1}E_t v_t + \dfrac{1}{(\theta + k)a_1}E_t u_t$

There are several points to note with reference to the policy set-up given by the target rule (28) and equations (30) – (32) which complete the description of monetary policy. First, if we proceed to solve the model for y_t and p_t, compute their variances and substitute into the loss function (12), we can optimize to find the optimal . This value will be the same value as in (25) where p_t and y_t are observed. This follows from the certainty equivalence property of the model due to the quadratic loss function and additive disturbances. Second, while the optimal value of theta is unchanged and depends only on and k, all the parameters of the model and variances of the shocks enter the policy rule via the formation of expectations. These expectations are a necessary component of the policy rule – a *target rule* (28). Finally, substitution of the optimal

value of into the reaction function (32) for the target rule yields the implicit instrument rule corresponding to the target rule (28). This results in

$$33. \quad i_t = \frac{1}{a_1} E_t v_t + \frac{\mu k}{a_1 (1 + \mu k^2)} \ E_t u_t$$

which is optimal explicit instrument rule derived earlier. With the same information assumptions, the three policies - - an explicit instrument rule; a combination policy, and a target rule are identical.

4. THE NEW KEYNESIAN MODEL

The debate about instrument versus target rules has taken place mostly within the New Keynesian model. Here we use a simple version of that model that has been termed the *canonical form* to examine alternative specifications of the optimal policy rule.

A. **The Case where the Inflation Rate and Output Gap are Currently Observed**

The canonical version of the New Keynesian model is given by the following structural equations

$$34. \quad y_t = -a_1 (i_t - E_t \pi_{t+1}) + E_t y_{t+1} + v_t$$

$$35. \quad \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t$$

where y_t is now the output gap and π_t is the rate of inflation. The other symbols are as defined in previous models. The policymaker's model's loss function is given by

$$36. \quad E[L_t] = V(y_t) + \mu V(\pi_t)$$

Equation (34) is the standard forward-looking *IS* equation and equation (35) is a forward-looking Phillips curve. The loss function can be justified by a pragmatic argument that inflation and the output gap are recognized policy objectives. Alternatively, Rotemberg and Woodford (1999) provide a more formal justification of (36) as an approximation of a utility-based loss function.

<u>The Optimal Target Rule</u>

First consider a simple case where the disturbances are white noise processes and the discount factor (β) is set to one. Additionally, we restrict our attention to a target rule.[5]

37. $y_t + \pi_t = 0$

This form of a target rule represents *simple* commitment, where as Clarida, Gali and Gertler (1999, p. 1678) explain this restricts "the form of the policy rule to the general form that arises in equilibrium under discretion." We consider a broader class of globally optimal rules in the next section.

Substitution of (34) and (35) into the target rule (37) and solution for implicit instrument rule yields

38. $r_t = i_t - E_t \pi_{t+1} = \dfrac{1}{(\theta + \kappa) a_1} E_t \pi_{t+1} + \dfrac{1}{a_1} (E_t y_{t+1} + v_t) + \dfrac{1}{(\theta + \kappa) a_1} u_t$

where r_t, the real interest rate, will be assumed to be the policy instrument.

Substituting (38) into the *IS* relationship (34), using the result in the Phillips curve (35) and the fact that with white noise processes

$E_t(\pi_{t+1}) = E_t(y_{t+1}) = 0$

yields

39. $\pi_t = \dfrac{\theta}{\theta + \kappa} u_t$

40. $y_t = \dfrac{1}{\theta + \kappa} u_t$

Putting these solutions into the loss function (36) and minimizing over θ yields

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[5] We will continue to use θ as the parameter in the target rule.

41. $$\theta^* = \frac{1}{\mu\kappa}$$

The optimal target rule depends only upon the relative weight given to inflation in the policymakers loss function (μ) and the parameter giving the terms of the output gap – inflation tradeoff (κ)

The Optimal Explicit Instrument Rule

Consistent with the information assumption under the target rule, namely that the current state of the economy is observed, the optimal instrument rule is of the form

42. $r_t = \gamma_1 v_t + \gamma_1 u_t$

where constant terms in the model are ignored and again the error terms are assumed to be white noise processes.[6]

Combining (42) with the *IS* equation (34) and the Phillips curve (35), noting that with white noise disturbances $E_t \pi_{t+1} = E_t y_{t+1} = 0$, yields the solutions for the output gap and inflation rate

43. $$y_t = (1 - a_1\gamma_1)v_t - a_1\gamma_2 u_t$$

44. $$\pi_t = \kappa(1 - a_1\gamma_1)v_t + (1 - \kappa a_1\gamma_2)u_t$$

Computing the variances of the output gap and rate of inflation, substituting these values into the loss function (36), and minimizing yields the optimal γ_s

45. $$\gamma_1^* = \frac{1}{a_1} \quad \gamma_2^* = \frac{\mu\kappa}{a_1(1+\mu k^2)}$$

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[6] As with the target rule, in this section we restrict the analysis to the class of rules typically considered in analysis of policy under discretion. Globally optimal rules under commitment are considered in the next section. With white noise disturbances, equation (42) is the optimal form of the instrument rule under discretion or simple commitment.

