

A Simple Model of Tax-Favored Retirement Accounts

András Simonovits*

Institute of Economics, Hungarian Academy of Sciences
also Department of Economics, CEU
and Mathematical Institute, Budapest University of Technology
Budapest, Budaörsi út 45, Hungary, 1112
e-mail: simonov@econ.core.hu
August 22, 2009

* I express my debt to Á. Horváth, J. Gács, P. Holtzer, J. Király, E. Kovács, Á. Matits, P. Sipos and J. Tóth for our discussions; to Jiri Král and J. Szűcs for the information on the Czech and the Hungarian tax-favored retirement accounts, respectively, while A. Szijártó for redirecting my attention to welfare maximization. P. Benczúr deserves a special acknowledgment for his suggestion that in my model, the Cobb–Douglas utility function should be replaced by a Constant Relative Risk Averse (CRRA) one. I am indebted to H. Fehr for suggesting a radical streamlining of the previous version, M.-L. Leroux for her comments and to S. Valdés–Prieto for calling my attention to Homburg (2006). The author is solely responsible for the content of the paper. This research has received generous financial support from OTKA K 67853.

A Simple Model of Tax-Favored Retirement Accounts

András Simonovits

Abstract

To defend myopic workers against themselves, the government introduces a mandatory system but to help savers, it adds tax-favored retirement accounts. In a very simple model, we compare three extreme systems: (i) the pure mandatory system, (ii) the asymmetric system, where only the savers participate in the voluntary system, (iii) the symmetric system, where both types participate proportionally to their wages. The symmetric voluntary system is welfare-superior to the asymmetric one as well as to the pure mandatory system, which in turn are close to each other.

JEL code: H55, D91

Key words: mandatory pensions, tax-favored retirement accounts, voluntary contributions, subsidies.

1. Introduction

In most developed countries, in addition to the mandatory (funded and/or unfunded, public or private) pension system, a voluntary pension system exists, providing tax and contribution subsidies. The voluntary pension system is formed by *tax-favored retirement accounts*. In the default case, these subsidized savings cannot be withdrawn until the owner retires. The proponents of such systems justify these subsidies like this: a mandatory system does not and need not ensure high enough pensions, and the mostly partially myopic (for short, myopic) workers must be made interested in raising their old-age incomes through a voluntary system. The opponents are afraid that these subsidies are poorly targeted, mostly subsidize the well-paid savers, while worsening the burden of the others by increasing the *tax expenditures*. Up to now these tax expenditures have generally been quite low, thus they may be neglected, but under a possible contraction of the mandatory system they may become much higher. In this paper, we will discuss the issue in a very simple model. Since there are no other taxes in the model, we will write earmarked taxes rather than tax expenditures, pretending that a special tax finances the subsidies. Following Feldstein (1987, Part I), we consider only two types: the *myope* (L) with a low discount factor and the *saver* (H) with a high discount factor. (In fact, with Feldstein, the myopes are fully myope and the savers are fully savers with discount factors 0 and 1, respectively.) To avoid having four types, we assume that the myopes' earning is less than or equal to the savers'.

We consider three simple systems: (i) the pure mandatory system, without voluntary system but with forced savings for the myopes; (ii) a mandatory system and the asymmetric voluntary system, with only the savers's participations; (iii) a mandatory system and the symmetric voluntary system, with both types participating proportionally to their wages. Assuming that in both pension systems, the benefits are proportional to contributions, the individually optimal decisions are easy to calculate, opening the door for further investigation. We posited a utilitarian social welfare, without discounting of future utility (cf. Feldstein, 1987). Our main numerical results are as follows: The symmetric voluntary system is superior to the asymmetric one as well as to the pure mandatory system, which in turn are close to each other.

Starting with the Hungarian experiences, it should be emphasized that the Hungarian mandatory system is quite generous, replacing about 60–70% of the lifetime net wages, up to the triple of the average wage. In addition, the voluntary system is also generous: the current ceiling (on the sum of employee's and employer's contributions) is about 30% of the average gross wage and the matching rate varies between 30–50%. Nevertheless, the participation is quite modest, about 1/3 of the work force, while the average voluntary contribution is about 3.6% of the average wage. This is especially low if dormant accounts are taken into account (Matits, 2008). Our tentative results support those who criticize the Hungarian voluntary system for having too high ceilings and concentrated subsidies.

Turning to the international experience, let us underline that most pension systems deviate from the Hungarian system in a very important dimension: the mandatory or the voluntary system is progressive. For example, the US and the Czech mandatory systems as well as the German and the Czech voluntary systems are progressive. A proper evaluation of such systems needs modified models.

