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James M. Malcomson

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Abstract

This paper investigates relational incentive contracts with continuous, privately-observed agent types that are persistent over time. With fixed agent types, full separation is not possible when continuation equilibrium payoffs following revelation are on the Pareto frontier of attainable payoffs. This result is related to the ratchet effect in that: (1) a type imitating a less productive type receives an information rent and (2) with full separation, one imitating a more productive type receives the same future payoff as that more productive type. However, the reason for (2) is fundamentally different than with the ratchet effect. It arises from the dynamic enforcement requirement in relational contracts, not from the principal having all the bargaining power, and applies whatever the distribution between principal and agent of the future gains from the relationship (that is, whatever the point on the Pareto frontier). This result extends to sufficiently persistent types under certain conditions.

JEL-Code: C730, D820, D860.

Keywords: relational incentive contracts, private information, ratchet effect, dynamic enforcement.

James M. Malcomson
All Souls College
University of Oxford
United Kingdom - OX1 4AL Oxford
james.malcomson@economics.ox.ac.uk

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1 Introduction

Relational incentive contracts with non-contractible effort have proved insightful for analysing a variety of economic relationships. For applications, see Malcomson (1999) on employment and Malcomson (2013) on supply relationships. In many of these relationships, agents of different types are pooled in groups, with those in each group persistent over time and all treated the same despite differences between them. Employees are placed in grades, with those in a grade all paid the same. Toyota, as described by Asanuma (1989), places its suppliers into a small number of categories that receive differential treatment. This paper shows that persistent pooling is fundamental to relational incentive contracts with privately-observed, continuous and persistent agent types.

Pooling of such types arises from the *ratchet effect* in dynamic models of procurement, see Laffont and Tirole (1988). There it occurs when a principal is legally constrained from committing to contract terms for future periods, even those conditioned only on outcomes that can be contracted on when those future periods arrive, and makes "take it or leave it" offers that extract all future rent if the agent's type is revealed. Because a more productive agent receives an informational rent from pretending to be a less productive one, full separation of types is not possible. The constraint on committing to future contract terms is appropriate for sovereign bodies that cannot commit their successors, and for regulators who are not permitted to do so. But it is less appropriate for private sector principals. Pooling of privately-observed, continuous but non-persistent types arises from *dynamic enforcement* in the hidden information relational incentive contract model of Levin (2003). With a relational contract, parties make payments conditioned on non-contractible outcomes only if their payoffs from having the relationship continue are sufficient to make that worthwhile, which imposes a constraint on rewards. Full separation of privately-observed types is still possible but is sub-optimal if efficient effort for all types is not attainable. Because types are *iid* draws each period, there is no systematic persistence of a particular agent in a particular pool.

The present paper combines insights underlying the ratchet effect and dynamic enforcement to show that full separation of continuous, privately-observed agent types that are unchanging over time is not possible in a relational incentive contract when the parties cannot commit themselves to behave sub-optimally in the future. In the model, agent type affects the cost of supplying non-contractible effort to the principal. It is unchanging over time and privately observed by the agent. This framework extends Shapiro and Stiglitz (1984) to private information about the agent worker's disutility of effort and to continuous, not just binary, effort choice, as in MacLeod and Malcomson (1989). The inability to commit to future contract terms is purely informational. Thus, the result does not depend on legal constraints on committing to future contract terms that are in principle contractible and so is just as applicable to private as to public sector principals. It also does not depend on the principal making "take it or leave it" offers. It depends only on efficient effort being unattainable and future payoffs if the agent's type is revealed being on the feasible Pareto frontier. It thus significantly extends the set of circumstances under which persistent types are necessarily pooled.

Other related papers include Yang (2013), who considers persistent types that are private information, but allows for just two for which full separation is possible. Kennan (2001) and Battaglini (2005) also analyse revelation of two persistent types that are private information but without non-contractible effort. Athey and Bagwell (2008) analyse a model of collusion between firms in an oligopoly in which cost shocks are both private information and persistent. But collusion between firms has very different characteristics from employment or supply relationships. In particular, only one side of the market participates in the relational contract and monetary payments are not used because they make breach of antitrust rules more apparent. Finally, MacLeod and Malcomson (1988) analyse relational incentive contracts with a continuum of persistent, privately-observed agent types that are partitioned into separate pools. That paper, however, imposes restrictions on rewards and punishments that are not imposed here.

The structure of the paper is as follows. Section 2 sets out the model. Section 3 derives incentive compatibility conditions for the agent and the principal in a relational contract. Section 4 derives equilibrium conditions for relational contracts and characterises optimal continuation equilibria following full revelation of the agent's type. Section 5 establishes that full separation of all agent types is not possible when future actions attain the feasible Pareto frontier. Section 6 discusses the related literature in more detail, Section 7 extension to agent types that may change over time. Section 8 contains concluding remarks. Proofs of propositions are in an appendix.

2 Model

A principal uses an agent to perform a task in each of a potentially infinite number of periods. The principal's payoff in period t if matched with the agent is $e_t - w_t$, where $e_t \in [0, \overline{e}]$ is the agent's effort and w_t the payment to the agent in period t. Effort e_t cannot be verified by third parties, so a legally enforceable agreement for performance is not possible. The principal's payoff for a period not matched with the agent is $\underline{v} \geq 0$.

The agent's payoff in period t if matched with the principal is $w_t - c(e_t, a)$, where $c(e_t, a)$ is the cost of effort e_t to agent type $a \in [\underline{a}, \overline{a}]$, with a observed privately by the agent. Agent type is distributed $F(a)$, with $dF(a) > 0$ everywhere. The agent's payoff

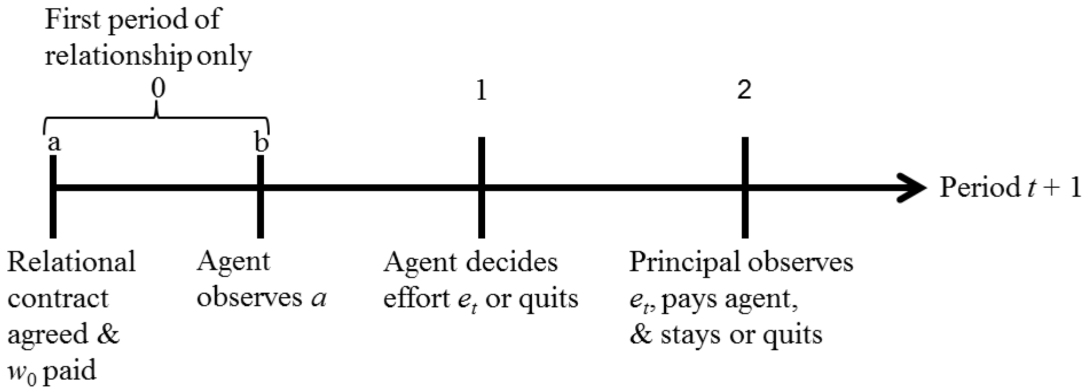

Figure 1: Timing of events in period t

for a period not matched with the principal is $\underline{u} \geq 0$, with $\underline{u}+\underline{v} > 0$. Principal and agent have the same discount factor δ. The function c has the following standard properties.

Assumption 1 *For all $a \in [\underline{a}, \overline{a}]$:* (1) $c(0,a) = 0$*;* (2) *for all $\tilde{e} \in [0, \overline{e}]$, $c(\tilde{e}, a)$ is twice continuously differentiable, with $c_1(\tilde{e}, a) > 0$, $c_2(\tilde{e}, a) \leq 0$ with strict inequality for $\tilde{e} \in (0, \overline{e}]$, $c_{11}(\tilde{e}, a) > 0$, and $c_{12}(\tilde{e}, a) < 0$; and* (3) $c_1(0,a) < 1$ *and* $c_1(\overline{e}, a) > 1$. *Moreover, $c(\tilde{e}, \underline{a}) > \tilde{e} - (\underline{u}+\underline{v})$ for all $\tilde{e} \in [0, \overline{e}]$.*

The timing of events in period t is shown in Figure 1. In the first period of the relationship ($t = 1$), the parties first decide (at stage 0a) whether to agree a relational contract (to be formally defined shortly) and, if they do, make initial payment w_0. Then the agent (at stage 0b) observes a. The other stages are the same for all t. At stage 1, the agent either incurs effort e_t or ends the relationship. At stage 2, the principal observes e_t, pays the agent and decides whether to continue the relationship.

As in MacLeod and Malcomson (1989) and Levin (2003), payment has a fixed component $\underline{w}_t$ conditioned only on the relationship being continued by both parties for period t (and not on effort at t). It also has a bonus component $w_t - \underline{w}_t$ that can be conditioned on the agent's effort in period t but is not legally enforceable because effort is unverifiable. The magnitude and sign of $\underline{w}_t$ are unrestricted (negative $\underline{w}_t$ requires the agent to pay the principal) but, to avoid a decision by the agent at stage 2 of whether to accept the bonus, $w_t - \underline{w}_t$ is restricted to being non-negative. (This restriction does not restrict the set of payoffs attainable with equilibrium relational contracts.)

Let $h_t = h_{t-1} \cup (e_{t-1}, w_{t-1})$, for $t \geq 2$, with $h_1 = \{w_0\}$, denote the commonly observed history at stage 1 of period t conditional on the relationship not having ended before then. At that stage, the agent can condition actions on (a, h_t). A strategy σ^a for the agent consists of a decision rule for whether to accept w_0, a decision rule $\gamma_t(a, h_t) \in \{0,1\}$ for each t for whether to continue the relationship at stage 1, and an effort choice $e_t(a,h_t)$ for each t conditional on continuation. At stage 2 of period t, the principal can condition actions on (h_t,e_t). A strategy σ^p for the principal consists of a decision rule for whether to pay w_0, a decision rule $\beta_t(h_t,e_t) \in \{0,1\}$ for each t for whether to continue the relationship at stage 2, and a payment choice $w_t(h_t,e_t)$ for each t conditional on continuation. Formally, a *relational contract* is a w_0, a $\underline{w}_t(h_t)$ for each h_t and t, and a strategy pair (σ^p,σ^a). To avoid the measurability details that can arise with mixed strategies when action spaces are continuous (see Mailath and Samuelson (2006, Remark 2.1.1)), attention is restricted to pure strategies.[1]

The joint payoff gain to the principal and the agent from being matched in period t conditional on a is $s(e_t,a) = e_t - c(e_t,a) - (\underline{u}+\underline{v})$. *Efficient* effort $e^*(a)$ maximises this joint gain. Under Assumption 1, $e^*(a) \in (0,\bar{e})$ for all a and is uniquely given by

$$s_1(e^*(a),a) = 0. \tag{1}$$

3 Incentive compatibility

Consider first incentive compatibility for the agent. Let $A_t(h_t)$ denote the set of agent types a with history h_t at t. For a best response effort, the payoff gain $U_t(a,h_t)$ to agent type $a \in A_t(h_t)$ from continuing the relationship at stage 1 of period t given history h_t is

$$U_t(a,h_t) = \max_{\tilde{e}\in[0,\bar{e}]} \Bigg\{ -c(\tilde{e},a) - \underline{u} + \underline{w}_t(h_t) + \beta_t(h_t,\tilde{e}) \Bigg[w_t(h_t,\tilde{e}) - \underline{w}_t(h_t) + \delta \max\Big\{0, U_{t+1}(a,(h_t,\tilde{e},w_t(h_t,\tilde{e})))\Big\}\Bigg]\Bigg\}. \tag{2}$$

(Explicit dependence of payoff gains on the contract is suppressed to avoid cumbersome notation.) The interpretation is as follows. Agent type a who continues the relationship for period t and chooses effort $\tilde{e}$ incurs cost of effort $c(\tilde{e},a)$, forgoes utility $\underline{u}$ available if not matched with the principal, and receives payment $\underline{w}_t(h_t)$. For $\beta_t(h_t,\tilde{e}) = 1$, the principal continues the relationship and pays the bonus $w_t(h_t,\tilde{e}) - \underline{w}_t(h_t)$. In that case, the agent receives payoff gain from the future of $U_{t+1}(a,(h_t,\tilde{e},w_t(h_t,\tilde{e})))$ if this is non-negative, so continuing is worthwhile. For $\beta_t(h_t,\tilde{e}) = 0$, the principal ends the relationship, in which case paying a bonus is never a best response.