The optimal explicit instrument rule [(42) with the optimal γ_s inserted] is identical to the implicit reaction function corresponding to the optimal target rule (38) in the previous subsection, once we insert $\theta^* = \dfrac{1}{\mu k}$ and $E_t y_{t+1} = E_t \pi_{t+1} = 0$.

B. The Case of Less than Complete Information about the Output Gap and Inflation

It is obviously unrealistic to assume that the current state of the economy is perfectly observable. Feasible versions of the optimal target and instrument rules will employ optimal conditional expectations of the target variables or realizations of the error terms. The feasible target rule would be

46. $\theta^* E_t y_t + E_t \pi_t = 0$

The optimal explicit instrument rule is

47. $r_t = \gamma_1^* E_t v_t + \gamma_2^* E_t u_t$

The θ^* and γ_s^* will be the same values as in (41) and (45), respectively. This follows from the certainty equivalence properly of the model.

One possible informational assumption is that financial market variables are observed and can be used to form the conditional expectations in (46) and (47). A variant of this assumption would be that the model contained a portfolio balance schedule, e.g., an *LM* equation. But the presence of observable financial variables in addition to money and of other information (or indicator) variables leads to the general approach set out in Svensson and Woodford (2003), (2004). The solution of this information problem leads to conditional expectations of the output gap and inflation rate that depend on the parameters of the model, variances of structural shocks: and of the error terms attached to information variables as estimates of the state variables.

C. <u>Globally Optimal Rules: The Timeless Perspective</u>

Previously in this section attention was confined to the class of rules that arise in considering policy under discretion. Specifically, these rules do not allow for policies to manage expectations in forward-looking models. For policy under commitment this case has been termed simple commitment. In this section we consider target and instrument rules that are globally optimal in the sense that they allow for managing expectations for future periods. For target rules, Woodford has termed this the *timeless perspective*.

<u>The Optimal Target Rule</u>

The discussion of optimal policy from the timeless perspective necessitates casting the policy problem into an intertemporal framework. The policymaker minimizes an intertemporal loss function that includes the target variables: the output gap and the rate of inflation

48. $\displaystyle \operatorname*{Min}_{\pi, y} \ E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j}^2 + \mu \pi_{t+j}^2) \quad 0 \le \beta \le 1$

The parameter β is the discount factor. As in the previous section, the respective target value for the output gap and the rate of inflation is zero. The policymaker minimizes the above loss function with respect to the target variables subject to the constraint imposed by the Phillips Curve (35), rewritten here

49. $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t$

Hence the Lagrangean for the policy problem becomes:

50. $\displaystyle \Gamma_t = E_t \{ \sum_{j=0}^{\infty} \beta^j [(y_{t+j}^2 + \mu \pi_{t+j}^2) + \lambda_{t+j} (\beta \pi_{t+j+1} + \kappa y_{t+j} + u_{t+j} - \pi_{t+j})] \}$

The global or timeless perspective indicates that the policymaker ignores the optimizing condition for the rate of inflation that prevails in period *1*. Accordingly, combining the first-order condition for inflation *(t=2,3,....)* with that for the output gap *(t=1,2,3.....)* yields the target rule that guides optimal policy from a timeless perspective:[7]

51. $$\frac{1}{\mu\kappa}(y_t - y_{t-1}) + \pi_t = 0$$

In this target rule the "past" - represented by the lagged output gap – matters for setting optimal policy in the present. The relative weight on the *change* in the output gap in (51) is the same as the relative weight on the output gap proper that guides policy under simple commitment (equation 37).