Among the large number of US studies, we single out the following ones: Poterba et

al. (1996) estimate that the introduction of tax-favored retirement accounts significantly increase total savings, while Engen et al. (1996) find the opposite. Trying at a synthesis, Hubbard and Skinner (1996) guess that both trends are present but the positive trend outweighs the negative. Note that all the three studies identify savings and social welfare; further, concentrate on the former rather than on the latter. Bernheim (1999) gives an excellent survey on the topic. Love (2007) analyze the impact of the age, the matching rate, the vesting policies and the withdrawal penalties on the participation rate. Baily and Kirkegaard (2009, p. 10) emphasize that “[t]he value of the tax breaks given to pensioners is very high in the US ... 1% of the GDP.” Börsch-Supan et al. (2008) study the reform of the German system. OECD (2005) provides a useful overview.

Modeling the much more complex British system, Sefton et al. (2008) ask the following question: what is the impact of the introduction of pension credit on other pension savings? According to their model, there was only a small increase, because the increase in the pension savings of the lower-paid induced by the pension credit was almost counterbalanced by the decrease in the pension savings of the higher-paid.

Even more complex models are used by Imrohoroglu et al. (1998) and Fehr et al. (2008). The latter emphasize the uncertainty of earning paths and longevity, and quantify the reduced quality of insurance following the setting up voluntary pension system. Admitting the virtues of these complex models, we still hope that our toy model has its own advantage of being transparent.

This approach is orthodox, because it heavily relies on time-consistency: as there is no new information, the workers do not change their saving behavior with the passage of time. Less orthodox models (e.g. Laibson, 1998; Diamond and Köszegi, 2003) employ the hyperbolic discounting when explaining and evaluating the voluntary pension system. To give a simple example: some workers plans to pay monthly voluntary contributions of 10 units during 480 months to get additional pension benefit of 20 units during 240 months. But he immediately realizes that if he skips the first month voluntary contribution, then his monthly benefit is only reduced by 0.046 units, therefore he may safely skip the first month. But what happens if he goes on in the second, third etc. month?

Using behavioral economics, Choi et al. (2004) also find a quite unorthodox behavior: if the default option is changed, and the new employees are automatically enrolled into a pension fund, from which they can opt-out, then a much higher share will stay in the voluntary system than in the original default. Being partial equilibrium models, the latter models neglect the tax burden of such schemes.

Homburg (2008) considers the problem of rational prodigals, and argues for wage taxes and saving subsidies as a second-best solution.

The structure of the remainder of the present paper is as follows: 2. The model framework. 3. Analytical results. 4. Numerical illustrations. 5. Conclusions.

2. The model framework

In this Section, we outline the model framework. First we determine the optimal voluntary contributions and savings chosen by the individual workers, then we define the welfare provided by various mandatory and voluntary systems.

Maximizing individual utility

We shall make the following extreme, nevertheless meaningful assumptions. The population and the economy are stationary, traditional saving does not yield interest. Every young-aged individual works and every old-aged individual is retired. Every worker is employed for a unit time period and every pensioner enjoys his retirement for a period of length μ , $0 < \mu < 1$. (In practice, the more one earns, the longer he lives on average; and the retirement age depends on the pension system, but here we neglect these relations.) Most existing systems superfluously differentiate between employer's and employee's mandatory contributions, but we assume a unified mandatory contribution. Contrary to practice, we prefer the total wage cost w to gross wages (their difference is the employer's contribution) and we calculate on its basis. Thus we assume that a worker with wage w pays a positive mandatory contribution τw , at least up to a ceiling $w_x > 0$. (The ceiling on the mandatory contributions will not play any role in this paper, but we display it, to stress its importance in reality, namely the higher the ceiling on mandatory contributions, the lower is the proper ceiling on voluntary contributions.) In addition, the worker with wage w pays an *earmarked tax* θw into the budget, financing the voluntary pensions.

In addition to his wage, the worker has another parameter called *discount factor*: δ . We assume that some type (w, δ) prefers additional benefits over the mandatory ones, therefore he pays a *voluntary contribution* r over the mandatory contribution, where $r \in [0, r_x]$, and $r_x \geq 0$ is the *ceiling* on voluntary contribution. The government matches the voluntary contribution r according to a *matching-voluntary contribution* function $a(r)$. Note that this system is equivalent to another one, where part of the voluntary contribution is returned directly to the worker. (Indeed, if the government immediately returns a from the extended voluntary contribution r , then this is equivalent to another system where the voluntary contribution is only $r - a$ but the government adds a to the account.)

The pension paid as a life annuity consists of two terms: the earnings-related mandatory benefit $b(w)$ and the voluntary pension $[r + a(r)]/\mu$. (As a matter of fact, voluntary pensions are seldom paid as life annuity, but this is irrelevant, because we do not discuss the distribution of consumption within the retirement period.)

Finally, there are types for whom even the maximal voluntary contribution r_x and the corresponding maximal subsidy a_x are insufficient. These types can *traditionally save* an additional sum, denoted by $s \geq 0$. We assume that the efficiency of this traditional saving is the same as that of the mandatory system, i.e. the corresponding life annuity is s/μ . Note that for an optimizing individual, $s > 0$ implies $r = r_x$!

