Published in: *FinanzArchiv* 65.2009, S. 51-72

IS TAX PROGRESSION GOOD FOR EMPLOYMENT? EFFICIENCY WAGES AND THE ROLE OF THE PRE-REFORM TAX STRUCTURE ***

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First submission August, 21 2008
Revised version: February 17, 2009

Abstract

Within an efficiency wage framework, we study the effects of two revenue-neutral tax reforms that change the progressivity of the labour tax system. A revenue-neutral increase in both the wage tax and tax exemption and a revenue-neutral change in the composition of labour taxation towards the tax with the smaller tax base will lead to the same results: they moderate wages, workers' effort, effective labour input and aggregate output. Whether employment rises or falls, however, depends in both reforms on the magnitude of the pre-reform total tax wedge. We show that this ambiguity stems from the effect tax progression has on the marginal revenue changes of tax and tax exemption changes. This budgetary effect determines the result in the same way in both tax reforms and turns out to be the crucial force in determining the impact the degree of tax progressivity has on employment.

Keywords: efficiency wages, tax progression, structure of labour taxation.
JEL classification: H22, J41, J48.

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*** Comments by Sven Wehke and two anonymous referees are gratefully acknowledged. The authors thank the Research Unit of Economic Structures and Growth (RUESG), financed by Academy of Finland, Yrjö Jahnsson Foundation, Bank of Finland and Nokia Group, for financial support. Koskela also thanks Academy of Finland (grant No. 1117698) for further financial support and Freie Universität Berlin for great hospitality.

1. Introduction

This paper analyses in an efficiency wage framework the impact of tax progession on individual effort and aggregate employment. In perfectly functioning labour markets, labour taxation only distorts the labour market and lowers employment but does not create involuntary unemployment. When labour market imperfections lead to wage rates above market clearing levels, involuntary unemployment occurs and labour taxes normally aggravate unemployment by widening the gap between the labour cost and the opportunity cost of labour. But it is not only the magnitude of the tax wedge that matters. The specific structure of labour taxation, in particular the degree of labour tax progression, is also of great importance. The way in which tax progression affects unemployment, however, crucially depends on the particular underlying labour market imperfection.

The impact of tax progression is well understood within the wage bargaining and the search and matching framework. An increase in tax progression leads to wage moderation and is good for employment (see, e.g., Koskela and Vilmunen 1996, Holm and Koskela 1996, Pisssarides 1998, Koskela and Schöb 1999 and Heijdra and Ligthart 2009), because a higher marginal tax keeping the average tax constant works like a penalty on wage increases. By contrast, the results for efficiency wage models where the threat of being laid off encourages individual working are still mixed. Wage moderation has a positive effect on labour demand but a negative effect on individual labour effort and thus workers' productivity. Hoel (1990) was the first to analyze the overall effect of tax progression in such a framework. He shows that a higher marginal income tax rate, which leaves the average tax level unchanged at the initial equilibrium wage rate, will decrease the gross wage and unemployment (see also Goerke 1999)**.** Fuest and Huber (1998) show that, for a rise in tax progression such that the tax burden per worker is the same in the old and new equilibrium, the result might be reversed. Using the Shapiro and Stiglitz (1984) shirking model, Pissarides (1998), in turn, does not find any effect of the tax structure on the wage rate. This is because the individuals' effort decisions in this model are discrete so that workers either shirk or do not shirk. When effort is a continuous variable, however, Sorensen (1999) indicates that higher tax progression induces wage moderation and lowers both unemployment and work effort (also see Picard and Toulemonde 2003, who derive a similar result in a generic model that allows the analysis of different types of labour market imperfections). All these results do not carry over to models where workers differ in their productivity. A tax reform that raises marginal tax rates at all income levels and increases (decreases) average taxes at high (low) income levels may lead to higher gross wages and unemployment (see Andersen and Rasmussen 1999). Rasmussen (2002) shows that in the long run with free entry and exit of firms when aggregate employment is determined by the zero-profit condition, changes in profits may imply that higher wage tax progressivity will negatively affect employment if the marginal tax rate is high enough. These results cast doubts whether tax progression is always good for employment in an efficiency wage framework.

In this paper, we focus on revenue-neutral changes in the degree of tax progression in an efficiency wage model where homogenous workers choose their optimal work effort level continuously. In doing so, we would like to contribute to the literature in the following two ways. First, we want to highlight the role of the governmental budget in determining the impact tax progression has on employment. The degree of tax progression not only affects gross wages and workers' reservation wages, but it also affects the way in which the government can substitute wage taxes for payroll taxes or increase tax allowances in a revenue-neutral way. To see this, consider a revenue-neutral tax reform that raises both the marginal wage tax and the tax exemption by initially keeping the average wage rate constant. This leads to wage moderation. The higher the total marginal tax wedge, the more this wage moderation reduces tax revenues and the less the government can raise the workers' tax exemption. For any given increase of the marginal tax rate, the effort enhancing effect thus decreases with the total tax wedge. A lower effort level decreases the workers' labour productivity and demand becomes smaller at any given wage rate. If the revenue-neutral rise in the tax exemption becomes very small, the initial positive employment effect may be reversed.

The second important point we want to stress in this paper is to show that different ways of altering the degree of tax progression yields the same results. This is important as the composition of wage and payroll taxes often changes due to labor tax reforms and the degree of tax progression. The impact of the composition of wage and payroll taxes on progressivity have not yet been analyzed in an efficiency wage framework, while results from union bargaining models already exist.[1] Koskela and Schöb (1999) show that when tax bases for wage and payroll taxes are equal, it does not matter who *de jure* pays the tax on labour. In this case the total tax wedge, i.e. the sum of wage and payroll taxes, is sufficient to specify the distortion due to labour taxation. But this equivalence result ceases to hold when the tax bases are not equal because of tax exemptions (see also Koskela and Schöb 2002). A revenue-neutral restructuring of labour taxes towards the narrower tax base then decreases the gross wage and boosts employment. In this paper, we will ask whether these findings concerning the impacts of differences in the structure of labour taxation hold in an efficiency wage framework with non-discrete work effort choice. In particular, we are interested in whether tax progression *per se* or the specific way in which it is achieved matters for the determination of its employment effects.

We proceed as follows. In section 2, we develop a model framework that mirrors the main stylized facts of labour taxation and provide comparative statics of tax parameters on wage setting, work effort, labour demand and aggregate output. In section 3 we then analyze tax-revenue-neutral changes in tax progression by varying the marginal wage tax and the tax exemption and show the similarities to changes in tax progression via a change in the composition of wage and payroll taxes. Section 4 discusses what happens when the tax reform also changes the reservation wage of workers. Finally we present concluding comments.