With $c_2 \le 0$, $U_t(a,h_t)$ is non-decreasing in a, so there is a lowest agent type $\alpha_t(h_t) \in$

[1] The timing used here has each party make decisions at only one stage in each period, which simplifies the analysis by avoiding having to keep track of the parties' payoffs at other stages within a period. A party's payoff from continuing the relationship is, however, at its lowest at its decision stage, so allowing a party to end the relationship at other stages would not affect individual rationality. Having the principal make the stay or quit decision simultaneously with the agent would make mutual quitting always a best response pair but would not affect the maximum sustainable effort or the set of equilibrium payoffs.

$A_t(h_t)$ that continues the relationship for period t given history h_t which satisfies

$$\begin{aligned} U_t(\alpha_t(h_t), h_t) &\geq \max\left[0, \underline{w}_t(h_t) - \underline{u}\right], \text{ for all } h_t, t, \\ U_t(a, h_t) &\leq 0, \text{ for } a < \alpha_t(h_t), \text{ all } h_t, t, \\ \alpha_t(h_t) &= \min a \in A_t(h_t), \text{ if } \underline{w}_t(h_t) > \underline{u}. \end{aligned} \tag{3}$$

The last of these is because, if $\underline{w}_t(h_t) > \underline{u}$, all agent types in $A_t(h_t)$ can guarantee payoff gain $\underline{w}_t(h_t) - \underline{u} > 0$ by continuing the relationship at t, setting $e_t = 0$ and quitting at $t+1$. For notational convenience define, for a given relational contract,

$$A_t^+(h_t) = \{a \mid a \in A_t(h_t), a \geq \alpha_t(h_t)\}, \text{ for all } h_t, t. \tag{4}$$

$$\begin{aligned} \tilde{U}_t\left(a', a, h_t\right) = -c\left(e_t\left(a', h_t\right), a\right) - \underline{u} + \underline{w}_t(h_t) + \beta_t\left(h_t, e_t\left(a', h_t\right)\right) \Big[w_t\left(h_t, e_t\left(a', h_t\right)\right) \\ - \underline{w}_t(h_t) + \delta \max\left\{0, U_{t+1}\left(a, \left(h_t, e_t\left(a', h_t\right), w_t\left(h_t, e_t\left(a', h_t\right)\right)\right)\right)\right\} \Big], \\ \text{for all } a, a' \in A_t^+(h_t), \text{ all } h_t, t. \end{aligned} \tag{5}$$

$A_t^+(h_t)$ is the set of a with history h_t who continue the relationship at t, $\tilde{U}_t(a', a, h_t)$ the maximand in (2) for agent type a choosing effort for type a', so $\tilde{e} = e_t(a', h_t)$.

Proposition 1 *Necessary conditions for decision rules for agent types $a \in A_t(h_t)$ in a relational contract to be best responses are, for all t,*

$$\gamma_t(a, h_t) = \begin{cases} 1, & \text{if } a \geq \alpha_t(h_t), \\ 0, & \text{otherwise;} \end{cases} \tag{6}$$

$$\begin{aligned} \tilde{U}_t(a, a, h_t) - \tilde{U}_t\left(a, a', h_t\right) \geq U_t(a, h_t) - U_t\left(a', h_t\right) \\ \geq \tilde{U}_t\left(a', a, h_t\right) - \tilde{U}_t\left(a', a', h_t\right), \text{ for all } a, a' \in A_t^+(h_t). \end{aligned} \tag{7}$$

These conditions are also sufficient if the continuation contracts following deviation to $e_t \neq e_t(a', h_t)$ for any $a' \in A_t^+(h_t)$ are the same as the continuation contract for $e_t = e_t(\alpha_t(h_t), h_t)$ except that (1) the principal pays no bonus at t ($w_t(h_t, e_t) = \underline{w}_t(h_t)$) and (2) the payment $\underline{w}_{t+1}(h_t \cup (e_t, \underline{w}_t(h_t)))$ is such that agent type $\alpha_t(h_t)$ would receive non-positive payoff gain from continuing the relationship at stage 1 of period $t+1$ ($U_{t+1}(\alpha_t(h_t), h_t \cup (e_t, \underline{w}_t(h_t))) \leq 0$).

That (6) defines a best response follows from the specification for $\alpha_t(h_t)$ in (3). The other results in Proposition 1 are related to results familiar from mechanism design for one-period models. A one-period model corresponds to $\delta = 0$ so, from (5), the left-most and right-most terms in (7) become just $c(e_t(a,h_t),a') - c(e_t(a,h_t),a)$ and $c(e_t(a',h_t),a') - c(e_t(a',h_t),a)$, respectively. For that case, it is standard to divide all terms in (7) by $a' - a$ and take the limit as $a' \to a$ to get a condition on the derivative $c_2(e_t(a,h_t),a)$ that is used to construct the difference between the payoffs of different types and also, given $c_{12} < 0$, to establish the requirement that $e_t(a,h_t)$ is non-decreasing in a. Here the additional terms in $\tilde{U}_t(a',a,h_t)$ take account of the future consequences from $t+1$ on of agent type a choosing the effort corresponding to type a' at t. The derivative formulation is less useful here because, for relevant continuation contracts, the additional terms in $\tilde{U}_t(a',a,h_t)$ are not differentiable in a at $a' = a$.

If the agent's performance were verifiable, deviation to effort that is not on the equilibrium path for any agent type could be deterred by a sufficiently large monetary penalty. With unverifiable performance (as here), the worst penalty that can be imposed on the agent is zero payoff gain following such a deviation because the agent can always quit. As in Abreu (1988), this penalty gives the largest set of equilibria. Conditions (6) and (7) are then not only necessary for best responses but also sufficient. Ending the relationship is, however, inefficient when a mutually beneficial relationship is possible. In Levin (2003), the same penalty is achieved without the relationship ending by a continuation contract following deviation the same as that with no deviation except that the agent pays the principal just enough to give the agent zero payoff gain from continuation. Money payments provide transferable utility with no efficiency loss. That approach is more complicated here because the principal may not know the agent's type and so the payment required to give the agent zero payoff gain from continuation following deviation is not common knowledge. Proposition 1, however, shows that a weaker requirement suffices to ensure that conditions (6) and (7) are sufficient, specifically that the payment following deviation at t by an agent with history h_t is such that the lowest agent type with that history continuing the relationship (formally $\alpha_t(h_t)$) receives zero payoff gain from continuation. With this continuation contract, higher a continue to receive a strictly positive payoff gain from continuation following deviation but that is not sufficient to make deviation worthwhile.

For the principal, let $P_t(a,(h_t,e_t))$ denote the payoff gain from continuing the relational contract with agent type a at stage 2 of period t given history (h_t,e_t), conditional on paying the bonus $w_t(h_t,e_t) - \underline{w}_t(h_t)$.

Proposition 2 *Suppose the continuation contracts following the principal's deviation to $w_t \neq w_t(h_t,e_t)$ are the same as that for $w_t = w_t(h_t,e_t)$ except that the payment $\underline{w}_{t+1}(h_t \cup (e_t,w_t))$ is such that the principal receives non-positive payoff gain from con-*

tinuing the relationship at stage 2 of period t when paying $w_t = \underline{w}_t(h_t)$. *Then best response decision rules for the principal are, for all* h_t, e_t *and* t,

$$\beta_t(h_t, e_t) = \begin{cases} 1, & \text{if } E_{a|h_t,e_t}\left[P_t(a,(h_t,e_t))\right] \geq 0, \\ 0, & \text{otherwise;} \end{cases} \tag{8}$$

if $w_t(h_t, e_t) - \underline{w}_t(h_t) > 0$, *pay* $w_t(h_t, e_t)$ *if and only if* $\beta_t(h_t, e_t) = 1$; *otherwise, pay* $w_t(h_t, e_t) = \underline{w}_t(h_t)$.

Most of this result follows directly from the definition of $P_t(a,(h_t,e_t))$. The principal does not deviate to a bonus smaller than specified in the relational contract because that would trigger a continuation contract with a fixed wage component in the next period leaving the principal no gain from the deviation. Because the principal's type is common knowledge, the payment required for this is also common knowledge.

For stage 0a of the first period of the relationship, neither party has information about the agent's type beyond its initial distribution. The agent starts a relational contract only if the initial payoff gain U_0 satisfies

$$U_0 \equiv w_0 + \int_{\alpha_1(h_1)}^{\overline{a}} U_1(\tilde{a}, h_1)\, dF(\tilde{a}) \geq 0. \tag{9}$$

The principal starts a relational contract only if the expected payoff gain from starting the relationship given the initial distribution of a, denoted P_0, satisfies $P_0 \geq 0$.

4 Equilibrium relational contracts

4.1 Equilibrium concept

A natural minimum equilibrium requirement for a strategy pair in this game is that it is a perfect Bayesian equilibrium. The contracting literature for finite horizons typically also imposes that contracts are renegotiation-proof, in the sense that it is not possible for the parties to renegotiate a contract at any stage to one that both prefer. But the standard renegotiation-proofness concepts in the literature on infinite horizon games are defined only for games without private information about types. The approach adopted here is, therefore, to require renegotiation-proofness only for continuation equilibria that follow full revelation of the agent's type, at which point there is no longer private information. As standard in that literature, conditions for renegotiation-proofness are specified as part of the equilibrium concept rather than derived from an explicit renegotiation game.

Definition 1 *An* equilibrium with optimal continuation *is a perfect Bayesian (PB) equilibrium in pure strategies for which equilibrium-path continuation equilibria following full revelation of the agent's type have payoffs at stage 1 of each period on the Pareto frontier of subgame perfect equilibria of the full information game for that agent type.*

Two requirements underlie Definition 1. First, for a the only agent type in $A_t(h_t)$, the principal interprets any action from t on not on the equilibrium path for type a as a deviation by type a rather than revise his belief about the agent's type. This corresponds to the condition *NDOC* ("Never Dissuaded Once Convinced") in Osborne and Rubinstein (1990, p. 96). It ensures that, from t on, the parties are engaged in a game of perfect information. Second, the continuation equilibria from t on have payoffs on the Pareto frontier of subgame perfect equilibria of that perfect information game.