The instantaneous consumption of a worker and of a pensioner are, respectively

$$c = w - \tau w - \theta w - r - s \quad \text{and} \quad d = b(w) + [r + a(r) + s]/\mu.$$

(Both c and d are positive. Of course, the instantaneous old-age consumption d means a lifetime pensioner consumption μd .)

We turn to the individual optimization. The *subjective* lifetime utility function of type (w, δ) consists of two terms: (i) the utility $u(\cdot)$ of instantaneous worker consumption c and (ii) the utility $\mu\delta u(d)$ of the pensioner's instantaneous consumption d . Here δ is the discount factor, $0 < \delta < 1$. In sum:

$$\hat{Z}(w, \delta, c, d) = u(c) + \mu\delta u(d).$$

The individual determines the pair (voluntary contribution, saving) $[r(w, \delta), s(w, \delta)]$ by maximizing his lifetime utility $\hat{Z}(w, \delta, c, d)$ under the lifetime budget constraint. Partly for the sake of simplicity, partly for bounded rationality, we assume that each worker takes the earmarked tax rate as given, i.e. does not consider the impact of his or others' choices. Substituting the consumption equations into \hat{Z} , provides subjective utility in another form:

$$Z(w, \delta, r, s) = u(w - \tau w - \theta w - r - s) + \mu\delta u(b(w) + [r + a(r) + s]/\mu).$$

The worker determines his *optimal* voluntary contribution \tilde{r} and saving \tilde{s} by taking the partial derivatives with respect to decisions r and s . (To avoid lengthy notations, we shall rarely use tilde for the optimum.) We must take into account the possibility of corner solutions. We assume that $b(w)$ and $a(r)$ are increasing concave functions, at least in the intervals $w_m \leq w \leq w_x$ and $0 \leq r \leq r_x$, respectively, where w_m is the minimal wage. Moreover, $b(0) \geq 0$ and $a(0) = 0$. To minimize the number of cases, for the time being, we assume that $b(w)$ and $a(r)$ are smooth functions. Here are the cases to be distinguished:

Zero voluntary contribution, zero saving, $r = 0, s = 0$:

$$Z'_r(w, \delta, 0, 0) = -u'(c) + \delta u'(d)[1 + a'(0)] \leq 0.$$

Positive voluntary contribution below ceiling, zero saving, $0 < r < r_x, s = 0$:

$$Z'_r(w, \delta, r, 0) = -u'(c) + \delta u'(d)[1 + a'(r)] = 0.$$

Maximal voluntary contribution, zero saving, $r = r_x, s = 0$:

$$Z'_s(w, \delta, r_x, 0) = -u'(c) + \delta u'(d) \leq 0.$$

Maximal voluntary contribution, positive saving, $r = r_x, s > 0$:

$$Z'_s(w, \delta, r_x, s) = -u'(c) + \delta u'(d) = 0.$$

Macro framework

In our model, workers have two characteristics: w and δ . We assume that their joint probability distribution is given by $(f_i)_{i=1}^I$ (possibly $i = (j, k)$) on the grid-points of the rectangle $w_m \leq w \leq w_x$ and $\delta_m \leq \delta \leq \delta_x$.

We assume that the mandatory contribution covers the mandatory pension expenditure, while the earmarked tax finances the subsidies. In formula:

Balance of the mandatory pensions

$$\sum_{i=1}^I f_i [\tau w_i - \mu b(w_i)] = 0.$$

Balance of the voluntary transfers

$$\sum_{i=1}^I f_i [\theta w_i - a(r(w_i, \delta_i))] = 0,$$

where $T_i = a(r(w_i, \delta_i)) - \theta w_i$ is the voluntary transfer *received* by type i . We also need the total savings, i.e. the aggregate traditional savings plus the aggregate voluntary savings, including the matching:

$$S = \sum_{i=1}^I f_i [s(w_i, \delta_i) + r(w_i, \delta_i) + a(r(w_i, \delta_i))].$$

Social welfare function

We also assume that the country is managed by a benevolent government which selects among various systems as to maximize an appropriately defined social welfare function. First of all, it removes discounting, and replaces subjective with *objective* utility functions:

$$U(w_i, \delta_i, c_i, d_i) = u(c_i) + \mu u(d_i).$$

(Note that U is independent of δ_i but to signal the second characteristic of the individual in aggregation, we still keep δ_i .)

The utilitarian social welfare function is the average of the individual objective utility functions, taken at the optima:

$$V = \sum_{i=1}^I f_i U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i).$$

If the government has a more egalitarian preference, it can choose a strictly concave scalar–scalar function ψ , and rely on a generalized utilitarian social welfare function:

$$V = \sum_{i=1}^I f_i \psi(U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i)).$$

The government looks for a mandatory contribution rate τ , an earmarked tax rate θ , and a pair of benefit and matching functions $b(\cdot), a(\cdot)$, which maximize the social welfare function under the budget constraints or more modestly, it selects among various systems on the basis of social welfare.