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[1] Koskela and Schöb (1999), Picard and Toulemonde (2001, 2003), and Heijdra and Ligthart (2009) analyze marginal tax reforms in which wage taxes substitute for payroll taxes in different labor market models but only Picard and Toulemonde (2003) consider tax progressivity in an efficiency wage framework. They show that, in general, a revenue-neutral shift of a tax on firms to a tax on workers has an incidence on employment (see their Proposition 2); but in their model tax progressivity, however, is only analyzed in the context of varying wage tax progression (see their Proposition 3).

2. Model framework and comparative statics

To start with, we specify the time sequence of decisions, and then analyze the decision of workers on their work effort, the optimal wage setting and labour demand of firms. Finally, based on these private decisions, we analyze the effects of two distinct revenue-neutral tax reforms that allow the government to alter the degree of tax progression.

2.1 Time sequence of decisions

When firms decide on the wage rate w they pay their workers and on the employment level L, they take the tax policy as given and assume that they cannot influence the tax parameters. The government therefore behaves as a Stackelberg leader by setting three tax instruments. To raise revenues, the government can employ either a payroll tax s or a wage tax t. Both tax rates are constant in relation to the respective tax base. In addition, the government can affect the degree of tax progression by granting a tax exemption a that reduces the tax base for the wage rate t to $(w-a)L$. In the presence of a positive tax exemption a, the marginal tax rate t exceeds the average tax rate $t(1-a/w)$ so that the tax system is linearly progressive.[2] The net-of-tax wage is given by $w^n = (1-t)w + ta$. While the wage taxes are progressive in all OECD countries, the payroll taxes (see section 3), i.e. the social security contributions paid by employers, are approximately proportional. We therefore abstract from an additional tax exemption for the payroll tax so that the tax base for the payroll tax is wL. The gross wage rate, i.e. the labour cost, is then given by $w^g = w(1+s)$.

We can study two ways in which the government can alter tax progression without changing tax revenues. It can directly affect the *wage tax progression* by increasing the wage tax rate and increasing the tax exemption accordingly. Alternatively, it can increase the wage tax and lower the payroll tax. Such a change in the *structure of labour taxation* also affects the overall tax progression as it changes the shares of the progressive wage tax and the proportional payroll tax.

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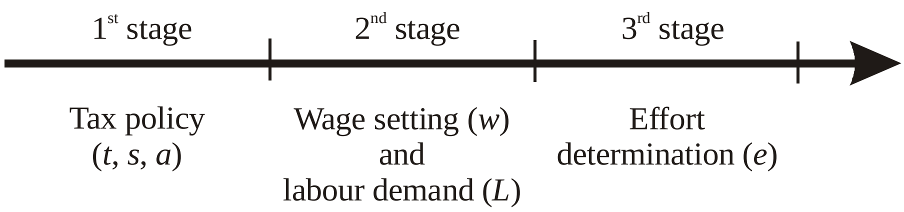
[2] For a seminal paper about tax progression, see Musgrave and Thin (1948), and for another elaboration, see e.g. Lambert (2001, chapters 7-8).

When the government announces its tax policy in the first stage, firms decide on the wage rate *w* and the employment level *L* in the 2nd stage. In this process, they cannot perfectly monitor the individual work effort *e* of their workers. As effort increases the disutility of working, workers have an incentive to shirk, but this incentive can be offset by paying higher wages since this raises the penalty for shirking workers who are caught and fired. On-the-job workers can decide upon their work effort in the 3rd stage. The time sequence of decisions is shown in Figure 1. In what follows, we proceed by using backward induction and start our analysis with the 3rd stage of the game, in which the wage rate, employment and tax parameters are already determined.

Figure 1: Sequence of decisions



2.2 Effort determination

Each worker provides one unit of labour and decides about effort *e* in the 3rd stage by taking the tax policy, wage setting and labour demand as given. Since effort cannot be fully controlled by firms, they can set a standard effort that we normalize to one. If workers meet this standard, their jobs are secure, but if they shirk by providing less effort, firms can fire them. However, effort cannot be monitored perfectly. The employment probability ρ can thus be described by a minimum function. For effort lower than the standard, we assume, for analytical convenience, an iso-elastic probability function of employment $\rho(e) = e^d$ where $d > 0$ denotes the (constant) employment probability elasticity of effort.[3] The employment

[3] We exclude the case where $d = 0$ because in this case, the job would be secure even without providing effort and total output would fall to zero. This would lead firms to set a wage rate equal to zero. Both employment supply and demand would then be indetermined. Furthermore, note that if the detection probability should be concave in effort, we would have to assume $d \leq 1$.

probability rises with effort for $e < 1$ and is 1 for a higher effort level, so that we have the employment probability function $\rho \equiv \min(1, e^d)$ and the probability of being laid off is $1 - \min(1, e^d)$. The parameter d is increasing in both monitoring intensity and monitoring efficiency. Low values of d make it less risky for workers to shirk, while $d \to +\infty$ implies perfect monitoring and the firing of all workers who do not meet the working effort standard.

We consider a representative risk-neutral worker with a specific utility function V^w that is additively separable and quasi-linear,

$$V^w = \min(1, e^d)[w^n - g(e)] + \left(1 - \min(1, e^d)\right)b = e^d\left[w^n - g(e)\right] + (1 - e^d)b\,, \tag{1}$$

where b denotes the workers' reservation wage, which, of course, is from the viewpoint of the worker net of taxes, and, for the time being, assumed to be exogenous (for further elaboration see section 4), and $g(e) = \alpha e^{1/\alpha}$, $0 < \alpha < 1$, denotes the convex disutility of effort. Working time per worker is fixed and normalized to unity. In what follows we only focus on the interesting case $e < 1$.