The motivation for this second requirement is the following. If payoffs are on the Pareto frontier, one party must lose from renegotiation. Thus the requirement is sufficient for there to be no alternative continuation equilibrium that both prefer. Moreover, for any continuation equilibrium not on the Pareto frontier, there exists a continuation equilibrium on the Pareto frontier that both parties prefer. Thus the requirement is also necessary for there to be no alternative continuation equilibrium both prefer.[2]

For the concepts of renegotiation-proofness in Farrell and Maskin (1989), continuation equilibria are required to themselves be renegotiation-proof, not just subgame perfect as in Definition 1. That restricts the "off the equilibrium path" punishments used to sustain continuation equilibria. For the game here, the difference is actually immaterial. It follows from an argument in Goldlücke and Kranz (2013, Section 4.3) that the same set of equilibrium-path payoffs can be sustained with punishment payoffs that are also on the Pareto frontier at each subsequent decision node and hence correspond to strong perfect equilibria (and so also strong renegotiation-proofness in the sense of Farrell and Maskin (1989)). Thus the same conclusions hold with both minimal and maximal reasonable restrictions on "off the equilibrium path" continuation equilibria.

Definition 1 imposes no renegotiation-proofness requirement when the agent's type has not been fully revealed. To demonstrate the non-existence of equilibria with full revelation of types this is an advantage. No decision node following full revelation of the agent's type can be followed by one with private information about that type. Thus any concept of renegotiation-proofness imposed on continuation equilibria with private information about type cannot expand the set of equilibria under Definition 1. So, if no equilibrium with full revelation of types exists under Definition 1, none exists with *any*

[2]If the parties were able to commit to a sub-optimal continuation equilibrium, that would in general affect the extent of separation possible in previous periods. But such commitment seems inappropriate for parties who, as here, cannot commit not to renegotiate.

additional restriction on continuation equilibria with private information about types.

For simplicity, when only one agent type has history h_τ at τ in an equilibrium with optimal continuation, the continuation equilibrium for that history is referred to as an *optimal continuation equilibrium for* h_τ. Also, in describing equilibria, the history argument is omitted where that does not result in ambiguity; for pure strategy equilibria, h_t at each t is fully determined by the relational contract and the agent's type.

4.2 Equilibrium conditions

In equilibrium, the parties' payoffs must be consistent with the total output produced. Let $S_t^i(a)$ denote the joint gain to the principal and the agent (also called the *surplus*) from continuing the relationship at stage $i = 1,2$ of period t for type a for a given relational contract. These two measures can be defined recursively as

$$S_t^1(a) = e_t(a) - c(e_t(a), a) - \underline{u} - \underline{v} + \beta_t(e_t(a))\, S_t^2(a), \quad \text{for all } a, t; \tag{10}$$

$$S_t^2(a) = \delta \gamma_{t+1}(a)\, S_{t+1}^1(a), \quad \text{for all } a, t. \tag{11}$$

The joint gain to starting a relational contract is

$$S_0 = \int_{\alpha_1}^{\overline{a}} S_1^1(a)\, dF(a). \tag{12}$$

A necessary condition for a relational contract to start is that $S_0 \geq 0$. Moreover, provided $S_0 \geq 0$, there is always a w_0 such that the agent's and the principal's initial payoff gains U_0, given by (9), and P_0 are both non-negative. Equilibrium requires that the agent receives that part of the joint gain not received by the principal. It follows from (2) that

$$U_t(a) = -c(e_t(a), a) - \underline{u} + \underline{w}_t + \beta_t(e_t(a)) \left[S_t^2(a) - P_t(a) \right], \quad \text{for all } a, t. \tag{13}$$

This condition is the *budget balance constraint* from which the dynamic enforcement constraint in Levin (2003) is derived.

4.3 Optimal continuation equilibria

Proposition 3 *Suppose agent type a is the only agent type with history h_τ at τ.*

1. *There exists a subgame perfect continuation equilibrium for h_τ for which the relationship continues if*

$$\max_{\tilde{e} \in [0, \overline{e}]} \left[\delta \tilde{e} - c(\tilde{e}, a) \right] \geq \delta \left(\underline{u} + \underline{v} \right). \tag{14}$$

2. *For a satisfying (14), an optimal continuation equilibrium for h_τ has, for all $t \geq \tau$, stationary effort $e_t(a) = e(a)$ that satisfies*

$$\delta e(a) - c(e(a), a) - \delta (\underline{u} + \underline{v}) \geq 0. \tag{15}$$

Moreover, for any continuation payoff gains $P_t(a) \geq 0$ and $U_t(a) \geq 0$ for $t \geq \tau$ consistent with the budget balance constraint (13) and independent of t, there exists an optimal continuation equilibrium for h_τ with $w_t(e(a))$ and $\underline{w}_t$ independent of t that has those continuation payoff gains.

3. *If (14) is satisfied for type a but efficient effort $e^*(a)$ does not satisfy (15), an optimal continuation equilibrium for h_τ has effort $e(a)$ the highest that satisfies (15) with equality, $P_t(a) = 0$, $U_t(a) = \underline{w}_t - \underline{u} \geq 0$ and*

$$c(e(a), a) = S_t^2(a), \qquad \text{for all } t \geq \tau. \tag{16}$$

Part 1 of Proposition 3 gives a condition for continuation of a relationship with known type a to be an equilibrium. Part 2 shows that effort in an optimal continuation equilibrium is stationary and satisfies (15). Stationary effort follows essentially from Levin (2003, Theorem 2) that, if an optimal contract exists, there are stationary contracts that are optimal. Necessity of (15) can be seen as follows. When the principal and agent type a continue the relationship at each date along an equilibrium path, $\gamma_t(a) = \beta_t(e(a)) = 1$ for all $t \geq \tau$ from (6) and (8). Then, from (10) and (11),

$$S_t^2(a) = \frac{\delta}{1-\delta} \left[e(a) - c(e(a), a) - \underline{u} - \underline{v} \right], \quad \text{for all } t \geq \tau. \tag{17}$$

Combined with the budget balance constraint (13), this gives

$$\delta e(a) - c(e(a), a) - \delta (\underline{u} + \underline{v}) = (1-\delta) \left[U_t(a) + \underline{u} - \underline{w}_t + P_t(a) \right], \quad \text{for all } t \geq \tau. \tag{18}$$

With the agent's type revealed to be a, continuation of the relationship requires $U_t(a) \geq \max[0, \underline{w}_t - \underline{u}]$ and $P_t(a) \geq 0$ from (3) and (8), so the right-hand side of (18) must be non-negative. Thus (15) is necessary. Part 2 of Proposition 3 also establishes that, for any stationary effort $e(a)$ that can be sustained as an optimal continuation equilibrium, there exist payments that distribute the joint gain in any way consistent with individual rationality. The reason can be seen from (18), which $w_t(e(a))$ enters only through the payoff gains $U_t(a)$ and $P_t(a)$ and cancels out in their sum. By changing $w_t(e(a))$, these payoff gains can, for given $e(a)$, range from $U_t(a) = \underline{w}_t - \underline{u}$ to $P_t(a) = 0$ without changing the value of the square bracket on the right-hand side. Moreover, $\underline{w}_t$ can be set equal to $\underline{u}$, so $U_t(a) = 0$ is also possible.[3]

Efficient effort for a is $e^*(a)$ defined by (1). If this satisfies (15), any continuation equilibrium on the Pareto frontier has efficient effort because that maximizes the joint gain to be distributed between the parties. If efficient effort does not satisfy (15), the Pareto frontier is obtained with $e(a)$ at the highest level that does, in which case (15) holds with equality, as specified in Part 3 of Proposition 3. Denote by $\hat{\alpha}$ the lowest a for which (14) is satisfied. It follows that, for any $a \geq \hat{\alpha}$ the only type with its history, effort in an optimal continuation equilibrium is[4]

$$\hat{e}(a) = \begin{cases} e^*(a)\,, \text{ if } e^*(a) \text{ satisfies (15);} \\ \max e(a) \text{ that satisfies (15) with equality, otherwise;} \end{cases} \quad \text{for } a \in [\hat{\alpha}, \overline{a}]\,. \quad (19)$$

Part 3 of Proposition 3 also establishes that, when efficient effort is not attainable, the bonus is set to make $P_t(a) = 0$. A higher bonus makes it possible to induce higher effort. So, when efficient effort is unattainable, it is optimal to have the bonus at the highest level consistent with the principal continuing the relationship. That requires the principal's future payoff gain from continuing the relationship by paying the bonus to be zero. The agent's payoff gain is $U_t(a) = \underline{w}_t - \underline{u}$. This is the lowest payoff gain consistent with the agent incurring the required effort because the agent can guarantee payoff gain of at least $\underline{w}_t - \underline{u}$ by putting in no effort at t and ending the relationship in period $t+1$ even when the principal pays no bonus. With any greater payoff gain, it would be possible to induce higher effort. The shares of the joint gain are determined by $\underline{w}_t$. For $\underline{w}_t - \underline{u} = S_t^1(a)$, $U_t(a) = S_t^1(a)$, so the agent receives all the joint gain at stage 1 of period t. For lower $\underline{w}_t$, the principal receives some of the joint gain at that stage (even though $P_t(a)$, which is measured at stage 2 of period t, is zero). For $\underline{w}_t = \underline{u}$, the principal receives all the joint gain. Because the joint gain can be shared in any

[3]To see why Proposition 3 is robust to the changes in timing discussed in footnote 1, let $P_t(a)$ be measured at stage 1 of period t. Then the budget balance constraint (13) becomes, for $\beta_t(e(a)) = \gamma_{t+1}(a) = 1$,

$$U_t(a) = -c(e_t(a), a) - \underline{u} + \underline{w}_t + \left[S_t^2(a) + w_t(e(a)) - \underline{w}_t - \delta P_{t+1}(a)\right].$$

For $e_t(a) = e(a)$ and with (17), which is unaffected by the change in timing, this changes (18) to

$$\delta e(a) - c(e(a), a) - \delta\left(\underline{u} + \underline{v}\right) = (1-\delta)\left[U_t(a) + \underline{u} - \underline{w}_t + \delta P_{t+1}(a) - \left(w_t(e(a)) - \underline{w}_t\right)\right].$$

With this timing, continuation of the relationship requires $U_t(a) \geq \max[0, \underline{w}_t - \underline{u}]$ and also $\delta P_{t+1}(a) \geq w_t(e(a)) - \underline{w}_t$ because otherwise the principal will not pay the bonus $w_t(e(a)) - \underline{w}_t$. So, by the same argument as for the timing in the text, (14) and (15) apply to the revised timing. The only change to the proposition under the revised timing is to Part 3, for which $P_{t+1}(a) = [w_t(e(a)) - \underline{w}_t]/\delta$. This change does not affect the results that follow.

[4]Critical for separating optimal effort from distribution is that the possibility of monetary payment makes the game one of transferable utility. But monetary payments are central to the model. Without them, the agent would never incur effort, so the only equilibrium would have the relationship never start.

proportions in this way, it is in the interests of both parties to choose a continuation equilibrium for h_τ that satisfies (19), independently of how the additional joint gain is divided between them (and hence of relative bargaining power).

5 Separation of continuing types

This section establishes the central result that there exists no equilibrium with optimal continuation that separates all types for whom a mutually beneficial relationship is possible. The next result is a first step.