3. Analytical results

In this Section we shall outline some preliminaries, and then compare three systems mentioned in the Introduction: (i) the pure mandatory system, (ii) the asymmetric system, where only the savers participate in the voluntary system, (iii) the symmetric system, where both types contribution rates are equal.

Preliminaries

We shall work with homogeneous linear benefit and matching functions with ceilings. Bounded homogeneous linear benefit–wage-function

$$b(w) = \beta \min(w, w_x),$$

where $\beta > 0$ is the gross replacement ratio.

Bounded homogeneous linear matching–voluntary contribution function

$$a(r) = \alpha \min(r, r_x),$$

where r_x is the voluntary contribution's ceiling, α is the matching rate, $a_x = \alpha r_x$ is the subsidy's ceiling. Then $a(r) = \min(\alpha r, a_x)$.

In the continuation, it is useful to apply a simple utility function, namely CRRA: $u(c) = \sigma^{-1} c^\sigma$, where $\sigma < 0$. As a special limiting case ($\sigma = 0$), Cobb–Douglas: $u(c) = \log c$ can also be very useful.

Since $u'(c) = c^{\sigma-1}$, therefore for the interior optimal consumption pair with matching, we have

$$c^{\sigma-1} = \delta(1 + \alpha)d^{\sigma-1}, \quad \text{i.e.} \quad d = [\delta(1 + \alpha)]^{1/(1-\sigma)}c.$$

We shall need the ratio of the optimal old- and young-age consumption:

$$\gamma(\delta, \alpha) = [\delta(1 + \alpha)]^{1/(1-\sigma)},$$

With this notation, the interior optimum condition reduces to

$$d = \gamma(\delta, \alpha)c.$$

For the homogeneous linear case, the balance equations are also simple: for example, $\mu\beta = \tau$.

As a start, we shall first analyze the pure mandatory pension system (unaccompanied by a voluntary pension system).

Theorem 1. *Consider a pure mandatory pension system with a contribution rate τ implied by the corresponding discount factor δ° . Then the optimal consumption pairs and traditional saving are*

$$c^\circ = \frac{w}{1 + \mu\gamma(\delta^\circ, 0)}, \quad d^\circ = \frac{\gamma(\delta^\circ, 0)w}{1 + \mu\gamma(\delta^\circ, 0)}, \quad s^\circ = \frac{\mu[\gamma(\delta, 0) - \gamma(\delta^\circ, 0)]_+ w}{(1 + \mu\gamma(\delta^\circ, 0))(1 + \mu\gamma(\delta, 0))},$$

where x_+ is the positive part of the real number x : $x_+ = x$ if $x \geq 0$, 0 otherwise.

Proof. If the government chooses a discount factor $\delta^\circ < 1$, then the corresponding mandatory contribution rate is

$$\tau = \frac{\mu\gamma(\delta^\circ, 0)}{1 + \mu\gamma(\delta^\circ, 0)}$$

(dropping $^\circ$ from τ). The type (w, δ) will then choose the subjectively optimal consumption pair and traditional saving given above. (For discount factors lower than the mean, there would be no traditional saving at all.) ■

Example 1. If the government sets its discount factor to 1, (first-best solution), then

$$c^* = \frac{w}{1 + \mu}, \quad d^* = \frac{w}{1 + \mu}, \quad \tau^* = \frac{\mu}{(1 + \mu)}.$$

If the mandatory contribution rate is too high, implying little or no traditional saving, then the workers may restrain their labor supply or underreport their actual earnings.

If the mandatory contribution rate is too low, then workers with low discount factor will have unacceptably low old-age consumption. As a compromise, the government sets a medium mandatory contribution rate and introduces a voluntary pension system, the subsidy of which is financed by an earmarked tax rate θ , which covers the resulting subsidies: $\theta = \alpha\bar{r}$, where average wage is taken as unity. The government's hope is that at least some type will increase its total saving.

Inserting the consumption functions into the optimality conditions, after rearrangement, for any given θ , we obtain an optimum for each case. Four cases are to be distinguished.

Theorem 2. For any given earmarked tax rate θ , the optimal solutions are as follows:

Zero voluntary contribution, zero saving if

$$\beta > \gamma(\delta, \alpha)(1 - \tau - \theta).$$

Positive voluntary contribution, zero saving:

$$r = \frac{\gamma(\delta, \alpha)(1 - \tau - \theta) - \beta}{\gamma(\delta, \alpha) + \mu^{-1}(1 + \alpha)}w.$$

Maximal voluntary contribution, zero saving

$$\frac{\gamma(\delta, 0)(1 - \tau - \theta) - \beta}{\gamma(\delta, 0) + \mu^{-1}}w \leq r_x < \frac{\gamma(\delta, \alpha)(1 - \tau - \theta) - \beta}{\gamma(\delta, \alpha) + \mu^{-1}(1 + \alpha)}w,$$

Maximal voluntary contribution, positive saving

$$r = r_x \quad \text{and} \quad s = \frac{\gamma(\delta, 0)(1 - \tau - \theta)w - \beta w - [\gamma(\delta, 0) + \mu^{-1}(1 + \alpha)]r_x}{\gamma(\delta, 0) + \mu^{-1}}.$$

Proof. We discuss the four cases one after the other.

