Central bank independence and the monetary instrument problem *

Stefan NIEMANN† Paul PICHLER‡ Gerhard SORGER§

This version: October 2011

Abstract

We study the monetary instrument problem in a dynamic non-cooperative game between separate, discretionary fiscal and monetary policy makers. We show that monetary instruments are equivalent only if the policy makers' objectives are perfectly aligned; otherwise an instrument problem exists. When the central bank is benevolent while the fiscal authority is short-sighted relative to the private sector, excessive public spending and debt emerge under a money growth policy but not under an interest rate policy. Despite this property, the interest rate is not necessarily the optimal instrument.

JEL *classification*: E52, E61, E63
Keywords: Monetary instrument problem; central bank independence; dynamic game; optimal fiscal and monetary policy; lack of commitment

*We thank Elisabeth Beckmann, Alejandro Cunat, Alain Gabler, Christian Ghiglino, Michael Krause, and audiences at the University of Essex, the University of Linz, and the Swiss National Bank for helpful comments. We also acknowledge helpful comments from referees who have assessed an earlier version of this paper. The views expressed in this paper are solely the responsibility of the authors and do not necessarily reflect the views of the Oesterreichische Nationalbank.

†Department of Economics, University of Essex. E-mail: sniem@essex.ac.uk
‡Corresponding author. Oesterreichische Nationalbank, Economic Studies Division, Otto-Wagner-Platz 3, 1090 Wien, Austria. Tel.: +43-1-40420-7214. E-mail: paul.pichler@oenb.at
§Department of Economics, University of Vienna. E-mail: gerhard.sorger@univie.ac.at

1 Introduction

We study the monetary instrument problem in a dynamic non-cooperative game between separate fiscal and monetary policy makers who choose policies in a discretionary fashion. We show that, unless the policy makers' objectives are perfectly aligned, the central bank faces an instrument problem. Its choice of strategic variable – *money growth rate* versus *nominal interest rate* – has a critical impact on the equilibrium allocation implemented under discretionary policy. In particular, the instrument choice affects how well the central bank can accomplish its policy objectives. The underlying mechanism is fundamentally different from the mechanisms previously studied in the literature on the monetary instrument problem.

In our benchmark environment, policy makers face a conflict of interest. They agree on desirable allocations today but disagree about the *intertemporal* trade-offs inherent in policy making. The central bank is benevolent whereas the fiscal authority is impatient relative to society (i.e., it discounts future utility at a higher rate). This is a realistic scenario, given the ample evidence on frictions inherent in fiscal decision making that induce governments to follow short-sighted policies.[1]

The monetary instrument choice strongly affects the dynamic distortions arising from fiscal impatience. Under a money growth policy, fiscal impatience leads to excessive public spending and debt. By contrast, under an interest rate policy, no spending and debt biases emerge; fiscal impatience has actually no effect at all on the eventual equilibrium allocation. This clear-cut result obtains because, in the economy under consideration, the nominal interest rate fully determines private agents' future asset portfolio, which, in turn, fully determines future policies and hence future distortions. For a given interest rate, the fiscal authority's dynamic optimal policy problem thus boils down to the *static* problem of minimizing current distortions. In this situation, the fiscal rate of time-preference becomes irrelevant, while the central bank can balance current and future distortions perfectly in line with its own objectives.

Despite this property the optimal choice of monetary instrument is not trivial. We show

[1]In political business cycle models, electoral uncertainty typically gives rise to strategic myopia: realizing that it might be replaced by a government with different partisan preferences, an incumbent government has an incentive to follow relatively short-sighted policies (Persson and Tabellini, 1999). Malley, Philippopoulos, and Woitek (2007) provide empirical evidence for the U.S. that electoral uncertainty actually induces policies that resemble the behavior of an impatient fiscal policy maker.

that situations exist where money growth policies are optimal because fiscal impatience has *positive* welfare effects; by counteracting the monetary time inconsistency problem, it moves the equilibrium allocation under discretion closer to the second-best (Ramsey) allocation. Such situations, however, arise for empirically implausible environments, and hence interest rate polices are typically optimal. This is consistent with the fact that independent central banks in most developed economies use the interest rate as their primary policy instrument.

Our paper contributes to a large literature on the monetary instrument problem. This literature started with the seminal paper by Poole (1970), who shows in a simple IS-LM framework that the desirability of money growth versus interest rate rules depends on the sources and relative volatilities of macroeconomic shocks. In a rational expectations framework, Sargent and Wallace (1975) and McCallum (1981) argue that the optimality of a monetary instrument hinges on whether it can implement a unique equilibrium. These aspects of the instrument choice have been further explored in various environments (see, among others, Canzoneri, Henderson, and Rogoff, 1983; Carlstrom and Fuerst, 1995; Collard and Dellas, 2005; King and Wolman, 2004; Dotsey and Hornstein, 2008). Moreover, several recent contributions have investigated how the optimal instrument choice interacts with fiscal policy (e.g., Benhabib, Schmitt-Grohe, and Uribe, 2001; Schabert, 2006, 2010). A common feature of all these papers is that stochastic disturbances give rise to the monetary instrument problem because policy rules based on different intermediate targets differ with respect to the stabilization properties or their effects on equilibrium (in)determinacy. By contrast, we present an analysis of the monetary instrument problem in a *deterministic* environment, where the instrument problem arises from the interaction between separate, strategically optimizing fiscal and monetary policy makers.

Our approach to modeling optimal fiscal and monetary policies differs in several ways from the one commonly adopted in the literature.[2] There, a monolithic Ramsey planner chooses among all allocations that are consistent with a market equilibrium and commits to this choice over time. Strategic interaction between separate policy makers is absent, the question of how policy makers can induce desirable allocations through their available instruments is not addressed, and optimal policies are generically time-inconsistent. These are important omissions, given that fiscal and monetary policies in most countries today are determined by distinct fis-

[2]Prominent contributions include Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991), Schmitt-Grohe and Uribe (2004), Siu (2004), and Khan, King, and Wolman (2003).

cal and monetary policy makers, who have no direct control over allocations, and who lack commitment power. Our model seeks to address these institutional considerations.

We formulate the policy problem as a game between successive governments and analyze Markov-perfect equilibrium policies, following a recent literature in dynamic macroeconomics. This literature analyzes optimal taxation and public spending in non-monetary economies (Klein and Rios-Rull, 2003; Ortigueira, 2006; Klein, Krusell, and Rios-Rull, 2008) and fiscal-monetary policy interactions under a single monolithic authority (Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008; Martin, 2009, 2010). Recent papers by Adam and Billi (2008, 2010) and Niemann (2011) study discretionary fiscal and monetary policies chosen by separate authorities. They explore the desirability of monetary conservatism in a dynamic game framework but abstract from the instrument choice problem.

The rest of this paper is organized as follows. Section 2 describes the economic environment. Section 3 analyzes the policy game and establishes our main analytical results. Section 4 examines two numerical examples to illustrate the non-trivial welfare implications of the monetary instrument choice. Section 5 discusses the robustness of our findings. Section 6 concludes. All proofs are relegated to the Appendix.

2 Model

Our model is a variant of the cash-in-advance economy studied by Nicolini (1998). We choose this model for two reasons. First, it is a convenient benchmark as its optimal policy prescriptions under a monolithic policy maker are well understood.[3] Second, it arguably constitutes the simplest possible environment to study the monetary instrument problem under separate fiscal and monetary policy makers.

[3]Nicolini (1998) has studied optimal policy under the assumption of commitment, while Ellison and Rankin (2007), Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), and Martin (2009) have recently analyzed discretionary policy in very similar frameworks.

2.1 The private sector

There is a continuum (of measure one) of private households. Households are identical, i.e., they have identical preferences and identical asset endowments. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, g_t) - \alpha n_t], \tag{1}$$

where c_t and g_t denote consumption of a private and public good, n_t denotes labor effort, $\beta \in (0, 1)$ is a time-preference factor, $\alpha > 0$ is the (constant) marginal disutility of labor, and u is an instantaneous utility function that satisfies standard properties.[4]

The representative household enters period t with M_t units of fiat money and B_t government bonds. Each bond delivers one unit of money when it matures at the end of period t. The household supplies labor and consumes. Output is linear in labor, $y_t = n_t$. The household's flow budget constraint is

$$P_t n_t + M_t + B_t \geq P_t c_t + M_{t+1} + \frac{B_{t+1}}{1 + i_t}, \tag{2}$$

where P_t denotes the nominal price of the output good and i_t the net nominal interest rate on bonds issued in period t. Finally, the household is subject to a cash-in-advance constraint that requires private consumption purchases to be financed with cash carried over from the previous period (cf. Svensson, 1985),

$$M_t \geq P_t c_t, \tag{3}$$

and a no-Ponzi-scheme constraint,

$$\lim_{T \to \infty} \prod_{s=0}^{T-1} (1 + i_s)^{-1} B_T \geq 0. \tag{4}$$

[4]Specifically, we assume u to be continuous, increasing, and concave on its domain $\mathbb{R}^2_+$, and twice continuously differentiable, strictly increasing, and strictly concave on the interior of its domain.