The optimal individual effort level can be derived from the first-order condition $V_e^w = de^{d-1}(w^n - \alpha e^{1/\alpha} - b) - e^d e^{(1/\alpha)-1} = 0$. The worker chooses an effort level at which the expected utility loss of working harder, which occurs with probability e^d, equals the expected utility gain from an increased probability of staying in employment de^{d-1} and receiving the surplus $w^n - \alpha e^{1/\alpha} - b$ with $w^n = (1-t)w + ta$. This yields the following effort function

$$e = A(w^n - b)^{\alpha}\,, \tag{2}$$

where $A = (d/(1 + \alpha d))^{\alpha}$ is constant. We assume a concave effort function with respect to the difference between the net-of-tax wage rate and the workers' reservation wage so that we have $0 < \alpha < 1$. Effort is increasing in the net-of-tax wage rate, $e_{w^n} > 0$,[4] and decreasing in the reservation wage, $e_b < 0$. Furthermore, we have $e_t < 0$, because this reduces the penalty when

[4] In what follows, partial derivatives will be denoted by subscripts.

caught shirking, $e_w > 0$, and $e_a > 0$, as both a higher wage rate *w* and a higher tax exemption *a* increase the rent from being employed.[5]

2.3 Wage setting and labour demand

In the 2nd stage, each firm takes the tax parameters as given and decides on the wage rate *w* and labour demand L. In doing so, it takes into account how the representative worker will adjust work effort when the wage rate *w* changes. Production depends on effective labour input eL. For analytical convenience, we parametrize the production function for the representative firm as $f(eL) = \varepsilon^{-1}(eL)^{\varepsilon}$ with $0 < \varepsilon < 1$ denoting the revenue share of labour and $(1-\varepsilon)$ the profit share. Hence we have a concave production function in terms of effective labour input, i.e. $f'(eL) > 0$ and $f''(eL) < 0.$ The output price is normalized to unity and profits are defined by $\pi = f(eL) - w(1+s)L$. The first-order conditions in terms of L and w are $f'(eL) = w(1+s)/e$ and $f'(eL) = (1+s)/e_w$, so that we obtain the Solow condition

$$\frac{e_w w}{e} = 1, \tag{3}$$

(Solow 1979), according to which the wage elasticity of effort is equal to one, i.e. the optimal wage is set such that a one percent increase in the wage rate (and thus the production costs) leads to a one percent increase in output (at a given employment level). From the Solow-condition (3) we can derive an explicit solution for the optimal efficiency wage rate for $e_w = \alpha(1-t)e(w(1-t)+ta-b)^{-1} > 0$:

$$w = \frac{b-ta}{(1-t)(1-\alpha)}. \tag{4}$$

The comparative statics of the wage function shows that $sign(w_t) = sign(b-a)$ and $w_a < 0$. The Solow condition states that it is optimal for the firm to set the wage such that the relative change in the wage rate is equal to the relative change in effort. If $b = a$, the level of *t* has no

[5] We could allow for a more general utility function that is concave in terms of rents and convex in terms of disutility of effort so that we could include risk aversion. Qualitative results by using the standard HARA-type utility function (suggested originally by Merton 1971) are similar and are available upon request.

effect on the optimal wage rate. If $b > a$, however, a tax rate increase raises the impact a wage rate increase has on effort: the higher *t* is, the stronger the relative increase of $w^n - b$ due to a wage increase is. A higher tax exemption *a*, by contrast, makes working more attractive and therefore allows the firms to decrease the efficiency wage rate. Note that the payroll tax *s* does not affect wage determination. The labour demand function is given by

$$L = [w(1+s)]^{-\delta} e^{\delta-1}, \tag{5}$$

where $\delta \equiv 1/(1-\varepsilon) > 1$ and $\delta - 1 \equiv \varepsilon/(1-\varepsilon) > \varepsilon.$ The comparative statics of labour demand with respect to effort and the payroll tax are $L_e = L\,(\delta-1)e^{-1} > 0$ and $L_s = -\delta w[w(1+s)]^{-\delta-1} e^{\delta-1} = -\delta L(1+s)^{-1} < 0$, respectively. The wage tax and the tax exemption – levied on workers – only affect labour demand indirectly via the effort determination, and the payroll tax only affects labour demand directly via the gross wage rate. The total effect of a change in the wage rate *w*, however, influences labour demand in two different ways. There is a negative *direct* effect, $L_w = -\delta L w^{-1} < 0$, and a positive *indirect* effect of the wage rate via effort, $L_e e_w = (\delta-1)[w(1+s)]^{-\delta} e^{\delta-2}(\delta-1)e_w = (\delta-1)Le_w e^{-1}$. The former effect dominates, so that a higher wage rate *w* decreases labour demand. For the concave production function, the absolute value of the wage elasticity of labour demand is lower than in the case when wages do not affect effort. Inserting the Solow condition, the total wage elasticity of labour demand in the firm's profit maximum becomes

$$\frac{dL}{dw}\frac{w}{L} = -1. \tag{6}$$

Effective labour input eL and the wage bill $(1+s)wL$ remain constant due to a marginal wage increase and thus profit. This is a complementary condition to the Solow condition.

Having analyzed workers' and firms' behaviour with respect to changes in the tax parameters, we can now turn to the first stage. Rather than analyzing optimal tax systems, we consider small tax reforms in the first stage and focus on the effects of revenue-neutral changes in (i) wage tax progression and (ii) the structure of labour taxation on wage formation, effort determination, employment and output.