Next, substitute the Phillips Curve (setting $\beta = 1$) and the *IS* relation into the optimal policy rule and solve for the interest rate:

52. $$r_t = \frac{1}{a_1}(E_t y_{t+1} + v_t) + \frac{1}{a_1(\theta+\kappa)}(E_t\pi_{t+1} + u_t - \theta y_{t-1}) \qquad\qquad \theta^* = \frac{1}{\mu\kappa}$$

Equation (52) is the policymaker's reaction function (or implicit instrument rule) under the timeless perspective. The above equation is in turn substituted into the *IS* relation to obtain the reduced form equation for the output gap:

53. $$y_t = \frac{\theta}{\theta+\kappa} y_{t-1} - \frac{1}{\theta+\kappa}(E_t\pi_{t+1} + u_t)$$

Combining equation (53) with the policy rule (equation 51) yields the reduced form equation for the rate of inflation:

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[7] For a step-by-step derivation of the globally optimal target rule see Froyen and Guender (2007), McCallum and Nelson (2004) or Woodford (1999).

54. $\quad \pi_t = \dfrac{\kappa\theta}{\theta+\kappa} y_{t-1} + \dfrac{\theta}{\theta+\kappa}(E_t\pi_{t+1} + u_t)$

Application of the method of undetermined coefficients results in the final form equation for the output gap and the rate of inflation. For the case of white-noise disturbances, the output gap and the rate of inflation under policy from the timeless perspective evolve as follows:

55. $\quad y_t = \dfrac{1}{\tau} y_{t-1} - \dfrac{\mu\kappa}{\tau} u_t$

56. $\quad \pi_t = \dfrac{(\tau-1)}{\tau\mu\kappa} y_{t-1} + \dfrac{1}{\tau} u_t$

where $\quad \tau = 1 + \dfrac{\kappa^2\mu}{2} + \dfrac{\kappa\mu\sqrt{\kappa^2 + \dfrac{4}{\mu}}}{2} > 1$

After using (55) and (56) to eliminate the conditional expectations of the output gap and the rate of inflation in (52), we can restate the reaction function (implicit instrument rule as):

57. $\quad r_t = \dfrac{1}{a_1}(v_t + \Lambda(\mu\kappa u_t - y_{t-1}))$

where $\quad \Lambda = \dfrac{1}{\tau^2(1+\mu\kappa^2)}(\tau(\tau-1) - \mu\kappa^2)$

The implicit instrument rule has the interest rate respond mechanically to an *IS* disturbance. The policymaker's preference parameter conditions the interest rate response to the cost-push shock and to the lagged output gap.

The Optimal (Explicit) Instrument Rule

Inspection of (57) reveals that the implicit instrument rule responds to the shocks of the model economy and the lagged output gap. The lagged output gap appears in the

implicit instrument rule because it forms part of the optimal target rule. The question arises of why (or how) policymaker choosing an explicit instrument rule would arrive at this specification. Why would the rule under simple commitment (42, with 45 inserted) not still be followed?

The target rule is what Woodford (2003) terms a "higher level" description of policy. It comes directly from the first-order conditions derived in the previous subsection. The choice of the information set for the instrument rule is less clear cut.

In particular, the new Keynesian model we are considering contains no lagged endogenous variables and the shocks are white noise processes. So there is no obvious reason for an inertial component in the explicit instrument rule. Still, a number of papers [e.g. Turnovsky (1980), Weiss (1980), Canzoneri, Henderson and Rogoff (1983)] consider instrument rules with inertial elements. Moreover, in these papers the role of the inertial elements is, as in the case of the target rule under the timeless perspective, to influence the formation of expectations. We return to the issue of advantages of a "higher level" policy formulation in Section 5. here we simply assume that the explicit instrument rule responds to the name information set as the globally optimal target rule.

This point of departure yields the following specification of the explicit instrument rule:

58. $r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_3 y_{t-1}$

The above explicit instrument rule is substituted into the *IS* relation. The resulting equation is then combined with the Phillips Curve. Applying the method of undetermined coefficients allows us to solve the model for the variances of the endogenous variables:

59. $\quad V(y_t) = \dfrac{\phi_{12}^2 \sigma_u^2}{1 - \phi_{13}^2}$

60. $\quad V(\pi_t) = (\phi_{22}^2 + \dfrac{\phi_{23}^2 \phi_{12}^2}{1 - \phi_{13}^2})\sigma_u^2$

where $\quad \phi_{12} = -\dfrac{a_1\gamma_2}{1 - \phi_{13}} \qquad\qquad \phi_{13} = \dfrac{1 + \sqrt{1 + a_1\gamma_3}}{2} \qquad\qquad \phi_{11} = \phi_{21} = 0$

$$\phi_{22} = -\kappa a_1\gamma_2(\dfrac{\phi_{13} - a_1\gamma_3}{(1 - \phi_{13})^2} + 1) + 1 \qquad\qquad \phi_{23} = \kappa(\dfrac{\phi_{13}^2 - a_1\gamma_3}{1 - \phi_{13}})$$

and

61. $\quad y_t = \phi_{11}v_t + \phi_{12}u_t + \phi_{13}y_{t-1}$

62. $\quad \pi_t = \phi_{21}v_t + \phi_{22}u_t + \phi_{23}y_{t-1}$

The policymaker's objective can then be stated as follows:

63. $\quad \underset{\gamma_1,\gamma_2,\gamma_3}{\text{Min}} \ E[L_t] = V(y_t) + \mu V(\pi_t)$

The optimal settings of the policy parameters that minimize the above expected loss

function are:

$$\gamma_1^* = \dfrac{1}{a_1} \qquad \gamma_2^* = \dfrac{(1 - Z)^3(3 + Z)\kappa\mu}{8a_1(1 + 2a_1\gamma_3 - Z + 2\kappa^2\mu)} \qquad \gamma_3^* = \dfrac{2\mu\kappa^2 - (X - \mu\kappa^2)(1 + \mu\kappa^2)}{2a_1}$$

$$Z = \sqrt{1 + 4a_1\gamma_3} \qquad X = \kappa\sqrt{\mu(4 + \mu\kappa^2)}$$

It remains to be seen whether the γ_i^* $i = 1,2,3$ produce the optimal response to the *IS*

shock, the cost-push, and the lagged output gap that emerges under the optimal target

rule. Given the complexities of the coefficients in the two rules, the performance check is

best done by numerical analysis. The results of this exercise appear in the top two panels

of Table I.[8] Two important results stand out. First, the explicit instrument rule (labeled

Case I) elicits the same response to the shocks and lagged output gap as the implicit

instrument rule under the target rule (the top panel). Second, as a direct consequence, the

variances of the endogenous variables and of the policy instrument are the same under

both types of rules. In sum, the optimal explicit instrument rule replicates the behavior of

the target variables and the policy instrument under the target rule. There is no difference

between the two approaches as long as the specification of the optimal explicit instrument

rule relies on the same information set as the target rule approach.

5. KEY ISSUES

Previous sections of this paper illustrate the equivalence of optimal target and

instrument rules is a number of widely used models. This section considers possible

reasons why one class of rules might still be preferred to another. Taking a monetary

policy rule as a prescribed guide for the conduct of monetary policy, we mean by

"equivalence" that within each of the models considered the two types of policy rules are

equivalent if they result in the same policies actions by a central bank. Along other

dimensions there may be relative advantages to one type of rule. The other dimensions

we consider are robustness, transparency, and the "higher order" of one type of rule. We

begin with a different type of comparison that of optimal target rules with simple (or

suboptimal) instrument rules. This issue has received attention most often with regard to

so-called Taylor rules.

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[8] The remaining three panels on the table will be discussed in Section 5.

A. **What is Wrong with Taylor Rules?**

This sub section heading is taken from Svensson's (2003) paper. Our analysis leads to a fairly simple answer which does not seem contradictory to Svensson's. Taylor's rule and other simple rules will in many model's not be optimal instrument rules because they do not efficiently exploit all available information. Svensson (2001) provides an example of this in the context of an open-economy model. A Taylor rule responds to only a small subset of the models 11 state variables. The optimal reaction function responds to all of them.[9]

Moreover, even in models where inflation and the output gap are the only state variables, it is as Svensson (1999, p. 608) points out "usually inefficient to let the instrument respond to target variables, compared to letting the instrument respond to the determinants of the target variables." In the forward-looking new Keynesian model in Section 4.A, for example, the implicit instrument rule derived from the optimal target rule (equation 38) responds to the current values of the *IS* and Phillips curve disturbances. The *IS* disturbance is simply offset and the response to the Phillips curve disturbance results in the optimal trade-off in the response of the output gap and inflation rate. A Taylor type rule for the model is

$$(38') \quad r_t = \bar{r} + \lambda_1\left(\pi_t - \pi^*\right) + \lambda_2 y_t$$

With this rule to completely offset the effect of the *IS* disturbance requires $\lambda_2 \quad \infty$. This rules out the optimal response to the Phillips curve disturbance. There is less information

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[9] As an illustration of this point, in Ball (1999b), which is a simple backward-looking New Keynesian model, the optimal reaction function is a Taylor-type rule. In Ball (1999a), an open-economy version of the same model, the exchange rate is an additional information variable and the optimal policy becomes an MCI (monetary conditions index) which contains the real exchange rate.

content in the realized value of the target variables than in the realized values of the individual shocks.