2.2 The public sector

The government consists of two authorities, a fiscal authority and a monetary authority (central bank). The authorities' preferences can differ from the representative household's preferences, i.e., policy makers might not be benevolent. Moreover, the fiscal authority's preferences might not be perfectly aligned with the central bank's preferences, giving rise to a conflict of interest between the two. We focus on the case where this conflict is *intertemporal* in nature. Specifically, we assume that both authorities share the household's instantaneous utility function u but need not share its time-preference factor β.[5] Instead, the fiscal authority discounts future utility with the factor β^F and the monetary authority with the factor β^M. Accordingly, the period-t fiscal authority's preferences are given by

$$\sum_{s=t}^{\infty}(\beta^F)^{s-t}[u(c_s, g_s) - \alpha n_s], \tag{5}$$

while the monetary authority's preferences are

$$\sum_{s=t}^{\infty}(\beta^M)^{s-t}[u(c_s, g_s) - \alpha n_s]. \tag{6}$$

The fiscal authority purchases output and transforms it into a public good g_t at a one-to-one rate. Its outlays are financed by seignorage income received from the monetary authority and by issuance of new debt. For simplicity there are no taxes.[6] The consolidated flow budget constraint of the public sector is

$$P_t g_t + \bar{B}_t = \bar{M}_{t+1} - \bar{M}_t + \frac{\bar{B}_{t+1}}{1 + i_t}, \tag{7}$$

where $\bar{M}_t$ denotes cash in circulation and $\bar{B}_t$ the amount of public debt outstanding at the start of period t. The fiscal authority seeks to maximize (5) by appropriate choices of government spending, g_t, and issuance of nominal debt, $\bar{B}_{t+1}$. The monetary authority seeks to maximize (6) by appropriate choices of the interest rate, i_t, and the money stock, $\bar{M}_{t+1}$. Each authority must respect the consolidated flow budget constraint (7) and market clearing conditions.

[5]We discuss the implications of this assumption in Section 5.

[6]As we show in Section 5, the introduction of distortionary taxes does not affect our central results.

In particular, the public supply of bonds and money must meet the private demand at the prevailing interest rate.

2.3 The first-best allocation

To establish a normative benchmark, let us briefly describe the first-best allocation (c^*, g^*, n^*) in the economy under consideration. This allocation maximizes the objective function (1) subject to the aggregate resource constraint

$$c_t + g_t = n_t. \tag{8}$$

It is characterized by the necessary and sufficient first-order optimality conditions

$$u_c(c_t, g_t) = u_g(c_t, g_t) = \alpha \tag{9}$$

and the constraint (8).[7] Note that a benevolent policy maker who could directly choose the allocation would choose (c^*, g^*, n^*). However, policy makers in our model cannot directly choose the allocation, but they must decentralize it via fiscal and monetary policies. Examining the private-sector response to these policies is thus a key step in the formulation of the optimal policy problem.

2.4 The private-sector equilibrium

The representative household takes prices $(P_t)_{t=0}^{\infty}$ and policies $(g_t, i_t, \bar{B}_{t+1}, \bar{M}_{t+1})_{t=0}^{\infty}$ as given. Starting from an initial asset position (M_0, B_0), it chooses $(c_t, n_t, M_{t+1}, B_{t+1})_{t=0}^{\infty}$ to maximize (1) subject to the constraints (2), (3) and (4). The corresponding Lagrangian function is

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, g_t) - \alpha n_t + \lambda_t \left(P_t n_t + M_t + B_t - P_t c_t - M_{t+1} - \frac{B_{t+1}}{1+i_t} \right) + \nu_t \left(M_t - P_t c_t \right) \right\},$$

[7] The notation u_j denotes the partial derivative of u with respect to variable j; for the derivatives of other functions with several arguments we use an analogous notation. For functions of only one variable, we use a prime to denote the derivative.

where λ_t and ν_t are non-negative multipliers. The first-order conditions are

$$\begin{aligned}
0 &= u_c(c_t, g_t) - (\lambda_t + \nu_t)P_t, \\
0 &= -\alpha + \lambda_t P_t, \\
0 &= -\lambda_t + \beta(\lambda_{t+1} + \nu_{t+1}), \\
0 &= -\lambda_t \frac{1}{1+i_t} + \beta\lambda_{t+1}.
\end{aligned}$$

Eliminating λ_t and ν_t and introducing $\pi_{t+1} = P_{t+1}/P_t - 1$, these conditions imply

$$\beta(1+i_t) = 1 + \pi_{t+1}, \tag{10}$$

$$\alpha(1+i_t) = u_c(c_{t+1}, g_{t+1}). \tag{11}$$

Equation (10) shows that the gross real interest rate is constant and equal to $1/\beta$ under optimal household behavior. Equation (11), when compared to (9), shows that a positive interest rate is distortionary. Specifically, if $i_t > 0$, there is a wedge between the marginal utility of (next period's) consumption and the marginal cost of producing the output good. This wedge reflects the opportunity cost of holding money; it originates from the cash-in-advance constraint that forces households to carry money from t to $t+1$ in order to consume in period $t+1$. Clearly, private agents do not hold more money than necessary whenever the nominal interest rate is positive, i.e., the cash-in-advance constraint (3) binds whenever $i_{t-1} > 0$.[8]

For characterizing the private-sector equilibrium it is useful to rewrite the optimality conditions (10)-(11) and the constraints (2), (3) and (7) in terms of the normalized variables $b_t = B_t/\bar{M}_t$, $\bar{b}_t = \bar{B}_t/\bar{M}_t$, $m_t = M_t/\bar{M}_t$, and $p_t = P_t/\bar{M}_t$. This normalization by the aggregate money stock imposes stationarity (cf. Cooley and Hansen, 1991). Note that the two asset market clearing conditions

$$\bar{B}_t = B_t \qquad \text{and} \qquad \bar{M}_t = M_t \tag{12}$$

[8]When $i_{t-1} = 0$, on the other hand, the opportunity cost of holding money vanishes and the agents are indifferent about holding financial wealth in the form of money or in the form of bonds. In order to have a well-defined money demand function also in this case, we simply assume that the agents hold the minimal amount of money that is consistent with optimal behavior even if $i_{t-1} = 0$. In other words, we assume that the cash-in-advance constraint holds with equality for all $t \geq 1$.

imply $b_t = \bar{b}_t$ and $m_t = 1$. Moreover, the cash-in-advance constraint holding with equality implies $p_t = 1/c_t$. Denoting the money growth rate by $\mu_t = \bar{M}_{t+1}/\bar{M}_t - 1$, the conditions characterizing a private-sector equilibrium are

$$1+\mu_t = \frac{\beta u_c(c_{t+1}, g_{t+1}) c_{t+1}}{\alpha c_t}, \tag{13}$$

$$1+i_t = \frac{u_c(c_{t+1}, g_{t+1})}{\alpha}, \tag{14}$$

$$g_t + (1+b_t)c_t = \beta c_{t+1}\left(\frac{u_c(c_{t+1}, g_{t+1})}{\alpha} + b_{t+1}\right), \tag{15}$$

$$c_t + g_t = n_t, \tag{16}$$

together with the no-Ponzi-scheme constraint (4).

3 The policy game

In our framework fiscal and monetary policies are chosen by separate authorities in a discretionary fashion. The authorities in period t can choose period-t policy variables, but they cannot control policy variables for the future $\{t+1, t+2, ...\}$. Following the standard approach in the literature, we model this by assuming that there exist different incarnations (selves) of the fiscal and monetary authorities in each time period; accordingly, each authority takes the policy rules of its current opponent and all future authorities as given.

Optimal policy corresponds to a Nash equilibrium in the game between authorities. Since this game is dynamic, strategies can in principle depend on its entire history. Without any restriction on strategies the model would have *'embarrassingly many equilibria'* (cf. Albanesi, Chari, and Christiano, 2003, p. 715) and its optimal policy prescriptions would be impossible to assess. Therefore, and following the common approach in the literature, we restrict attention to Markov strategies, which depend only on a minimal payoff-relevant state vector. Inspection of (13)-(16) shows that the minimal state vector in our economy consists of only a single state variable, the debt-to-money ratio b_t. Markov-perfect equilibrium policy rules will thus take the form $i_t = \mathcal{I}(b_t)$, $g_t = \mathcal{G}(b_t)$, $\mu_t = \mathcal{M}(b_t)$, *etc.*

Regarding the interaction between fiscal and monetary authorities *within* a given time period, we consider three different institutional scenarios. In the first one both authorities are benevolent and cooperate. This is the standard framework in the literature[9] and will serve mainly as a benchmark for comparison. In the second and third scenario the two authorities' preferences are not perfectly aligned; the fiscal authority is impatient relative to the monetary authority and the private sector, and policies are implemented in a non-cooperative fashion. The difference between scenarios 2 and 3 lies in the choice of monetary instrument. In the second scenario the central bank chooses the *money growth rate* μ_t, and the fiscal authority simultaneously chooses expenditures g_t; each authority takes the other authority's policy action, the future authorities' policy rules, and the private-sector equilibrium response to policies as determined by (13)-(16) as given. By contrast, in the third scenario the central bank chooses the *interest rate* i_t rather than the money growth rate.[10]

3.1 Optimal policy under full cooperation

We first examine the case where fiscal and monetary policies are chosen by benevolent policy makers who fully cooperate. To lay out the optimal policy problem we first need to introduce some more notation. Let *future* fiscal and monetary policies, i.e., policies from period $t+1$ onwards, be governed by the rules $\{\mathcal{G}, \mathcal{M}\}$[11] and let the *future* private-sector decision rules induced by these policies be denoted by $\{\mathcal{C}, \mathcal{B}, \mathcal{N}, \mathcal{I}\}$. Finally, let $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$ be the *current* private-sector decision rules when period-t policies are (g_t, μ_t) and future policies are governed by $\{\mathcal{G}, \mathcal{M}\}$.