Proposition 4 *Consider period t of an equilibrium relational contract with optimal continuation for which $[\underline{a}_t, \overline{a}_t] \subseteq A_t^+(h_t)$ and $\hat{e}(a) < e^*(a)$ for $a \in [\underline{a}_t, \overline{a}_t]$.*

1. *For $a, a' \in [\underline{a}_t, \overline{a}_t]$ with $a > a'$ both fully separated from all other types in $[\underline{a}_t, \overline{a}_t]$ at t, $\lim_{a \to a'} [e_t(a, h_t) - e_t(a', h_t)]$ is bounded below by some $\varepsilon > 0$.*
2. *The equilibrium relational contract does not separate all $a \in [\underline{a}_t, \overline{a}_t]$ at t.*

Proposition 4 considers relational contracts with optimal continuation that separate at t agent types in an interval with the same history at t (including $t = 1$ when all agent types necessarily have the same history) for which efficient effort is not attainable in the continuation equilibrium. Part 1 establishes that, to separate type a from type $a' < a$ in period t, the effort of type a at t must be discretely greater than that of a' as $a \to a'$. It follows that, as established in Part 2, it is not possible to separate at t all types in an interval $[\underline{a}_t, \overline{a}_t]$ who have the same history h_t because a monotone function defined on an interval cannot have a continuum of jumps.

Why does full separation require a discrete jump in effort? Fully separating a from $a' < a$ for given $e_t(a', h_t)$ requires finding an effort $\check{e}$ such that, if $e_t(a, h_t)$ is set equal to $\check{e}$, a prefers $\check{e}$ and a' prefers $e_t(a', h_t)$. For a and a' both fully separated under the conditions of the proposition, $e_\tau(a, h_\tau) = \hat{e}(a) < e^*(a)$ and $e_\tau(a', h_\tau) = \hat{e}(a') < e^*(a')$ for $\tau > t$. By Proposition 3, this implies payoff gain $U_\tau(a) = \underline{w}_\tau - \underline{u} \geq 0$ for all $\tau > t$. But $a' < a$ choosing $\check{e}$ in period t can guarantee the same payoff gain $\underline{w}_{t+1} - \underline{u}$ at $t+1$ as a by continuing the relationship for $t+1$ (so forgoing outside opportunity with payoff $\underline{u}$) and collecting the fixed wage $\underline{w}_{t+1}$, but delivering no effort (so receiving no bonus at $t+1$) and quitting for $t+2$.[5] So the difference in payoff between a and a' if both choose $\check{e}$ at t is just the difference in payoff in period t itself, $c(\check{e}, a') - c(\check{e}, a)$. In contrast, by

[5]Allowing negative bonuses would not alter this conclusion because type $a' < a$ would not pay a negative bonus in period $t+1$ if intending to quit for $t+2$.

imitating a' from t on, a can obtain an additional payoff gain over a' amounting to $c(\hat{e}(a'),a') - c(\hat{e}(a'),a)$ for each period from $t+1$ on, in addition to the difference in the cost of effort in period t, $c(e_t(a',h_t),a') - c(e_t(a',h_t),a)$. From (7) in Proposition 1, a necessary condition for a and a' both to choose separation at t is then that $\check{e}$ satisfies

$$
\begin{aligned}
& c(\check{e},a') - c(\check{e},a) \\
& \quad \geq U_t(a,h_t) - U_t(a',h_t) \geq c(e_t(a',h_t),a') - c(e_t(a',h_t),a) \\
& \qquad\qquad + \frac{\delta}{1-\delta}\left[c(\hat{e}(a'),a') - c(\hat{e}(a'),a)\right]. \qquad (20)
\end{aligned}
$$

(When the principal continues the relationship, $\beta_t(.) = 1$.) Now consider $a \to a'$ for given $e_t(a',h_t)$. By Assumption 1, $c(\tilde{e},a)$ is differentiable, and hence continuous, in a, so the expressions in (20) before the first inequality and after the last inequality both go to zero as $a \to a'$ and thus $\lim_{a \to a'-} U_t(a,h_t) = U_t(a',h_t)$. For $\check{e}$ to be such that a marginally above a' prefers $\check{e}$, but a' prefers $e_t(a',h_t)$, when the difference in payoff goes to zero as a goes to a', the derivative with respect to a of the expression before the first inequality in (20) must be no less than the derivative of the expression after the last inequality when both are evaluated at a'. Thus $\check{e}$ must satisfy

$$
-c_2(\check{e},a') \geq -c_2(e_t(a',h_t),a') - \frac{\delta}{1-\delta} c_2(\hat{e}(a'),a'). \qquad (21)
$$

By Assumption 1, $c_2(\tilde{e},a) < 0$ for $\tilde{e} \in (0,\overline{e}]$ and $c_{12}(\tilde{e},a) < 0$. Applied to (21), the former implies $c_2(\check{e},a')$ more negative than $c_2(e_t(a',h_t),a')$ by a discrete amount. The latter then implies $\check{e} = e_t(a,h_t)$ greater than $e_t(a',h_t)$ by a discrete amount. Because (7), and hence (20), are necessary conditions for a best response choice between actions that are on the equilibrium path for some agent type with history h_t, the only "off the equilibrium path" beliefs on which Proposition 4 depends are those underlying Definition 1 concerning optimal continuation for a fully separated type.[6]

Critical to this argument is that an optimal continuation equilibrium for a, if fully separated, is on the Pareto frontier because it is this that requires $U_{t+1}(a) = \underline{w}_{t+1} - \underline{u}$. When efficient effort is unattainable, effort is necessarily below the efficient level so to be on the Pareto frontier requires effort at the highest level consistent with dynamic enforcement. The critical constraint for this is that the required effort must not make the agent worse off than continuing the relationship for $t+1$ (so forgoing outside opportunity with payoff $\underline{u}$) and collecting the fixed wage $\underline{w}_{t+1}$, but delivering no effort (so

[6] A referee has asked whether it makes a difference if minimal effort is $\underline{e} > 0$ with positive cost that is decreasing with type. That adds the term $\delta\left[c(\underline{e},a') - c(\underline{e},a)\right]$ to the left-hand side of (20) and hence $-\delta c_2(\underline{e},a')$ to the left-hand side of (21). The conclusion still follows because, with $c_2(\tilde{e},a), c_{12}(\tilde{e},a) < 0$, $-c_2(\underline{e},a') < -c_2(\hat{e}(a'),a')/(1-\delta)$.

receiving no bonus at $t+1$) and quitting for $t+2$, thus obtaining payoff gain $\underline{w}_{t+1} - \underline{u}$ at $t+1$. If sticking to the contract gives payoff gain $U_{t+1}(a) > \underline{w}_{t+1} - \underline{u}$, a higher level of effort would have been consistent with dynamic enforcement, so the continuation equilibrium would not have been on the Pareto frontier. But a lower type $a' < a$ taking the action for a at t can also attain payoff gain $\underline{w}_{t+1} - \underline{u}$ at $t+1$ by continuing the relationship for $t+1$, delivering no effort and quitting for $t+2$. Thus type a separating fully at t receives no higher payoff from $t+1$ on than $a' < a$ would by imitating a at t. This applies however the future gain from being on the Pareto frontier is divided between principal and agent. The division of that gain is determined by $\underline{w}_{t+1}$ — changing the value of this traces out the whole Pareto frontier. In particular, agent type a is not restricted to zero payoff gain from continuing the relationship because $\underline{w}_{t+1}$ can be strictly greater than $\underline{u}$.

Also critical to the argument is that, with lower cost of effort, $a > a'$ choosing the effort for a' at t obtains a higher payoff than a' in every future period by continuing to choose the effort for a' — an informational rent. To induce a, but not a', to prefer the effort for a at t, the difference in payoff between them from that effort must be sufficient to offset the informational rent. Money is equally valuable to both, so payments at t do not generate a difference in payoff from choosing the same effort. Thus the difference in payoff must come through effort at t, which has lower cost for a than for $a' < a$. Specifically, the effort for a at t must be sufficiently much higher than that for a' that a prefers the effort for a to that for a' but a' does not. That is what (20) ensures.

The remaining step in the argument is that, to separate all types on an interval, the conditions for separation must be satisfied as a approaches a'. As that happens, the difference in payoffs between them must approach zero (an implication of (20)) because the cost of effort is continuous in type and so a' would not choose a different effort from a for a close enough to a' if there was a discrete jump in payoff between the efforts. Thus, to ensure a prefers the separating effort $\check{e}$, but a' does not, as a increases above a', the difference in payoffs between a and a' from choosing $\check{e}$ must increase faster with a than that from choosing $e_t(a', h_t)$. That corresponds to the expression before the first inequality in (20) increasing faster with a than the expression after the last inequality. The former increases with just the difference in current period effort cost because a' receives the same future payoff as a from choosing $\check{e}$. The latter increases not just with the difference in current period effort cost but also with the future informational rent. This gives condition (21) on the derivatives with respect to a evaluated at a'. With a cost of effort function that has continuous derivatives, the inequality can be satisfied only with a discrete upward jump in effort between a' and a.

Proposition 4 applies to an interval of pooled agent types and hence to all agent types in the first period of a relationship. The next result extends Proposition 4 to the whole relationship.

Proposition 5 *If there is more than one agent type $a \in [\underline{a}, \overline{a}]$ for which a mutually beneficial relational contract is possible, there exists no equilibrium with optimal continuation that continues the relationship for all those types and fully separates them.*

When there is more than one agent type for which a mutually beneficial relational contract is possible, Assumption 1 ensures that there is an interval of agent types a for which efficient effort does not satisfy (15) and, hence, $\hat{e}(a) < e^*(a)$. By Proposition 4, it is not possible to separate in one period all such agent types with the same history. That applies for any number of periods as long as the continuation equilibrium retains an interval of types with the same history. Such equilibria are not, however, the only possible continuation equilibria with pooling. Laffont and Tirole (1988) describe, in the context of a two-period procurement model, continuation equilibria that exhibit infinite reswitching in which actions that generate the same outcome are chosen by different types, but never by neighbouring types. That is, for any two types choosing the same action, there is always some intermediate type that chooses an action that generates a different outcome. Sun (2011) shows that, in the two-period procurement model, such continuation equilibria are not optimal. In the relational contract model used here, contracts with infinite reswitching are no more effective at achieving full separation with optimal continuation than are contracts with intervals of types that are pooled. So, as stated in Proposition 5, not all agent types for which a mutually beneficial relational contract is possible can be separated. The only restriction on continuation equilibria used to derive this result is that, conditional on full revelation of type a, effort for that type is $\hat{e}(a)$ thereafter. No restriction is imposed on effort in continuation equilibria for types that remain pooled.

6 Relationship to the literature and further discussion

With the ratchet effect in the dynamic procurement model of Laffont and Tirole (1988), it is also not possible to fully separate all of a continuum of privately-observed, persistent agent types. There, as here, agent type a receives future payoff no higher than $a' < a$ from choosing an action designed to reveal a's type. But there the reason is that the principal makes a "take it or leave it" contract offer in the subsequent period that gives a the same future payoff as quitting and a' could obtain that same future payoff by taking the action for a at t and actually quitting. Thus the mechanism is fundamentally different. With the ratchet effect, it is that the principal has all the bargaining power and so receives all the future gains from continuing the relationship once the agent's type is revealed. With the relational contract model of this paper, it is not the result of bargaining power. As already explained, it applies whatever the division of the future gain from continuing the relationship.