(i) Inserting equations $d = \beta w$ and $c = (1 - \tau - \theta)w$ into inequality $d > \gamma(\delta, \alpha)c$, yields

$$d = \beta w > \gamma(\delta, \alpha)(1 - \tau - \theta)w.$$

determining domain 1 in the (w, δ) -plane, regardless of the wage.

(ii) Inserting equations $d = \beta w + (1 + \alpha)r/\mu$ and $c = (1 - \tau - \theta)w - r$ into $d = \gamma(\delta, \alpha)c$, yields the optimal voluntary contribution, assuming $0 \leq r \leq r_x$, defining domain 2, depending on the wage.

(iii) Inserting the equations into the inequality yields $\gamma(\delta, 0)c \leq d < \gamma(\delta, \alpha)c$, defining domain 3.

(iv) Inserting equations $d = \beta w + [(1 + \alpha)r_x + s]/\mu$ and $c = (1 - \tau - \theta)w - r_x - s$ into equation $d = \gamma(\delta, 0)c$, yields the optimal saving. We must require $s \geq 0$, otherwise the worker would pay his voluntary contribution from credit. We have obtained domain 4. ■

Three systems

To compare the three pension systems (i)–(iii), we confine the in-depth analysis to the two-type case. Notation of types: L and H, relative frequencies f_L and f_H , wages w_L and w_H , and pensions $b_L = \beta w_L$ and $b_H = \beta w_H$ and with increasing discount factors: $0 < \delta_L < \delta_H < 1$. We shall call the types *myope* (L) and *saver* (H). Since typically the myopes' earning is less than or equal to the savers', we assume $w_L \leq w_H$. As a normalization, we also assume that the average wage is unity: $f_L w_L + f_H w_H = 1$. We assume that the government chooses its discount factor between the two types': $\delta_L < \delta^\circ < \delta_H$.

Pure mandatory system

We reformulate Theorem 1 for the two-type case.

Theorem 1.* *The optimal consumption pair and the traditional saving in the pure mandatory system are as follows:*

$$c_L^\circ = \frac{w_L}{1 + \mu\gamma(\delta^\circ, 0)}, \quad d_L^\circ = \frac{\gamma(\delta^\circ, 0)w_L}{1 + \mu\gamma(\delta^\circ, 0)}, \quad s_L^\circ = 0$$

and

$$c_H^\circ = \frac{w_H}{1 + \mu\gamma(\delta^\circ, 0)}, \quad d_H^\circ = \frac{\gamma(\delta^\circ, 0)w_H}{1 + \mu\gamma(\delta^\circ, 0)}, \quad s_H^\circ = \frac{\mu[\gamma(\delta, 0) - \gamma(\delta^\circ, 0)]w_H}{(1 + \mu\gamma(\delta^\circ, 0))(1 + \mu\gamma(\delta_H, 0))}.$$

Since d_L is too low, the government sets up tax-favored pension funds with a matching rate $\alpha > 0$, and ceiling $r_x > 0$ on the voluntary contributions. Rather than considering all the possibilities, we shall only discuss two special cases, to be called asymmetric and symmetric voluntary systems.

Asymmetric system

To simplify the calculations, first we assume that the matching rate α is so low that the myopes do not participate at the voluntary pensions: $\delta_L(1 + \alpha) \leq \delta^\circ$: *asymmetric system*. This is equivalent to

$$0 < \alpha \leq \alpha_L = \frac{\delta^\circ}{\delta_L} - 1.$$

There is another practical constraint: the savers do *not* pay too high voluntary contribution, i.e. their young-age consumption is higher than their old-age consumption: $c_H \geq d_H$, i.e. $\delta_H(1 + \alpha) \leq 1$. This is equivalent to

$$0 < \alpha \leq \alpha_H = \frac{1}{\delta_H} - 1.$$

We assume that $\alpha_L < \alpha_H$.

On the other hand, since $\delta^\circ < \delta_H$, the savers always contribute to the voluntary system. Let us assume that the ceiling is so high that the savers' voluntary contribution is lower than the ceiling: $0 < r_H < r_x$, i.e. $s_H = 0$. We formulate

Theorem 3. *If the matching rate is low enough: $0 < \alpha \leq \alpha_L$ and the ceiling is high enough:*

$$r_H(\alpha) = \frac{[\gamma(\delta_H, \alpha)(1 - \tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H, \alpha)(1 + f_H\alpha w_H) + (1 + \alpha)\mu^{-1}} < r_x,$$

then H 's optimal voluntary contribution is equal to $r_H(\alpha)$, while $s_H = 0$.