The authorities in period t choose g_t and μ_t in a cooperative way. They take future policies $\{\mathcal{G}, \mathcal{M}\}$ as given and anticipate that the private-sector equilibrium response to (current and future) policies is $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$. The optimal policy problem under cooperation thus takes the form

$$\max_{g_t, \mu_t} u(\hat{\mathcal{C}}(b_t, g_t, \mu_t), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mu_t) + g_t) + \beta V(\hat{\mathcal{B}}(b_t, g_t, \mu_t)),$$

[9]See Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) and Martin (2009), among others.

[10]Our analysis focuses on the monetary instrument problem and ignores the possibility of a fiscal instrument problem. Throughout, we will therefore assume that the fiscal authority directs the level of public goods provision, g_t, while nominal debt issuance $\bar{B}_{t+1}$ is determined by market clearing.

[11]Note that, equivalently, we could have assumed that future monetary policies are governed by the interest rate rule $\mathcal{I}$; it is straightforward to check that both instrument choices are equivalent under a cooperative government.

where the continuation value function V is defined recursively by

$$V(b) = u(\mathcal{C}(b), \mathcal{G}(b)) - \alpha(\mathcal{C}(b) + \mathcal{G}(b)) + \beta V(\mathcal{B}(b)). \tag{17}$$

Note that the government's budget constraint is satisfied by construction of the private-sector equilibrium decision rules.

Definition 1. *A Markov-perfect equilibrium under cooperation of two benevolent fiscal and monetary authorities is a set of functions* $\{\mathcal{M}, \mathcal{G}, V, \hat{\mathcal{C}}, \mathcal{C}, \hat{\mathcal{B}}, \mathcal{B}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$ *such that, for all* b_t *and all* $t \geq 0$,

(i) $\{\mathcal{M}(b_t), \mathcal{G}(b_t)\} = \arg\max_{\mu_t, g_t} u(\hat{\mathcal{C}}(b_t, g_t, \mu_t), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mu_t) + g_t) + \beta V(\hat{\mathcal{B}}(b_t, g_t, \mu_t))$,

(ii) $V(b_t) = u(\mathcal{C}(b_t), \mathcal{G}(b_t)) - \alpha(\mathcal{C}(b_t) + \mathcal{G}(b_t)) + \beta V(\mathcal{B}(b_t))$, *and*

(iii) $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$ *satisfy (13)-(16) for given* $\{\mathcal{M}, \mathcal{G}, \mathcal{C}\}$.

Note that this definition implies stationarity, i.e., $\mathcal{C}(b_t) = \hat{\mathcal{C}}(b_t, \mathcal{G}(b_t), \mathcal{M}(b_t))$ and $\mathcal{B}(b_t) = \hat{\mathcal{B}}(b_t, \mathcal{G}(b_t), \mathcal{M}(b_t))$. Note also that there may exist Markov-perfect equilibria where policy functions display discontinuities that are not rooted in the economy's fundamentals but are rather an artifact of the infinite horizon (Martin, 2010). In the present paper we abstract from such equilibria and focus on differentiable Markov-perfect equilibria. The following proposition characterizes the equilibrium outcome under cooperation of two benevolent policy makers.

Proposition 1. *A Markov-perfect equilibrium outcome* $(g_t, \mu_t, c_t, b_t, n_t, i_t)_{t=0}^{+\infty}$ *under cooperation of two benevolent fiscal and monetary authorities satisfies (13)-(16) and*

$$w_t^c = (1 + b_t) w_t^g, \tag{18}$$

$$w_t^g F'(b_{t+1}) = \beta c_{t+1} w_{t+1}^g, \tag{19}$$

where $w_t^c = u_c(c_t, g_t) - \alpha$, $w_t^g = u_g(c_t, g_t) - \alpha$, *and* $F(b_{t+1}) = \beta \mathcal{C}(b_{t+1})[\frac{u_c(\mathcal{C}(b_{t+1}), \mathcal{G}(b_{t+1}))}{\alpha} + b_{t+1}]$.

The proof is given in the Appendix. To understand the content of Proposition 1, note that w_t^c and w_t^g are the wedges between the marginal utility of consumption of private and public goods in period t and the marginal cost of producing these goods, while $F(b_{t+1})$ denotes the real gross revenue for the government in period t from issuing money, M_{t+1}, and bonds, B_{t+1}, to the households. Equation (18) characterizes the optimal provision of the public good. It shows that the marginal utility of private consumption exceeds the marginal utility of public consumption when outstanding debt is positive. This result is due to the cash-in-advance constraint on private consumption. Since private agents' nominal money holdings are predetermined, the cash-in-advance constraint implies that an increase in c must be accommodated by a decline in the price level p and, accordingly, an increase in the real value of government debt. This effect depresses consumption of the private good relative to consumption of the public good if outstanding debt is positive.

Equation (19) is the generalized Euler equation characterizing the dynamics of the endogenous state variable b. A marginal increase in liabilities b_{t+1} raises the consolidated government's gross real revenues by $F'(b_{t+1})$ and, in further consequence, increases current utility by $w_t^g F'(b_{t+1})$. Along the optimal allocation, this gain must be balanced by $\beta c_{t+1} w_{t+1}^g$, the discounted future utility loss due to the tighter budget constraint in period $t+1$. Finally, note that $F'(b_{t+1})$ in (19) contains the unknown policy functions $\mathcal{C}$ and $\mathcal{G}$ as well as their derivatives, making (19) a *generalized* Euler equation.

Evaluating (18) and (19) at the steady state yields the following characterization of the long-run level of debt induced by Markov-perfect equilibrium policies.

Corollary 1. *The steady state level of debt $\bar{b}$ in a Markov-perfect equilibrium under cooperation of two benevolent authorities satisfies*

$$\bar{b} = \frac{u_{cc}(\bar{c}, \bar{g})\bar{c}}{\alpha} \left(\frac{1}{\bar{\sigma}} - 1 \right) - \frac{u_{cg}(\bar{c}, \bar{g})\bar{c}}{\alpha} \frac{\mathcal{G}'(\bar{b})}{\mathcal{C}'(\bar{b})}, \tag{20}$$

where $\bar{c} = \mathcal{C}(\bar{b})$, $\bar{g} = \mathcal{G}(\bar{b})$, and $\bar{\sigma} = -\bar{c} u_{cc}(\bar{c}, \bar{g})/u_c(\bar{c}, \bar{g})$ is the elasticity of the marginal utility of private consumption with respect to c evaluated at the steady state.

Equation (20) shows that the steady state level of debt depends critically on the value of $\bar{\sigma}$, the cross derivative $u_{cg}(\bar{c}, \bar{g})$, and the relative curvature of $\mathcal{C}$ and $\mathcal{G}$. Specifically, if the utility function u is additively separable, $u_{cg}(\bar{c}, \bar{g}) = 0$, then the sign of the steady-state debt is solely determined by the value of $\bar{\sigma}$. Debt is positive, zero, or negative depending on whether $\bar{\sigma}$ is greater than, equal to, or smaller than one (cf. Martin, 2009).

3.2 Optimal non-cooperative policy under a money growth regime

We next examine Markov-perfect equilibrium policies when the fiscal and monetary authorities do not cooperate and the central bank uses the money growth rate as its instrument. Accordingly, the central bank chooses μ_t, taking as given the fiscal policy action g_t, future policies $\{\mathcal{G}, \mathcal{M}\}$, and the private-sector equilibrium response $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$. The central bank's optimization problem is

$$\max_{\mu_t} u(\hat{\mathcal{C}}(b_t, g_t, \mu_t), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mu_t) + g_t) + \beta^M V^M(\hat{\mathcal{B}}(b_t, g_t, \mu_t)),$$

where V^M is defined as in (17) but with the discount factor β replaced by β^M. Conversely, the fiscal authority chooses g_t, given μ_t, $\{\mathcal{G}, \mathcal{M}\}$, and $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$. It solves

$$\max_{g_t} u(\hat{\mathcal{C}}(b_t, g_t, \mu_t), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mu_t) + g_t) + \beta^F V^F(\hat{\mathcal{B}}(b_t, g_t, \mu_t)),$$

where V^F is defined as in (17) but with discount factor β^F. In a Markov-perfect equilibrium, the solution of the central bank's optimization problem must be a best response to the solution of the fiscal authority's problem and vice versa. Moreover, both must be best responses to future policies. Formally:

Definition 2. *A non-cooperative Markov-perfect equilibrium under a money growth regime is a set of functions* $\{\mathcal{M}, \mathcal{G}, V^M, V^F, \hat{\mathcal{C}}, \mathcal{C}, \hat{\mathcal{B}}, \mathcal{B}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$ *such that, for all* b_t *and all* $t \geq 0$,

(i) $\{\mathcal{M}(b_t)\} = \arg\max_{\mu_t} u(\hat{\mathcal{C}}(b_t, \mathcal{G}(b_t), \mu_t), \mathcal{G}(b_t)) - \alpha(\hat{\mathcal{C}}(b_t, \mathcal{G}(b_t), \mu_t) + \mathcal{G}(b_t)) + \beta^M V^M(\hat{\mathcal{B}}(b_t, \mathcal{G}(b_t), \mu_t))$,

(ii) $\{\mathcal{G}(b_t)\} = \arg\max_{g_t} u(\hat{\mathcal{C}}(b_t, g_t, \mathcal{M}(b_t)), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mathcal{M}(b_t)) + g_t) + \beta^F V^F(\hat{\mathcal{B}}(b_t, g_t, \mathcal{M}(b_t)))$,

(iii) $V^M(b_t) = u(\mathcal{C}(b_t), \mathcal{G}(b_t)) - \alpha(\mathcal{C}(b_t) + \mathcal{G}(b_t)) + \beta^M V^M(\mathcal{B}(b_t))$,

(iv) $V^F(b_t) = u(\mathcal{C}(b_t), \mathcal{G}(b_t)) - \alpha(\mathcal{C}(b_t) + \mathcal{G}(b_t)) + \beta^F V^F(\mathcal{B}(b_t))$, *and*

(v) $\{\hat{\mathcal{C}}, \hat{\mathcal{B}}, \hat{\mathcal{N}}, \hat{\mathcal{I}}\}$ *satisfy (13)-(16) for given* $\{\mathcal{M}, \mathcal{G}, \mathcal{C}\}$.