In the hidden information model in Levin (2003), types are *iid* draws each period, so all types are pooled at the start of each period and revelation of type does not affect future payoffs. An implication is that the second term after the right-hand inequality in (20) is zero — with type an *iid* draw, the agent receives no future informational rent from $t+1$ on from concealing type at t. Then (20) is satisfied by any effort function that is non-decreasing and, as a result, full separation is always possible. Pooling arises only when full separation is not optimal, so the reason for pooling is fundamentally different than with persistent types. The same applies to the one-period version of the procurement model of Laffont and Tirole (1988), where pooling occurs only when the distribution of agent types makes full separation not optimal. Full separation may not be optimal in Levin (2003) because dynamic enforcement restricts the spread of bonuses that are incentive compatible. That, in turn, restricts the spread of incentive compatible efforts that are available for separating types and pooling the most productive types is the optimal way to restrict the spread of efforts. That is different from the role of dynamic enforcement in the model of the present paper. Here dynamic enforcement ensures type a receives no higher future payoff from separation when the continuation equilibrium is on the Pareto frontier than $a' < a$ could obtain by imitating a. That is because, to induce the highest possible effort when incentives are limited by dynamic enforcement, an agent type that has been fully revealed receives payoff no higher than from choosing zero effort, which a' can also achieve with zero effort. It is this mechanism that replaces the principal's "take it or leave it" offers in Laffont and Tirole (1988).

In the present model, effort $\check{e}$ required by (21) to induce a to separate is bounded above by $\hat{e}(a)$ defined in (19). It follows that separation is more easily achieved with lower $e_t(a', h_t)$ and with lower $\hat{e}(a')$. The former illustrates the benefits of starting a relationship "small", as in Watson (1999) and Watson (2002). The latter illustrates a limitation that arises from the parties being unable to commit themselves to inefficient actions in the future. If the parties could commit to sub-optimal effort for type a' in period $t+1$, separation of types at t would be easier to achieve.

Proposition 4 applies to agent types for which efficient effort cannot be achieved following full revelation. Assumption 1 does not rule out efficient effort being attainable for some types. For them, effort in an optimal continuation equilibrium following full revelation is efficient. Moreover, it may not require the agent's payoff gain to equal that from zero effort so, for such types, the argument used to establish Proposition 5 does not go through.[7] But this is never the case for all types for which a mutually beneficial relational contract is possible. The following example illustrates the range of productive types for which efficient effort is unattainable.

Example 1 *Consider $c(e_t, a) = e_t^2/(2a)$. Efficient effort defined by (1) is then $e^*(a) = a$, so this specification identifies agent type with its output when effort is efficient. Once type is revealed, the lowest agent type for which a continued relational contract is feasible is that for which the maximum value on the left-hand side of (14) just equals the right-hand side and this type produces output $2(\underline{u} + \underline{v})$. For $\delta > 1/2$, the lowest type for which efficient effort is feasible is $(\underline{u} + \underline{v})2\delta/(2\delta - 1)$, which is also its output. Thus the ratio of the output at the upper end of the range for which a relational contract is feasible but not efficient to that at the lower end is $\delta/(2\delta - 1)$. For $\delta = 0.9$, that implies output at the upper end of the range is 12.5% higher than that at the lower end. As $\delta \to 1/2$, efficient effort becomes infeasible for any type even for $\overline{a} \to \infty$. For empirical applications, δ corresponds to the pure time-discount factor multiplied by the probability that the relationship does not end for exogenous reasons and by the probability that deviation by the agent is detected by the principal, so it may be considerably smaller than the pure time-discount factor.*

With Proposition 5 establishing that full separation of all agent types is not possible, an obvious question is what pattern of pooling is optimal. Two things in particular make the answer complicated. First, for at least minimal consistency, one needs to impose a renegotiation-proofness requirement for continuation equilibria with pooling of types. The complication with this is that the standard renegotiation-proofness concepts in the literature on infinite horizon games are defined only for games without private information about types and it is not obvious how to extend them appropriately to games with private information. Second is the sheer variety of possible patterns of separation. Changing the pattern in any one period in general affects more than one type and in a way that is not differentiable, so evaluating changes is not straightforward. Moreover, changing the pattern of separation in one period affects the possibilities for separation in subsequent periods, so what is optimal has to be considered over this dimension too. One thing is clear, however — the optimal pattern of separation is going to be sensitive to the distribution of types, so general results that apply to all the distributions experienced in practice are unlikely to be available.

[7] The conclusion still holds if the principal receives all the joint gain following separation at t, as a result of which $U_{t+1}(a) = 0$. But the reason is then the same as with the ratchet effect.

7 Changing agent types

The preceding analysis assumes that the agent's type is observed at stage 0b of the first period of the relationship and remains fixed thereafter. A natural question is whether similar results apply if the agent's type may change during the course of the relationship. This section considers agent type changes of the following form.

Assumption 2 *The agent's cost of effort e_t at t is $c(e_t, a_t)$ where, with probability $1-\pi$ for $\pi \in [0,1)$, $a_t = a_{t-1}$ and, with probability π, a_t is a random draw from the distribution $F(a)$ with support $a \in [\underline{a}, \overline{a}]$.*

The cases in which the principal observes, and does not observe, whether the agent's type has changed are both discussed below. The only change to the timing in the model in Figure 1 is that the agent observes a_t for period t immediately before deciding effort e_t at stage 1 of period t. The definition of an equilibrium with optimal continuation also needs to be extended to this case. The following is a natural extension of Definition 1.

Definition 2 *An* equilibrium with optimal continuation with changing agent type *is a perfect Bayesian (PB) equilibrium in pure strategies for which equilibrium-path continuation equilibria following full revelation of agent type a_t at t have payoffs at stage 2 of period t on the Pareto frontier of perfect Bayesian continuation equilibria for the principal and agent type a_t.*

The difference from Definition 1 is that continuation equilibria immediately following full revelation of the agent's type are required to be on the Pareto frontier of perfect Bayesian continuation equilibria, not just of subgame perfect continuation equilibria. Because this applies at a node at which the agent's type has been fully revealed, the relevant Pareto frontier is that between the principal and a known agent type at a node with symmetric, but incomplete, information. As with Definition 1, Definition 2 does not restrict continuation equilibria off the equilibrium path. Nor does it restrict continuation equilibria conditional on the agent's type changing at $t+1$. Thus it imposes only a rather weak additional requirement on perfect Bayesian continuation equilibria.

The implications of Assumption 2 and Definition 2 are discussed here informally. Critical to the argument in Section 5 that full separation is not possible when agent type is fixed is that type $a' < a$ taking an action intended to fully separate a at t can attain the same payoff as a at $t+1$. This property arises because, for optimal continuation for type a fully revealed at t to yield a future joint payoff on the Pareto frontier, effort for a at $t+1$ must be at the highest feasible level if efficient effort cannot be attained.

That requires a to be indifferent between choosing the required effort and choosing zero effort. But a' can then attain the same payoff as a at $t+1$ by choosing zero effort.

Under Assumption 2, type a_t fully revealed at t may change at $t+1$, so a continuation equilibrium at $t+1$ will in general specify a menu of equilibrium-path efforts for $t+1$ from which the agent chooses conditional on type at $t+1$. But type $a' < a_t$ taking an action intended to fully separate a_t at t can still attain the same payoff as a_t at $t+1$ if Definition 2 requires effort for type a_t fully revealed at t to be the maximum feasible conditional on type not changing at $t+1$, no matter what the outcome if type does change at $t+1$. The reason is as follows. Conditional on type not changing at $t+1$, type a' imitating a_t receives the same payoff as a_t would for the same reason as when type is fixed. Conditional on type changing at $t+1$, type a' imitating a_t also receives the same payoff as a_t because they *are* the same current type with the same history observed by the principal. Under Assumption 2, each possible type change occurs with the same probability for a' as for a_t. Thus the expectation of future payoffs at the beginning of $t+1$ is the same for a' as for a_t. Moreover, a_t imitating the action for $a' < a_t$ still receives an informational rent unless type changes (so with effective discount factor now $\delta(1-\pi)$). Thus the arguments used in Section 5 can be applied.

Moreover, provided the relationship is sufficiently productive given full revelation of type a_t, there certainly exists a perfect Bayesian continuation equilibrium with effort at $t+1$ for type a_t fully revealed at t the maximum feasible conditional on type not changing at $t+1$. Consider, for example, a continuation in which, for $\tau > t$, types $a_\tau \in [\underline{a}, a_t)$ either choose zero effort or end the relationship (in the former case, the principal may end the relationship if the probability of agent type rising to at least a_t is sufficiently low), and all types $a_\tau \in [a_t, \overline{a}]$ choose effort $\hat{e}(a_t, \pi)$ that is the maximum sustainable by a_t given the actions of agent types $a_\tau \in [\underline{a}, a_t)$ and the principal. With a_t preferring effort $\hat{e}(a_t, \pi)$ to zero effort, types $a_\tau \in [a_t, \overline{a}]$ do so too because they have lower cost of effort. Moreover, with $\hat{e}(a_t, \pi)$ the maximum effort feasible for a_t, it is a best response for types $a_\tau \in [\underline{a}, a_t)$ to either choose zero effort or end the relationship because of their higher cost of effort.

When efficient effort for a_t is not attainable, the only reason the Pareto frontier at stage 2 of period t might be attained by effort conditional on type not changing that is below the maximum feasible for a_t would be if this permitted a higher joint payoff conditional on type changing. If, when agent type changes at $t+1$, the principal receives a signal to that effect (though not of what type changes to), there is no need to reduce effort conditional on type not changing in order to get the highest joint payoff conditional on type changing. Then certainly to be on the Pareto frontier at stage 2 of period t as required by Definition 2 requires the agent's effort conditional on type not changing to be the highest feasible.

The case in which the principal receives no such signal is trickier. Let $S(e_{t+1}, a_t, \pi)$ denote the maximum joint payoff gain attainable at $t+1$ conditional on type changing at $t+1$ for given e_{t+1} for fully-revealed type a_t. Also let $\hat{e}_{t+1}(a_t, \pi) < e^*(a_t)$ for all $\pi \leq \pi^*$, with $\pi^* > 0$, denote the highest feasible effort conditional on type not changing given the joint payoff gain $S(\hat{e}_{t+1}(a_t, \pi), a_t, \pi)$ conditional on type changing. (It need not be for the stationary continuation equilibrium described in the previous paragraph.) Now consider the joint payoff gain at stage 2 of period t from deviating to $e_{t+1} = \hat{e}_{t+1}(a_t, \pi) - \Delta$, for $\Delta > 0$, conditional on type not changing. Because $s(e, a)$ is strictly concave in e, that joint gain is strictly less than

$$\delta \left\{ -(1-\pi) s_1(\hat{e}_{t+1}(a_t, \pi), a_t) \Delta + \pi \left[S(\hat{e}_{t+1}(a_t, \pi) - \Delta, a_t, \pi) - S(\hat{e}_{t+1}(a_t, \pi), a_t, \pi) \right] \right\}.$$

Even if the expression in square brackets is positive, it is certainly bounded, so the second term in this expression approaches zero as π approaches zero. The first term is strictly negative for $\hat{e}_{t+1}(a_t, \pi) < e^*(a_t)$ and $\Delta > 0$ by the specification of $e^*(a_t)$ in (1). So the whole expression is strictly negative for $\pi < \pi^*$ sufficiently close to zero. If the maximum value function $S(e_{t+1}, a_t, \pi)$ satisfies local left-sided Lipschitz continuity in e_{t+1} at $\hat{e}_{t+1}(a_t, \pi)$ for all $\pi < \pi^*$, the joint payoff gain is thus certainly always increased by reducing Δ for sufficiently small π. Because $S(e_{t+1}, a_t, \pi)$ is an integral of the joint payoff gain functions over all types for a distribution $F(a)$ with no mass points, local left-sided Lipschitz discontinuity in the joint payoff gain function at $\hat{e}_{t+1}(a_t, \pi)$ for isolated types is not a problem for this. To determine when the maximum value function $S(e_{t+1}, a_t, \pi)$ satisfies local left-sided Lipschitz continuity (for which it is sufficient that it is continuously differentiable, like maximum value functions for some stochastic dynamic programming problems, see Stokey and Lucas (1989)) is beyond the scope of this paper. But when it is, the conclusion is that, even if the principal does not know that type has changed, an equilibrium with optimal continuation with changing agent type has effort for a revealed type at the maximum sustainable for that type when π is sufficiently small. The arguments used in Section 5 can then still be applied.