Proof. The interior optimality condition holds for H : $d_H = \gamma(\delta_H, \alpha)c_H$.

Then the earmarked tax balance is very simple: $\theta = f_H\alpha r_H$. Therefore $c_H = (1 - \tau)w_H - (1 + \alpha f_H w_H)r_H$ and $d_H = \beta w_H + (1 + \alpha)r_H/\mu$. Substituting c_H and d_H into H 's optimum condition:

$$\beta w_H + (1 + \alpha)r_H/\mu = \gamma(\delta_H, \alpha)[(1 - \tau)w_H - (1 + f_H\alpha w_H)r_H].$$

After rearrangement, we have the voluntary contribution. ■

Remark. It is obvious that the bill of savers' 'perfection' is partly paid by the myopes:

$$c_L = \frac{w_L}{1 + \mu\gamma(\delta^\circ, 0)} - \alpha f_H r_H w_L < c_L^\circ \quad \text{and} \quad d_L = \frac{\gamma(\delta^\circ, 0)w_L}{1 + \mu\gamma(\delta^\circ, 0)} = d_L^\circ.$$

Typically the welfare provided by the asymmetric voluntary system is close to that of the pure mandatory one.

Symmetric system

Before discussing the third system, let us introduce type i 's voluntary contribution rate ρ_i : $r_i = \rho_i w_i$, $i = L, H$.

In comparison to the asymmetric system, it seems to be more appropriate if the government sets such a low ceiling and such a high matching rate that both voluntary contribution rates are equal: $\rho_L = \rho_H = \rho$ and H 's voluntary contribution reaches the ceiling: $\rho w_H = r_x$. We shall call this system *symmetric*.

Theorem 4. *If the matching rate is high enough: $\alpha_L < \alpha \leq \alpha_H$ and the voluntary contribution ratio is*

$$\rho = \frac{\gamma(\delta_L, \alpha)(1 - \tau) - \mu^{-1}\tau}{[\gamma(\delta_L, \alpha) + \mu^{-1}](1 + \alpha)},$$

then the voluntary ceiling $r_x = \rho w_H$ is consistent, while H 's traditional saving ratio is equal to

$$\frac{s_H}{w_H} = \frac{\gamma(\delta_H, \alpha)[1 - \tau - (1 + \alpha)\rho] - \mu^{-1}[\tau - (1 + \alpha)\rho]}{\gamma(\delta_H, \alpha) + \mu^{-1}} \geq 0.$$

Proof. In the symmetric system, $\theta = \alpha\rho$, hence L's optimum condition

$$\mu^{-1}[\tau + (1 + \alpha)\rho] = \gamma(\delta_L, \alpha)[1 - \tau - (1 + \alpha)\rho]$$

yields the optimal voluntary contribution rate, which in turn yields the voluntary ceiling.

To determine H's traditional saving, substitution into $d_H = \gamma(\delta_H, \alpha)c_H$ yields

$$\mu^{-1}\{\tau + (1 + \alpha)\rho\}w_H + s_H = \gamma(\delta_H, \alpha)\{[1 - \tau - (1 + \alpha)\rho]w_H - s_H\}.$$

Solving for s_H/w_H , gives the result. ■

For a high enough matching rate α , the ceiling r_x is positive and low enough to defend the pensioner L, without impoverishing the worker L. Finally, we compare the welfare values provided by the two voluntary systems and the pure mandatory system .

Conjecture 1. a) *The asymmetric voluntary system provides a similar social welfare as does the pure mandatory one.* b) *The asymmetric voluntary system provides a lower social welfare than does the symmetric one.*

Finally, we formulate an interesting corollary to Theorem 4, which outlines the equivalence between various combinations of mandatory and symmetric voluntary systems.

Corollary. *Under the condition of Theorem 4, there is a curve $(\delta^\circ, \alpha(\delta^\circ))$ in the interval $[\delta^\circ, \delta^\circ]$ such that the corresponding symmetric systems provide essentially the same solution.*

Proof. Let τ_0 be the mandatory contribution rate corresponding to δ_0° and α_0 be a feasible matching rate, with the corresponding voluntary contribution rate ρ_0 . Then in the vicinity of τ_0 there is a unique solution $\alpha(\tau)$ to the implicit equation

$$\tau + (1 + \alpha)\rho(\tau, \alpha) = \tau_0 + (1 + \alpha_0)\rho_0,$$

yielding the same optimal $(c_L, d_L, 0, c_H, d_H, s_H)$. ■

4. Numerical illustration

We continue our analysis with numerical illustrations. We assume that the time spent at retirement is half as long as that of working: $\mu = 0.5$. Basically we follow the logic of the previous section.