The equilibrium allocation under a money growth regime is characterized as follows.

Proposition 2. *A non-cooperative Markov-perfect equilibrium outcome* $(g_t, \mu_t, c_t, b_t, n_t, i_t)_{t=0}^{+\infty}$ *under a money growth regime satisfies (13)-(16) and*

$$\frac{w_t^c F'(b_{t+1})}{1+b_t} = \frac{\beta^M c_{t+1} w_{t+1}^c}{1+b_{t+1}} + \frac{\beta^M [w_{t+1}^c - (1+b_{t+1}) w_{t+1}^g] \mathcal{G}'(b_{t+1})}{1+b_{t+1}}, \tag{21}$$

$$w_t^g F'(b_{t+1}) + [w_t^c - (1+b_t) w_t^g] \frac{H'(b_{t+1}) c_t}{H(b_{t+1})} \tag{22}$$

$$= \beta^F c_{t+1} w_{t+1}^g + \beta^F [w_{t+1}^c - (1+b_{t+1}) w_{t+1}^g] \left(\frac{H'(b_{t+2}) c_{t+1}}{H(b_{t+2})} \mathcal{B}'(b_{t+1}) - \mathcal{C}'(b_{t+1}) \right),$$

where $H(b_{t+1}) = F(b_{t+1}) - \beta \mathcal{C}(b_{t+1}) b_{t+1}$ *is the equilibrium real money stock associated with the state* b_{t+1}*.*

Equilibrium conditions (21) and (22) take a more complicated form than their counterparts under full cooperation. This is because policy makers (generically) disagree on the optimal levels of current private and public consumption, and hence the equilibrium allocation does not satisfy $w_t^c = (1+b_t) w_t^g$. Intuitively, this friction reflects the fiscal spending bias, which distorts w_t^g away from $w_t^c/(1+b_t)$ if $\beta^F \neq \beta^M$. By contrast, when both authorities are benevolent, $\beta^M = \beta^F = \beta$, the equilibrium allocation under cooperation is an equilibrium also in the non-cooperative scenario. This shows that it is the strategic interaction between policy makers rather than a coordination problem that drives the differences between the cooperative and non-cooperative environments.

Corollary 2 below shows how the conflict of interest affects the long-run evolution of debt in our economy. Due to the complexity of the equilibrium conditions we are only able to provide analytical results for the case of a separable utility function.

Corollary 2. *Suppose that* $u(c, g) = \gamma \ln(c) + v(g)$*. The steady state level of debt in a non-cooperative Markov-perfect equilibrium under a money growth regime satisfies*

$$\bar{b} = \frac{\beta^M - \beta}{\beta} \frac{\bar{c}}{\mathcal{C}'(\bar{b})} + \frac{\beta^F - \beta^M}{\beta} \left[1 + \frac{\beta^F \bar{w}^c \mathcal{C}'(\bar{b})}{\beta^M \bar{w}^g \mathcal{G}'(\bar{b})} \right]^{-1} \frac{\bar{c}}{\mathcal{C}'(\bar{b})}. \tag{23}$$

It is instructive to compare this result with the corresponding result for the cooperative equilibrium. Given the utility function $u(c, g) = \gamma \ln(c) + v(g)$, equation (20) in Corollary 1implies $\bar{b} = 0$. Obviously, equation (23) coincides with this result in the case where both authorities are benevolent ($\beta^M = \beta^F = \beta$). When monetary and fiscal preferences are perfectly aligned but the policy makers are *not* benevolent ($\beta^M = \beta^F \neq \beta$), the steady state level of debt reflects the bias in preferences. In particular, impatient authorities induce positive government debt.[12] More generally, the second term on the right-hand side of (23) emerges only if the two authorities' objectives diverge, $\beta^F \neq \beta^M$. This term captures the effects of strategic interaction. A fiscal authority that is short-sighted relative to the monetary authority exerts further upwards pressure on the level of government debt.

3.3 Optimal non-cooperative policy under an interest rate regime

Finally, we examine non-cooperative Markov-perfect equilibrium policies when the central bank uses the interest rate as its instrument. The central bank chooses i_t, taking as given the fiscal policy action g_t, future policies $\{\mathcal{G}, \mathcal{I}\}$, and the private-sector equilibrium response to (g_t, i_t) as governed by $\{\tilde{\mathcal{C}}, \tilde{\mathcal{B}}, \tilde{\mathcal{N}}, \tilde{\mathcal{M}}\}$. The central bank's optimization problem is

$$\max_{i_t} u(\tilde{\mathcal{C}}(b_t, g_t, i_t), g_t) - \alpha(\tilde{\mathcal{C}}(b_t, g_t, i_t) + g_t) + \beta^M V^M(\tilde{\mathcal{B}}(b_t, g_t, i_t)),$$

where V^M is defined as in Section 3.2. The fiscal authority, in turn, solves

$$\max_{g_t} u(\tilde{\mathcal{C}}(b_t, g_t, i_t), g_t) - \alpha(\tilde{\mathcal{C}}(b_t, g_t, i_t) + g_t) + \beta^M V^M(\tilde{\mathcal{B}}(b_t, g_t, i_t)),$$

taking i_t, $\{\mathcal{G}, \mathcal{I}\}$, and $\{\tilde{\mathcal{C}}, \tilde{\mathcal{B}}, \tilde{\mathcal{N}}, \tilde{\mathcal{M}}\}$ as given. In analogy to the money growth scenario, the Markov-perfect equilibrium is defined as follows.

Definition 3. *A non-cooperative Markov-perfect equilibrium under an interest rate regime is a set of functions* $\{\mathcal{I}, \mathcal{G}, V^M, V^F, \tilde{\mathcal{C}}, \mathcal{C}, \tilde{\mathcal{B}}, \mathcal{B}, \tilde{\mathcal{N}}, \tilde{\mathcal{M}}\}$ *such that, for all* b_t *and all* $t \geq 0$,

(i) $\{\mathcal{I}(b_t)\} = \arg\max_{i_t} u(\tilde{\mathcal{C}}(b_t, \mathcal{G}(b_t), i_t), \mathcal{G}(b_t)) - \alpha(\tilde{\mathcal{C}}(b_t, \mathcal{G}(b_t), i_t) + \mathcal{G}(b_t)) + \beta^M V^M(\tilde{\mathcal{B}}(b_t, \mathcal{G}(b_t), i_t))$,

(ii) $\{\mathcal{G}(b_t)\} = \arg\max_{g_t} u(\tilde{\mathcal{C}}(b_t, g_t, \mathcal{I}(b_t)), g_t) - \alpha(\tilde{\mathcal{C}}(b_t, g_t, \mathcal{I}(b_t)) + g_t) + \beta^F V^F(\tilde{\mathcal{B}}(b_t, g_t, \mathcal{I}(b_t)))$,

(iii) $V^M(b_t) = u(\mathcal{C}(b_t), \mathcal{G}(b_t)) - \alpha(\mathcal{C}(b_t) + \mathcal{G}(b_t)) + \beta^M V^M(\mathcal{B}(b_t))$,

[12] In this context, note that one can show (given a separable utility function) that the consumption policy $\mathcal{C}$ is strictly decreasing in the neighborhood of a stable steady state.

(iv) $V^F(b_t) = u(\mathcal{C}(b_t), \mathcal{G}(b_t)) - \alpha(\mathcal{C}(b_t) + \mathcal{G}(b_t)) + \beta^F V^F(\mathcal{B}(b_t))$*, and*

(v) $\{\tilde{\mathcal{C}}, \tilde{\mathcal{B}}, \tilde{\mathcal{N}}, \tilde{\mathcal{M}}\}$ *satisfy (13)-(16) for given* $\{\mathcal{I}, \mathcal{G}, \mathcal{C}\}$.

Note that imposing Markov-perfect equilibrium policies on the private-sector equilibrium condition (14) reveals a crucial property of the decision rule $\tilde{\mathcal{B}}$. Specifically, in a Markov-perfect equilibrium, b_{t+1} is a function of only the nominal interest rate, $\tilde{\mathcal{B}}(b_t, g_t, i_t) = \mathrm{B}(i_t)$.[13] The fiscal authority's choice of g_t has therefore no effect on future economic activity and distortions. Its optimal policy problem degenerates to the static problem

$$\max_{g_t} u(\tilde{\mathcal{C}}(b_t, g_t, i_t), g_t) - \alpha(\tilde{\mathcal{C}}(b_t, g_t, i_t) + g_t),$$

that is, the fiscal authority chooses g_t such as to maximize *current* welfare (minimize current distortions). Its rate of time preference is thus completely irrelevant for public spending decisions and, accordingly, the equilibrium allocation is independent of β^F.

Proposition 3. *A non-cooperative Markov-perfect equilibrium outcome* $(g_t, \mu_t, c_t, b_t, n_t, i_t)_{t=0}^{+\infty}$ *under an interest rate regime satisfies (13)-(16) and*

$$w_t^c = (1 + b_t) w_t^g, \tag{24}$$

$$w_t^g F'(b_{t+1}) = \beta^M c_{t+1} w_{t+1}^g. \tag{25}$$

Comparing the equilibrium allocation under the interest rate regime with the allocation under the money growth regime shows that the central bank's choice of instrument matters. An allocation that satisfies equilibrium conditions (24)-(25) does not satisfy (21)-(22) except for the special case where $\beta^F = \beta^M$. Hence, we arrive at the following proposition.