The essential argument here is that, when efficient effort is not attainable, reducing effort conditional on type not changing below the maximum feasible results in a first-order loss in joint payoff conditional on type not changing. To incur that loss for a bounded potential gain conditional on type changing is thus never worthwhile if the probability of type changing is sufficiently small. But then, as with unchanging types, some lower type choosing the effort at t intended to fully reveal a higher type achieves the same payoff at the beginning of $t+1$ as that higher type. Given that the higher type obtains a strictly positive informational rent from imitating the lower type, there has to be a discrete upward jump between their efforts to induce revelation by the higher type. But there cannot be a discrete upward jump between efforts for all types on a continuum, which means full revelation of all types is not possible.

8 Conclusion

This paper combines insights from the ratchet effect in the literature on procurement and dynamic enforcement in the literature on relational incentive contracts to analyse relational incentive contracts when the agent's type is privately known by the agent and persistent over time. It differs from the relational contract models of Levin (2003) and MacLeod (2003), in which the agent's type is an *iid* random draw each period. Applied to employment, it generalizes the models of Shapiro and Stiglitz (1984) and MacLeod and Malcomson (1989) to private information about workers' disutility of effort. It also differs from the ratchet effect model of Laffont and Tirole (1988) in which the parties are legally constrained from committing to future contract terms and the principal makes "take it or leave it" contract offers.

The central result shown here is that, with continuous privately-observed agent types that are unchanging over time, relational contracts for which future actions are optimal (that is, are at any point on the Pareto frontier) once the agent's type is fully revealed cannot fully separate all types because full separation requires a discrete jump in effort between neighbouring types. This result extends to sufficiently persistent types under certain conditions. Thus, the ratchet effect result that some pooling of agent types is unavoidable applies even though the parties are not legally constrained from committing to future contract terms and the principal does not have the power to make "take it or leave it" contract offers. This result significantly extends beyond the traditional ratchet effect the set of circumstances under which full separation of types is not possible.

Appendix: Proofs

Proof of Proposition 1. That (6) defines a best response continuation rule follows from the specification of $\alpha_t(h_t)$ in (3).

Effort function $e_t(a,h_t)$ may not be a best response at t because agent type a prefers to deviate to either (1) $\tilde{e} = e_t(a',h_t) \neq e_t(a,h_t)$ for some $a' \in A_t^+(h_t)$ or (2) $\tilde{e} \neq e_t(a',h_t)$ for any $a' \in A_t^+(h_t)$. Incentive compatibility to the first type of deviation requires

$U_t(a,h_t) = \tilde{U}_t(a,a,h_t)$. That in turn requires

$$U_t(a,h_t) \geq \tilde{U}_t\left(a',a,h_t\right) = U_t\left(a',h_t\right) + \tilde{U}_t\left(a',a,h_t\right) - \tilde{U}_t\left(a',a',h_t\right), \quad \forall a,a' \in A_t^+(h_t),$$

and, with the roles of a and a' interchanged,

$$U_t\left(a',h_t\right) \geq \tilde{U}_t\left(a,a',h_t\right) = U_t(a,h_t) + \tilde{U}_t\left(a,a',h_t\right) - \tilde{U}_t(a,a,h_t), \quad \forall a,a' \in A_t^+(h_t).$$

These two conditions imply that (7) is necessary. They also imply (7) is sufficient to deter deviation to $\tilde{e} = e_t(a',h_t) \neq e_t(a,h_t)$ for $a' \in A_t^+(h_t)$.

Now consider deviation to $\tilde{e} \neq e_t(a',h_t)$ for any $a' \in A_t^+(h_t)$. Let

$$h'_{t+1} = h_t \cup (e_t(\alpha_t(h_t),h_t), w_t(h_t, e_t(\alpha_t(h_t),h_t))).$$

(This is the history at $t+1$ conditional on the agent choosing effort $e_t(\alpha_t(h_t),h_t)$ and the principal paying the corresponding bonus.) With the specified continuation contracts, $\beta_t(h_t,\tilde{e}) = 1$, $w_t(h_t,\tilde{e}) = \underline{w}_t(h_t)$ and

$$\underline{w}_{t+1}(h_t \cup (\tilde{e},\underline{w}_t(h_t))) \leq -\left[\tilde{U}_{t+1}\left(\alpha_t(h_t),\alpha_t(h_t),h'_{t+1}\right) - \underline{w}_{t+1}\left(h'_{t+1}\right)\right]. \qquad \text{(A.1)}$$

The payoff gain at stage 1 of period t to $a \geq \alpha_t(h_t)$ continuing the relationship while deviating to $\tilde{e}$ would, given (A.1), be

$$\begin{aligned}
&-c(\tilde{e},a) - \underline{u} + \underline{w}_t(h_t) \\
&\qquad + \delta\left[\tilde{U}_{t+1}\left(\alpha_t(h_t),a,h'_{t+1}\right) - \underline{w}_{t+1}\left(h'_{t+1}\right) + \underline{w}_{t+1}(h_t \cup (\tilde{e},\underline{w}_t(h_t)))\right] \\
&\quad \leq -c(\tilde{e},a) - \underline{u} + \underline{w}_t(h_t) + \delta\left[\tilde{U}_{t+1}\left(\alpha_t(h_t),a,h'_{t+1}\right) - \tilde{U}_{t+1}\left(\alpha_t(h_t),\alpha_t(h_t),h'_{t+1}\right)\right].
\end{aligned}$$

Choice of $\tilde{e}$ affects only the first term, so this payoff gain cannot be greater than for $\tilde{e} = 0$ so that $c(\tilde{e},a) = 0$. Thus, not deviating to $\tilde{e}$ is a best response if

$$U_t(a,h_t) \geq -\underline{u} + \underline{w}_t(h_t) + \delta\left[\tilde{U}_{t+1}\left(\alpha_t(h_t),a,h'_{t+1}\right) - \tilde{U}_{t+1}\left(\alpha_t(h_t),\alpha_t(h_t),h'_{t+1}\right)\right], \quad \text{for all } a \in A_t^+(h_t). \qquad \text{(A.2)}$$

For $a' = \alpha_t(h_t)$, (7) implies

$$U_t(a,h_t) \geq \tilde{U}_t(\alpha_t(h_t),a,h_t) - \tilde{U}_t(\alpha_t(h_t),\alpha_t(h_t),h_t) + U_t(\alpha_t(h_t),h_t), \quad \text{for all } a \in A_t^+(h_t). \qquad \text{(A.3)}$$

So, if the right-hand side of (A.3) is greater than that of (A.2), (7) is sufficient to deter $a \in A_t^+(h_t)$ from deviating to $\tilde{e} \neq e_t(a',h_t)$ for any $a' \in A_t^+(h_t)$.

Type $a \geq \alpha_t(h_t)$ imitating $\alpha_t(h_t)$ at t cannot receive a lower payoff than from continuing to imitate $\alpha_t(h_t)$ at $t+1$. So, from (2) and (5),

$$\tilde{U}_t(\alpha_t(h_t), a, h_t) \geq -c(e_t(\alpha_t(h_t), h_t), a) - \underline{u} + \underline{w}_t(h_t) + w_t(h_t, e_t(\alpha_t(h_t), h_t)) \\ - \underline{w}_t(h_t) + \delta \tilde{U}_{t+1}\left(\alpha_t(h_t), a, h'_{t+1}\right), \text{ for all } a \geq \alpha_t(h_t),$$

and this holds with equality for $a = \alpha_t(h_t)$. Thus

$$\tilde{U}_t(\alpha_t(h_t), a, h_t) - \tilde{U}_t(\alpha_t(h_t), \alpha_t(h_t), h_t) \\ \geq -c(e_t(\alpha_t(h_t), h_t), a) + c(e_t(\alpha_t(h_t), h_t), \alpha_t(h_t)) \\ + \delta\left[\tilde{U}_{t+1}\left(\alpha_t(h_t), a, h'_{t+1}\right) - \tilde{U}_{t+1}\left(\alpha_t(h_t), \alpha_t(h_t), h'_{t+1}\right)\right], \text{ for all } a \in A_t^+(h_t).$$

With $c_2(\tilde{e}, a) < 0$ for $\tilde{e} \in (0, \overline{e}]$, (A.3) therefore implies

$$U_t(a, h_t) \geq \delta\left[\tilde{U}_{t+1}\left(\alpha_t(h_t), a, h'_{t+1}\right) - \tilde{U}_{t+1}\left(\alpha_t(h_t), \alpha_t(h_t), h'_{t+1}\right)\right] \\ + U_t(\alpha_t(h_t), h_t), \quad \text{for all } a \in A_t^+(h_t),$$

and so, because (3) requires $U_t(\alpha_t(h_t), h_t) \geq \underline{w}_t(h_t) - \underline{u}$, (A.3) implies (A.2). ■

Proof of Proposition 2. From the definition of $P_t(a, (h_t, e_t))$, the principal does at least as well by continuing the relationship ($\beta_t(h_t, e_t) = 1$) and paying the bonus $w_t(h_t, e_t) - \underline{w}_t(h_t)$ as by ending it if $E_{a|h_t, e_t}\left[P_t(a, (h_t, e_t))\right] \geq 0$. Moreover, if ending the relationship at t ($\beta_t(h_t, e_t) = 0$), the principal clearly cannot gain by paying a bonus at t. Suppose the principal were to continue the relationship at t ($\beta_t(h_t, e_t) = 1$) but pay $w_t \neq w_t(h_t, e_t)$ when $w_t(h_t, e_t) - \underline{w}_t(h_t) > 0$. Under the specified continuation contract, the principal would receive non-positive payoff gain from continuation and so, given (8), would not make a greater payoff gain than from paying $w_t(h_t, e_t)$. ■

Definition 3 *A* stationary pooling continuation contract for h_τ *has the continuation contracts following deviation in Propositions 1 and 2 and, with the definition* $a_\tau^-(a) = \alpha_\tau(h_\tau)$ *for* $a \in A_\tau^+(h_\tau)$*:*

1. *every agent type* $a \in A_\tau^+(h_\tau)$ *choose the same effort* $e_t = e(a_\tau^-(a))$ *at* $t \geq \tau$*,*

2. *the principal continue the relationship at* $t \geq \tau$ *if* $e_t = e(a_\tau^-(a))$*.*