Table 1: Labour taxation in the OECD countries

Country	(1) Average wage tax	(2) Social security contributions paid by employee	(3) Marginal wage tax	(4) average wage tax rate progression	(5) Social security contributions paid by employer	(6) average payroll tax rate progression	(7) Tax exemption *a* in Euro	(8) Calculated *a/w*	(9) *Unemployment benefit* $\bar{b}$ in Euro	(10) Standardized unemployment rate 2004
Australia	24.3	0	31.5	7.2	6.0	0.0	6,421	22.9	n.a.	5.5
Austria	10.8	18.0	24.6	13.8	29.0	–0.2	10,943	56.1	3,706	4.8
Belgium	26.6	14.0	40.8	14.2	34.5	4.8	9,087	34.8	6,005	7.8
Canada	17.8	6.9	24.2	6.4	4.2	–7.0	6,598	26.4	3,991	7.2
Czech Republic	11.4	12.5	17.5	6.1	34.9	–0.1	3,670	34.9	2,527	8.3
Denmark	30.6	10.6	38.6	8.0	0.0	0.0	5,709	20.7	8,539	5.4
Finland	24.2	6.1	38.2	14.0	24.1	0.0	8,053	36.6	3,516	8.9
France	13.1	13.6	18.8	5.7	39.3	0.1	5,657	30.3	3,732	9.7
Germany	19.6	20.9	35.6	16.0	20.8	0.1	11,596	44.9	2,838	9.5
Greece	0.6	16.0	12.6	12.0	28.0	0.0	12,067	95.2	n.a.	10.5
Hungary	12.4	13.5	26.0	13.6	33.6	–3.2	3,708	52.3	1,134	5.9
Iceland	25.5	0.2	36.8	11.3	5.7	0.0	6,853	30.7	6,918	3.1
Ireland	10.6	5.0	21.0	10.4	10.8	0.0	9,909	49.5	4,802	4.5
Italy	18.6	9.2	34.9	16.3	33.1	0.1	9,004	46.7	3,084	8.0
Japan	5.9	11.6	11.3	5.4	12.6	0.2	10,926	47.8	5,487	4.7
Korea	2.2	7.1	11.0	8.8	8.9	0.2	19,471	80.0	3,894	3.7
Luxembourg	8.9	13.8	25.1	16.2	12.9	–0.6	14,913	64.5	8,318	4.8
Mexico	3.0	1.5	13.7	10.7	10.7	–2.2	5,211	78.1	n.a.	2.4
Netherlands	8.5	25.8	19.6	11.1	10.8	–5.7	14,138	56.6	7,490	4.6
New Zealand	20.7	0	33.0	12.3	0.0	0.0	7,711	37.3	n.a.	3.9
Norway	20.9	7.8	28.0	7.1	13.0	0.0	6,174	25.4	7,304	4.4
Poland	6.1	25.4	9.2	3.1	20.4	0.1	3,552	33.7	2,213	18.8
Portugal	5.6	11.0	14.0	8.4	23.8	0.0	5,731	60.0	1,624	6.7
Slovak Republic	7.9	13.4	16.5	8.6	35.6	–0.1	3,943	52.1	1,286	18.0
Spain	12.7	6.4	22.4	9.7	30.6	0.2	7,140	43.3	4,452	10.9
Sweden	24.0	7.0	28.9	4.9	32.7	0.0	3,241	17.0	3,058	6.4
Switzerland	9.8	11.1	18.4	8.6	11.0	–0.1	11,788	46.7	5,548	4.4
Turkey	15.4	15.0	17.6	2.2	21.4	0.0	1,508	12.5	0,000	9.5
United Kingdom	15.9	8.5	24.5	8.6	12.8	2.9	8,463	35.1	3,375	4.7
United States	16.5	7.7	21.3	4.8	7.7	0.1	5,770	22.5	1,280	5.5

Source: OECD (2004)

Legend: Tax rates are for the year 2004 for a single person with 100% of average wage. Column (4) shows the difference between marginal and average rate of income tax. Social security contributions paid by employees are assumed not to be subject to tax exemption. Social security contributions are marginal contributions. As an approximation it is assumed that for each country the tax schedule consists of a tax exemption and a constant marginal tax rate. We took the exchange rate between US Dollar and Euro as of December 31, 2004: 1 US Dollar = 0.73292 Euro. Social assistance levels do not include housing costs. Numbers of social assistance are from 2002 taken from OECD (2004), Benefits and Wages, OECD Indicators.

3. Revenue-neutral changes in wage tax progression

Tax progression can be affected by tax exemptions and by the structure of labour taxation when tax bases differ. Table 1 provides some information about wage taxes and payroll taxes in the OECD countries, calculated for an average productive worker who is not married. The first and second column report the average wage taxes and the social security contributions paid by the employees. The third column shows the marginal income taxes.

The *average wage tax progression* (see Lambert 2001, chapters 7-8) is reported in the fourth column. This rate states the difference between the marginal and the average income tax rate due to the tax exemption for wage taxes. The higher this difference, the more progressive wage taxation is. Table 1 shows that all OECD countries have a progressive wage tax system, though there are huge differences with the highest degree of progression being reported for Italy, with 16.3 percentage points, and the lowest one for Turkey, with only 2.2 percentage points.

Payroll taxes, reported in the fifth column, mainly consist of social security contributions paid by the employer. The *average payroll tax progression* in the sixth column is very small, and even negative for some countries because of some work-related social-security contributions that are not dependent on wage income. While the maximum difference between a country's average and marginal wage rate is above 16 percentage points, the difference between average and marginal payroll taxes are substantially lower in all countries, with a maximum difference below five percentage points. Thus, while we observe highly progressive wage tax systems, the payroll tax systems are approximately proportional.

In what follows, we focus on the analysis of a tax-revenue-neutral change in the composition of labour taxes, i.e. the progressivity of the wage tax system and then briefly discuss how these results carry over to changes in tax progression via changing the structure of labour taxation. The government budget is given by

$$G = \left[t(w-a) + sw\right]L . \qquad (7)$$

Revenue-neutrality is interpreted as keeping the total tax reveues from labour taxation constant at the level *G*. Tax progression increases when the difference between the total marginal tax wedge $t+s$ and the total average tax rate $(t+s)-ta/w$ increases. We call a revenue-neutral increase in both the wage tax rate *t* and the tax exemption *a* an increase in wage tax progression.

What is the combination of changes in the wage tax rate t and the tax exemption a combined with the incurred change in the wage rate w that will keep government tax revenues constant? Total differentiation of (7) gives $dG = G_t dt + G_a da + G_w dw = 0$. Taking into account the induced change in the wage rate $dw = w_t dt + w_a da$, this can be written as (see Appendix 1 for details)

$$dG = (G_t + G_w w_t)dt + (G_a + G_w w_a)da = 0\,. \tag{8}$$

If we are on the upward-sloping part of the Dupuit-Laffer curve for both the wage tax rate and the tax exemption, which means that the tax revenues increase in *t* and decrease in *a*, then we have $G_t + G_w w_t > 0$ and $G_a + G_w w_a < 0$. Since $G_w = taL/w > 0$ (see Appendix 1) and $w_t > 0$ (assuming $b > a$), sufficient conditions for this to hold are $G_t > 0$ and $G_a < 0$.

3.1 The effects of tax revenue-neutral change in marginal wage tax and tax exemption

Wages and Effort

Now we are prepared to explore the behavioural effects of tax revenue-neutral change in wage tax progression. After some calculations (delegated to Appendix 1), we obtain the effect of an increase in wage tax progression on the wage rate *w*:

$$\left.\frac{dw}{dt}\right|_{dG=0} = \frac{w_t + \dfrac{(w-a)}{t} w_a}{1 + w_a G_w G_a^{-1}}\,. \tag{9}$$

Due to our assumption of an upward-sloping Dupuit-Laffer curve, the denominator of (9) is positive. As for the numerator in (9), using the partial derivatives of (4) imply

$$w_t + \frac{(w-a)}{t} w_a = \frac{(1-\alpha)(b-a)-(w-a)(1-t)(1-\alpha)}{(1-t)^2(1-\alpha)^2} = -\frac{\alpha w}{(1-t)(1-\alpha)} < 0\,,$$

so that we obtain an unambiguous wage moderation effect of raising tax progression:

$$\left.\frac{dw}{dt}\right|_{dG=0} < 0\,.$$

If the firm lowers the wage rate, it benefits from lower wage costs but at the same time suffers from lower work effort, which reduces labour productivity. In the initial equilibrium, the firm sets the wage rate such that these two effects balance out at the margin. A revenue-neutral increase in wage tax progression implies that it becomes beneficial for firms to lower the wage rate because the effect on effort becomes smaller when the marginal tax rate increases.