Proposition 4. *The monetary instruments* μ *and* i *are equivalent if and only if the fiscal and monetary policy makers' (time-)preferences are perfectly aligned; generically the central bank faces an instrument problem.*

[13] Note that there could, in principle, exist multiple solutions to (14), but these solutions are generically isolated. As a matter of fact, in all our numerical examples studied below, equation (14) is satisfied by a unique value b_{t+1} at equilibrium policies.

The intuition behind this result is straightforward. The policy makers in our economy do not face a coordination problem and have identical information sets. When they also have identical preferences (and hence pursue the same objectives) the non-cooperative equilibrium resembles the cooperative solution, independent of the central bank's policy instrument. By contrast, if the policy makers' preferences are not perfectly aligned, they face a conflict of interest. This introduces strategic motives into the policy makers' interaction, and the central bank's choice of strategic variable affects the equilibrium outcome via its impact on the fiscal authority's optimal policy problem.

In the economy under consideration, the fiscal policy problem is dynamic under a money growth regime but becomes static under an interest rate regime; see the discussion above. This feature gives rise to a further central result of our paper.

Proposition 5. *Fiscal impatience affects the equilibrium allocation under a money growth regime but not under an interest rate regime.*

An important implication of Proposition 5 is that, independent of the fiscal time-preference rate, a benevolent central bank can use an interest rate peg to implement the same equilibrium allocation that would obtain under cooperation of two benevolent policy makers. One might think that this implies that the interest rate is the optimal instrument, at least when the central bank is benevolent. Note, however, that this intuitive reasoning presupposes that fiscal impatience has *adverse* welfare effects on the private sector. While this is probably true in most economic environments, it is not always guaranteed. Under lack of commitment, a non-benevolent policy maker might choose policies that eventually yield higher private-sector welfare than under a benevolent policy maker; one prominent example is Rogoff's (1985) conservative central banker. The underlying mechanism is that biased preferences may mitigate the policy maker's time-inconsistency problem, and thereby move the discretionary equilibrium allocation closer to the (second-best) Ramsey allocation.

4 Optimal instrument choice

In this section we further discuss the optimal choice of monetary instrument. Our goal is *not* to identify the optimal instrument in a realistically calibrated economy. Instead, we present two numerical examples that illustrate the non-trivial nature of the optimal instrument choice. In particular, our examples show that, even when the central bank is benevolent, the interest rate might *not* be the optimal instrument.

In both examples the utility function takes the form

$$u(c, g) = \frac{\left(c^{\gamma_1} g^{1-\gamma_1}\right)^{1-\gamma_2} - 1}{1 - \gamma_2},$$

with $\gamma_1 \in (0, 1)$ and $\gamma_2 > 0$. The parameter γ_1 measures the relative weight of private versus public consumption in the Cobb-Douglas aggregate $c^{\gamma_1} g^{1-\gamma_1}$. The parameter γ_2 determines the (constant) elasticity of intertemporal substitution with respect to this aggregate. Moreover, γ_2 pins down the sign of the cross partial derivative

$$u_{cg}(c, g) = \frac{\gamma_1(1 - \gamma_1)(1 - \gamma_2) \left(c^{\gamma_1} g^{1-\gamma_1}\right)^{1-\gamma_2}}{cg}.$$

Depending on the value taken by γ_2, c and g enter the utility function as substitutes ($\gamma_2 \geq 1$) or as complements ($\gamma_2 < 1$). Our first example considers $\gamma_2 = 1$, which implies the additively separable utility function $u(c, g) = \gamma_1 \log c + (1 - \gamma_1) \log g$. The second example assumes $\gamma_2 = 0.4$, such that consumption is relatively elastic over time, and private and public consumption are complementary goods.

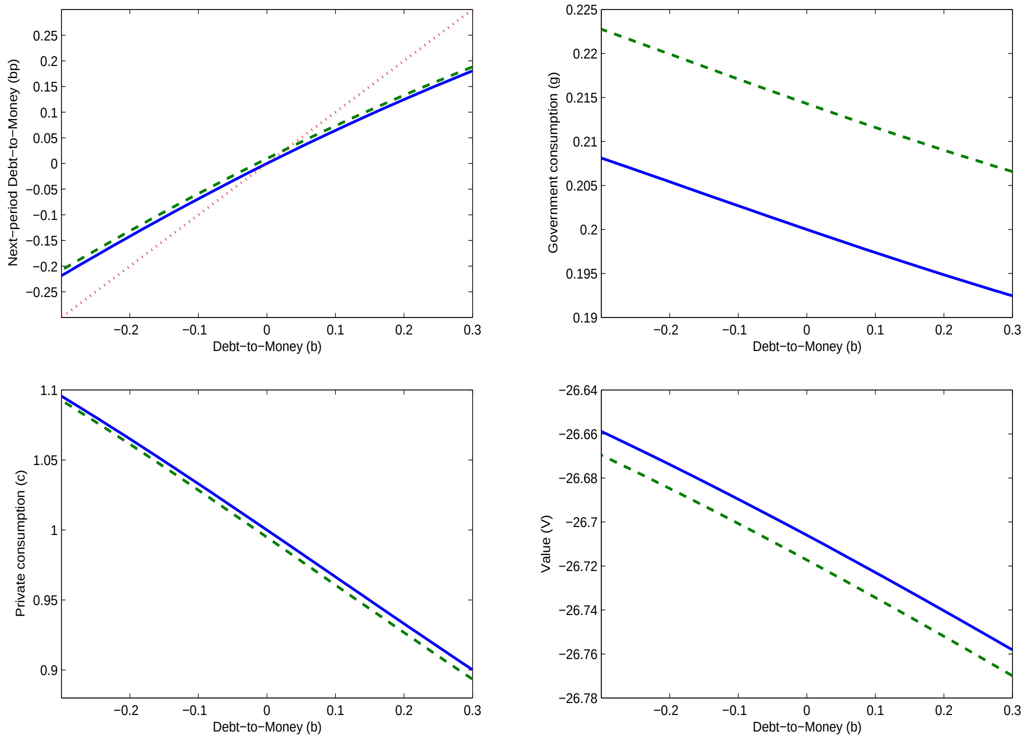
Since the nature of our numerical exercise is mainly illustrative, we choose values for the remaining parameters β, β^M, β^F, α, and γ_1 in a simple fashion. We set $\beta = 0.96$ corresponding to an annual real interest rate of close to 4%. The monetary authority is assumed to be benevolent, $\beta^M = \beta = 0.96$, while the fiscal authority is relatively impatient, $\beta^F = 0.8$. Finally, we choose $\alpha = 2/3$ and $\gamma_1 = 5/6$, thus normalizing steady-state consumption of private goods to $\bar{c} = 1$ and consumption of the public good to $\bar{g} = 1/5$. Given these parameters, we compute (high-order) polynomial approximations to the equilibrium policy functions $\{\mathcal{C}, \mathcal{G}, \mathcal{B}\}$ and the private-sector value function V using collocation projection methods as described in Judd (1992).[14]

[14]Since we use high-order polynomial approximations, the numerical accuracy of the approximated functions as measured by Euler equation errors is close to machine precision.

4.1 Example 1: A case for the interest rate

Observe from Corollary 1 that the equilibrium outcome under cooperation of benevolent policy makers features non-negative steady-state debt if $\gamma_2 \geq 1$. We consider this the empirically relevant scenario. The equilibrium policy and value functions for the case $\gamma_2 = 1$ are displayed in Figure 1.[15] Since the central bank is benevolent, the equilibrium under the interest rate

Figure 1: Equilibrium policy and value functions ($\gamma_2 = 1$)



Notes: The figure displays numerical approximations to the debt policy $\mathcal{B}(b)$ (top-left panel), the public consumption policy $\mathcal{G}(b)$ (top-right panel), the private consumption policy $\mathcal{C}(b)$ (bottom-left panel), and the value function $V(b)$ (bottom-right panel) under the interest rate policy (solid line) and the money growth policy (dashed line). The underlying parameters are $\beta = \beta^M = 0.96$, $\beta^F = 0.8$, $\alpha = 2/3$, $\gamma_1 = 5/6$, $\gamma_2 = 1$.

regime coincides with the equilibrium under cooperation of two benevolent authorities. In particular, this equilibrium is not affected by fiscal impatience which therefore has no welfare consequences under the interest rate regime. By contrast, the fiscal time-preference factor does influence equilibrium policies under the money growth regime. This property manifests itself

[15]We will not separately discuss results for the case $\gamma_2 > 1$. We have experimented with several values for γ_2 larger than one and found the results to be qualitatively similar to those for $\gamma_2 = 1$. For space considerations these results are omitted from the paper, but they are available upon request.

in a *higher* level of debt issuance, a *higher* level of public consumption, and a *lower* level of private consumption. Intuitively, since private and public consumption are substitutes, the high level of public spending decreases the marginal utility of private consumption, making it attractive for the private sector to decrease its consumption level and increase bond holdings. The overall welfare consequences of the fiscal bias are negative under the money growth regime. Accordingly, as shown in the bottom-right panel of Figure 1, private-sector welfare is lower under the money growth regime than under the interest rate regime (for all values of b). The optimal monetary instrument is thus the interest rate.