For the time being, we assume that every worker has a unit total wage and we vary the discount factor and the ceiling on mandatory contributions.

As a baseline case, we calculate the optimal consumption pairs plus the mandatory contribution rate for four corresponding discount factors. Each case has a name, two have an abbreviations: myope (L) and saver (H), and the other two have symbols: mean ($^\circ$) and government (*). Table 1 presents the optimal young- and old-age consumption and the saving.

Table 1. *Discounting and optimal consumption pair: no matching*

Type i	Discounting factor δ_i	Worker c o n s u m p t i o n c_i	Pensioner d_i	Pension- saving rate τ_i
Myopic (L)	0.15	0.838	0.324	0.162
Mean ($^\circ$)	0.2	0.817	0.365	0.183
Saver (H)	0.5	0.739	0.522	0.261
Government (*)	1	0.667	0.667	0.333

Remark: $w = 1$. We display $10 * U + 100$ rather than U .

Table 1 displays that the lower the discount factor, the higher is the worker consumption and the lower is the pensioner consumption, and the corresponding saving or mandatory contribution rate. (The value of the consumption ratio depends on the exponent of the utility function, σ . The higher the absolute value of σ , the higher is the ratio of the pensioner's consumption to the worker's.)

From now on we move on to the two-type case, with relative frequencies $f_L = 2/3$ and $f_H = 1/3$, wage rates $w_L = 1/2$ and $w_H = 2$, yielding $\bar{w} = 1$. We assume different discount factors $\delta_L = 0.15$, $\delta_H = 0.5$ (first and third rows in Table 1). The government chooses a compromise: $\delta^\circ = 0.175, 0.2, 0.225$, i.e. the corresponding medium mandatory contribution rate $\tau = 0.183$ (second row in Table 1). In the asymmetric as well as the symmetric system, the matching rate is the maximal: it is equal to α_L and α_H , respectively.

Table 2 compares the welfare levels of three systems: the pure mandatory system, a mandatory system completed by an asymmetric and a symmetric one, as the mean discount factor varies.

Table 2. *Social welfare in mandatory and voluntary systems*

Discount factor δ°	0.175	0.2	0.225
Pure mandatory	60.757	61.594	62.265
Asymmetric voluntary (Matching rate	60.619	61.321	61.861
Symmetric voluntary	0.167	0.333	0.5)
	63.659	63.659	63.659

In harmony with our conjecture, the social welfare provided by the pure mandatory and asymmetric voluntary systems are close too each other, and are dominated by the symmetric voluntary system.

Note that in harmony with Corollary 1, the social welfare achieved by the symmetric voluntary system is invariant to the value of mean discount factor, or equivalently, to the value of the mandatory contribution rate. In fact, raising δ° or τ , the matching rate α and the consistent voluntary ceiling r_x change, making the sum of the mandatory and voluntary pension contribution plus the earmarked tax invariant.

Following the viewpoint of some studies mentioned in the Introduction, Table 3 compares the total savings in the three systems.

Table 3. *Total savings in mandatory and voluntary systems*

Discount factor δ°	0.175	0.2	0.225
Pure mandatory	0.059	0.052	0.046
Asymmetric voluntary (Matching rate	0.070	0.073	0.076
Symmetric voluntary	0.167	0.333	0.5)
	0.073	0.063	0.054

Here the pure mandatory system falls short of both voluntary systems, while the symmetric system achieves greater total savings than does the asymmetric one for lower mean discount factors, and vice versa for higher factors. For $\delta^\circ = 0.175$, the two voluntary systems achieve almost the same total saving!

Finally, Table 4 displays a desaggregated picture for the medium case with government discount factor $\delta^\circ = 0.2$. The last column contains the efficiency of the system in terms of the pure mandatory one, where e defines the real number, by which multiplying the wages, the modified pure mandatory system becomes welfare equivalent to the voluntary system (either asymmetric or symmetric).

Table 4. *Comparison of mandatory and voluntary pensions*

Earning w_i	Voluntary contri- bution r_i	Traditi- onal saving s_i	Worker c o n s u m p t i o n c_i	Pensioner d i o n d_i	Volun- tary transfer T_i	Life- time utility U_i	Efficiency e
Pure mandatory system ($\alpha = 0$)							1
0.5	0	0	0.409	0.183	0	48.167	
2.0	0	0.157	1.478	1.045	0	88.447	
Asymmetric voluntary system ($\alpha = 0.333$)							0.993
0.5	0	0	0.399	0.183	-0.009	47.607	
2.0	0.165	0	1.433	1.170	0.018	88.750	
Symmetric voluntary system ($\alpha = 1$)							1.057
0.5	0.008	0	0.393	0.215	0	51.265	
2.0	0.032	0.092	1.478	1.045	0	88.447	

The pure mandatory system is only displayed as a benchmark, with relative efficiency 1. Note the unacceptably low old-age consumption of the myope: $d_L = 0.183$.