Lemma 1 *The following apply to stationary pooling continuation contracts for* h_τ*:*

1. *There exists a stationary pooling continuation contract for* h_τ *that is a PB continuation equilibrium for* h_τ *if and only if*

$$\delta e\left(a_\tau^-(a)\right) - c\left(e\left(a_\tau^-(a)\right), a\right) - \delta\left(\underline{u} + \underline{v}\right) \geq 0, \quad \textit{for all } a \in A_\tau^+(h_\tau), \tag{A.4}$$

or, equivalently,

$$S^2_{t-1}\left(a^-_\tau(a)\right) \geq c\left(e\left(a^-_\tau(a)\right), a^-_\tau(a)\right), \quad \textit{for all } a \in A^+_\tau(h_\tau) \textit{ and } t \geq \tau. \tag{A.5}$$

2. *Consider continuation payoff gains $P_t(a^-_\tau(a)) \geq 0$ and $U_t(a^-_\tau(a)) \geq 0$ consistent with (13) and independent of $t \geq \tau$. If $e(a^-_\tau(a))$ satisfies (A.4) and either $A^+_\tau(h_\tau) = A_\tau(h_\tau)$ or $U_t(a^-_\tau(a)) = 0$, there exists a stationary pooling continuation contract for h_τ with $w_t(e(a^-_\tau(a)))$ and $\underline{w}_t$ independent of $t \geq \tau$ that is a PB continuation equilibrium for h_τ with those continuation payoff gains.*

3. *To be a PB continuation equilibrium for h_τ, any stationary pooling continuation contract for h_τ for which $e(a^-_\tau(a))$ satisfies (A.4) with equality for $a = a^-_\tau(a)$ has $P_t(a^-_\tau(a)) = 0$, $U_t(a^-_\tau(a)) = \underline{w}_t - \underline{u} \geq 0$ and*

$$c\left(e\left(a^-_\tau(a)\right), a^-_\tau(a)\right) = S^2_{t-1}\left(a^-_\tau(a)\right), \textit{ for all } t \geq \tau. \tag{A.6}$$

Proof. Part 1: Necessity. For stationary pooling continuation contracts for h_τ, $S^2_{t-1}(a)$ is stationary, and $\gamma_t(a) = \beta_t(e(a)) = 1$, for all $a \in A^+_\tau(h_\tau)$ and all $t \geq \tau$. Then, from (10) and (11),

$$S^2_{t-1}(a) = \frac{\delta}{1-\delta}\left[e(a) - c(e(a), a) - \underline{u} - \underline{v}\right], \text{ for all } a \in A^+_\tau(h_\tau) \text{ and } t \geq \tau. \tag{A.7}$$

Used to substitute for $S^2_t(a)$ in the budget balance constraint (13), this gives

$$\begin{aligned}&\delta e(a) - c(e(a), a) - \delta\left(\underline{u} + \underline{v}\right) \\ &\qquad = (1-\delta)\left[U_t(a) + \underline{u} - \underline{w}_t + P_t(a)\right], \quad \text{for all } a \in A^+_\tau(h_\tau) \text{ and } t \geq \tau. \end{aligned} \tag{A.8}$$

All $a \in A^+_\tau(h_\tau)$ have the same history for $t \geq \tau$, so $P_t(a)$ is the same. Thus, from (8), continuation of the relationship requires $P_t(a) \geq 0$ for $a \in A^+_\tau(h_\tau)$ and $t \geq \tau$. Moreover, from (3) and Proposition 1, continuation by agent types $a \in A^+_\tau(h_\tau)$ implies

$$U_t(a) \geq \max\left[0, \underline{w}_t - \underline{u}\right], \quad \text{for all } a \in A^+_\tau(h_\tau) \text{ and } t \geq \tau. \tag{A.9}$$

Together with (A.8), these imply that, for $e(a^-_\tau(a))$ to be a PB continuation equilibrium at τ for all $a \in A^+_\tau(h_\tau)$, it must satisfy (A.4). (A.7) for $a = a^-_\tau(a)$ implies that (A.5) is equivalent to (A.4) for $a = a^-_\tau(a)$. That and $c_2(\tilde{e}, a) < 0$ for $\tilde{e} > 0$ imply (A.4) is satisfied for all $a \geq a^-_\tau(a)$ if it is satisfied for $a^-_\tau(a)$, so (A.5) and (A.4) are equivalent.

Sufficiency. Suppose $e(a^-_\tau(a))$ satisfies (A.4) and consider the stationary pooling continuation contract for h_τ that has $e_t(a) = e(a^-_\tau(a))$, $\gamma_t(a) = \beta_t(e(a^-_\tau(a))) = 1$,

$w_t(e(a_\tau^-(a))) = w$, and $\underline{w}_t = \underline{w}$ for all $a \in A_\tau^+(h_\tau)$ and $t \geq \tau$ for some w and $\underline{w}$ with $w \geq \underline{w}$. Under this continuation contract, payoff gains are stationary. It follows from (2) for the agent, and a corresponding calculation for the principal, that

$$U_t(a) = \frac{-c(e(a_\tau^-(a)),a) - \underline{u} + \underline{w} + (w - \underline{w})}{1-\delta}, \quad \text{for all } a \in A_\tau^+(h_\tau),\ t \geq \tau, \tag{A.10}$$

$$P_t(a) = \frac{-(w - \underline{w}) + \delta\left[e(a_\tau^-(a)) - \underline{v} - \underline{w}\right]}{1-\delta}, \quad \text{for all } a \in A_\tau^+(h_\tau),\ t \geq \tau. \tag{A.11}$$

With (A.4) satisfied, there certainly exist $w \geq \underline{w}$ such that $U(a) \geq \max[0, \underline{w} - u]$ and $P(a) \geq 0$ for all $a \in A_\tau^+(h_\tau)$, specifically when

$$c\left(e\left(a_\tau^-(a)\right), a_\tau^-(a)\right) + \underline{u} - \underline{w} \leq w - \underline{w} \leq \delta\left[e\left(a_\tau^-(a)\right) - \underline{v} - \underline{w}\right] \tag{A.12}$$

because, with $c_2(\tilde{e}, a) < 0$ for $\tilde{e} > 0$, $c(e,a)$ is decreasing in a. With the continuation contracts for deviation to $\tilde{e} \neq e(a_\tau^-(a))$ in Proposition 1, the only conditions for agent types $a_\tau^-(a)$ to continue the relationship for $t \geq \tau$ are those in (A.9). With $U_t(a)$ necessarily non-decreasing in a, this is sufficient to ensure that (A.9) is satisfied for all $a \in A_\tau^+(h_\tau)$ and, by Proposition 1, do not deviate to $\tilde{e} \neq e(a_\tau^-(a))$. That and the condition in (8) for it to be a best response for the principal to continue the relationship are thus satisfied for the specified stationary pooling continuation contract. Moreover, with the continuation contracts for deviation to $w_t \neq w$ in Proposition 2, it is a best response for the principal to pay w. Thus the specified stationary pooling continuation contract for h_τ is a PB continuation equilibrium for h_τ.

Part 2. The stationary pooling continuation contract specified in the proof of sufficiency for Part 1 has $w_t(e(a_\tau^-(a)))$ and $\underline{w}_t$ independent of $t \geq \tau$. Consider $\underline{w} = \underline{u}$. Then w satisfies (A.12) if

$$\underline{u} + c\left(e\left(a_\tau^-(a)\right), a_\tau^-(a)\right) \leq w \leq \underline{u} + \delta\left[e\left(a_\tau^-(a)\right) - \underline{v} - \underline{u}\right].$$

By choosing w appropriately between the upper and lower bounds in this, $U_t(a_\tau^-(a))$ and $P_t(a_\tau^-(a))$ specified by (A.10) and (A.11) can take on any non-negative values independent of $t \geq \tau$ that are consistent with (A.8) and thus (A.4). That establishes the result for $A_\tau^+(h_\tau) = A_\tau(h_\tau)$. For $A_\tau^+(h_\tau) \subset A_\tau(h_\tau)$, it may need to be that $U_t(a_\tau^-(a)) = 0$ for $a \in A_\tau^+(h_\tau)$ for $a \notin A_\tau^+(h_\tau)$ not to continue the relationship.

Part 3. For (A.4) to hold with equality for $a = a_\tau^-(a)$, the right-hand side of (A.8) must be zero for $a = a_\tau^-(a)$. With $P_t(a) \geq 0$ and (A.9), that requires $P_t(a_\tau^-(a)) = 0$ and $U_t(a_\tau^-(a)) = \underline{w}_t - \underline{u} \geq 0$. (A.6) follows from the equivalence of (A.4) and (A.5). ■

Proof of Proposition 3. The proof is in two steps. Step 1 shows that, if (14) is satisfied for a, there exists a subgame perfect continuation equilibrium for h_τ and establishes the proposition for continuation equilibria restricted to the class with stationary effort. Step 2 establishes that no continuation equilibrium with non-stationary effort can do as well, so any optimal continuation equilibrium for h_τ must have stationary effort.

Step 1. For a the only type in $A^+_\tau(h_\tau)$, a stationary continuation contract for h_τ is a stationary pooling contract for h_τ with just one type in the pool, so Lemma 1 applies with subgame perfection substituted for Bayesian perfection. If (14) is satisfied for a, there exists $e(a)$ that satisfies (15) and thus satisfies (A.4) for $a^-_\tau(a) = a$. By Part 1 of Lemma 1, (A.4) is sufficient for there to exist a stationary continuation contract for h_τ that is a subgame perfect continuation equilibrium for h_τ. That establishes Part 1.

By Part 1 of Lemma 1, (A.4) is also necessary. Thus, a continuation equilibrium for h_τ optimal in the class with stationary effort must satisfy (15), as specified in Part 2. To be on the Pareto frontier, it must also maximise $S^1_\tau(a)$ subject to (15). With $\gamma_\tau(a) = 1$, maximising $S^1_\tau(a)$ corresponds to maximising $S^2_{\tau-1}(a)$ in (11). $S^2_t(a)$ for $t \geq \tau - 1$, given by (A.7), is a continuous function of $e(a)$ to be maximised by choice of $e(a)$ from the compact set defined by (15), so an optimal $e(a)$ certainly exists. It is, moreover, independent of $w_t(e(a))$ and $\underline{w}_t$. Because $A^+_\tau(h_\tau) = A_\tau(h_\tau)$ for a the only type with history h_τ, Part 2 of Lemma 1 suffices to complete the proof of Part 2 of the proposition for the class of stationary continuation contracts for h_τ.

If efficient effort $e^*(a)$ satisfies (15), clearly that maximises $S^2_{\tau-1}(a)$ subject to (15). If not, $S^2_{\tau-1}(a)$ is maximised by the highest effort $\hat{e}(a)$ that satisfies (15) with equality. It then follows from Part 3 of Lemma 1 with $a^-_\tau(a) = a$ that a continuation contract for h_τ optimal in the class with stationary effort has $P_t(a) = 0$ and $U_t(a) = \underline{w}_t - \underline{u} \geq 0$ for all $t \geq \tau$, and has effort $\hat{e}(a)$ that satisfies (16) for all $t \geq \tau$. That establishes Part 3 of the proposition for this class of continuation contract for h_τ.