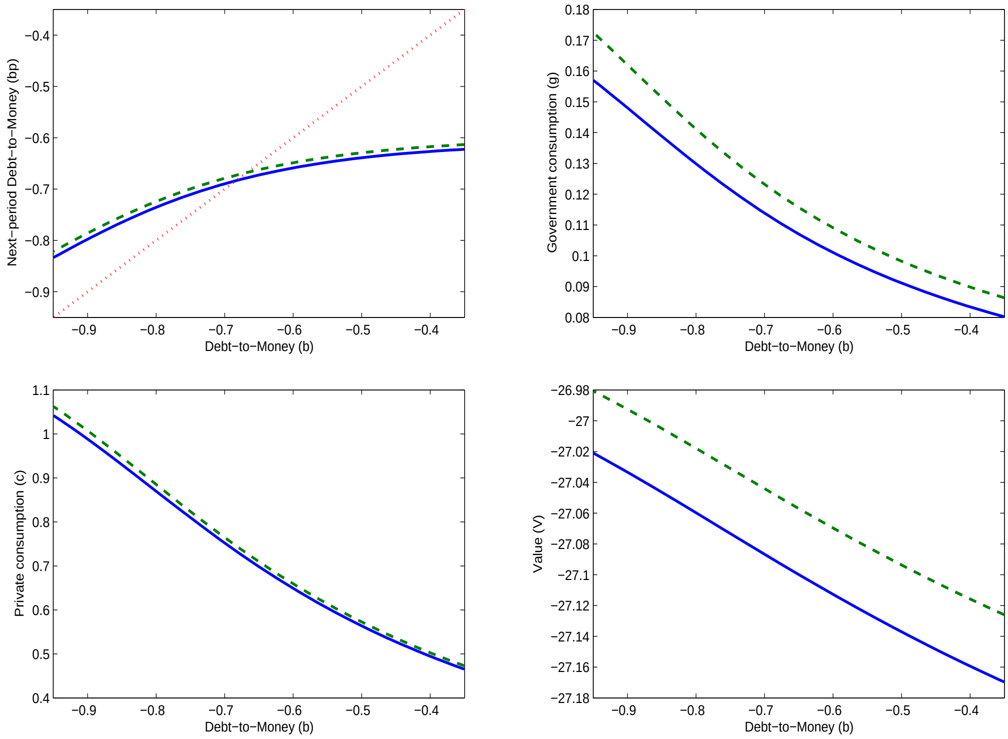
4.2 Example 2: A case for the money growth rate

We now consider the case where $\gamma_2 = 0.4$, such that private and public consumption enter the utility function as complements ($u_{cg}(c, g) > 0$). Figure 2 displays equilibrium policy and value functions. We observe that the money growth regime again leads to a *higher* level of public consumption and debt as compared to the interest rate regime. But in contrast to the case where $\gamma_2 = 1$, it now leads to a *higher* level of private consumption. Intuitively, this is due to the complementarity between private and public consumption. The high level of fiscal spending now *increases* the marginal utility of private consumption, making it attractive for the private sector to raise its consumption level (and increase labor supply accordingly). The overall welfare effects are positive: the equilibrium under the money growth regime features a *higher* level of private-sector welfare than under the interest rate regime. Accordingly, different from the example with $\gamma_2 = 1$, the optimal monetary instrument is the money growth rate.

The mechanism driving this result is the following. With $\gamma_2 = 0.4$, the intertemporal elasticity of substitution exceeds unity. Future consumption is more elastic than current consumption (whose elasticity is pinned down at unity due to the cash constraint), and the discretionary policy makers in every period face a recurrent incentive to increase current tax distortions relative to future distortions. As a consequence, the equilibrium allocation is characterized by a sub-optimally low level of consumption under discretionary policy, even when both authorities are benevolent.[16] In the present example, fiscal impatience counteracts this time-inconsistency

[16]To see this, consider a government with commitment power. When current consumption is relatively inelastic, this government would choose a policy plan that features higher distortions in the initial period than in later periods, and thus lower consumption in the initial period than in later periods (i.e., it would choose an

Figure 2: Equilibrium policy and value functions ($\gamma_2 = 0.4$)



Notes: The figure displays numerical approximations to the debt policy $\mathcal{B}(b)$ (top-left panel), the public consumption policy $\mathcal{G}(b)$ (top-right panel), the private consumption policy $\mathcal{C}(b)$ (bottom-left panel), and the value function $V(b)$ (bottom-right panel) under the interest rate policy (solid line) and the money growth policy (dashed line). The underlying parameters are $\beta = \beta^M = 0.96$, $\beta^F = 0.8$, $\alpha = 2/3$, $\gamma_1 = 5/6$, $\gamma_2 = 0.4$.

problem. It generates a public spending bias and, as private and public consumption are complements, induces a higher level of private consumption. Fiscal impatience thus moves the equilibrium allocation closer to the second-best Ramsey allocation. Allowing for this positive effect on the equilibrium allocation, the money growth rate is the optimal instrument.

5 Robustness

Our results so far have been derived for the arguably simplest framework amenable to studying the monetary instrument problem under dynamic, strategic policy interactions; we have chosen

increasing consumption path). Absent commitment, the incentive to choose a low level of current consumption is present in every period, and thus consumption will be sub-optimally low in every period. A detailed discussion of this mechanism is provided, among others, in Nicolini (1998) and Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

this simple framework to sharpen the discussion of our findings. In order to assess their robustness, we now consider a more general environment by introducing labor income taxes and credit goods, whose consumption by private agents is not subject to the cash constraint (3). This modification is particularly interesting because it allows to introduce taxation in a way that breaks the equivalence between monetary (inflation) and fiscal (labor income tax) policies with respect to the private-sector margins they distort.

We denote by c_t^1 and c_t^2 the consumption of cash and credit goods, respectively, and by τ_t the labor income tax rate chosen by the fiscal authority in period t. The monetary authority takes the current tax rate τ_t as parametrically given, and anticipates that future taxes will be set according to the rule $\mathcal{T}$ as in $\tau_{t+1} = \mathcal{T}(b_{t+1})$, *etc.*

Recall that the (qualitative) differences between the money growth and the interest rate regime in Section 3 emerge from the different private-sector decision rules $\hat{\mathcal{B}}$ and $\tilde{\mathcal{B}}$ the policy makers anticipate in the two scenarios. The equivalent functions for the cash-credit economy take the form $\hat{\mathcal{B}}(b_t, g_t, \mu_t, \tau_t)$ and $\tilde{\mathcal{B}}(b_t, g_t, i_t, \tau_t)$, whereby $b_{t+1} = \hat{\mathcal{B}}(b_t, g_t, \mu_t, \tau_t)$ solves[17]

$$1 + \mu_t = \frac{(1 - \tau_t)\beta u_c(\mathcal{C}^1(b_{t+1}), \mathcal{C}^2(b_{t+1}), \mathcal{G}(b_{t+1}))\mathcal{C}^1(b_{t+1})}{\alpha \hat{\mathcal{C}}^1(b_t, g_t, \mu_t, \tau_t)}, \tag{26}$$

and $b_{t+1} = \tilde{\mathcal{B}}(b_t, g_t, i_t, \tau_t)$ solves

$$1 + i_t = \frac{(1 - \mathcal{T}(b_{t+1}))u_c(\mathcal{C}^1(b_{t+1}), \mathcal{C}^2(b_{t+1}), \mathcal{G}(b_{t+1}))}{\alpha}. \tag{27}$$

Equation (27) shows that the function $\tilde{\mathcal{B}}(b_t, g_t, i_t, \tau_t)$ boils down to a function of the form $\mathrm{B}(i_t)$ also in this more general environment featuring credit goods and distortionary taxes. As the future state of the economy b_{t+1} continues to be completely determined by the nominal interest rate, the equilibrium outcome under an interest rate regime is again independent of the fiscal authority's time-preference factor.

The above exercise illustrates that the existence of a monetary instrument problem is generic in models of discretionary fiscal and monetary policy implemented by separate authorities. In this sense, Proposition 4 establishes a robust result. On the other hand, the irrelevance of the fiscal authority's time-preference factor under an interest rate regime (Proposition 5) is a less

[17] For a derivation of these conditions see, e.g., Section 3 in Martin (2009).

general phenomenon. The mechanics of our analysis indicate that this irrelevance is limited to environments where (i) the vector of endogenous state variables that can be manipulated over time is completely determined by the nominal interest rate, and where (ii) the policy authorities have aligned instantaneous utility functions such that their conflict of interest is dynamic rather than static in nature. In more general environments, specific properties of the equilibrium outcomes as well as the welfare implications of the monetary instrument choice may be different from those in the present environment. But there will typically be a monetary instrument problem, and the basic mechanism identified in this paper will apply.

6 Conclusions

We have studied the optimal choice of monetary instrument in a dynamic game between two separate fiscal and monetary policy makers. We have shown that instruments are equivalent only if the policy makers' preferences are perfectly aligned. Generically, there exists an instrument problem. Focussing on an environment where the fiscal authority is impatient relative to the monetary authority and society, we have shown that public spending and debt biases emerge under a money growth regime, but not under an interest rate regime. The interest rate is hence typically (that is, for environments with non-negative steady-state debt under benevolent cooperation) the optimal instrument, but there do also exist situations in which this is not the case.

There is a number of interesting aspects that could be explored further in more detail. From a normative perspective, it would be interesting to study the monetary instrument problem under different specifications of the policy makers' conflict of interest. For example, one might consider monetary conservatism, which creates a static conflict of interest between policy makers, and study the interaction between the optimal degree of conservatism and the optimal instrument choice. From a positive perspective, it would be interesting to examine the welfare consequences of the instrument choice in a quantitative framework, and study its implications for public spending and debt. Such analysis, however, requires a more elaborate model that is able to replicate the dynamics found in actual data. These issues are beyond the scope of the present paper and therefore left for future research.