A third inefficiency in the Taylor rule arises when we relax the assumption that the target variables are observed and assume that instrument can respond only to the expectations of the disturbances or target variables. The Taylor rule allows only a response to a past observation of the targets. The optimal (implicit) instrument rule, in this case equation (46), allows a response to all available information.[10]

There are, however, two points to keep in mind. First, the *optimal* explicit instrument rule for either information assumption (equations 42 and 47) is, as previously shown, identical to the implicit instrument rule for the optimal target rule. The comparison is with a simple instrument such as a Taylor rule. Second, Taylor (e.g., Taylor, 1993, 1999, 2009) asks whether a simple rule might not have resulted in better economic performance than resulted from actual policy. Taylor's proposal is in the spirit of Milton Friedman's (1968, p. 16) argument that in monetary policy formation, "the best is likely to be the enemy of the good." In contrast Svensson (2003) worries that the good might be enemy of the best.

B. Robustness

A key advantage cited by advocates of rules is greater robustness. Svensson (2002, p. 778) states that "The specific targeting rule is relatively robust, in that it only depends on the marginal tradeoffs between the target variables, that is, the derivatives of the loss function and aggregate-supply relation with respect to the target variables." Svensson (1998, p. 22) argued that "As concerns the difference between targeting rules

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[10] Here, by a Taylor rule we mean the type of simple instrument rule use by Taylor (1993), (1999) and analyzed by Svensson (2003). The term is used loosely in the literature with adjectives such as "forward looking." These modified Taylor-types rules may respond to additional information.

and instrument rules, the optimal instrument rules generally depend on all model parameters, whereas even rather specific targeting rules are arguably more model-robust than instrument rules in that they may depend on only parts of the model."[11] More generally Svensson (2005, p. 622) praises a target rule as " a compact, robust, structural and therefore practical representation of goal-directed monetary policy."

To appraise the robustness properties of target rules it is useful to consider separately the case where the target variables are observed and the case where they are not observed.

<u>Inflation and Output Gap are Observed</u>

If the target variables are observed, the optimal target rule is then given by

37. $\theta^* y_t + \pi_t = 0$

where $\theta^* = 1/\mu\kappa.$

The target rule depends only on the weight on inflation in loss function relative to the output gap and on the slope of the Phillips curve.

As in the case of earlier models such as Poole's (1970) combination policy (equation 6), the target rule (37) can be represented as the response of one observable to the other. In the case of the new Keynesian model:

$$y_t = -(\ )\pi_t$$

The output gap responds to the inflation rate. This obviously assumes both target variables are observable. The optimal policy problem is to see that output and inflation respond in optimal proportions to a Phillips curve shock.

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[11] See also Woodford (2003, p. 614).

A comparison to the aggregate demand-aggregate supply framework in section 3.B is helpful. Within that framework the optimal target rule (equation 23) specifies a policy response of output to price

$$(23') \quad y_t = -\frac{1}{\theta^*} p_t$$

Within the AS-AD framework, this can be seen as the policymaker moving the aggregate demand curve in response to a supply shock to achieve the optimal p-y combination. All demand-side shocks are simply offset. In this case and in the New Keynesian model it is therefore not surprising that policy might be robust to changes in the demand-side of the model. Within the target rule framework a change on the demand side - - an additional *IS* shock for example - - would affect the implicit instrument rule that implements the target rule (equation 38 in the New Keynesian model). This additional *IS* shock would also change the optimal explicit instrument rule (equation 42); the two rules are identical.

Still the robustness of even the target rule itself is limited. The marginal tradeoffs between the target variables, on which the parameter of the target rule depends, will involve all the parameters of the demand side of the model in several not unrealistic cases. In the open economy the presence of a direct exchange rate channel in the Phillips curve, will result in the optimal θ depending on all the parameters in the model (see Froyen and Guender (2007, pp. 245-253).[12] The presence of a cost channel for monetary policy (see, for example, Ravenna and Walsh, 2006) will also result in the target rule parameter (θ^*) depending on the demand-side of the model.

Inflation and the Output Gap not Observed

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[12] A direct exchange rate channel in the Phillips curve is a property of some open-economy models and not others. This channel is present in Ball (1999a) and Svensson (2000) but not in Clarida, Gali and Gertler (2002) or Gali and Monacelli (2005).