To determine the impact on labour demand and work effort, we have to derive the change in the gross and net-of-tax wage rate, respectively. As we keep the payroll tax constant, a fall in the wage rate w also lowers the gross wage rate $w(1+s)$. The change in the net-of-tax wage rate is given by:

$$\left.\frac{dw^n}{dt}\right|_{dG=0} = -(w-a) + t\left.\frac{da}{dt}\right|_{dG=0} + (1-t)\left.\frac{dw}{dt}\right|_{dG=0}. \tag{10}$$

If the tax reform did not change the wage rate, increasing tax progression would leave the net-of-tax wage rate unaffected.[6] But when the wage rate falls, there are two negative effects on the net-of-tax wage rate. First, there is the immediate direct effect of wage moderation. Second, there is an indirect budgetary effect. As wage moderation lowers tax revenues, the government has to reduce the extent of the increase in the tax exemption a relative to the potential increase dw/dt. This further reduces the net-of-tax wage rate, so that the total effect is unambiguously negative:

$$\left.\frac{dw^n}{dt}\right|_{dG=0} = \left[(1-t) - t\frac{G_w}{G_a}\right]\left.\frac{dw}{dt}\right|_{dG=0} < 0. \tag{10a}$$

[6] This can be seen from substituting (A4) from Appendix 1 in (10) and setting $\left. dw/dt \right|_{dG\,=\,0} = 0$.

Effort depends positively on the net-of-tax wage, i.e. $e_{w^n} > 0$, so that we can immediately deduce that workers' effort also falls:

$$\left.\frac{de}{dt}\right|_{dG=0} = e_{w^n} \left.\frac{dw^n}{dt}\right|_{dG=0} < 0\,. \tag{11}$$

These findings are summarized in

> **Proposition 1:** *An increase in the revenue-neutral wage tax progression leads to wage moderation that reduces both the gross wage rate and the net-of-tax wage rate. A fall in the net-of-tax wage rate reduces individual work effort and thus negatively affects labour productivity.*

Employment and output

Labour demand depends both on the gross wage and effort. Firms will lower the gross wage but also face a lower labour productivity. Thus there are two countervailing effects on labour demand. From the total differential of employment $dL = (L_w + L_e e_w)dw + L_e e_t\; dt + L_e e_a\; da$ and by using the revenue-neutral change in the tax exemption (7), we obtain

$$dL = \left(L_e e_t - L_e e_a \frac{G_t}{G_a} \right) dt + \left(L_w + L_e e_w - L_e e_a \frac{G_w}{G_a} \right) dw. \tag{12}$$

The first term equals the employment effect of a revenue-neutral tax reform when the wage rate does not change. This term is zero (see Appendix 2 for the calculations) because a revenue-neutral change in tax parameters without a change in the wage rate would not alter the net-of-tax wage rate. If the efficiency wage rate does not change, both effort and employment do not change. The employment effect thus only depends on the induced wage rate change, so that we have:

$$\left.\frac{dL}{dt}\right|_{dG=0} = \left(L_w + L_e e_w - L_e e_a \frac{G_w}{G_a} \right) \left.\frac{dw}{dt}\right|_{dG=0}. \tag{13}$$

To interpret this result and to sign the effect, we rewrite (13) in the following way:

$$sign\left.\frac{dL}{dt}\right|_{dG=0} = -sign\left(\frac{L_w + L_e e_w}{L_e e_a} - \frac{G_w}{G_a}\right) = -sign\left(\frac{\frac{dL}{dw}}{\frac{dL}{da}} - \frac{G_w}{G_a}\right) = -sign\left(-\frac{(1-t)}{(\delta-1)t} - \frac{G_w}{G_a}\right).$$

The first ratio of the right-hand side indicates the relative impact the wage rate and the tax exemption have on employment. Let us assume that we increase the wage rate by one percent and the tax exemption by the same absolute amount. The effect of the wage rate, consisting of a direct effect via the gross wage and an opposing indirect effect via raising effort on employment, is then -1. The change in the tax exemption only increases effort. However, the effect of an equal-size increase in the tax exemption on the net-of-tax wage rate is only $t/(1-t)$ of the effect of a wage rate increase. Multiplying by the effort elasticity of labour $(\delta-1)$ thus yields the total effect of the above increase in the tax exemption: $dL/da \cdot w/L = (\delta-1)t/(1-t)$.

By how much can we actually change the wage rate and the tax exemption when we consider that revenue-neutrality depends on the marginal tax revenues? The higher the tax revenue of a tax instrument, the lower is its revenue-neutral adjustment. If G_w is large, which is the case the stronger the total tax wedge $s+t$ is, a low absolute value of G_a, which is the case when t is low, requires a large reduction in the tax exemption to compensate for the induced effect.

The relative magnitudes can be seen best by inserting $-G_w/G_a = (t+s)/t$ in equation (13). This yields the following condition:

$$\left.\frac{dL}{dt}\right|_{dG=0} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \text{as} \quad \frac{1-t}{(\delta-1)t} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{t+s}{t} \quad \Leftrightarrow \quad \frac{1-t}{t+s} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \delta - 1. \tag{14}$$

If the total tax wedge $s+t$ becomes very high, the wage moderating effect requires a higher downward adjustment of the tax exemption. If this already has a strong impact on effort (which depends on the technology parameter $(\delta-1)$), it becomes very likely that employment will fall. The adverse effect of tax progression on the budget adjustment requirements then outweigh the wage moderating effect on the gross wage.