The mandatory system with an asymmetric voluntary system only makes things a little bit worse because the matching rate is too low to help the myope ($\alpha = 1/3$) and the ceiling on voluntary contribution is high enough ($r_x = 0.165$) to allow the saver to appropriate the benefits. The earmarked tax rate creates a net transfer from the myopes to the savers. The young-age consumption of the former slightly diminishes, just to help raise the savers' old-age consumptions. The pure mandatory system can

achieve the same social welfare as the asymmetric voluntary system with 0.7 percent lower wages.

The mandatory system with a symmetric voluntary system redresses the injustice: the matching rate is raised to 1, while the ceiling is lowered to 0.032. Now the myope's old-age consumption raises to $d_L = 0.215$. The pure mandatory system can achieve the welfare of symmetric voluntary system by increasing wages by 5.7 percent.

5. Conclusions

We have constructed a simple model, where in addition to the contribution-based mandatory system, there is a tax-favored retirement system, financed from earmarked taxes. The voluntary contribution and the traditional saving are determined by the workers maximizing their subjective utility functions, while the corresponding earmarked tax rate and the ceiling on voluntary contributions are calculated by the government. In our "general equilibrium" model, we have done the first theoretical and numerical calculations. Our proportional tax-favored system with high ceiling and low matching is poorly targeted, when the mandatory system is also proportional and generous: it helps just those who do not need this help, asymmetry. It is socially more attractive to diminish radically the ceiling and enhance the matching: symmetry. The results seem to be acceptable but a lot of further analytical arguments and numerical trials are needed to confirm our tentative deductions. For progressive mandatory or voluntary systems, the evaluation will be different.

References

- Baily, M.N. and Kirkegaard, J.F. (2009): *US Pension Reform: Lessons from other Countries*, Petersons Books.
- Bernheim, B. D. (1999): Taxation and Saving, NBER WP 7061, *Auerbach, A.J. and Feldstein, M. (2002): Handbook of Public Economics*, Amsterdam, Elsevier.
- Börsch-Supan, A.; Reil-Held, A. and Schunk, D. (2008): “Saving Incentives, Old-age Provision and Displacement Effects: Evidence from the Recent German Pension Reform”, *Journal of Pension Economics and Finance* 7, 295–319.
- Choi, J., Laibson, D., Madrian, B. and Metrick, A. (2004): “For Better or Worse: Default Effects and 401(k) Saving Behavior”, Wise, D. ed. (2004): *Perspectives in the Economics of Aging*, Chicago, University of Chicago Press 81–121.
- Diamond, P. and Köszegi, B. (2003): “Quasi-hyperbolic Discounting and Retirement”, *Journal of Public Economics*, 87, 1839–1872.
- Engen, E. M., Gale, W. G. and Scholz, J. (1996): “The Illusory Effects of Savings Incentives on Saving”, *Journal of Economic Perspectives* 10:4, 111–138.
- Fehr, H., Habermann, C. and Kindermann, F. (2008): “Tax-Favored Retirement Accounts: Are they Efficient in Increasing Savings and Growth?”, *FinanzArchiv, Public Finance Analysis* 64, 171–198.
- Feldstein, M. S. (1987): “Should Social Security Means Tested?”, *Journal of Political Economy* 95, 468–484.
- Homburg, S. (2006): “Coping with Rational Prodigal: A Theory Social Security and Saving Subsidies”, *Economica* 73, 47–58.
- Hubbard, R. G. and Skinner, J. S. (1996): “Assessing the Effectiveness of Saving Incentives”, *Journal of Economic Perspectives* 10:4, 73–90.
- Imrohoroğlu, A., Imrohoroğlu, S and Joines, D. H. (1998): “The Effect of Tax-favored Retirement Accounts on Capital Accumulation”, *American Economic Review* 88, 749–768.
- Laibson, D. (1998): “Life-Cycle Consumption and Hyperbolic Discount Functions”, *European Economic Review* 42, 861–871.
- Love, D. A. (2007): “What can the Life-cycle Model Tell us about 401(k) Contributions and Participation?”, *Journal of Pension Economics and Finance* 6, 147–185.
- Matits, Á. (2008): “Voluntary Pension Funds: The Third Pillar”, Gál, R., Iwasaki, I. and Széman, Zs. eds. (2008): *Assessing Intergenerational Equity*, Budapest, Akadémiai Publisher, 111–135.
- OECD (2005): *Tax-Favored Retirement Saving*, OECD Economic Studies, 39, Paris.
- Poterba, J. M., Venti, S. F. and Wise, D. A. (1996): “How Retirement Savings Program Increase Savings”, *Journal of Economic Perspectives* 10:4, 91–112.
- Sefton, J., van de Ven, J. and Weale, M. (2008): “Means Testing Retirement Benefits: Fostering Equity or Discouraging Saving?”, *Economic Journal*, 118, 556–590.