The case where the target variables are observed should not be pursued too far. It is not reality. This case is considered because with the class of model we have been considered certainty equivalence holds.[13] When the current state of the economy is not perfectly observable, the optimal target rule is

46. $\theta^* E_t y_t + E_t \pi_t = 0$

Here any change in the model disturbances or parameters, even if it did not affect θ^*, would affect the target rule via an effect on the optimal conditional expectations of the target variables. The optimal explicit instrument rule

47. $r_t = \gamma_1^* E_t v_t + \gamma_2^* E_t u_t$

would be altered in an analogous way. It is hard to see how the two frameworks differ in robustness. A new shock such as a credit market shock would directly affect the explicit instrument rule (47) while changing the target rule (46) via the optimal expectations of the targets.

Svensson and Woodford (2003) highlight the separation principle "according to which the selection of the optimal policy (the optimization problem) and the estimation of the current state of the economy (the estimation or signal-extraction problem) can be treated as separate problems. But policy formulation consists of both problems. Robustness of a policy rule must mean robustness of both aspects of the policy rule (46 or 47). Isolating a part of the rule, the choice of θ, still may be useful for the next issue we consider – transparency.

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[13] Svensson and Woodford (2003) extend certainly equivalence and the resulting "separation principle" to linear rational expectations models with forward-looking variables.

C. Transparency

There seems no doubt that the desire for effective communication with the public – commonly referred to as transparency – has been a goal of central banks in moving to inflation targeting. A target rule such as (equation 37) conveys an important part of the monetary policy rule to the public in a compact form. It tells the public the policymaker's marginal rate of substitution between the policy goals. How does this compare with the transparency of instrument rules?

If the comparison is with a simple instrument rule such as a Taylor rule, the target rule is likely to be more transparent. A Taylor rule for example specifies the relative weight for the policy objectives but it seems unlikely that a central bank would commit to fixed weights in such a simple rule. The policy responses are likely to be state dependent.

A rule specifying the tradeoffs between target variables is especially transparent if one target, the inflation rate, has the dominant role in the loss function. As $\mu \to \infty$, the policymaker simply announces the target inflation rate and policy is transparent. This fact, is probably sufficient to explain the movement to inflation targeting by many central banks that wanted to be transparent and to "get the mean right" for inflation. A targeting rule such as equation (37) can still be transparent if there are multiple goals for the central bank as long as the central bank is willing to specify the relative weight put on the competing goals (θ^*).

If the comparison over the dimension of transparency is between a central bank following a target rule and one following an optimal instrument rule, the situation is different. The optimal instrument rule is derived from the policymaker's loss function which can be communicated to the public. The additional information conveyed by the

target rule is the tradeoff policymakers believes they faced (κ) in pursuing the two targets.

In practice it does not seem that inflation targeting central banks communicate the perceived tradeoff between multiple goals. The gain in transparency is rather the commitment to a target range for an announced primary goal of policy. Critics of inflation targeting such as Benjamin Friedman (2004, 2008) argue that for a central bank interested in stabilization of real variables as well as price stability, inflation targeting leads to "obfuscation and opacity" and possibly a "distorted set of policy objectives" if the central bank believes its own rhetoric.

D. **A Higher-Order Policy Rule**

The last issue we consider is a more technical, or model-based one. Discussing a target rule, Woodford (2003, p. 533) states that "A rule of this kind represents a 'higher-level' description of policy than an explicit specification of the instrument setting in each possible state of the world …" The targeting rule as Svensson (1999, p. 617) explains "can be expressed as a system of equations representing a first-order condition for a minimum of the loss function." The targeting rule is structural and specifies an equation that "the targets must fulfill." Higher level then refers to closeness to the policymaker's loss function; a rule closely based on optimality given model structure.

In part, this question of higher-order rules is related to robustness. Because it is only a step removed from the loss function, the target rule will be robust to some changes at more removed levels that would have an impact on an instrument rule. These issues have been addressed in Section 5.B.

Where this issue of higher-level policy rules has additional implications is with consideration of globally optimal policy rules in Section 4.C in particular concerning the "timeless perspective" and role of policy inertia in optimal policy.

As explained there, the target rule approach by focusing on the first-order conditions leads directly to the role of an inertial component in monetary policy. Traditional applications of the instrument variable approach do not so clearly define a role for a reaction to past deviations of the output gap from its optimal level. Papers in the literature that consider optimal instrument variables have taken into account the role that policy inertia might have in stabilization (e.g. Turnovsky (1980), Weiss (1980), Canzoneri, Henderson and Rogoff (1983). The choice of an inertial rule is this framework can be considered more ad hoc - - not coming directly from the optimization process.

A general solution procedure for rational expectations models going back to Muth (1961) begins by postulating a solution of the model containing an infinite distributed lag on the disturbances in the model along with any trends or constant terms. Following this procedure to choose the optimal instrument rule would in some models also lead to the inclusion of lagged responses to model errors. This is the procedure followed by Turnovsky (1980) which led him to a rule that allows for *prospective* monetary policy and thus a response of the instrument, in his model the money supply, to lagged disturbances.