The effects of the revenue-neutral change in the wage tax progression on *effective labour* input eL and therefore on output are *a priori* unclear because an increase in tax progression has a negative effect on effort and an ambiguous effect on employment. The total differential for (eL) is

$$\left.\frac{d(eL)}{dt}\right|_{dG=0} = \left(L\left.\frac{de}{dt}\right|_{dG=0} + e\left.\frac{dL}{dt}\right|_{dG=0} \right)\cdot \left.\frac{dw}{dt}\right|_{dG=0} . \tag{15}$$

Solving (15) by using the results derived before (see Appendix 2), we obtain:

$$\left.\frac{d(eL)}{dt}\right|_{dG=0} = -\frac{G_w\left[e_a + L_e e_a\right]}{G_a}\cdot \left.\frac{dw}{dt}\right|_{dG=0} < 0 \cdot \tag{16}$$

The effort effect of higher tax progression is unambiguously negative, while the employment effect is *a priori* ambiguous but, according to equation (16), the first effect dominates. The direct effect of a wage rate change on labour demand would exactly compensate for lower work effort, but the indirect effect that forces the government to reduce the extent of the tax exemption increase further lowers effort, employment and, therefore, effective labour input.

In summing up the findings of this subsection, we can conclude with

Proposition 2: *An increase in the revenue-neutral wage tax progression raises (lowers) employment when the ratio of net-of-tax wage and revenue share of the wage is higher (lower) than the effort elasticity of labour demand. Effective labour input and output fall unambiguously.*

Note that the effect on effective labour input and output hinges on the functional form chosen and may not hold in general.

3.2 Revenue-neutral changes in the composition of wage and payroll taxes

Alternatively tax progression can be affected by changes in the composition of wage and payroll labour taxation because such a reform changes the progressivity when the tax bases are different, i.e. $a > 0$. Increasing the wage tax on the narrow tax base and reducing the payroll tax on the brorader tax base raises the marginal tax at a given average tax rate and thus increases the degree of tax progression. Taking the total differential of the government budget constraint (7) with respect to wage tax, payroll tax and gross efficiency wage gives $dG = G_t dt + G_s ds + G_w dw = 0$. The change in the efficiency wage depends only on changes in the wage tax rate, $dw = w_t dt$, so that we have

$$\left.\frac{ds}{dt}\right|_{dG=0} = -\frac{G_t + G_w w_t}{G_s}. \tag{17}$$

If we are on the upward-sloping part of the Dupuit-Laffer curve, we have $G_t + G_w w_t > 0$ (which always holds when $G_t > 0$ and $b > a$) and $G_s > 0$, so that a revenue-neutral increase in *t* implies a lower payroll tax *s*.

The payroll tax borne by the employer does not affect the efficiency wage rate firms choose [cf. equation (4)]. The wage rate therefore is only affected by the wage tax rate, i.e. $dw/dt|_{dG=0} = w_t$. Irrespectively of the sign of w_t, the net-of-tax wage falls and thus also effort.

The total differential of employment with respect to wage rate, payroll tax rate and wage tax rate can be written as $dL = (L_w + L_e e_w)dw + L_e e_t\, dt + L_s ds$. By using the revenue-neutral change in labour taxation (17), we obtain

$$dL = \left(L_e e_t - L_s \frac{G_t}{G_s} \right) dt + \left(L_w + L_e e_w - L_s \frac{G_w}{G_s} w_t \right) dw. \tag{18}$$

A change in the composition of wage and payroll taxes leads to both direct and indirect effects. The two direct effects reinforce each other. Both the increase in *t* and *w* (note that we have $dw = w_t dt$) will have a negative net effect on labour demand. The indirect effects work via the government budget. The term G_t/G_s is the weight of the positive direct effect $-L_s$ in the first bracket term and determines how much the effect due to an increase in *t* is offset. Since the wage rate also increases, the second indirect effect works in exactly the same qualitative way as the first indirect effect, whereby G_w/G_s in the second bracket term is the weight of the positive indirect effect $-L_s$.

It turns out that condition (14) is also relevant when tax progression is affected by the change in the composition of labour taxes if $a > 0$. Furthermore, irrespectively of the way in which tax progression is raised, effective labour input and thus output will fall. Only if the tax bases are equal, i.e. $a = 0$, neither effort nor labour is affected by the revenue-neutral change in the composition of wage and payroll taxes because tax progression is unaffected in this special case (see Appendix 3 for the calculations).

In summing up the findings of this subsection, we can conclude with

Proposition 3: *A revenue-neutral change in the composition of labour taxation that raises tax progression (i.e. $a > 0$) increases (decreases) employment when the ratio of the net-of-tax wage and the revenue share of the wage is higher (lower) than the effort elasticity of labour demand. Effective labour input and output fall unambiguously. When the tax bases are equal, effort, employment and output do not change.*

The intuition for $a > 0$ is similar to the intuition for Proposition 2 and will not be repeated here. This result shows that in the case of equal tax bases, the structure of labour taxation does not matter in terms of employment in the efficiency wage framework. The same result holds in the union bargaining framework without efficiency considerations (see Koskela and Schöb 1999). Furthermore, both tax reforms increase tax progression and yield similar effects with respect to effort, labour demand, effective labour input and output. This indicates a systematic pattern of how the degree of tax progression actually affects the labour market and production.

4. The role of the reservation wage

So far we have assumed a constant reservation wage b. The reservation wage normally depends on the labour market conditions and is endogenously determined in general equilibrium. However, very little is known about the exact relationship. For instance, Bewley (1999) interviewed more than 300 business executives, labour leaders, professional recruiters and advisors to the unemployed and concluded that the workers' morale is important for workers performance but that workers' morale depends on being treated fairly *within* firms –

for instance by paying "fair" wages according to some established internal pay structure. An exogenous *b* may be thus a good approximation.[7]

By contrast, Agell and Bennmarker (2003, 2007) report from a random survey of Swedish human resource managers that two-thirds of their respondents believe that an increase in external wages is detrimental to workers' effort: "Most Swedish managers indicate that *both* internal and external wages are important considerations in the local wage bargain." (Agell and Bennmarker 2003, p. 25). Thus, the reservation wage *b* in our model should reflect the labour market conditions, namely aggregate unemployment and average wages.