Step 2. Now consider whether it is possible to achieve as high or higher $S^2_{\tau-1}(a)$ with a continuation equilibrium that has non-stationary effort. It cannot be if efficient effort $e^*(a)$ is attainable because efficient effort is stationary, non-stationary effort must depart from efficient effort for some $t \geq \tau$ and that must lower $S^2_{\tau-1}(a)$. Consider, therefore, a continuation equilibrium that is optimal from the class with stationary effort and that has $\hat{e}(a) < e^*(a)$ and joint payoff gain $\hat{S}^2(a)$ for $t \geq \tau - 1$. From Levin (2003, Theorem 2), no non-stationary continuation equilibrium can achieve $S^2_t(a) > \hat{S}^2(a)$ for $t \geq \tau - 1$, so any optimal continuation equilibrium must satisfy the budget balance constraint (13) with $S^2_t(a) = \hat{S}^2(a)$. To achieve $S^2_t(a) = \hat{S}^2(a)$ with non-stationary effort, it must be that $e_t(a) > \hat{e}(a)$ for some $t \geq \tau$. By step 1, an optimal continuation equilibrium with stationary effort $\hat{e}(a) < e^*(a)$ has $P_t(a) = 0$ and $U_t(a) = \underline{w}_t - \underline{u}$ for all $t \geq \tau$. It is thus not possible to satisfy (13) with $S^2_t(a) = \hat{S}^2(a)$, $P_t(a) \geq 0$, $U_t(a) \geq \underline{w}_t - \underline{u}$ (as required by (3)) and $\beta_t(\hat{e}(a)) = 1$ for all $t \geq \tau$ with $e_t(a) > \hat{e}(a)$ for any $t \geq \tau$. Thus, any optimal continuation equilibrium for h_τ must have $e_t(a) = \hat{e}(a)$ for all $t \geq \tau$. ■

Proof of Proposition 4. Part 1. With optimal continuation for a and a' fully separated at t, their efforts from $t+1$ on are $\hat{e}(a)$ and $\hat{e}(a')$ respectively, with a's payoff from $t+1$ on $\underline{w}_{t+1}(h_t, e_t(a,h_t), w_t(h_t, e_t(a,h_t))) - \underline{u}$ by Part 3 of Proposition 3. By choosing effort $e_t(a,h_t)$ at t, $a' < a$ could attain payoff at least $\underline{w}_{t+1}(h_t, e_t(a,h_t), w_t(h_t, e_t(a,h_t))) - \underline{u}$ from $t+1$ on by setting $e_{t+1} = 0$ and quitting from $t+2$. By choosing the effort for a' from t on, $a > a'$ could attain payoff from $t+1$ on

$$\tilde{U}_{t+1}(a', a, h_{t+1}) = \tilde{U}_{t+1}(a', a', h_{t+1}) + \frac{1}{1-\delta}\left[c(\hat{e}(a'), a') - c(\hat{e}(a'), a)\right].$$

With these future payoffs, the necessary condition (7) for a to prefer $e_t(a,h_t)$ and a' to prefer $e_t(a',h_t)$ at t becomes

$$\begin{aligned} & c(e_t(a,h_t), a') - c(e_t(a,h_t), a) \\ & \quad \geq U_t(a,h_t) - U_t(a',h_t) \geq c(e_t(a',h_t), a') - c(e_t(a',h_t), a) \\ & \qquad + \frac{\delta}{1-\delta}\left[c(\hat{e}(a'), a') - c(\hat{e}(a'), a)\right]. \end{aligned} \tag{A.13}$$

By Assumption 1, $c(\tilde{e}, a)$ is continuously differentiable. So, by the Mean Value Theorem, there exist $a_1, a_2, a_3 \in (a', a)$ such that

$$\begin{aligned} c(e_t(a,h_t), a) - c(e_t(a,h_t), a') &= c_2(e_t(a,h_t), a_1)(a - a') \\ c(e_t(a',h_t), a) - c(e_t(a',h_t), a') &= c_2(e_t(a',h_t), a_2)(a - a') \\ c(\hat{e}(a'), a) - c(\hat{e}(a'), a') &= c_2(\hat{e}(a'), a_3)(a - a'). \end{aligned}$$

Use of these in (A.13) and division by $a - a' > 0$ gives, in the limit as $a \to a'$,

$$-c_2(e_t(a,h_t), a') \geq -c_2(e_t(a',h_t), a') - \frac{\delta}{1-\delta} c_2(\hat{e}(a'), a') \tag{A.14}$$

because $a_1, a_2, a_3 \in (a', a)$. By Assumption 1, $c_2(\tilde{e}, a) < 0$ for $\tilde{e} \in (0, \overline{e}]$ and $c_{12}(\tilde{e}, a) < 0$. It follows from (A.14) that $e_t(a,h_t)$ must have an upward jump discontinuity at $a = a'$.

Part 2. From Part 1, for full separation $e_t(a,h_t)$ must have an upward jump discontinuity at every $a \in [\underline{a}_t, \overline{a}_t]$. It must, therefore, be monotone. But for a monotone function defined on an interval, the set of jump continuities is at most countable, which results in a contradiction because the set of $a \in [\underline{a}_t, \overline{a}_t]$ is uncountable. ■

Proof of Proposition 5. Under Assumption 1, if there is more than one agent type for which a mutually beneficial relational contract is possible, there exists a non-degenerate interval of such types $[a^-, a^+]$, with $a^+ > a^-$, for which $\hat{e}(a) < e^*(a)$ for all $a \in [a^-, a^+]$. Proposition 4 shows that no optimal continuation equilibrium for h_t can fully separate at t all types a in a non-degenerate interval $[\underline{a}_t(h_t), \overline{a}_t(h_t)] \subseteq [a^-, a^+]$ with history h_t. This necessarily applies to $t = 1$. It also applies to any subsequent period in which an interval of types has the same history. The other possibility is that, at some t, $a \in [\underline{a}_t(h_t), \overline{a}_t(h_t)]$ are separated into pools that do not include an interval of types, so every pooled type is separated from its immediate neighbours, with full separation occurring only in some later period. A necessary condition for $a, a' \in [\underline{a}_t(h_t), \overline{a}_t(h_t)]$ to be separated into different pools at t is that there exists an optimal continuation equilibrium for h_t with $e_t(a, h_t) \neq e_t(a', h_t)$ for which (7) is satisfied. Let $k(a)$ denote the number of periods after t for which a has payoff gain along an optimal continuation equilibrium path strictly greater than $\underline{w}_\tau(h_\tau) - \underline{u}$ for $t < \tau \leq t + k(a)$, which must be finite if a is eventually to be fully separated because, by Part 3 of Proposition 3, the payoff gain to a once fully separated at τ is $\underline{w}_\tau(h_\tau) - \underline{u}$. Consider the deviation for a' of imitating $a > a'$ until $t + k(a)$, putting in no effort at $t + k(a) + 1$ and quitting in $t + k(a) + 2$. Similarly, consider the deviation for a of imitating a' indefinitely. For these deviations by any $a, a' \in [\underline{a}_t(h_t), \overline{a}_t(h_t)]$ not to yield greater payoffs than the equilibrium path, (7) requires

$$
\begin{aligned}
&-\sum_{i=0}^{k(a)} \delta^i \left[c(e_{t+i}(a), a) - c\left(e_{t+i}(a), a'\right)\right] \\
&\qquad \geq U_t(a, h_t) - U_t\left(a', h_t\right) \geq -\sum_{i=0}^{k(a')} \delta^i \left[c\left(e_{t+i}\left(a'\right), a\right) - c\left(e_{t+i}\left(a'\right), a'\right)\right] \\
&\qquad\qquad\qquad\qquad - \frac{\delta^{k(a')+1}}{1-\delta} \left[c\left(\hat{e}\left(a'\right), a\right) - c\left(\hat{e}\left(a'\right), a'\right)\right], \\
&\qquad\qquad\qquad\qquad\qquad \text{for } a, a' \in [\underline{a}_t(h_t), \overline{a}_t(h_t)], a > a', \quad \text{(A.15)}
\end{aligned}
$$

By Assumption 1, $c(\tilde{e}, a)$ is continuously differentiable in a. So, by the Mean Value Theorem, there exist $a_{t+i}, a'_{t+i}, a'' \in (a', a)$ such that

$$
\begin{aligned}
c(e_{t+i}(a), a) - c\left(e_{t+i}(a), a'\right) &= c_2(e_{t+i}(a), a_{t+i})\left(a - a'\right) \\
c\left(e_{t+i}\left(a'\right), a\right) - c\left(e_{t+i}\left(a'\right), a'\right) &= c_2\left(e_{t+i}\left(a'\right), a'_{t+i}\right)\left(a - a'\right) \\
c\left(\hat{e}\left(a'\right), a\right) - c\left(\hat{e}\left(a'\right), a'\right) &= c_2\left(\hat{e}\left(a'\right), a''\right)\left(a - a'\right).
\end{aligned}
$$

Use of these in (A.15), division by $a - a' > 0$, and re-arrangement gives the requirement

$$
-\sum_{i=0}^{k(a)} \delta^i c_2(e_{t+i}(a), a_{t+i}) + \sum_{i=0}^{k(a')} \delta^i c_2\left(e_{t+i}\left(a'\right), a'_{t+i}\right)
$$

$$\geq -\frac{\delta^{k(a')+1}}{1-\delta} c_2\left(\hat{e}\left(a'\right), a''\right), \quad \text{for } a, a' \in \left[\underline{a}_t\left(h_t\right), \overline{a}_t\left(h_t\right)\right], a > a'. \quad \text{(A.16)}$$

For a' to be separated from its immediate neighbours at t, (A.16) must hold in the limit as $a \to a'$. With $a_{t+i}, a'_{t+i}, a'' \in (a', a)$, in the limit as $a \to a'$, (A.16) becomes

$$-\lim_{a \to a'} \sum_{i=0}^{k(a)} \delta^i c_2\left(e_{t+i}(a), a\right) + \sum_{i=0}^{k(a')} \delta^i c_2\left(e_{t+i}\left(a'\right), a'\right) \geq -\lim_{a \to a'} \frac{\delta^{k(a)+1}}{1-\delta} c_2\left(\hat{e}\left(a'\right), a\right). \quad \text{(A.17)}$$

Define

$$h(a) = -\sum_{i=0}^{k(a)} \delta^i c_2\left(e_{t+i}(a), a\right).$$

Then (A.17) implies

$$\lim_{a \to a'} h(a) - h\left(a'\right) \geq -\lim_{a \to a'} \frac{\delta^{k(a)+1}}{1-\delta} c_2\left(\hat{e}\left(a'\right), a\right). \quad \text{(A.18)}$$

Because $c_2(\tilde{e}, a) < 0$ for $\tilde{e} > 0$, the right-hand side of (A.18) is strictly positive for $k(a)$ finite, so $h(a)$ must have an upward jump discontinuity at a'. But for all agent types to be separated from their immediate neighbours, (A.18) must hold for all $a' \in \left(\underline{a}_t\left(h_t\right), \overline{a}_t\left(h_t\right)\right]$. Thus, for full separation to occur, $h(a)$ must have an upward jump discontinuity at every $a \in \left(\underline{a}_t\left(h_t\right), \overline{a}_t\left(h_t\right)\right]$. Such a function is certainly monotone, so the set of such continuities is at most countable, a contradiction because the set of a in $\left(\underline{a}_t\left(h_t\right), \overline{a}_t\left(h_t\right)\right]$ is uncountable. ■

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