This leads us to examine the degree to which inclusion of lagged disturbances in the instrument rule leads it to mimic the optimal target rule under the timeless perspective. The results of this exercise are shown in the lower three panels of Table 1, labeled Case 2-4. Recall that Case 1 simply shows that an instrument rule including the

current shocks and lagged output gap can duplicate the performance of the optimal target rule.

In these panels, Case 2 is an explicit instrument rule that responds to the current and the lagged cost-push shock in addition to the current-period demand shock. The stabilization performance of this instrument rule is clearly worse than the optimal instrument rule. The welfare loss amounts to 4.6 percent.[14] The primary reason for the sub-par performance is the weak response of the instrument to the current cost-push shock. This suboptimal adjustment of the policy instrument brings about a stabilization bias. Compared to the optimal instrument rule, application of the explicit instrument rule reduces the variance of the output gap – from *0.02139* to *0.0044* - but only at the expense of increasing the variance of the rate of inflation from *0.83471* to *0.89108*. The variance of the policy instrument is approximately the same under both instrument rules. Cases 3 and 4 consider explicit instrument rules that respond to shocks further in the past. Several findings are noteworthy. As the number of lagged shocks in the explicit instrument rule increases

- the policymaker reacts more emphatically to the current-period cost push shock
- the stabilization bias decreases, i.e. the variance of the output gap increases but the variance of inflation decreases
- the welfare loss decreases: 3.6 percent for 5 lags and 2.7 percent for 9 lags.

Implementing monetary policy via an explicit instrument rule that draws on past realizations of the cost-push shock activates the expectations channel of monetary policy in much the same manner as optimal policy under a target rule. The extent to which an

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[14] The welfare loss is calculated as $\dfrac{E[\,L_t^{Case\,i}\,] - E[\,L_t^{OPT}\,]}{E[\,L_t^{OPT}\,]}\,x100 \quad i=2,3,4.$

explicit instrument rule can replicate the outcomes associated with the target rule
improves with the number of lagged cost-push shocks included.

In sum, previous studies of optimal instrument rules have taken into account an
expectations channel that has features in common with the channel under the timeless
perspective. The criteria for choosing such an inertial instrument rule are more of a trial
and error process that with the target rule approach.

6. CONCLUSION

We believe that a comparison of optimal instrument rules with optimal target
rules clarifies the issues in the instrument –target rule debate. This comparison exploits
the equivalence of the two types of optimal rules to limit the issues that stem from the
inherent properties of the rules. In the models we consider the target rule approach leads
to an implicit instrument rule that is equivalent to the optimal explicit instrument rule. By
this we mean that a central bank following either of these rules would take the same
actions.

This fact narrows the range of advantages one approach can have over the other.
Relative advantages still might exist. We discuss such advantages along the dimensions
of robustness, transparency and the "higher order "or more "structural" nature of the
target rule procedure.

Many issues in the literature arise from a comparison of target rules with simple
not optimal explicit instrument rules. Our approach leads to the conclusion that many
advantages claimed for the target rule approach are because these simple instrument rules
are suboptimal instrument rules not because they are instrument rules per se. A Taylor
rule will, for example, in general fail to exploit all available relevant information because

of its simplicity not because it is an instrument rule. A Taylor rule might in practice be a "good" rule but that requires a historical or other empirical type of examination. A simple target rule such as "the inflation forecast should equal 2%" might be a good rule but in general would not be optimal.

But is the examination of optimal instrument rules of interest in itself? McCallum and Nelson (2004, p. 600) agree with Svensson's statement that commitment to optimal instrument rule is "completely impractical" and "has no advocates." The optimal instrument rule will in general be complex and will change with any changes in the parameters of the model and shocks that affect the model equations. We have seen in Section 5 that the same is true of the optimal target rule once the unobservable nature of the target variables is recognized.

Is the optimal explicit instrument rule then relevant? We would argue that it is the closest formulation in the literature to the policy followed by the U.S. Federal Reserve now and for the past 30 years. This is a regime where the federal funds rate responds at each FOMC meeting to the optimal forecasts of the state variables. The Federal Reserve has in recent years tried to increase the transparency of the process by providing more information about these forecasts and thus indirectly about the Open Market Committee members' tradeoffs among policy goals. The Federal Reserve has followed a process much closer to our description of the optimal instrument rule than to a Taylor rule. If the issue is whether the Federal Reserve would be better off as an inflation targeter, it is a comparison with the explicit optimal instrument rule that is relevant. The issues relevant to this comparison are those in Section V: B.C.D. Simple instrument rules are irrelevant.