Depending on the way in which these labour market conditions affect the reservation wage, the total effect may increase or reduce the "first-round" or short-run effects we have analyzed in section 3. Since so little is known about the functional form, we only present a heuristic argument how the general equilibrium effects are altered when the first-round effect yields a zero employment effect. A change in the reservation wage raises the wage, i.e. the partial derivative of (4) yields $w_b > 0$. The effect on effort is positive because the positive wage effect outweighs the negative direct effect, so that

$$\frac{de}{db} = e_b + e_w w_b = \frac{\alpha e}{(b - ta)} > 0 . \tag{19}$$

The employment effect can be also signed. It is strictly negative as we have

$$\frac{dL}{db} = L_w w_b + L_e \frac{dL}{de} = -\frac{L(\delta(1-a)+\alpha)}{(b-ta)} < 0 . \tag{20}$$

A special case, frequently used in the literature, assumes that an unemployed worker faces the probability $1-u$ of being employed in another industry and the probability u of remaining unemployed. Formally, the net-of-tax reservation wage is then given by

$$b = (1-u)(\overline{w}(1-t) - \alpha \overline{e}^{1/\alpha} + ta) + u\overline{b} , \tag{21}$$

(see e.g. Layard, Nickell and Jackman 1991, pp. 100-101), where $\overline{w}$ denotes the average wage rate and $\overline{e}$ the optimal effort level at the average wage rate. For similar firms in various

[7] In similar line, Campbell and Kamlani (1997) report that workers mainly compare their wages with their own past wages, the wages of other workers *within* the firm, and with firms' profits.

industries, the average wage rate equals the wage rate each single firm sets, i.e. $w = \overline{w}$, and optimal effort will be the same for all workers $e = \bar{e}$. For simplicity we set $\bar{b} = 0$. The total effect on *b* is (see Appendix 4 for the calculations):

$$db = \left[w^n - g(e)\right]\frac{dL}{dt} + \frac{1}{(1+\alpha d)} L \frac{dw^n}{dt}. \tag{22}$$

How do results change when the initial employment effect according to (14) is equal to zero? In this case, the first term in (22) vanishes and the reservation wage unambiguously falls. This reinforces the first-round effects on the net wage income and effort. From (20) it follows that *cet. par.* employment then rises in equilibrium after the reservation wage adjusted. If the initial employment effect is positive, we have two countervailing effects. But as long as $db \leq 0$, which will be the case for parameter values that ensure sufficiently small values of $dL/dt|_{dG=0}$, the first-round employment effect will be reinforced. But when $db > 0$ becomes sufficiently large, the general equilibrium effects may eventually overcompensate the first-round effects. When the initial employment effect is negative according to (14), the general equilibrium effects on effort and the net wage income are stronger than the first-round effects while the employment effect may become positive (close to $dL/dt|_{dG=0} = 0$) or may be less negative.

5. Concluding remarks

The structure of labour taxes, i.e. payroll and wage taxes, in OECD countries varies considerably due to different tax rates and different regulations concerning tax allowances with respect to wage and payroll taxes. Wage taxation in OECD countries is progressive, although the degree of progressivity varies across countries. In the case of payroll taxes, the difference between marginal and average payroll taxes is very small, i.e. we observe approximately proportional payroll tax systems in most OECD countries. For these stylized facts, we studied the impacts of two different tax-revenue-neutral changes in wage tax progression. First, we analyzed the revenue-neutral tax reform where both the wage tax rate and the tax exemption were increased so that *wage tax progression* increased. Second, we compared these findings with a rise in tax progression due to a change in the composition of labour taxation towards the tax with the lower tax base.

Our analysis shows that when the wage tax system becomes more progressive, this leads to wage moderation and to a fall in workers' effort. Whether employment rises or falls depends on the pre-existing tax system relative to the labour demand elasticity in terms of work effort (see equations (14), (22)), because the magnitude of the total tax wedge affects the way in which the government can influence workers' effort in a revenue-neutral way. The increase in the wage tax *cet. par.* raises the gross wage and lowers labor demand. The larger the tax wedge, the greater the fall in tax revenues due to the induced wage moderation and the smaller the revenue-neutral rise in the tax exemption or the cut in payroll taxes, which *ceteris paribus* both increase labor demand. This budgetary effect affects the result in the same way in both tax reforms discussed in this paper and is thus the crucial force in determining the impact the degree of tax progressivity has on employment.

Appendix 1

The total differential for (6) gives $dG = G_t dt + G_a da + G_w dw = 0$. Using

$$G_w = \left[t + s + (tw - ta + sw)\frac{(L_w + L_e e_w)}{L} \right] L = \frac{ta}{w} L > 0, \tag{A1}$$

we can write the total differential as $dG = 0 \Leftrightarrow G_t dt + G_a da + taw^{-1}\, Ldw = 0$. Inserting the respective partial derivatives of labor and effort, applying $e_t t = -(w-a)e_a$ and assuming that the tax revenues are positively related to the wage tax rate and negatively related to the tax exemption according to the upward-sloping Dupuit-Laffer curve, we can determine the sign of G_t and G_w as follows:

$$G_t = L\left[w - a + (w - a + \frac{s}{t}w)\frac{L_e e_t\, t}{L} \right] = L\left[w - a + (w - a + \frac{s}{t}w)\frac{(\delta - 1)e_t\, t}{e} \right] > 0, \tag{A2}$$

$$G_a = tL\left[-1 + (w - a + \frac{s}{t}w)\frac{L_e e_a}{L} \right] = \frac{-t}{(w-a)} G_t < 0. \tag{A3}$$

Using the expressions (A2) and (A3), we can now determine the revenue-neutral change in the tax exemption when the wage tax rate is increased marginally:

$$da\Big|_{dG=0} = \left(\frac{w-a}{t}\right)dt - \frac{G_w}{G_a}\,dw. \tag{A4}$$

Substituting the RHS of (A4) for da in the total differential $dw = w_t dt + w_a da$ yields (9).

Appendix 2

Substituting (A4) for da in the total differential $dL = (L_w + L_e e_w)dw + L_e e_t dt + L_e e_a da$ gives

$$\frac{dL}{dt}\bigg|_{dG=0} = L_e e_t + \frac{(w-a)}{t}L_e e_a + \left(L_w + L_e e_w - \frac{L_e e_a G_w}{G_a}\right)\left(\frac{dw}{dt}\bigg|_{d\bar{G}=0}\right). \tag{A5}$$

Using the partial derivatives of the employment and effort functions, we obtain

$$L_e e_t + \frac{(w-a)}{t}L_e e_a = \frac{L(\delta-1)}{e}\left[e_t + \frac{(w-a)}{t}e_a\right] = 0\,.$$

Using (A1) and (A3), we obtain after some further manipulations

$$\frac{dL}{dt}\bigg|_{dG=0} = -\frac{L}{w}\left(\frac{e - w(1+\frac{s}{t})(\delta-1)e_a}{e - w(1+\frac{s}{t})(\delta-1)e_a + a(\delta-1)e_a}\right)\left(\frac{dw}{dt}\bigg|_{d\bar{G}=0}\right), \tag{A6}$$

where $e - w(1+s/t)(\delta-1)e_a + a(\delta-1)e_a = -eG_a > 0$. From the effort determination (2) we have $e - w(1+s/t)(\delta-1)e_a = e\left[w^n - b\right]^{-1}\left[w(1-t) + ta - b - w(t+s)\alpha(\delta-1)\right]$ so that

$$\frac{dL}{dt}\bigg|_{dG=0}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\text{ as }\; w(1-t) + ta - b\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} w(t+s)\alpha(\delta-1)\,. \tag{A7}$$

Substituting the efficiency wage equation (4) for w in (A7) gives condition (14).