A Proofs

Proof of Proposition 1. Under the assumptions of Proposition 1, the policy makers choose g_t and μ_t while (c_t, b_{t+1}, i_t, n_t) are determined by the private-sector equilibrium conditions (13)-(16). Note that (13) and (15) characterize the optimal private sector response (c_t, b_{t+1}) to the policies g_t and μ_t, while (14) and (16) deliver i_t and n_t, respectively, for given $(g_t, \mu_t, c_t, b_{t+1})$.

Given that future policies follow $\{\mathcal{G}, \mathcal{M}\}$, and that these policies induce $\mathcal{C}$, we can rewrite (13) and (15) in the form

$$\begin{aligned} 1+\mu_t &= \frac{H(b_{t+1})}{c_t}, \\ g_t + (1+b_t)c_t &= F(b_{t+1}), \end{aligned}$$

where

$$\begin{aligned} H(b) &= \frac{\beta u_c(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}(b)}{\alpha}, && \text{(A.1)} \\ F(b) &= \beta \mathcal{C}(b)\left[\frac{u_c(\mathcal{C}(b), \mathcal{G}(b))}{\alpha} + b\right]. && \text{(A.2)} \end{aligned}$$

The private-sector equilibrium decision rules $\hat{\mathcal{C}}(b_t, g_t, \mu_t)$ and $\hat{\mathcal{B}}(b_t, g_t, \mu_t)$ must satisfy

$$\begin{aligned} 1+\mu_t &= \frac{H(\hat{\mathcal{B}}(b_t, g_t, \mu_t))}{\hat{\mathcal{C}}(b_t, g_t, \mu_t)}, && \text{(A.3)} \\ g_t + (1+b_t)\hat{\mathcal{C}}(b_t, g_t, \mu_t) &= F(\hat{\mathcal{B}}(b_t, g_t, \mu_t)) && \text{(A.4)} \end{aligned}$$

for *all* admissible values of (b_t, g_t, μ_t). Note that we can therefore differentiate (A.3)-(A.4) with respect to any of the variables (b_t, g_t, μ_t). Taking derivatives with respect to μ_t and g_t, we get

$$\begin{aligned} 1 &= \frac{H'(b_{t+1})\hat{\mathcal{B}}_\mu c_t - H(b_{t+1})\hat{\mathcal{C}}_\mu}{(c_t)^2}, && \text{(A.5)} \\ (1+b_t)\hat{\mathcal{C}}_\mu &= F'(b_{t+1})\hat{\mathcal{B}}_\mu, && \text{(A.6)} \\ 0 &= H'(b_{t+1})\hat{\mathcal{B}}_g c_t - H(b_{t+1})\hat{\mathcal{C}}_g, && \text{(A.7)} \\ 1 + (1+b_t)\hat{\mathcal{C}}_g &= F'(b_{t+1})\hat{\mathcal{B}}_g, && \text{(A.8)} \end{aligned}$$

where $c_t = \hat{\mathcal{C}}(b_t, g_t, \mu_t)$ and $b_{t+1} = \hat{\mathcal{B}}(b_t, g_t, \mu_t)$. Note that the arguments (b_t, g_t, μ_t) have been omitted for the sake of notational simplicity. Finally, for later purposes, note that (A.7) and (A.8) together imply

$$\hat{\mathcal{C}}_g = \left[\frac{F'(b_{t+1})H(b_{t+1})}{H'(b_{t+1})c_t} - (1+b_t)\right]^{-1}. \tag{A.9}$$

Recall that the problem of the benevolent period-t government is given by

$$\max_{g_t,\mu_t} u(\hat{\mathcal{C}}(b_t, g_t, \mu_t), g_t) - \alpha(\hat{\mathcal{C}}(b_t, g_t, \mu_t) + g_t) + \beta V(\hat{\mathcal{B}}(b_t, g_t, \mu_t)).$$

The first-order conditions to this problem are

$$\begin{aligned} 0 &= w_t^c \hat{\mathcal{C}}_g + w_t^g + \beta V'(b_{t+1})\hat{\mathcal{B}}_g, \\ 0 &= w_t^c \hat{\mathcal{C}}_\mu + \beta V'(b_{t+1})\hat{\mathcal{B}}_\mu. \end{aligned}$$

Using (A.7) and (A.9), the first condition can be written as

$$-\beta V'(b_{t+1}) = w_t^g F'(b_{t+1}) + [w_t^c - (1+b_t)w_t^g]\frac{H'(b_{t+1})c_t}{H(b_{t+1})}.$$

Using (A.6), the second condition can be written as

$$-\beta V'(b_{t+1}) = \frac{w_t^c}{1+b_t}F'(b_{t+1}). \tag{A.10}$$

Together, these equations imply

$$0 = [w_t^c - (1+b_t)w_t^g]\left[\frac{F'(b_{t+1})}{1+b_t} - \frac{H'(b_{t+1})c_t}{H(b_{t+1})}\right].$$

Using (A.7) and (A.8), it can be shown that the second term in square brackets is zero only if $\hat{\mathcal{B}}_g = \infty$, which is ruled out in a differentiable Markov-perfect equilibrium. Hence, the equilibrium must feature $w_t^c = (1+b_t)w_t^g$ as required by (18). Finally, note that as V satisfies

(17) for all b_t, its derivative satisfies

$$V'(b_t) = w_t^c \mathcal{C}'(b_t) + w_t^g \mathcal{G}'(b_t) + \beta V'(b_{t+1}) \mathcal{B}'(b_t). \tag{A.11}$$

Moreover, (15) evaluated for equilibrium policies, $\mathcal{G}(b_t) + (1 + b_t)\mathcal{C}(b_t) = F(\mathcal{B}(b_t))$, implies $\mathcal{B}(b_t) = F^{-1}(\mathcal{G}(b_t) + (1 + b_t)\mathcal{C}(b_t))$ and thus

$$\mathcal{B}'(b_t) = \frac{\mathcal{G}'(b_t) + \mathcal{C}(b_t) + (1 + b_t)\mathcal{C}'(b_t)}{F'(b_{t+1})}. \tag{A.12}$$

Using (A.10) and (A.12) in (A.11), we have

$$V'(b_t) = -w_t^g c_t. \tag{A.13}$$

Equations (A.10) and (A.13) imply (19). This completes the proof of the proposition. □

Proof of Corollary 1. Recall that the function F is defined by (A.2). Differentiating this equation with respect to b, we get

$$F'(b) = \beta \mathcal{C}'(b) \left[\frac{u_c(\mathcal{C}(b), \mathcal{G}(b))}{\alpha} + b \right] + \beta \mathcal{C}(b) \left[\frac{u_{cc}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}'(b) + u_{cg}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{G}'(b)}{\alpha} + 1 \right],$$

which can equivalently be written as

$$F'(b) = \beta \mathcal{C}(b) \left\{ 1 + \frac{\mathcal{C}'(b) b}{\mathcal{C}(b)} + \frac{u_{cc}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}'(b)[1 - 1/\sigma(b)] + u_{cg}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{G}'(b)}{\alpha} \right\}$$

with $\sigma(b) = -\mathcal{C}(b) u_{cc}(\mathcal{C}(b), \mathcal{G}(b))/u_c(\mathcal{C}(b), \mathcal{G}(b))$. Using this expression for $F'(b)$, we can rewrite (19) as

$$\frac{w_{t+1}^g}{w_t^g} = 1 + \frac{\mathcal{C}'(b_{t+1}) b_{t+1}}{c_{t+1}} + \frac{u_{cc}(c_{t+1}, g_{t+1})\mathcal{C}'(b_{t+1})[1 - 1/\sigma(b_{t+1})] + u_{cg}(c_{t+1}, g_{t+1})\mathcal{G}'(b_{t+1})}{\alpha}.$$

In a steady state this equation simplifies to (20). □

Proof of Proposition 2. Under a money growth regime, the authorities' first-order conditions can be derived in analogy to the cooperative scenario. In particular, it is straightforward to show that the central bank's first-order condition reads

$$-\beta^M V^{M\prime}(b_{t+1}) \quad = \quad \frac{w_t^c}{1+b_t} F'(b_{t+1}), \tag{A.14}$$

while the fiscal authority's first-order condition reads

$$-\beta^F V^{F\prime}(b_{t+1}) \quad = \quad w_t^g F'(b_{t+1}) + [w_t^c - (1+b_t)w_t^g] \frac{H'(b_{t+1})c_t}{H(b_{t+1})}. \tag{A.15}$$

Note that the derivatives of the value functions satisfy

$$V^{M\prime}(b_t) \quad = \quad w_t^c \mathcal{C}'(b_t) + w_t^g \mathcal{G}'(b_t) + \beta^M V^{M\prime}(b_{t+1})\mathcal{B}'(b_t), \tag{A.16}$$

$$V^{F\prime}(b_t) \quad = \quad w_t^c \mathcal{C}'(b_t) + w_t^g \mathcal{G}'(b_t) + \beta^F V^{F\prime}(b_{t+1})\mathcal{B}'(b_t). \tag{A.17}$$

Eliminating the derivatives $V^{M\prime}$ and $V^{F\prime}$ in (A.14) and (A.15) by use of (A.16), (A.17) and (A.12), we arrive at the equilibrium conditions (21) and (22). This completes the proof. □