Optimal instrument and target rulers share many features. Foremost, they lead to the same actions. Moreover both make use of optimal forecasts of the target variables. An explicit loss function is central to both frameworks and potentially is transparent in both. Section V considers they way the two frameworks may still differ. Central to these is the target rule itself , in the New Keynesian model equation (37) or (46) depending on the informational assumption. There is no direct counterpart to this piece of the optimal policy framework in the instrument rule approach. The target rule has an advantage in communicating the tradeoff between the policy goals to the public in a compact way. Including this piece in the optimal policy framework also helps to define the form of the optimal instrument rule. Balanced against this is the fact that central banks with multiple goals may be unwilling to specify this tradeoff. Instead they may specify only an inflation target therefore losing the transparency advantage to the approach. The target rule approach may also place too much emphasis on the tradeoff between the target variables in the presence of Phillips curve shock and too little on the stabilization of aggregate demand in the face of *IS* and financial market shocks.

Table I: Target vs Instrument Rules from the Timeless Perspective: $a_1 = .5, \kappa = .05, \sigma_u^2 = .9, \sigma_v^2 = .9, \mu = 1$				
Target Rule				
$r_t = c_v v_t + c_u u_t + c_y y_{t-1}$ $$c_v = \frac{1}{a_1}, c_u = \frac{(\tau(\tau-1)-\mu\kappa^2)\mu\kappa}{a_1\tau^2(1+\mu\kappa^2)}, \quad c_y = -\frac{(\tau(\tau-1)-\mu\kappa^2)}{a_1\tau^2(1+\mu\kappa^2)}$$ $c_v = 2, \quad c_u = .004639, \quad c_y = -.092775$	$V(y_t)$ .021396	$V(\pi_t)$ .834715	$E(L_t^{OPT})$ .856111	$V(r_t)$ 3.60020
Instrument Rules				
Case 1: $r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_3 y_{t-1}$ $$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = \frac{(1-Z)^3(3+Z)\kappa\mu}{8a_1(1+2a_1\gamma_3-Z+2\kappa^2\mu)} = .004639 \quad \gamma_3^* = \frac{2\mu\kappa^2-(X-\mu\kappa^2)(1+\mu\kappa^2)}{2a_1} = -.092779$$ $Z = \sqrt{1+4a_1\gamma_3} \qquad X = \kappa\sqrt{\mu(4+\mu\kappa^2)}$	$V(y_t)$ .021397	$V(\pi_t)$ .834716	$E(L_t)$ .856114	$V(r_t)$ 3.60013
Case 2: $r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1}$ $$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = \frac{\kappa^3\mu^2}{a_1(1+3\kappa^2\mu+\kappa^4\mu^2)} = .00024 \quad \gamma_4^* = \frac{\kappa\mu}{a_1(1+3\kappa^2\mu+\kappa^4\mu^2)} = .09925$$	$V(y_t)$ .004444	$V(\pi_t)$ .891084	$E(L_t)$ .895528	$V(r_t)$ 3.60886
Case 3: $r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1} + \gamma_5 u_{t-2} + \gamma_6 u_{t-3} + \gamma_7 u_{t-4} + \gamma_8 u_{t-5}$ $$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = .001199 \quad \gamma_4^* = .000955 \quad \gamma_5^* = .000714$$ $\gamma_6^* = .000475 \quad \gamma_7^* = .000237 \quad \gamma_8^* = .094972$	$V(y_t)$ .012538	$V(\pi_t)$ .874453	$E(L_t)$ .886991	$V(r_t)$ 3.60812
Case 4: $r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1} + \gamma_5 u_{t-2} + \gamma_6 u_{t-3} + \gamma_7 u_{t-4} + \gamma_8 u_{t-5} + \gamma_9 u_{t-6} + \gamma_{10} u_{t-7} + \gamma_{11} u_{t-8} + \gamma_{12} u_{t-9}$ $\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = .002039 \quad \gamma_4^* = .001799 \quad \gamma_5^* = .001565 \quad \gamma_6^* = .001334 \quad \gamma_7^* = .001106 \quad \gamma_8^* = .000882$ $\gamma_9^* = .000659 \quad \gamma_{10}^* = .000438 \quad \gamma_{11}^* = .000219 \quad \gamma_{12}^* = .087671$	$V(y_t)$ .018792	$V(\pi_t)$ .860658	$E(L_t)$ .87945	$V(r_t)$ 3.60693

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