Using the equations (11) and (A5), we can rewrite (15) as follows (using $e_{w^n} = e_w(1-t)^{-1} = e_a t^{-1}$)

$$\frac{d(eL)}{dt}\bigg|_{dG=0} = \left\{L\left(e_w - \frac{e_a G_w}{G_a}\right) + e\left(L_w + L_e e_w - L_e e_a\frac{G_w}{G_a}\right)\right\}\left(\frac{dw}{dt}\right)_{dG=0}$$

Using partial derivatives of the employment and effort functions, the common term $Le_w + e(L_w + L_e e_w)$ can be written as

$$Le_w + e(L_w + L_e e_w) = \frac{eL}{w}\left[\frac{e_w w}{w} + \frac{dL}{dw}\frac{w}{L}\right] = 0\,. \tag{A8}$$

Inserting these, we obtain condition (16).

Appendix 3

Taking the total differential of government budget constraint (6) with respect to wage tax, payroll tax and gross efficiency wage gives $dG = 0 = G_t dt + G_s ds + G_w dw$, whereby

$$G_s = L\left[w + (t(w-a) + sw)\frac{L_s}{L}\right] \qquad \text{(A9)}$$

Using $\frac{L_s}{L} = -\frac{\delta}{1+s}$, we obtain for the first term of the RHS of equation (21)

$$\left(L_e e_t - L_s \frac{G_t}{G_s}\right) = \left[1-(t+s-(ta/w))\frac{\delta}{1+s}\right]^{-1} \frac{L(w-a)}{w(1+s)(1-t)}\left[1+s-(t+s)\delta\right] \qquad \text{(A10)}$$

The second RHS term of equation (21) is

$$\left(L_w + L_e e_w - L_s \frac{G_w}{G_s} w_t\right) = \left[1-(t+s-(ta/w))\frac{\delta}{1+s}\right]^{-1}\left(-\frac{L}{w(1+s)}\left[1+s-(t+s)\delta\right]\right). \qquad \text{(A11)}$$

Using equation (18) implies

$$\left(L_w + L_e e_w - L_s \frac{G_w}{G_s}\right)\left(\left.\frac{dw}{dt}\right|_{dG=0}\right) =$$

$$\left[1-(t+s-(ta/w))\frac{\delta}{1+s}\right]^{-1}\left(-\frac{L}{w(1+s)}\left[1+s-(t+s)\delta\right]\right)\left(\left.\frac{dw}{dt}\right|_{dG=0}\right) = \qquad \text{(A12)}$$

$$\left[1-(t+s-(ta/w)\frac{\delta}{1+s}\right]^{-1}\left(-\frac{L\left[1+s-(t+s)\delta\right]}{w(1+s)(1-t)}\left(w-\frac{a}{1-\alpha}\right)\right)$$

Finally, combining (A10) and (A12) yields

$$sign\left.\frac{dL}{dt}\right|_{dG=0} = sign\left(\frac{L\left[1+s-(t+s)\delta\right]}{w(1+s)(1-t)}\frac{a\alpha}{1-\alpha}\right) = sign\left[1+s-(t+s)\delta\right], \text{ as } a>0. \qquad \text{(A13)}$$

This gives the same condition as in (14). Concerning effective labour input and output we have

$$L\left.\frac{de}{dt}\right|_{dG=0} = -\frac{\alpha eL}{w^n - b}\frac{\alpha a}{1-a} = -\frac{eL}{w(1-t)}\frac{\alpha a}{1-a} < 0. \qquad \text{(A14)}$$

and thus

$$L\left.\frac{de}{dt}\right|_{dG=0} + e\left.\frac{dL}{dt}\right|_{dG=0} = \frac{eL}{w(1-t)}\frac{\alpha a}{1-a}\left[\frac{1+s-(t+s)\delta}{1+s-(t+s)\delta+(ta/w)\delta} - 1\right] < 0 \text{ as } a>0. \qquad \text{(A15)}$$

Appendix 4

Using $w^n - b = \alpha w(1-t)$ from (4), and rewriting the disutility of effort as

$$\alpha e^{1/\alpha} = \frac{d\alpha}{(1+\alpha d)}(w^n - b) = \frac{d\alpha^2}{(1+\alpha d)} w(1-t) \qquad \text{(A16)}$$

Using (A16), we can specify the total effect on the reservation wage:

$$\begin{aligned}
db &= b_L \frac{dL}{dt} + b_{w^n} \frac{dw^n}{dt} + b_e \frac{de}{dt} = \left[w^n - \alpha e^{1/\alpha}\right]\frac{dL}{dt} + L\frac{dw^n}{dt} - Le^{(1/\alpha)-1}\frac{de}{dt} \\
&= \left[w^n - \alpha e^{1/\alpha}\right]\frac{dL}{dt} + L\left[1 - e^{(1/\alpha)-1} e_{w^n}\right]\frac{dw^n}{dt} \\
&= \left[w^n - \alpha e^{1/\alpha}\right]\frac{dL}{dt} + \frac{L}{(w^n - b)}\left[w^n - b - \alpha e^{1/\alpha}\right]\frac{dw^n}{dt} \\
&= \left[w^n - \alpha e^{1/\alpha}\right]\frac{dL}{dt} + \frac{1}{(1+\alpha d)} L \frac{dw^n}{dt} \qquad \text{(A17)} \\
&= \left[w^n - \alpha e^{1/\alpha}\right]\left(L_w + L_e e_w - L_e e_a \frac{G_w}{G_a}\right)\frac{dw}{dt} + \frac{1}{(1+\alpha d)} L \left[(1-t) - t\frac{G_w}{G_a}\right]\frac{dw}{dt} \\
&= \frac{\left[w^n - \alpha e^{1/\alpha}\right]}{w}\left(-1 - (\delta - 1)\frac{t}{(1-t)}\frac{G_w}{G_a}\right) L \frac{dw}{dt} + \frac{1}{(1+\alpha d)} L \left[(1-t) - t\frac{G_w}{G_a}\right]\frac{dw}{dt}.
\end{aligned}$$

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