Proof of Corollary 2. When the utility function has the form specified in the corollary, then it follows that $F'(b) = \beta[\mathcal{C}(b) + b\mathcal{C}'(b)]$ and $H'(b) = 0$. Substituting these expressions into (21)-(22) and evaluating at the steady state, we get

$$\beta\bar{w}^c[\bar{c} + \bar{b}\mathcal{C}'(\bar{b})] = \beta^M \left\{\bar{c}\bar{w}^c + [\bar{w}^c - (1+\bar{b})\bar{w}^g]\mathcal{G}'(\bar{b})\right\},$$
$$\beta\bar{w}^g[\bar{c} + \bar{b}\mathcal{C}'(\bar{b})] = \beta^F \left\{\bar{c}\bar{w}^g - [\bar{w}^c - (1+\bar{b})\bar{w}^g]\mathcal{C}'(\bar{b})\right\}.$$

These two equations can equivalently be written as

$$\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}'(\bar{b}) = (\beta/\beta^M)[\bar{c} + \bar{b}\mathcal{C}'(\bar{b})] - \bar{c},$$
$$\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}'(\bar{b})\left[\beta^M + \beta^F \frac{\bar{w}^c\mathcal{C}'(\bar{b})}{\bar{w}^g\mathcal{G}'(\bar{b})}\right] = \bar{c}(\beta^F - \beta^M).$$

Using the first of these equations to eliminate the term $\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}'(\bar{b})$ from the second equation, we obtain after simple rearrangements equation (23). □

Proof of Proposition 3. Under the assumptions of Proposition 3, the fiscal authority chooses g_t, the central bank chooses i_t, and the variables $(c_t, b_{t+1}, \mu_t, n_t)$ are determined by the private-sector equilibrium conditions (13)-(16). Note that (14) and (15) characterize the optimal private sector response (c_t, b_{t+1}) to the policies g_t and i_t, while (13) and (16) deliver μ_t and n_t, respectively, for given (g_t, i_t, c_t, b_{t+1}).

Given that future policies follow $\{\mathcal{G}, \mathcal{I}\}$, and that these policies induce $\mathcal{C}$, we can rewrite (14) and (15) in the form

$$\begin{aligned} 1 + i_t &= K(b_{t+1}), \\ g_t + (1 + b_t)c_t &= F(b_{t+1}), \end{aligned}$$

where

$$K(b) = \frac{u_c(\mathcal{C}(b), \mathcal{G}(b))}{\alpha},$$

and F is defined as in (A.2). The private-sector equilibrium decision rules must satisfy

$$\begin{aligned} 1 + i_t &= K(\tilde{\mathcal{B}}(b_t, g_t, i_t)), \\ g_t + (1 + b_t)\tilde{\mathcal{C}}(b_t, g_t, i_t) &= F(\tilde{\mathcal{B}}(b_t, g_t, i_t)), \end{aligned}$$

for all admissible values of (b_t, g_t, i_t). Hence,

$$1 = K'(b_{t+1})\tilde{\mathcal{B}}_i, \quad (A.18)$$

$$0 = K'(b_{t+1})\tilde{\mathcal{B}}_g, \quad (A.19)$$

$$(1 + b_t)\tilde{\mathcal{C}}_i = F'(b_{t+1})\tilde{\mathcal{B}}_i, \quad (A.20)$$

$$1 + (1 + b_t)\tilde{\mathcal{C}}_g = F'(b_{t+1})\tilde{\mathcal{B}}_g. \quad (A.21)$$

Note that equations (A.18) and (A.19) imply $\tilde{\mathcal{B}}_g = 0$, and (A.21) then implies $\tilde{\mathcal{C}}_g = -1/(1+b_t)$.

The first-order conditions associated with the fiscal and monetary optimal policy problems are given by

$$\begin{aligned} 0 &= w_t^c \tilde{\mathcal{C}}_g + w_t^g + \beta^F V^{F'}(b_{t+1})\tilde{\mathcal{B}}_g, \\ 0 &= w_t^c \tilde{\mathcal{C}}_i + \beta^M V^{M'}(b_{t+1})\tilde{\mathcal{B}}_i. \end{aligned}$$

Using $\tilde{\mathcal{B}}_g = 0$ and $\tilde{\mathcal{C}}_g = -1/(1+b_t)$, the fiscal condition boils down to (24). Using (A.20), the monetary condition reads

$$-\beta^M V^{M\prime}(b_{t+1}) \quad = \quad w_t^c \frac{F'(b_{t+1})}{1+b_t}.$$

Finally, using the envelope condition to substitute for the derivative of the value function, we arrive at (25). This completes the proof. □

Proof of Proposition 4. The proof follows trivially from Propositions 2 and 3. □

Proof of Proposition 5. The proof follows trivially from Propositions 2 and 3. □

References

Adam, K., and R. M. Billi (2008): "Monetary conservatism and fiscal policy," *Journal of Monetary Economics*, 55(8), 1376–1388.

——— (2010): "Distortionary fiscal policy and monetary policy goals," Mimeo, University of Mannheim.

Albanesi, S., V. V. Chari, and L. J. Christiano (2003): "Expectation Traps and Monetary Policy," *Review of Economic Studies*, 70(4), 715–741.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001): "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91(1), 167–186.

Canzoneri, M. B., D. W. Henderson, and K. S. Rogoff (1983): "The Information Content of the Interest Rate and Optimal Monetary Policy," *The Quarterly Journal of Economics*, 98(4), 545–66.

Carlstrom, C. T., and T. S. Fuerst (1995): "Interest rate rules vs. money growth rules a welfare comparison in a cash-in-advance economy," *Journal of Monetary Economics*, 36(2), 247–267.

Chari, V. V., L. J. Christiano, and P. J. Kehoe (1991): "Optimal Fiscal and Monetary Policy: Some Recent Results," *Journal of Money, Credit and Banking*, 23(3), 519–39.

Collard, F., and H. Dellas (2005): "Poole in the New Keynesian model," *European Economic Review*, 49(4), 887–907.

Cooley, T. F., and G. D. Hansen (1991): "The Welfare Costs of Moderate Inflations," *Journal of Money, Credit and Banking*, 23(3), 483–503.

Diaz-Gimenez, J., G. Giovannetti, R. Marimon, and P. Teles (2008): "Nominal Debt as a Burden on Monetary Policy," *Review of Economic Dynamics*, 11(3), 493–514.

Dotsey, M., and A. Hornstein (2008): "On the implementation of Markov-perfect interest rate and money supply rules: global and local uniqueness," Discussion paper.

Ellison, M., and N. Rankin (2007): "Optimal monetary policy when lump-sum taxes are unavailable: A reconsideration of the outcomes under commitment and discretion," *Journal of Economic Dynamics and Control*, 31(1), 219–243.

Judd, K. L. (1992): "Projection methods for solving aggregate growth models," *Journal of Economic Theory*, 58(2), 410–452.

Khan, A., R. G. King, and A. L. Wolman (2003): "Optimal Monetary Policy," *Review of Economic Studies*, 70(4), 825–860.

King, R. G., and A. L. Wolman (2004): "Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria," *The Quarterly Journal of Economics*, 119(4), 1513–1553.

Klein, P., P. Krusell, and J.-V. Rios-Rull (2008): "Time-Consistent Public Policy," *Review of Economic Studies*, 75(3), 789–808.

Klein, P., and J.-V. Rios-Rull (2003): "Time-consistent optimal fiscal policy," *International Economic Review*, 44(4), 1217–1245.

Lucas, R. J., and N. L. Stokey (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12(1), 55–93.

Malley, J., A. Philippopoulos, and U. Woitek (2007): "Electoral uncertainty, fiscal policy and macroeconomic fluctuations," *Journal of Economic Dynamics and Control*, 31(3), 1051–1080.

Martin, F. (2009): "A Positive Theory of Government Debt," *Review of Economic Dynamics*, 12(4), 608–631.

——— (2010): "Government Policy in Monetary Economies," Discussion paper, University of Pennsylvania, Department of Economics.

McCallum, B. T. (1981): "Price level determinacy with an interest rate policy rule and rational expectations," *Journal of Monetary Economics*, 8(3), 319–329.

Nicolini, J. P. (1998): "More on the time consistency of monetary policy," *Journal of Monetary Economics*, 41(2), 333–350.

NIEMANN, S. (2011): "Dynamic monetary-fiscal interactions and the role of monetary conservatism," *Journal of Monetary Economics*, doi:10.1016/j.jmoneco.2011.03.008.

ORTIGUEIRA, S. (2006): "Markov-Perfect Optimal Taxation," *Review of Economic Dynamics*, 9(1), 153–178.

PERSSON, T., AND G. TABELLINI (1999): "Political economics and macroeconomic policy," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1 of *Handbook of Macroeconomics*, chap. 22, pp. 1397–1482. Elsevier.

POOLE, W. (1970): "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," *The Quarterly Journal of Economics*, 84(2), 197–216.

ROGOFF, K. S. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *The Quarterly Journal of Economics*, 100(4), 1169–89.

SARGENT, T. J., AND N. WALLACE (1975): "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy*, 83(2), 241–54.

SCHABERT, A. (2006): "Central Bank Instruments, Fiscal Policy Regimes, and the Requirements for Equilibrium Determinacy," *Review of Economic Dynamics*, 9(4), 742–762.

——— (2010): "Monetary policy under a fiscal theory of sovereign default," *Journal of Economic Theory*, 145(2), 860–868.

SCHMITT-GROHE, S., AND M. URIBE (2004): "Optimal fiscal and monetary policy under sticky prices," *Journal of Economic Theory*, 114(2), 198–230.

SIU, H. E. (2004): "Optimal fiscal and monetary policy with sticky prices," *Journal of Monetary Economics*, 51(3), 575–607.

SVENSSON, L. E. O. (1985): "Money and Asset Prices in a Cash-in-Advance Economy," *Journal of Political Economy*, 93(5), 919–44.